Development of a strain-gradient plasticity behaviour law for use in machining processes

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Abstract: A mean to better understand the complex phenomena involved in the cutting process is to better qualify the behaviour law used in the simulation of machining processes (analytical and finite element modelling).The aim of this paper is to present the choices made regarding the behaviour law. Our study is based on a strain-gradient plasticity theory with at first a brief state-of-the-art review, we then concentrated our efforts on the strength and limitations of such theories in the context of machining. Gurtin's work on strain gradient plasticity shows the most promising insight toward our work. We finally present what our initiatives, regarding this existing work, will be in order to present a road-map for our future work.

Key words: Strain gradient, plasticity, finite elements, machining, thermodynamics.

Nomenclature:

ñ	Density
q	Heat flux vector
W_{int}	Virtual power of the "internal forces"
å	Strain tensor
ó	Stress tensor
ç	Defect micro-stress
$ó^p$	Plastic stress tensor
h^p	Stress tensor
ø	Free energy
T	Temperature
s	Specific entropy
G	Burgers Tensor

1- Introduction

Previous studies [CD1] have shown, that during cutting processes and more particularly during the chip formation, the presence of complex phenomena has been observed by measuring the moments at the tip of the tool. The analysis of the tool-work piece-chip interface shows the existence of four zones, as shown in Fig. 1. The focus of our study is in relation to the primary and the secondary shear zones.

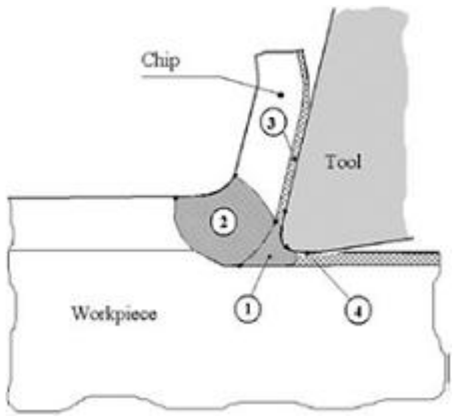

Figure 1 : Characteristic zones of the cutting process.

During chip formation, shear stresses are generated in the primary and secondary shearing zone because of the displacement of material. The micrograph (Fig. 2) shows a non-linear strain evolution. Therefore, strain modelling must take into account the non-linear evolution of strain. Thus, a suitable theory has to be used to model the observed phenomena. The classical theory of the continuous media is the most used theory in mechanics; nevertheless it is not sufficient in this case as substantial rotation phenomena cannot be expressed with the definitions of the displacement.

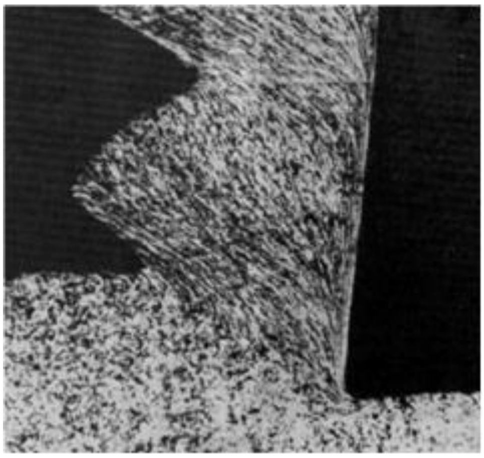

Figure 2 : Chip formation micrograph.

An option would be the couple stress theory considered in [C2]. The media was considered to be comprised of micro-particles self rotating regardless of the displacement of the surrounding media. Furthermore, they were considered perfectly rigid and therefore this theory is not sufficiently representative of the phenomena studied here.

A representative theory is the strain gradient theory, which is a generalization of the classical theory of the continuous media. The displacement description gives the possibility to introduce the strain of the media, as in the classical theory, and the strain gradient. The strain gradient enables the modelling of rotational strain phenomena.

2- Strain gradient plasticity

This method was developed in the 1960s by Mindlin in [M1], Toupin in [T1] and Germain in [G1]. These works form the basis of this theory, but they only considered linear elasticity. Fleck and Hutchinson in [F1] developed the case for plasticity, and the viscoplasticity case was treated by Gurtin in [G2].
In the context of machining we need to develop a theory capable of taking into account phenomena not always found in other research areas. We first need to take into account micro-rotations inside the studied material. We also have to consider working with finite strains as displacements can be considered rather large. We finally need to consider the effect of temperature on the behaviour law of the material. As a result we decided to focus our work on developing a thermodynamically consistent finite deformation strain gradient theory accounting the temperature as a variable of the free energy.

Our work is based on and is the continuation of the work done by different authors. Gurtin in [G3] and [G4] worked on a gradient theory which accounts for Burgers vector and more particularly for dissipations due to plastic spin in [G3]. On the other hand, finite deformation developments of the strain gradient theory were done by different authors such as Polizzotto in [P1] and Lele in [L2]. Polizzotto presents in his work a constitutive model thermodynamically consistent for finite deformations. Lele develops a finite deformation viscoplastic strain gradient theory based on Gurtin's work on strain gradient plasticity and viscoplasticity. Finally, our work was influenced by articles done by Voyiadjis [V1] and Abu Al-Rub [A1] where they focus their work on developing a thermodynamically consistent strain-gradient formulation.

3- Current and future developments

In the section, the complete architecture of the current and future developments will be presented. It will allow to have a better grasp of the purpose of each steps required to development a behaviour law for use in machining.

3.1 – Kinematics

In our developments of a gradient theory fit for a use in machining we decided to fully account for the Burgers vector in our strain-gradient theory; therefore we need to involve the plastic-rotation field in the constitutive theory.

3.2 - Principle of Virtual Powers

In order to determine the correct form balances in our strain gradient theory, we shall use the principle of virtual power. In the external expenditure of power, we have to account for the surface traction, volume forces and a micro-traction. For the internal power expenditure in a similar manner than Gurtin in [G3], we account for expenditure from elastic stress, micro-stress, and defect micro-stress. We obtain an equation similar to the following for the internal power expenditure:

$$W_{\text{int}} = \int_D (\sigma : \dot{\varepsilon} + \sigma^p : \dot{h}^p + \eta : \nabla \times \dot{h}^p) \quad (1)$$

3.3 – Thermodynamics

With the final objective of determining a flow rule for materials used in machining, thermodynamics provide all the necessary information. We use the Clausius-Duhem inequality which derives from the first and second principles of thermodynamics. We can therefore obtain the following equation:

$$\sigma : \dot{\varepsilon} + \sigma^p : \dot{h}^p + \eta : \nabla \times \dot{h}^p - \rho \cdot (\dot{\Psi} + s \cdot \dot{T}) - \frac{q \cdot \nabla T}{T} \leq 0 \quad (2)$$

3.4 – Constitutive theory and Flow rule

In the continuing effort to account for the complex phenomena observed in machining we have to base the theory on an energetic constitutive equation (Eq. 3) that accounts for elastic strain energy, energy associated with the Burgers vectors and temperature.

$$\Psi = \Psi(\varepsilon, G, T) \quad (3)$$

Taking the time derivative of Eq. 3 with respect to its internal state variables and using Eq. 2 we obtain Eq. 4.

$$\left\{\sigma - \rho\frac{\partial\Psi}{\partial\varepsilon}\right\} : \dot{\varepsilon} + \left\{\eta - \rho\frac{\partial\Psi}{\partial G}\right\} : \dot{G} + \sigma^{p} : \dot{h}^{p} - \left[\rho\frac{\partial\Psi}{\partial T} : \dot{T} + s \cdot \dot{T} + \frac{q \cdot \nabla T}{T}\right] \geq 0 \quad (4)$$

We then obtain, using a classical hypothesis permitting to cancel terms in the inequality independently, three state laws (Eq. 5, 6 and 7) and two dissipation equations (Eq. 8 and 9):

$$\sigma = \rho\frac{\partial\Psi}{\partial\varepsilon} \quad (5)$$

$$\eta = \rho\frac{\partial\Psi}{\partial G} \quad (6)$$

$$s = -\rho\frac{\partial\Psi}{\partial T} \quad (7)$$

$$-\frac{q \cdot \nabla T}{T} \geq 0 \quad (8)$$

$$\sigma^{p} : \dot{h}^{p} \geq 0 \quad (9)$$

Equation 7 represents the thermal dissipation per unit of volume and equation 8 represents the plastic dissipation.

We can then choose a form for the free energy and therefore fully express the different equations to obtain the flow rule.

4- Conclusion and Perspectives

During cutting processes, complex phenomena arise and thus the behaviour rule of the material must be suitable to the phenomena. The strain gradient theory is the most suitable to describe the complete behaviour of the material and more particularly the behaviour within the primary and secondary shear zones.
In this paper we presented the complete architecture of the development of a flow rule for use in machining using the strain gradient theory. The different steps and choice of states variables and preliminary equations were shown. Consequently, the complete developments must be done in order to fully express the behaviour law.
Following that study, a 3D finite element implementation will be done in order to test the behaviour law and to confront it to experiments representative of the cutting process. We plan to use the results from the simulation to reverse engineer the different material parameters used in the behaviour law.

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