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On Cardinal Numbers and Self Adjusting $\mathcal{M}-$sets. A Mathematical Letter.

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Introduction

This is a technical letter where we outline certain considerations involving cardinal numbers. Notations and terminology follow similarly from our previous article[1]. The mechanistic approach to the logical development of arithmetic in our previous article had proven so useful, that we wish here to extend its application to cardinality. Our Approach involves a mechanistic approach similar to that of our previous article with no reference to formal set theory[2].

Outline

We will again form our analysis on $\mathcal{B}_i$ as these can be paired off with the naturals N. A self adjusting $\mathcal{M}-$set is best seen via the following example:

If it were required, via the existing mechanisms surrounding arithmetic operations, that a representation to be formed expressing $\frac{1}{11}$*, is met with recursion, as can be seen from the multiplication* $1 \approx (0.1)(11) = 1.1$*. As such, the only means of removing the* 0.1 *in* 1.1 *via the multiplicative mechanism is by adjusting the term to which* 11 *is multiplied. i.e* $(0.011)(11)$*. To remove the* 0.01 *we multiply* $(0.01011)..(0.0101011)$ *and so on.*

This necessary recursion or 'adjusting' of the $(0,1)$ symbols is informally defined to be a self adjusting mechanism.

There are two ways in which it seems possible to form an irrational number.

A) Via operations on N.

B) Via Construction.

The famous diagonalisation argument of Cantor can be seen as a form of constructing irrationals.

To see the mechanisms at play in the formation of rationals we introduce the following notation for a number followed by a decimal value :

$$\Im := n : \overline{P\{p\}}$$

P here denotes the decimal portion of the number and $\{p\}$, a value indicating the current position of the decimal if n, P were combined. From this one can define a rational number as :

$$\sum_{p \in pos(P)}^{M} n : \overline{P\{p\}} \tag{1}$$

[1]With fond memories of a true friend and advisor without whom many endeavors would not have been possible. For Dr.W.E.Meyer, Many thannks.

Where it is necessarily required that M be finite. This tells us that in order for $\Im$ to be rational, it is required that $\forall p_i, p_{i+1} \in pos(P)$ there is a finite maximum ℓ such that $p_{i+1} - p_i \leq \ell$. For if this were not the case then no finite summation of the form (1) would be capable of forming a natural number. This brings us to the conclusion that The only other mutually exclusive case of mechanisms forming P is the case where no finite ℓ exists, specifically, no maximum measure between elements $p_i, p_{i+1} \in pos(P)$ exists.This set will form the set of irrational numbers, as a **finite** summation of the form (1) will not be capable of reducing this mechanistic set to an integer or rational.

We wish now to consider all possible partitions one can form on the set $\Im_{\mathcal{R}}$. By doing this one can perhaps study better the cardinality associated with the partition sets.
We now have a clear definition in terms of $\ell-$measure, the procedure for forming irrational sets. Using this, one can form partitions in many different ways and it is the manner in which this can be done that is of interest. This is because the nature of such partitions (as we shall see) gives indication to the types of mappings possible with the naturals.

Before we proceed on with forming partitions, consider the following. as was demonstrated by Cantor, algebraic fields are in one to one corrospondance with the naturals. Given our new insights, a question is, what property of the reals might enable these to map to the naturals? Given the nature of irrational numbers as described above, it is easy to see that only multiplications between the irrationals can form rationals, as the multiplication itself constitutes an infinite summation.
furthermore, it is trivial to see that, forming a partition via any steady change in $prog\mathcal{I}$ will result in a one to one mapping with the naturals, as such, the partition formed for this not to be the case must have the exact properties as those of the set $\Im_{\mathcal{R}}$

The above considerations brings us to this one single method of forming a partition that is not of the previous types.
Any infinite set of the form :

$$\{p_o \in pos((I))\}|prog(p_o)|\forall \ell_i \in \ell, \exists \ell_k \in \ell|\ell_k > \ell_i \tag{2}$$

forms an irrational number. A partition P of such a set which is also of the same type, would be all elements belonging to a set of the type (2) such that $\forall \ell'_i \in P, \ell_i > \ell'_i$. However due to the progression of ℓ' being divergent, both sets are actually the same.
Another way of forming a similar partition P_g is by leaving gaps in the set P. i.e. a Set of the form with ℓ forming between alternate pairs of p_j.
i.e. We have a progressive set of one symbols that are of the type (2), and only between pairs of such symbols, we allow the arbitrary introduction of other one

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symbols.

$$0 \quad |1[0101..]| \quad 0 \quad 1 \quad 0 \quad |1[111100]| \quad 1 \quad 0 \quad 0 \quad |1|$$

Figure 1. Expresses the concept of 'gap-type-irrational numbers'. The vertical bars indicate where arbitrary progressions of one symbols are allowed referred to as the **Bar-Symbols**. The extent to which one symbols are allowed to be adjoined to these bar symbols is known as the limit-measure $\jmath$, and is smaller than the at least one minus the position of the one symbol that follows and is associated with $prog(\mathcal{I})$ i.e. of the position symbols of the gap irrational number in concern.

Let $pos(Q_1), pos(Q_2)$ be two sets of infinite positions of one symbols associated with two Rational numbers such that one is greater than the other. For this to be the case, out of the list of all symbols in each set, the series of symbols followed by the decimal position must be equal and have a point where one symbol has position closer to the decimal position than that of the other. The remainder of the symbols to infinity is irrelevant.
Using our blueprint for the creation of gap type irrational numbers, we can select for the initial part, the same set of one symbols as that of the two rationals, up until the first difference in position of the one symbols. From this point on we can include the one symbol that is deviated further away and beginning with a one symbol, one position before the position that follows in the rational in concern, follow the gap $\ell-$measures for gap-irrationals. Since these are arbitrary and can be formed in infinitely many ways between rationals, it can be deduced that an infinite set of gap irrationals exist between any two rationals, furthermore constructing non gap-like irrationals where the ℓ-measures co-exist with bar-symbol positions and between appropriate gaps of any two such gap irrational that exists between the rationals in concern are also infinitely possible showing from the nature of the constructions themselves that a one-to-one mapping is not possible between these two types of irrationals and between the integers and gap-type irrationals.

This leads to believe that cardinalities C may exist of the form:

$$\aleph_0 < C < 2^{\aleph_0}$$

At this point this is merely conjecture with some indication that this might be so. We will discuss this further in a follow up article.

References

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