

Dual-hop Spatial Modulation (Dh-SM)

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Abstract—In this paper, we introduce Dual-hop Spatial Modulation (Dh-SM). We look at the effect that Dh-SM has on the required signal to noise ratio (SNR) at the destination and how it can help alleviate the multi-hop burden in the system. Initial bit-error-ratio (BER) results comparing the performance of Dh-SM with orthogonal decode-and-forward (DF) are presented where Dh-SM is shown to have up to a 10 dB SNR advantage.

I. INTRODUCTION

Spatial modulation (SM) is a recently proposed approach to multiple-input-multiple-output (MIMO) systems which entirely avoids inter-channel interference (ICI) and requires no synchronisation between the transmit antennas, while achieving a spatial multiplexing gain [1]. This is performed by mapping a block of information bits into a constellation point in the signal and spatial domains [2]. In SM, the number of information bits, k , that are encoded in the spatial domain is directly related to the number of transmit antennas N_t ; in particular $N_t = 2^k$. This means that the number of transmit antennas must be a power of two unless fractional bit encoding is used [3]. It should also be noted that SM is shown to outperform other MIMO schemes in terms of bit-error-ratio (BER) [2]. The presented work proposes the use of SM as a possible relaying technique.

The basic relaying problem can be reduced to the simple inability of a single transmitter to reach its intended target with the necessary signal to noise ratio (SNR). There are several approaches to this problem. On one hand, the orthogonal amplify-and-forward (AF) utilises the relay antenna as a simple amplifier. Any signal received by the relay at time instance t_1 is amplified and retransmitted at instance t_2 forming a non-regenerative system. On the other hand, the orthogonal decode-and-forward (DF) algorithm decodes the received signal at the relay, then re-encodes and retransmits this information establishing a regenerative system. Outage probabilities, mutual information calculations and transmit diversity bounds for AF and DF relaying are derived in [4] with end to end performance being considered in [5]. Taking into consideration the above relaying protocols, the use of SM is proposed to provide additional power and capacity gains over the non-cooperative AF and DF systems. Let us assume a basic system as shown in Fig. 1, where transmissions are carried out at 2 bits/s/Hz in the signal domain and only a single transmit antenna is active at the source. In orthogonal AF and

DF two time slots are needed for the relevant information to reach the destination node, effectively halving the source-destination spectral efficiency to 1 bit/s/Hz. Dual-hop Spatial Modulation (Dh-SM) can partially mitigate this effect. While maintaining a fixed signal constellation, Dh-SM can utilise the spatial domain to transmit additional information bits. Since the receiver decodes the channel used for the transmission, it can determine the transmitting antenna and, in so doing, decode the bits used to activate the particular antenna. This serves to increase the source to destination spectral efficiency *i.e.* almost halving the multihop burden as will be explained in Section II. Alternatively, since some of the data in SM is transmitted in the spatial domain, a lower order modulation scheme can be used for signal domain transmission which in turn leads to a lower transmit power requirement. This is a unique advantage that Dh-SM has when compared to all other relaying systems and results in a decreased bit error ratio at the destination for the same transmit power *i.e.* Dh-SM increases the coding gain of the system.

In [6–8], analytical bounds for the BER performance of SM are derived. Each work considers the channel and the signal symbol as a joint input variable and averages across the channel to achieve a closed form solution. In this work, besides introducing Dh-SM, we validate the results of this work with the union bound based approach originally presented in [9, Eq. (8)].

In the remainder of the paper we introduce the system model in Section II, provide the theoretical framework in Section III, show and discuss the numerical results in Section IV and conclude the paper in Section V.

II. SYSTEM MODEL

In the following work we assume a three node scenario as shown in Fig. 1. While AF, DF and Dh-SM utilise a single transmit antenna at any instance, Dh-SM requires that the transmitter has more than one transmit antennas. Since in this work we seek to characterise the behaviour of SM in a dual-hop scenario, we compare its performance in terms of BER to non-cooperative DF. The source broadcasts a signal constellation symbol, x . The received signal is given by: $y_j = h_{ij}x + \eta$, where j is the index of the receive and i is the index of the transmit antenna, h_{ij} is the channel coefficient of the link between the active antenna i and the receiving

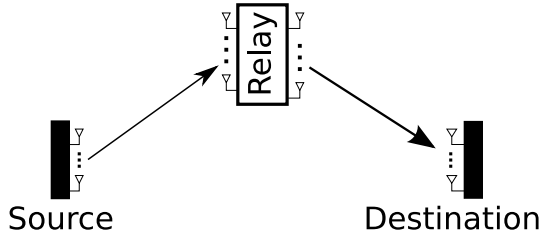


Fig. 1. Dual-hop spatial modulation

antenna j , η is the additive white Gaussian noise (AWGN) described by $\mathcal{N}(0, \sigma^2)$ with $\sigma^2 = E_x [|x|^2] / (\bar{\gamma}_{ij})$ where $\bar{\gamma}_{ij}$ is the average SNR of the link between nodes i and j and $E_x[\cdot]$ is the expectation with respect to the set of signal constellation points. The estimated symbol at the relay in the DF system using maximum-ratio-combining (MRC) is given by:

$$x_{\text{est}}^\ell = \frac{(\mathbf{h}_{k\ell}^i)^\dagger \bar{\mathbf{y}}_{k\ell}}{(\mathbf{h}_{k\ell}^i)^\dagger \mathbf{h}_{k\ell}^i} \quad (1)$$

$(\cdot)^\dagger$ denotes the complex conjugate and $\mathbf{h}_{k\ell}^i = [h_{i1} \dots h_{iN_r^\ell}]$ is a vector composed of the single tap channel coefficients from antenna i on the transmitting entity k to the receiving entity ℓ with N_r^ℓ number of receive antennas. The transmitting entity is either the source, s , or the relay, r , while the receiving entity is the relay or destination, d *i.e.* $k \in \{s, r\}$ and $\ell \in \{r, d\}$. The vector $\bar{\mathbf{y}}_{k\ell}$ is comprised of the symbols at each of the receive antennas at node ℓ . Finally, x_{est}^ℓ is passed through a maximum likelihood (ML) detector to obtain the original bit sequence.

The basic idea of SM is to map blocks of information bits into two information carrying units [2]: i) a symbol, chosen from a complex signal-constellation diagram, and ii) a unique transmit-antenna index, chosen from the set of transmit-antennas in the antenna-array *i.e.* the spatial-constellation diagram. The working principle of SM is exemplified in Fig. 2.

Throughout this paper, we consider a ML decoder, which computes the Euclidean distance between the received signal $\bar{\mathbf{y}}_{k\ell}$ and the set of all possible received signals, selecting the closest one [6]:

$$(x_{\text{est}}, n_t^k) = \underset{\{x_j, n_i\}}{\text{argmin}} \left\{ \|\bar{\mathbf{y}}_{k\ell} - x_j \mathbf{h}_{k\ell}^i\|_F^2 \right\} \\ \forall x_j \in \mathcal{X} \quad \forall i \in \{1 \dots N_t^k\}$$

where the pair (x_{est}, n_t^k) is formed from the estimated symbol x_{est} emitted from antenna n_t^k at node k , x_j is the current symbol being evaluated from the set of possible constellation points \mathcal{X} , N_t^k is the number of available transmit antennas on node k and $\|\cdot\|_F$ is the Frobenius norm.

A. Example

Let us again consider the system presented in Fig. 1, but now with a single source to relay transmit antenna and four relay to destination transmit antennas. In this case 2 bits can be sent in the spatial domain and 2 more in the signal domain on the relay to destination link. The use of SM on the relay to

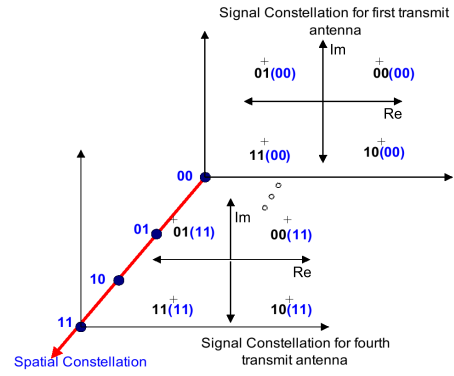


Fig. 2. If we wish to transmit four bits, the first two bits define the spatial-constellation point which identifies the active antenna, while the remaining two bits determine the signal-constellation point that is to be transmitted.

destination link enables the system to operate at 4 bits/s/Hz on that link. Since the system remains unchanged in the source to relay link, the source can transmit to the relay in the first two time slots a total of 4 bits. The relay can then transmit those 4 bits to the destination in the third slot by making use of the spatial domain. The use of Dh-SM results in 4 bits going from the source to the destination in three time slots and an end-to-end average spectral efficiency of 1.33 bits/s/Hz; a 33% improvement over standard AF and DF.

Alternatively, Dh-SM can be used to improve the bit-error-ratio of the system by transmitting a lower order modulation signal-symbol. We quantify the coding gains in Section IV-B.

III. ANALYTICAL MODELLING

The scenario presented in Fig. 1 represents a well known orthogonal relaying system. One major distinction is the use of multiple transmit and receive antennas at the relay. The end-to-end performance of a two-hop wireless communication system with non-regenerative (AF) and regenerative (DF) relays over a Rayleigh-fading channel is presented in [5]. The authors develop a closed form expression for the average BER of AF given in terms of the system's moment generating function (MGF). The system performance in terms of outage probability and BER demonstrate that DF systems perform better than AF at both low and high average SNRs in a Rayleigh fading environment. For this reason, we limit our comparison and only look at Dh-SM's performance relative to DF. In this work we aim to show that the use of spatial modulation in a fixed relay system provides a significant coding gain.

Starting from the system model presented in Section II, for the case of DF, the bit error probability averaged over the independent channel SNRs can readily be expressed as shown in (2), since the signal undergoes two stages of decoding [10],

$$P_b(E_{\text{sd}}) = P_b(E_{\text{sr}}) + P_b(E_{\text{rd}}) - 2P_b(E_{\text{sr}})P_b(E_{\text{rd}}). \quad (2)$$

$P_b(\cdot)$ is the average BER with average energy $E_{k\ell}$ between nodes k and ℓ . In the decode and forward case, the overall bit error ratio for a dual-hop system is a function of the individual links, meaning that if a system performs better in terms of BER

on the individual links, it will also perform better for the dual-hop when we consider that both $P_b(E_{sr})$ and $P_b(E_{rd})$ must be less than $1/2$. To show this we look at the directional derivative of $P_b(E_{sd})$ with respect to $P_b(E_{sr})$ and $P_b(E_{rd})$. We can define a unit vector $\vec{u} = \langle \alpha, \beta \rangle$ where α and β are non-negative coefficients defining the direction of the derivative.

$$\nabla_{\vec{u}} P_b(E_{sd}) = \alpha (1 - 2P_b(E_{rd})) + \beta (1 - 2P_b(E_{sr})). \quad (3)$$

Looking at (3) we see that the function is monotonically increasing with respect to the individual error probabilities since $\{P_b(E_{sr}), P_b(E_{rd})\} \in [0, 1/2]$. We now look at the expressions for these error probabilities. It should be noted that since the overall system error depends solely on the error of the individual links, we proceed to analyse the error expressions for the arbitrary k to ℓ link.

The BER of an M quadrature amplitude modulation (QAM) across multiple fading channels is given in (4) where M is the size of the QAM constellation and N_r^ℓ depends on which link is considered. The generalized expression for the average BER of a single link between nodes k and ℓ using QAM modulation and Gray coding is given by (4) [11].

$$P_b(E_{k\ell}) \cong A \sum_b \frac{\sqrt{M}/2}{\pi} \int_0^{\pi/2} M_{\bar{\gamma}_{k\ell}} \left(\frac{c}{2 \sin^2(\theta)} \right)^{N_r^\ell} d\theta \quad (4)$$

$$A = 4 \left(\frac{\sqrt{M} - 1}{\sqrt{M} \log_2(M)} \right) \quad c = -\frac{3 \log_2(M)(2b - 1)^2}{M - 1}$$

$M_{\bar{\gamma}}(s)$ is the moment generating function of the fading channel with $\bar{\gamma}$ being the average SNR at the receiver. The moment generating functions for different channel fading models can be found in [11]. In particular, we look at a Rayleigh fading channel and $M_{\bar{\gamma}}(s)$ is given by:

$$M_{\bar{\gamma}}(s) = \frac{1}{1 + s\bar{\gamma}}$$

Similarly, because Dh-SM is in principle a DF system, its BER can be represented by (2), where the individual error probabilities are those of individual SM links.

The BER of SM using the optimal detector can be bounded using union bound methods and is given by (5):

$$P_b(E_{k\ell}) \leq \mathbf{E}_{\mathbf{H}} \left[\sum_{\hat{x}, \hat{n}_t} \text{PEP}(x, n_t, \hat{x}, \hat{n}_t) \right] \quad (5)$$

where we define $\text{PEP}(x, n_t, \hat{x}, \hat{n}_t)$ to be the pairwise error probability between the symbol x emitted from antenna n_t being detected as symbol \hat{x} emitted by antenna \hat{n}_t . $\mathbf{E}_{\mathbf{H}}[\cdot]$ represents the expectation of the system with respect to the channel. Given this formulation, the symbol based union bound for (5) can be expressed as (6), where work in [12] shows the tightness of this approach.

$$P_b(E_{k\ell}) \leq \sum_{x=1}^M \sum_{\hat{x}=1}^M \sum_{n_t=1}^{N_t^k} \sum_{\hat{n}_t=1}^{N_t^\ell} \frac{\text{PEP}_{k\ell}(x, n_t, \hat{x}, \hat{n}_t)}{2(MN_t^k - 1)}. \quad (6)$$

In [9, Eq. (8)] the pairwise error probability conditioned on the channel between the two communicating nodes is given as:

$$\text{PEP}_{k\ell}(x, n_t, \hat{x}, \hat{n}_t) = Q \left(\sqrt{\frac{\|\mathbf{h}_{k\ell}^{n_t} x - \mathbf{h}_{k\ell}^{\hat{n}_t} \hat{x}\|^2}{2\sigma^2}} \right) \quad (7)$$

where the symbol x is transmitted from antenna n_t .

Given this analytical modelling, we proceed to analyse the performance of Dh-SM and DF in terms of their BER.

IV. NUMERICAL ANALYSIS

The aim of this section is to compare the performance of Dh-SM with conventional relaying utilising M-QAM under a variety of conditions. In particular, the presented results depict the different behaviour of the system when:

- the number of transmit antennas is changed at the source,
- the number of transmit antennas is changed at the relay,
- the number of receive antennas is changed at the relay,
- the number of receive antennas is changed at the destination and,
- under non-symmetric channel gain conditions.

A. Simulation Setup

A frequency-flat Rayleigh fading channel with no correlation between the transmitting antennas and additive white Gaussian noise is assumed. MRC in combination with ML detection is used in the DF system, with the ML detector in [6] being used for Dh-SM. Perfect channel state information (CSI) is assumed at the receiving node, with no CSI at the transmitter. Only one of the available transmit antennas at the source and relay nodes of Dh-SM is active at any transmitting instance. Since part of the data is encoded in the spatial domain, Dh-SM uses a lower order modulation symbol but the energy per symbol is equivalent to that in the DF system.

B. Results

In the legend on each figure, $\text{An}(k, \ell)$ represents the analytical BER on the link between nodes k and ℓ , while $\text{Sim}(k, \ell)$ is the simulation result. To describe the behaviour of DF we use (4), whereas (6) bounds the behaviour of Dh-SM. Throughout the discussion we refer to the coding gain as the difference between the SNR levels of the two systems required to reach the same BER. We begin by looking at the effects of additional antennas at the transmitter. In particular, Fig. 3 shows that when the source to relay and relay to destination channel conditions are comparable, Dh-SM exhibits over a 2 dB coding gain when compared to DF. It should be noted that as any DF system, Dh-SM is susceptible to bottlenecks. In this case, the two transmit antennas at the relay and the two receive antennas at the relay and destination limit the system's performance. The effect of the number of receive antennas is quantified below. Indeed the addition of two more antennas at the source increases the advantage of Dh-SM only by about 0.4 dB and, more noticeably, the addition of 28 more transmit antennas results in a mere 0.6 dB gain. Similarly, looking at Fig. 4, we

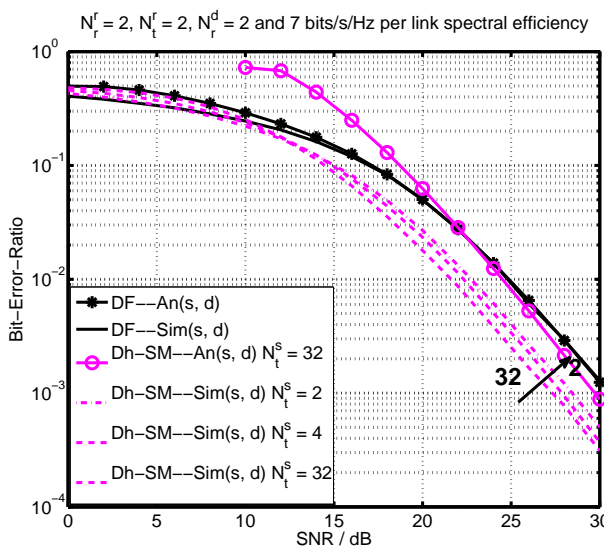


Fig. 3. Average bit error ratio when $\bar{\gamma}_{sr} = \bar{\gamma}_{rd}$. The average source to destination spectral efficiency is equal to 3.5 bits/s/Hz. The arrow on the figure indicates the progression of the curves going from $N_t^s = 32$ as the leftmost dashed curve to $N_t^s = 2$ as the rightmost dashed curve.

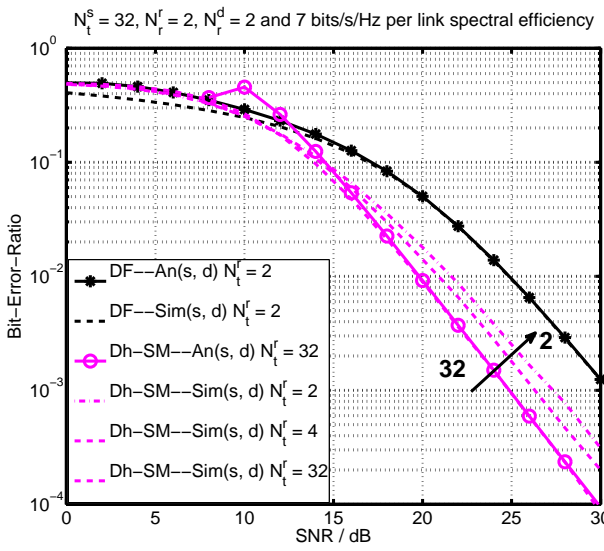


Fig. 4. Average bit error ratio when $\bar{\gamma}_{sr} = \bar{\gamma}_{rd}$. The average source to destination spectral efficiency is equal to 3.5 bits/s/Hz. The arrow on the figure indicates the progression of the curves going from $N_t^s = 32$ as the leftmost dashed curve to $N_t^s = 2$ as the rightmost dashed curve.

can see that if we remove one of the bottlenecks *i.e.* operate with 32 transmit antennas at the source, the coding gain of 3.5 dB achieved with only 2 transmit antennas at the relay is marginally increased by about 0.5 dB with 4 transmit antennas. A further 1 dB coding gain is achieved by using 32 transmit antennas at the relay. It must be noted that systems with more available antennas still exhibit better performance in terms of BER, although the coding gains achieved with every additional antenna are diminishing.

We now investigate the behaviour of Dh-SM with respect to the number of receive antennas. As Fig. 5 shows, Dh-SM

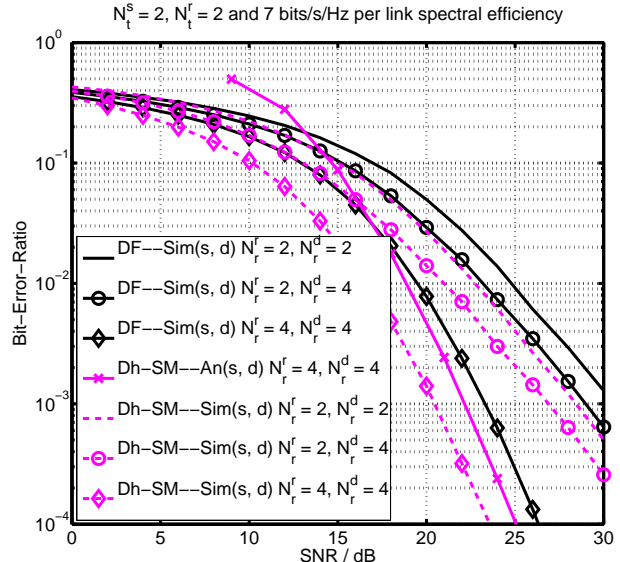


Fig. 5. Average bit error ratio when $\bar{\gamma}_{sr} = \bar{\gamma}_{rd}$. The average source to destination spectral efficiency is equal to 3.5 bits/s/Hz.

performs better as more receive antennas are added to the system, irrespective of which node they are added to. Despite the minor coding gains observed, the system is still limited by its worse performing hop in terms of the BER since the diversity of the overall system is $\min\{N_r^r, N_r^d\}$. Looking at Fig. 5, we can see that Dh-SM has a 2 dB gain with respect DF when there are 2 receive antennas at both the relay and the destination. When there are 2 and 4 receive antennas at the relay and destination nodes respectively, Dh-SM has a 2.5 dB gain with respect to DF, *i.e.* the performance of Dh-SM improves by 0.5 dB with the addition of 2 receive antennas at the destination. Similarly, when there are 4 antennas at each of the nodes, Dh-SM gains an additional 0.5 dB which results in a 3 dB better performance compared to DF. Similar to the effects observed with the number of transmit antennas, systems with more receive antennas still exhibit better performance in terms of BER, although the coding gains achieved with every additional antenna are diminishing. It should be noted that all coding gains observed in the Dh-SM system are in addition to the diversity gains resulting from the increase in the number of receive antennas experienced by both systems.

With the effect of the number of antennas analysed, Fig. 6 shows the effects of the non-symmetric channel gains, *i.e.* $\bar{\gamma}_{sr} \neq \bar{\gamma}_{rd}$. Since Dh-SM relies on sequential detection, from Fig. 6 we can see that both systems exhibit better performance when the average received SNR on the source to relay link is greater than the average received SNR on the relay to the destination link. As can be seen, however, the difference in the average received SNR merely causes a shift in the performance of the entire system. This means that the effects observed in the above discussion extend to arbitrary channel gain conditions and system geometry, given that on the channel vectors, $\mathbf{h}_{k\ell}^i$, from transmitting node k to receiving node ℓ are sufficiently distinct from each other.

With the individual effects of the number of transmit and

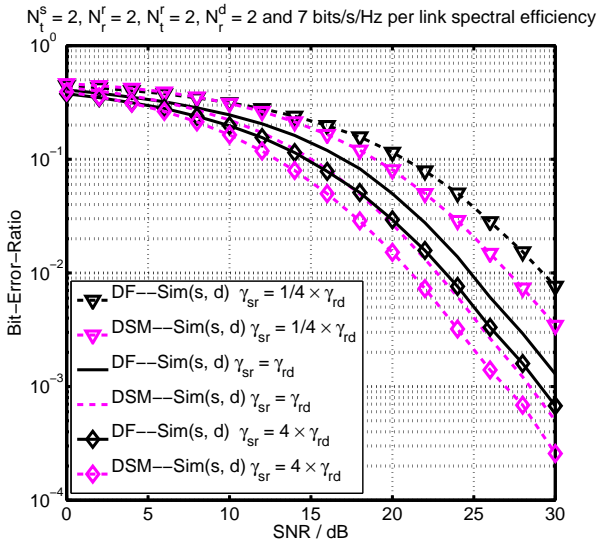


Fig. 6. The average source to destination spectral efficiency is equal to 3.5 bits/s/Hz.

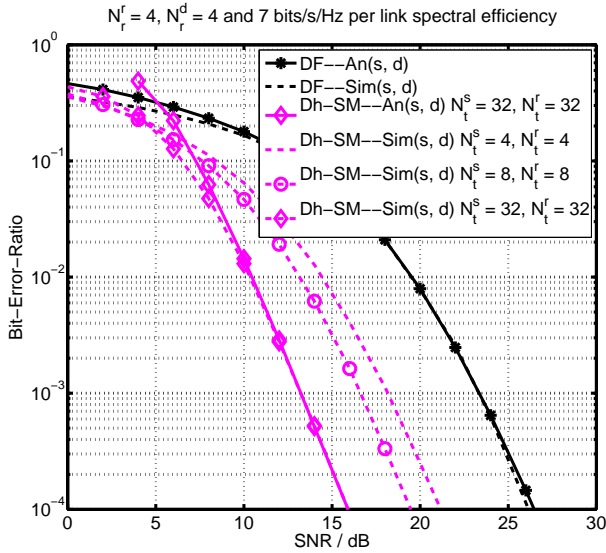


Fig. 7. Average bit error ratio when $\tilde{\gamma}_{sr} = \tilde{\gamma}_{rd}$. The average source to destination spectral efficiency is equal to 3.5 bits/s/Hz.

receive antennas analysed, Fig. 7 shows that Dh-SM can exhibit between 5 and 10 dB gains compared to DF when the number of receive antennas at both the relay and the destination is increased to 4 and $N_t^s = N_t^d$. These gains are larger than those presented in Fig. 3 and Fig. 4, due to the greater number of receive antennas at both the relay and destination, which provide a significant increase in spatial diversity for both systems and additional coding gains for Dh-SM *i.e.* all bottlenecks have been removed from the system.

V. CONCLUSION

In this work the application of SM in a dual-hop, non-cooperative scenario is considered. In Dh-SM the spatial domain is utilised to transmit extra information bits which help alleviate the multihop burden. Dh-SM is also shown to

provide coding gains by lowering the number of bits sent in the signal domain. The union bound method is used to bound the BER behaviour of SM and provide a good estimate for the potential performance of Dh-SM.

It is demonstrated that the application of SM in a relaying scenario results in better end-to-end system performance when compared to non-cooperative DF. The coding gain is increased as the number of transmit antennas is increased at either the source or relay nodes by about 2 dB. Furthermore, the coding gain is also increased with the number of receive antennas added at either the relay or destination nodes resulting in gains of around 3 dB when compared to DF. From this, it can be seen that Dh-SM has the potential to provide substantial spectral efficiency and coding gains in future wireless relay networks.

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REFERENCES

- [1] R. Mesleh, H. Haas, Y. Lee, and S. Yun, "Interchannel Interference Avoidance in MIMO Transmission by Exploiting Spatial Information," in *Proc. of the 16th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 1, Berlin, Germany, 11-14 Sep. 2005, pp. 141-145.
- [2] R. Mesleh, H. Haas, S. Sinanović, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228 - 2241, July 2008.
- [3] N. Serafimovski, M. D. Renzo, S. Sinanović, R. Y. Mesleh, and H. Haas, "Fractional Bit Encoded Spatial Modulation (FBE-SM)," *IEEE Commun. Lett.*, vol. 14, no. 5, pp. 429-431, May 2010.
- [4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [5] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over rayleigh-fading channels," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1126 - 1131, Nov. 2003.
- [6] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial Modulation: Optimal Detection and Performance Analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545-547, 2008.
- [7] T. Handte, A. Muller, and J. Speidel, "BER analysis and optimization of generalized spatial modulation in correlated fading channels," in *Vehicular Technology Conference Fall (VTC Fall-2009)*, Sep. 2009, pp. 1 - 5.
- [8] A. Younis, N. Serafimovski, R. Mesleh, and H. Haas, "Generalized Spatial Modulation," in *Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, 2010.
- [9] M. D. Renzo and H. Haas, "Performance analysis of spacial modulation," in *5th International ICST Conference on Communications and Networking in China*, August 2010.
- [10] R. M. Gagliardi, *Introduction to Communication Engineering*, 2, Ed. Wiley-Interscience, 1988.
- [11] M. K. Simon and M. Alouini, *Digital Communication over Fading Channels*, 2nd ed., ser. Wiley series in telecommunications and signal processing. John Wiley & Sons, Inc., 2005, ISBN: 978-0-471-64953-3.
- [12] M. Di Renzo and H. Haas, "A General Framework for Performance Analysis of Space Shift Keying (SSK) Modulation for MISO Correlated Nakagami-m Fading Channels," *IEEE Trans. Commun.*, vol. 58, no. 9, pp. 2590 - 2603, Sep. 2010.