Compromise based evolutionary multiobjective optimization algorithm for multidisciplinary optimization

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Abstract

Multidisciplinary Design Optimization deals with engineering problems composed of several sub-problems -called disciplines- that can have antagonist goals and thus require to find compromise solutions. Moreover, the sub-problems are often multiobjective optimization problems. In this case, the compromise solutions between the disciplines are often considered as compromises between all objectives of the problem, which may be not relevant in this context. We propose two alternative definitions of the compromise between disciplines. Their implementations within the well-known NSGA-II algorithm are studied and results are discussed.

1 Introduction

The design and optimization of complex engineering systems, such as aircrafts, cars or boats, require the collaboration of many engineering teams from different disciplines. These problems are referred to as Multidisciplinary Design Optimization (MDO). This work deals with a class of MDO problems, where each discipline is a Multiobjective Optimization Problem (MOP – see [3] for notations and definitions). For instance, the design of a wing of an airplane involves two strongly coupled disciplines: aerodynamics and structure. Both may have several objectives to achieve. For example, the minimization of the drag and the maximization of the lift for aerodynamics and the minimization of the weight and of the deflection for the structure. The goal of such a MDO problem is to propose compromise solutions to the decision maker between the disciplines. In other contexts, compromises solutions have to be found between several possible scenarios (see for example [1] for an application to risk management). A compromise solution can be understood as a preferred solution for the decision problem.

2 Context and goal

Classical MDO methods [9] do not consider problems where the disciplines have multiple objectives each. Recently, Evolutionary Multiple Objective (EMO) optimization methods such as EM-MOGA [10], COSMOS [11] and others (see [7] for a description of these methods), were designed to solve optimization problems where each discipline has multiple objectives. But these approaches consider the problem as a unique MOP composed with all the objectives of the disciplines optimized simultaneously. Hence, the solutions found by those methods are the efficient solutions of the following problem:

$$\min_{x \in X}(f_{1,1}(x), \ldots, f_{1,p_1}(x), \ldots, f_{q,1}(x), \ldots, f_{q,p_q}(x)) \tag{1}$$

where q is the number of disciplines and p_i the number of objectives of the i^{th} discipline. The function $f_{i,j}$ represents the j^{th} objective of the i^{th} discipline. We will assume that all the disciplines are defined on the same decision space.

We will note C_0 the first compromise which consists in solving the MOP described eq. 1. Unfortunately, C_0 does not give relevant solutions in MDO problems, as illustrated by the follwing didactic example composed by two disciplines. Each discipline has two objectives to minimize: $f_{1,1}$ and $f_{1,2}$ for the first discipline and $f_{2,1}$ and $f_{2,2}$ for the second one. Let $f_1(x) = (f_{1,1}(x), f_{1,2}(x))$ and $f_2(x) = (f_{2,1}(x), f_{2,2}(x))$ be respectively the objective functions of the first and the second discipline, and $X = \{x_a, x_b, x_c\}$ the feasible set of solutions in the decision space. The performances of these solutions are reported in Table 1.

Table 1: The performances of the feasible solutions by f_1 (resp. f_2) defined for Discipline 1 (resp. Discipline 2).

	Discipline 1		Discipline 2	
x	$f_{1,1}(x)$	$f_{1,2}(x)$	$f_{2,1}(x)$	$f_{2,2}(x)$
x_a	1	3	2	1
x_b	3	2	1	3
x_c	2	1	3	2

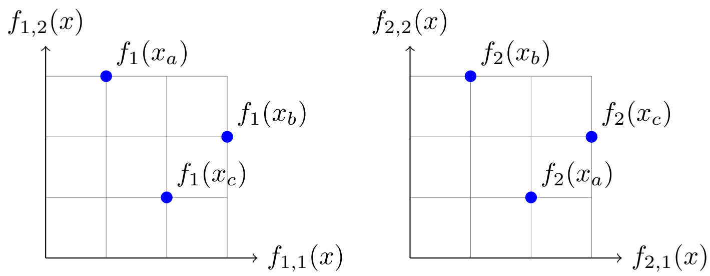

Figure 1: The objective spaces of the two disciplines of the didactic minimization problem: discipline 1 is on the left and discipline 2 on the right.

Considering independently the disciplines, we have two efficient solutions by discipline (see Fig. 1): x_a and x_c in the first discipline, and x_a and x_b in the second one. As x_a is an efficient solution in both disciplines, x_a is expected to be the only efficient compromise solution of this problem and reported as the unique output of the algorithm. But if we consider the four objectives problem (eq. 1), all the feasible solutions are efficient because their 4-dimensions performance vectors are mutually non-dominated: the C_0 compromise solution set is thus X. Indeed, solving the MOP can be interpreted as finding solutions between

Table 2: Ranks of the points in each discipline: $r_1(x)$ (resp. $r_2(x)$) is the rank of x in the discipline 1 (resp. discipline 2).

x	$r_1(x)$	$r_2(x)$
x_a	1	1
x_b	2	1
x_c	1	2

objectives without taking into account the grouping of the objectives within the disciplines. For instance, x_a would be preferred over x_b in the first discipline and over x_c in the second one but this information is missing in the C_0 compromise. We propose two compromises which are more relevant in this context: C_1 and C_2.

3 Computing a compromise

3.1 C_1: Ordering the solutions in disciplines

The first idea we propose – also presented in [8] – is to transform each disciplinary MOP into a single objective using a measure of solutions quality inside each discipline.

Ranking procedures are frequently used in multiobjective genetic algorithms for evaluating the fitness of the solutions. Goldberg [6] first introduced the rank to bias the selection operator based on the evaluations of the objective function. This idea has then been used by Srinivas and Deb [12] in the NSGA algorithm in order to sort the individuals for a multiobjective problem. Other ranking procedures have been proposed for multiobjective genetic algorithms, such as the one proposed by Fonseca and Fleming [5]. Definition 1 presents the rank as it is defined in ordered sets theory. This definition is the same as the one used by Srinivas and Deb.

Definition Let $O = (E, \leq)$ be an ordered set. For $e \in E$, the rank $r(e)$ of e is defined as follows:

- $r(e) = 1$, if e is a minimum.
- $r(e) = n$, if the elements of rank $< n$ have been assigned and e is a minimum in the ordered set $P \backslash \{x \in P : r(x) < n\}$.

The compromise C_1 consists in using the rank as the only objective of each discipline. The corresponding multidisciplinary optimization problem can be formulated as the following MOP:

$$\min_{x \in X}(r_1(x), \ldots, r_q(x)) \tag{2}$$

where $r_i(x)$ represents the rank of x in the i^{th} discipline. The efficient solutions of eq. 2 are the C_1-compromise solutions.

For instance, in the didactic example introduced in the previous section (Table 1, Fig. 1), each point is ranked in each discipline separately. The results are presented in Tab 2. Considering these ranks as performances of the solutions in each discipline, we can define a compromise solution as a non-dominated ranks vector. There is only one non-dominated rank vector $(1, 1)$, so the only C_1-compromise solution to this problem is x_a.

Unfortunately, a compromise based on ranks has an important drawback if implemented within an EMO algorithm because of its stochastic behavior: the order relation between two performance vectors depends on the spread of the performance vectors of the other individuals in the population. Thus, as the population evolves, the performance vectors move. The Fig. 2 shows that on the first chart, $f_i(x_a)$ is ranked 1 and $f_i(x_c)$ is ranked 2, whereas $f_i(x_a)$ is ranked 3 and $f_i(x_c)$ is ranked 2 on the second chart. Between the first and the second chart, only two points, $f_i(x_d)$ and $f_i(x_e)$ have been added, but it results in a reverse of the order of $f_i(x_a)$ and $f_i(x_c)$: $f_i(x_c)$ was greater than $f_i(x_a)$ on the first chart whereas it is less than $f_i(x_a)$ on the second one.

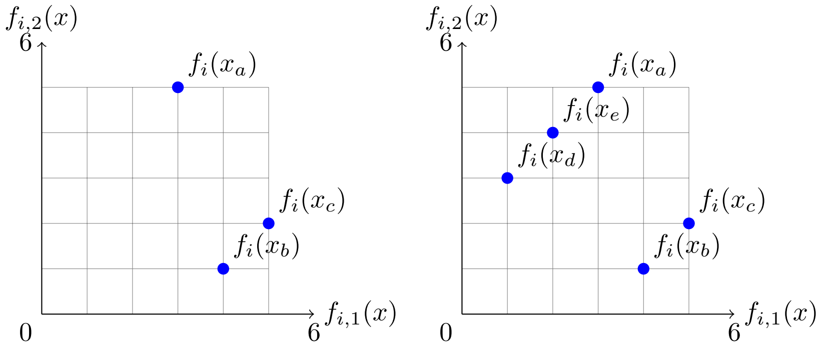

Figure 2: Using the rank to define an order relation between elements of a discipline i, the order relation depends on the other elements: the rank of $f_i(x_a)$ is less than the rank of $f_i(x_c)$ in the left chart whereas this relation is reversed in the right chart.

3.2 C_2: Taking into account incomparability in the disciplines

Another approach is to take into account the incomparabilities in the dominance relation: an element $f(x)$ dominates $f(x')$ if $f(x)$ dominates $f(x')$ in at least one discipline, and $f(x')$ never dominates $f(x)$:

$$f(x) \preceq f(x') \Longleftrightarrow \begin{cases} \exists i \in \{1, \ldots, n\} & f_i(x) \leq_i f_i(x') \\ \nexists j \in \{1, \ldots, n\} & f_j(x') \lneq_j f_j(x) \end{cases} \quad (3)$$

where $\leq_i$ is the componentwise order in the discipline i, and $\lneq_j$ is the Pareto-dominance relation in the discipline j. Let call C_2 the compromise such that the compromise solutions are such that their performance vectors in each disciplines are non-dominated according to the transitive closure of the $\preceq$ relation.

In the didactic example presented Fig. 1, let $f(x) = (f_1(x), f_2(x))$. The point $f(x_a)$ dominates $f(x_c)$ and $f(x_c)$ dominates $f(x_b)$ in one discipline each, and there are no other domination relations. So we have $f(x_a) \preceq f(x_c) \preceq f(x_b)$. Thus, x_a is the only solution to this problem. On this example, the solutions of C_2 are equivalent to C_1. Moreover, this compromise is expected to be less sensitive when implemented within an EMO algorithm, because it does not introduce new order relations in the disciplines depending on the spread of the population.

4 EMO-MDO algorithm

The proposed algorithm, called EMO-MDO (Algorithm 1), is a generalization of the NSGA-II algorithm [2]. Its particularity lies in the ranking procedure which can be computed on any preordered set[1], and not only the performance vectors of the individuals. It allows to modelize different kind of compromises such as C_0, C_1 and C_2. In particular, EMO-MDO used with C_0 is identical to NSGA-II. It also shares its genetic operators: real coded genes, SBX crossover (`crossover` routine), polynomial mutation (`mutation` routine), crowded binary constrained tournament selection (`selection` routine) and non-constrain-dominated sorting procedure with crowding (`generation` routine) [2].

Algorithm 1 EMO-MDO

Require: compromise c
Ensure: solution population
  population ← `initialization()`
  preorder ← `compromise`(c,population)
  ranks ← `compute_ranks`(preorder)
  for $i = 1$ to max_iteration **do**
    parents ← `selection`(population,ranks)
    offspring ← `mutation`(`crossover`(parents))
    population ← population ∪ offspring
    preorder ← `compromise`(c,population)
    ranks ← `compute_ranks`(preorder)
    population ← `generation`(population,ranks)
  end for
  return population

The initial population of size N is created from solutions randomly picked from the feasible set. The *preorder* between the individuals is then computed by the `compromise` function according to the population *pop* and the compromise $c \in \{C_0, C_1, C_2\}$. This *preorder* is used to compute the *ranks* of the individuals in the *population.* A *parents* population is selected from *pop* according to their *ranks.* `Crossover`s and `mutation`s are performed to create an *offspring* population. The *offspring* is then added to the current population *pop* to create a $2 \times N$ population on which the compromise relation is computed. The `generation` function selects the new individuals which will belong to the next generation.

[1]Ordered sets which are not necessarily antisymmetric.

5 Numerical experiments

A series of experiments were performed on the EMO-MDO in order to study its behavior on C_0, C_1 and C_2 compromises.

5.1 Test problem

This test problem comes from Engau and Wiecek [4]. It is composed of two bi-objective disciplines defined on the feasible set X. Here, all the feasible solutions are efficient for C_0 compromise.

$$D_1 \begin{cases} \min\limits_{(x_1,x_2)\in X} f_{1,1}(x_1,x_2) = (x_1-2)^2 + (x_2-1)^2 \\ \min\limits_{(x_1,x_2)\in X} f_{1,2}(x_1,x_2) = x_1^2 + (x_2-3)^2 \end{cases} \tag{4}$$

$$D_2 \begin{cases} \min\limits_{(x_1,x_2)\in X} f_{2,1}(x_1,x_2) = (x_1-1)^2 + (x_2+1)^2 \\ \min\limits_{(x_1,x_2)\in X} f_{2,2}(x_1,x_2) = (x_1+1)^2 + (x_2-1)^2 \end{cases} \tag{5}$$

with $X = \{(x_1,x_2) \in \mathbb{R}^2 : x_1^2 - x_2 \ \leq\ 0, x_1 + x_2 - 2 \ \leq\ 0, -x_1 \ \leq\ 0\}$

5.2 Test protocol

Two reference sets of solutions are computed by filtering the solutions satisfying the compromises from a sampling of the feasible set of decision variables. The first sampling corresponds to a uniform discretization of the feasible set, whereas the second sampling is a simple random sampling. For each test, 300 solutions are evaluated.

The three compromises have been compared with the filtering methods and the EMO-MDO algorithm. The latter has been performed in two conditions: with or without the phenotypic diversity procedure enabled (crowding) in the `selection` and the `generation` functions.

The EMO-MDO algorithm as been run with the following combinations of population sizes and iterations numbers: 10x50, 20x15, 20x20, 50x50 and 50x500. Three simulations of each combination have been done to verify the stability of the algorithm.

From these experiments, we want to compare the solutions found by the genetic algorithms to the reference solutions, and to compare C_1 and C_2 compromises to C_0. As far as we know, there is no quality measure of the solutions of such problems as they exist in the field of evolutionary multiobjective genetic algorithms.

5.3 Results and discussion

The crossover and mutation probabilities are respectively 0.8 and 0.2. The η and σ parameters for the SBX crossover and mutation are respectively 2 and 4. Other set of parameters have been tested and they produce comparable results :

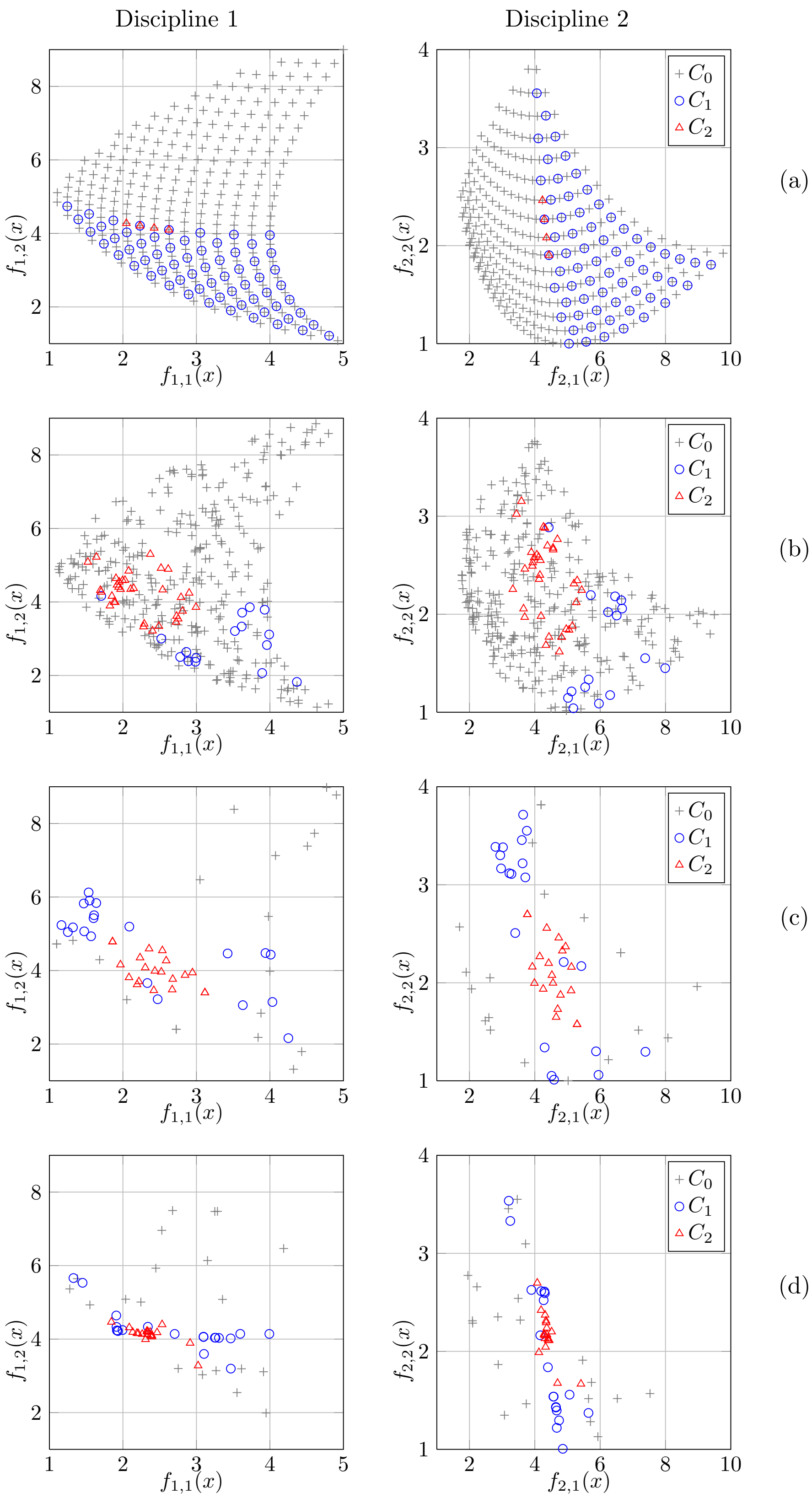

Figure 3: Results of the test problem using compromise C_0, C_1 and C_2 with filtering on uniform sampling (a), and simple random sampling (b) of 300 points. Results with EMO-MDO with phenotypic distance enabled (c), and disabled (d). Population size is 20, and 15 iterations were performed.

- We observe that the compromise solutions found with C_1 with the uniform sampling (figure 3-a) are uniformly distributed in an area approximately equal to the half of the image of the feasible set.
- A random sampling of the feasible set (figure 3-b) does not give the same results: the compromise solutions still belong to the same area but are not spread uniformly. This shows that the solutions of the compromise C_1 are dependent to the spread of the population as stated in section 3.
- The C_2 solutions are more largely spread with the simple random sampling than the uniform discretization (figure 3-a,b). Nevertheless, the solutions are in the same area, and this spread decrease as the sampling size increase.
- The EMO-MDO algorithm did not converge to the expected area when used with C_1 (figure 3-c,d). This is more salient when phenotypic diversity is disabled (figure 3-d).
- The EMO-MDO algorithm converged to the expected area when used with C_2 (figure 3-c,d).
- The phenotypic diversity of NSGA-II is not adapted to this kind of problem: many solutions have the same performances within a discipline (figure 3-c). Nevertheless, phenotypic diversity still improves the solutions: they are less diversifyied when disabled (figure 3-d).

We also observed that the compromise C_2 is less sensitive to the algorithm parameters than the compromise C_1.

6 Conclusions and on-going work

MDO problems are often composed of disciplines which have several objectives each. The compromise solutions are then often defined as the efficient solutions of the MOP (C_0). This can be interpreted as a compromise between the objectives instead of a compromise between the disciplines.

We propose a first compromise (C_1) based on the preferences of the disciplines using the ranks of the solutions. Unfortunately, this does not suit to an EMO algorithm because ranks will not change uniformly as the population evolves. To overcome this problem, we propose another compromise (C_2) allowing a non total order of the preferences of the disciplines. C_0 C_1 and C_2 have been implemented into an EMO algorithm. C_2 gives satisfying results and is less sensitive to the algorithm parameters than C_1.

The proposed compromises have been defined on a simplified class of MDO problems without local variables and coupling functions between disciplines. Our future research will extend the compromises to more complex problems.

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