Modeling Financial Crises Mutation

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Abstract

The recent financial turmoils in Latin America and Europe have led to a concatenation of several events from currency, banking and sovereign debt crises. This paper proposes a multivariate dynamic probit model that encompasses the three types of crises 'currency, banking and sovereign debt' and allows us to investigate the potential causality between all three crises. To achieve this objective, we propose a methodological novelty consisting of an exact maximum likelihood method to estimate this multivariate dynamic probit model, extending thus Huguenin, Pelgrin and Holly (2009).

Using a large sample of data for emerging countries, which experienced financial crises, we find that mutations from banking to currency (and vice-versa) are quite common. More importantly, the trivariate model turns out to be more parsimonious in the case of the two countries which suffered from the 3 types of crises. These findings are strongly confirmed by a conditional probability and an impulse-response function analysis, highlighting the interaction between the different types of crises and advocating hence the implementation of trivariate models whenever it is feasible.

J.E.L. Codes: *C33, F37*

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1 Introduction

Since the tulipmania,[1] economic literature has recorded numerous turmoils affecting the foreign exchange market (currency crisis), the banking market (banking crisis) and the government foreign debt (sovereign debt market). Nevertheless, recent episodes have proved that most of the time crises do not remain restricted to a single market, but tend to spill-over into another one. Analyzing the crisis events over a period of a hundred years in a sample of 56 countries, Bordo et al. (2001) have shown that the $ex-post$ probability of twin banking and currency crises has strongly increased since WWII. Similarly, using data back to the XIX century, Kaminsky and Reinhart (2008) present evidence of a strong connection between debt cycles and economic crises in an analysis of both cross-country aggregates and individual country histories.

Nevertheless, some historical events showed that bidirectional feed-back between crises was not always sufficient to get an exhaustive picture of a turmoil. For example, the Ecuadorian crisis in 1999 affected first the banking sector, subsequently impacted simultaneously the Sucre[2] and the country public finance. More recently, the European crisis emerged as a banking distress succeeding the collapse of the U.S. real estate bubbles. It took a sovereign debt dimension when some European countries (as Greece, Ireland, Portugal), penalized by the recessive consequences of the banking credit crunch or by the public safety plans set up to stabilize the financial system, came close to default. A third dimension is now reached with the increase in volatility of the Dollar/Euro exchange rate as well as the rumors over a split of the Euro area.

The balance sheet approach provides the theoretical framework to analyse the potential spill-over from one crisis to another. Using such an accounting framework, Rosenberg et al. (2005) and, more recently, Candelon and Palm (2010) show how balance sheets are linked across sectors. Consecutively, the transmission of a shock from one country's economy to that of another country will become visible in their balance sheet. The financial crisis takes then another shape.

It appears thus evident that an accurate financial crisis model has to take the mutability of a crisis into account. In a seminal paper, Glick and Hutchinson (1999) model twin crises and assess the extent to which each type of crisis provides information about the likelihood of the other one. Their approach relies in a first step on individual models for currency and banking crises. In a second step, the global model is estimated by using the instrumental variables method so as to tackle the potential endogeneity bias. Implemented on a pooled sample of 90 industrial and developing countries over the $1975-1997$ period, they find that

1. Kindelberger (2000) calls this event the first financial crisis listed in history. It has affected the Dutch tulip market in 1636.
2. The Ecuadorian currency has been replaced by the U.S. dollar on March, 13, 2000.

the twin crisis phenomenon was most common in financially liberalized emerging markets during the Asian crisis. Nevertheless, from a methodological point of view, the use of a two-step approach is not free of criticism with respect to the endogeneity problem. Moreover, the use of a panel framework, driven by the shortness of the time dimension, will require some degree of homogeneity among countries. Finally, Glick and Hutchinson (1999) do not consider sovereign debt crises, focusing exclusively on twin crises.

The issues addressed and the approach adopted in this paper differ from those studied by Pesaran and Pick (2007) who propose a canonical simultaneous equation system and investigate the conditions under which contagion can be distinguished from interdependence (between countries). While our model could be extended to study contagion between countries, we use a dynamic multivariate probit model to study the mutation of crises of one type into crises of another type within a country.

This paper extends Glick and Hutchinson (1999)'s study in several ways: First, it considers a multivariate dynamic probit model that encompasses the three types of crises (currency, banking and sovereign debt), thus allowing to investigate not only the potential mutation from currency to banking crises or vice versa but also the mutability of them into a sovereign debt crisis and vice-versa. Second, this paper introduces a methodological novelty by proposing an exact maximum likelihood approach to estimate this multivariate dynamic probit model. In a related study Dueker (2005) estimates a dynamic qualitative VAR model of business cycle phase using simulation based methods [3] However, as shown by Huguenin, Pelgrin and Holly (2009) for a static model, a multivariate probit model cannot be accurately estimated using simulation methods. Its estimation requires hence to derive an exact maximum-likelihood function. We thus generalize the univariate dynamic probit model developed by Kauppi and Saikkonen (2008) to a multivariate level and we derive its exact likelihood, allowing to obtain converging and efficient parameter estimates. Third, applied to a large sample of emerging countries, we show that in the bivariate case mutations of a banking crisis into a currency crisis (and vice-versa) have been quite common, confirming hence Glick and Hutchinson (1999)'s results. More importantly, for the two countries (Ecuador and South Africa) which suffered from the 3 types of crises, the trivariate model turns out to be more parsimonious, thus supporting its implementation anytime when it is feasible.

The rest of the paper is organized as follows. Section 2 presents a multivariate dynamic probit model. In section 3 we describe the Exact Maximum Likelihood method as well as some numerical procedures to estimate the multivariate dynamic probit model. In section 4, the multivariate dynamic probit model is estimated for 17 emerging countries in its bivariate (twin crises) or trivariate form.

3. See McFadden (1989) or Chib et Greenberg (1998) who proposed the simulation-based Bayesian and non-Bayesian estimation by MCMC of correlated binary data using the multivariate probit model.

2 A Multivariate Dynamic Probit Model

In this section we propose and describe a multivariate dynamic probit model, that allows us to identify and characterize by different means crisis mutation phenomena among several markets.

Consider M latent continuous variables $y^*_{m,i,t}$ representing the pressure on the market m in country i, $i \in 1, ..., I$ at time period $t \in 1, ...T$. A crisis is assumed to occur if the pressure variable $y^*_{m,i,t}$ exceeds a threshold, fixed here to zero. The observed variable $y_{m,i,t}$ takes the value 1 if a crisis occurs on market m, in country i at period t and the value 0 otherwise. For simplicity, the country index is removed in the sequel of the paper as the model is estimated separately for each country. The term 'market' refers to the banking sector, the market for public debt and the foreign currency market. It is used as a synonym for the type of crisis. Denote by y^*_t and y_t the $M \times 1$ vectors with elements $y^*_{m,t}$ and $y_{m,t}$ respectively.

In line with Kauppi and Saikkonen (2008), consider the stochastic process y_t (M-variate) and x_t (K-variate), where y_t is a vector of binary variables taking on the values zero and one and x_t is a vector of explanatory variables. Define $F_t = \textbf{no } \sigma \textbf{ here}[(y'_s, x'_s)'|s \leq t]$, as the information set available at time t. Assume that conditional on F_{t-1}, y_t has an M-variate Bernoulli distribution with probability p_t.

$$y_t|F_{t-1} \sim B(p_t). \tag{1}$$

We consider a multivariate conditional probit specification by assuming that the elements y_t are generated by

$$y_{m,t} = \mathbb{1}(y^*_{m,t} > 0), \tag{2}$$

with $\mathbb{1}(.)$ being the indicator function and

$$y^*_{m,t} = \pi_{m,t} + \epsilon_{m,t}, \tag{3}$$

where $\pi_{m,t}$ represents the expected value of $y^*_{m,t}$ that may depend on covariates $x_{m,t}$ that may vary across markets, countries and time. In vector notation, we have

$$y^*_t = \pi_t + \epsilon_t, \tag{4}$$

with $\pi_{m,t}$ and $\epsilon_{m,t}$ being the m^{th} elements of π_t and ϵ_t respectively and $\epsilon_t|F_{t-1}$ being i.i.d. multivariate normally distributed

$$\epsilon_t|F_{t-1} \sim IIN(0, \Omega). \tag{5}$$

A typical element $\omega_{m,m'}$ of Ω denotes the conditional covariance between $\epsilon_{m,t}$ and $\epsilon_{m',t}$, given the information set F_{t-1}.

The $M \times 1$ vector y_t^* is related to the conditional probability p_t through the common cumulative distribution function of ϵ_t, $\Phi(.)$:

$$p_t = Pr(y_t^* \geq 0|F_{t-1}) = P(-\epsilon_t \leq \pi_t|F_{t-1}) = \Phi(\pi_t). \tag{6}$$

Recall, however, that our objective is to scrutinize the potential crisis mutation among three particular markets, namely currency, banking and sovereign debt. Accordingly, without any loss of generality, henceforth we restrict our attention to the trivariate form of the model. In this case $y_{m,t}$ denotes the type of crisis, $m \in \{1, 2, 3\}$, with $M = 3$.

Hence, crises can be modeled assuming that π_t in (4) is determined as follows:

$$\pi_t = \alpha + Bx_{t-1} + \Delta y_{t-1} + \Gamma\pi_{t-1}. \tag{7}$$

$y_t = (y_{1,t}, y_{2,t}, y_{3,t})'$, $\pi_t = (\pi_{1,t}, \pi_{2,t}, \pi_{3,t})'$, and $\alpha = (\alpha_1, \alpha_2, \alpha_3)'$. B is a $3 \times k$ matrix and Δ and Γ are 3×3 matrices. If the exogenous variables are specific to the type of crisis so there is no common cause among them. Then $x_{t-1} = (x'_{1,t-1}, x'_{2,t-1}, x'_{3,t-1})'$, $x_{m,t-1}$ is a $(k_m \times 1)$ vector of explanatory variables corresponding to the m^{th} dependent variable at time $t-1$ and B is block-diagonal with the typical block on the diagonal being the row vector b'_m of slope coefficients corresponding to $x_{m,t-1}$. Similarly we could also allow the dynamics to be crisis-specific by assuming that Δ and Γ are diagonal matrices. Obviously, when there are covariates in common for some crises, the number of variables in x_{t-1}, k, will be smaller than $\sum_{m=1}^{3} k_m$, and B will not be block-diagonal.

Denote by $\theta_m = (\alpha_m; b_m; \delta_m; \gamma_m)'$ the vector of parameters for equation m, that is the stacked m^{th} lines of α, B, Δ and Γ and by $\theta = (\theta'_1; \theta'_2; \theta'_3)'$ all intercepts and slope coefficients in (7). Finally, we assume that the error term has a trivariate normal distribution. The disturbances ε are i.i.d. with a trivariate normal distribution of zero mean and covariance matrix $V(\varepsilon) = I_T \otimes \Omega$, where Ω is a covariance matrix given by:

$$\Omega = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 \\ \rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix}, \tag{8}$$

where $\rho_{m,m'}$ represent the correlation coefficients.

In this multivariate framework several ways to perceive the mutation mechanism can be designed, each of which implying restrictions in (7).

1. Unobserved common factors can be taken into account through the contemporaneous inter-market dependence of the innovation terms ($\omega_{mm'} \neq 0$).

2. The unobservable latent variable $y^*_{m,t}$ depends on past unobservable pressure values of other markets $y^*_{m',t-s}$ for $m \neq m'$. It implies that $\Delta = 0$. In such a case, $\pi_{m,t}$ depends only on the indexes $\pi_{m',t-s}$, $s > 0$, which can be interpreted as a mutation phenomenon.
3. The unobservable latent variable $y^*_{m,t}$ on a specific market may depend on past crisis/calm periods on other markets. Formally, the pressure index $y^*_{m,t}$ depends on past values of the observable variable $y_{m',t}$, where $m \neq m'$ and thus the column m' of Δ is different from 0.
4. It is possible to combine the two previous cases, assuming that $y^*_{m,t}$, depends on both the latent variable $y^*_{m',t}$, and past crisis/calm periods, $y_{m',t}$, on other markets.
5. Crises dynamics can be modeled by considering that the latent variable $y^*_{m,t}$ depends on its lagged values, $i.e.$ $\pi_{m,t}$ depends on $\pi_{m,t-1}$ via the matrix Γ.

Model (7) is a multivariate extension of the dynamic probit model recently proposed by Kauppi and Saikkonen (2008). This new specification enables us to compute not only marginal crisis probabilities, $\Pr(y_m = 1|y^*_m) = \Phi(y^*_m)$, as it is usually done in the literature, but also joint and conditional probabilities, $i.e.$ $\Pr(y_1 = 1, y_2 = 1, y_3 = 1|y^*) = \Phi_3(y^*)$, and $\Pr(y_m = 1|y^*_{m'}) = \Phi_3(y^*)/\Phi_2(y^*_{m'})$, for $m, m' \in \{1, 2, 3\}$, where Φ, Φ_2 and Φ_3 represent the univariate, bivariate and trivariate normal cumulative distribution functions respectively.

Remarks:

Matrices Δ and Γ provide useful information about the dynamics of crises, in particular about their persistence and mutation (causality). On the one hand, the diagonal terms of Γ specify the persistence of each crisis. These parameters correspond to a first order autoregressive representation of the latent variable. An increase in the pressure index during a certain period is always transmitted to the next period, hence linearly increasing the probability of a turmoil. The closer they are to 1, the more persistent the crisis episode will be. It is noticeable that the diagonal elements of this matrix will be constrained to be strictly inferior to 1. We exclude the case where the latent variable $y^*_{m,t}$ follows a random walk, which would be empirically counter-intuitive as financial crises cannot be apprehended as persistent events.

At the same time, the diagonal terms of Δ also deliver information about crises persistence but that is somewhat different from that inferred from Γ. Indeed, they indicate to what extent the probability of occurrence of a crisis depends on the regime prevailing the period before. In this situation we observe the existence of threshold effects, as a tumultuous period lasts more than one spell only if the pressure index soars sufficiently to exceed a threshold which sets off a crisis in the previous period.

Altogether, we can distinguish between a linear crisis persistence, captured through the diagonal terms of Γ, and a non-linear, threshold-based one, apprehended by the diagonal terms of Δ.

On the other hand, mutation is taken into account in the off-diagonal elements of the two matrices Γ and Δ. These Granger-causal effects between the three crises play a key role in the mutation of crises. As in the analysis of crises persistence, both a linear and a non-linear, threshold-effect transmission can be identified. A significant off-diagonal γ element shows that no sooner the pressure index on a specific market rockets than the index on another market rises. By contrast, a δ term reveals the presence of crisis transmission only if the corresponding pressure index is high enough to generate a crisis on the other market. In other words, if $\gamma_{1,2} > 0$ a high money market pressure at time $t-1$ increases the probability of a currency turmoil at t, whereas if $\delta_{1,2} > 0$ the probability of a currency turmoil increases at t as a consequence of a banking crisis that occurred the period before.

3 Exact Maximum Likelihood Estimation

The exact maximum likelihood estimator for the multivariate dynamic probit model cannot be obtained as a simple extension from the univariate model. For this reason, the simulated maximum likelihood method is generally considered. Nevertheless, Holly, Huguenin and Pelgrin (2009) prove that it leads to a bias in the estimation of the correlation coefficients as well as in their standard deviations. Therefore, they advocate the exact maximum likelihood estimation. Since the correlations between the crisis binary variables, i.e. the contemporaneous transmission channels from one crisis to another one, constitute our main focus, asymptotic unbiased estimation of the correlations is of importance here and it calls for an explicit form of the likelihood. This section deals with this objective.

3.1 The Maximum Likelihood

Let us first notice that to identify the slope and covariance parameters, we impose that the diagonal elements of Ω to be standardized, i.e. equal to one. Following Greene (2002), the full information maximum-likelihood (FIML) estimates are obtained by maximizing the log-likelihood $\text{LogL}(Y|Z;\theta,\Omega)$, where θ is the vector of identified parameters and Ω is the covariance matrix. Under the usual regularity conditions [4] (Lesaffre and Kauffmann, 1992),

4. If the parameters θ are estimated while the correlation coefficients are assumed constant, the log-likelihood function is concave. In this case the MLE exists and it is unique. Nevertheless, when θ and ρ are jointly estimated (as in our model), the likelihood function is not (strictly) log-concave as a function of ρ. Thus, the MLE exists only if the log-likelihood is not identically $-\infty$ and $E(z^T z|\rho)$ is upper semi-continuous finite and not identically 0. Furthermore, if no $\theta \neq 0$ fulfills the first order conditions for a maximum, the MLE of (θ, ρ) for the multivariate probit model exists and for each covariance matrix not on the boundary

the likelihood is given by the joint density of observed outcomes:

$$\mathrm{L}(y|z,\theta;\Omega)=\prod_{t=1}^{T}L_t(y_t|z_{t-1},\theta;\Omega), \tag{9}$$

where $y_t = (y_{1,t}, y_{2,t}, y_{3,t})'$ and $y = [y_1, ..., y_T]$. The individual likelihood $L_t(.)$ is given in Lemma 1 as it is a well known result in the literature.

Lemma 1. *The likelihood of observation t is the cumulative density function, evaluated at the vector w_t of a 3-variate standardized normal vector with a covariance matrix $Q_t\Omega Q_t$:*

$$L_t(y_t|z_{t-1},\theta;\Omega)=Pr(y_1=y_{1,t},y_2=y_{2,t},y_3=y_{3,t})=\Phi_{3,\varepsilon_t}(w_t;Q_t\Omega Q_t), \tag{10}$$

where Q_t is a diagonal matrix whose main diagonal elements are $q_{m,t} = 2y_{m,t} - 1$ and thus depends on the realization or not of the events ($q_{m,t} = 1$ if $y_{m,t} = 1$ and $q_{m,t} = -1$ if $y_{m,t} = 0$, $\forall\ m \in \{c, b, s\}$). Besides, the elements of the vector $w_t = [w_{1,t}, ..., w_{3,t}]$ are given by $w_{m,t} = q_{m,t}\pi_{m,t}$ (for a complete proof of Lemma 1, see Appendix 1).

Thus, the FIML estimates are obtained by maximizing the log-likelihood:

$$\mathrm{LogL}(y|z,\theta;\Omega)=\sum_{t}^{T}\mathrm{Log}\Phi_{3,\varepsilon}(w_t;Q_t\Omega Q_t) \tag{11}$$

with respect to θ and Ω [5].

3.2 The Empirical Procedure

The main problem with FIML is that it requires the evaluation of high-order multivariate normal integrals while existing results are not sufficient to allow accurate and efficient evaluation for more than two variables (see Greene, 2002, page 714). Indeed, Greene (2002) argues that the existing quadrature methods to approximate trivariate or higher-order integrals are far from being exact. To tackle this problem in the case of a static probit, Huguenin, Pelgrin and Holly (2009) decompose the triple integral into simple and double integrals, leading to an Exact Maximum Likelihood Estimation (EML) that requires computing double integrals. Most importantly, they prove that the EML increases the numerical accuracy of both the slope and covariance parameters estimates, which outperform the maximum simulated likelihood method (McFadden,1989) which is generally used for the estimation of multivariate probit models. Therefore, we extend the decomposition proposed by Huguenin,

of the definition interval, the MLE is unique.

5. Besides, we tackle the autocorrelation problem induced by the binary crisis variable by considering a Gallant correction for the covariance matrix.

Pelgrin and Holly, (2009) in the case of our multivariate dynamic model so as to obtain a direct approximation of the trivariate normal cumulative distribution function.

The EML log-likelihood function is given by:

$$\text{LogL}(y|z,\theta;\Omega) = \sum_{t=1}^{T} \text{Log}\left[\prod_{m=1}^{3} \Phi(w_{m,t}) + G\right], \tag{12}$$

where $\Phi(w_t)$ is the univariate normal cumulative distribution function of w_t. Indeed, the log-likelihood function depends on the product of the marginal distributions (w_t) and the correction term G which captures the dependence between the m events analyzed.

The maximum likelihood estimators $\{\widehat{\theta};\widehat{\Omega}\}_{EML}$ are the values of θ and Ω which maximize (12):

$$\{\widehat{\theta};\widehat{\Omega}\}_{EML} = \text{Arg}\max_{\theta;\Omega} \sum_{m=1}^{3} \text{LogL}(.), \tag{13}$$

with $L(.)$ given in (11).

Under the regularity conditions of Lesaffre and Kaufman (1992), the EML estimator of a multivariate probit model exists and is unique. Besides, the estimates $\{\widehat{\theta};\widehat{\Omega}\}_{EML}$ are consistent and efficient estimators of the slope and covariance parameters and are asymptotically normally distributed. It is worth noting that in a correctly specified model for which the error terms are independent across the m equations the EML function corresponds to $\sum_{t=1}^{T}\prod_{m=1}^{3}\Phi(w_{m,t})$, since the probability correction term G in eq. (12) tends toward zero.

We present here only the results for a bivariate and a trivariate model, which we shall use in the empirical analysis. Further details are provided in Appendices 1-3:

$$\Phi_2(w_t;Q_t\Omega Q_t) = \Phi(w_{1,t})\Phi(w_{2,t})\frac{1}{2\pi}\int_{0}^{\rho_{12}} \exp\left(-\frac{1}{2}\frac{w_{1,t}^2 + w_{2,t}^2 - 2w_{1,t}w_{2,t}}{1-\lambda_{12}^2}\right)\frac{d\lambda_{12}}{\sqrt{1-\lambda_{12}^2}} \tag{14}$$

for a bivariate model and

$$
\begin{aligned}
\Phi_3(w_t; Q_t\Omega Q_t) &= \prod_{m=1}^{3} \Phi(w_{m,t}) + G \\
&= \Phi(w_{1,t})\Phi(w_{2,t})\Phi(w_{3,t}) \\
&+ \Phi(w_{3,t}) \int_0^{\rho_{12}} \phi_2(w_{1,t}, w_{2,t}; \lambda_{12}) d\lambda_{12} \\
&+ \Phi(w_{2,t}) \int_0^{\rho_{13}} \phi_2(w_{1,t}, w_{3,t}; \lambda_{13}) d\lambda_{13} \\
&+ \Phi(w_{1,t}) \int_0^{\rho_{23}} \phi_2(w_{2,t}, w_{3,t}; \lambda_{23}) d\lambda_{23} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, 0)}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, 0, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\lambda_{23} \\
&+ \int_0^{\rho_{13}} \int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; 0, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} \\
&+ \int_0^{\rho_{12}} \int_0^{\rho_{13}} \int_0^{\rho_{23}} \frac{\partial^3 \phi_3(\dddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t} \partial w_{2,t} \partial w_{3,t}} d\lambda_{12} d\lambda_{13} d\lambda_{23}
\end{aligned}
\qquad (15)
$$

for a trivariate model, where ρ are the non-diagonal elements of the $Q_t\Omega Q_t$ matrix and λ are the non-diagonal elements of a theoretical 2×2 matrix and respectively a 3×3 matrix in which one of the correlation coefficients is null. Moreover, $\dot{w}_t$ is a vector of indices obtained by changing the order of the elements to $(w_{2,t}, w_{3,t}, w_{1,t})$. Similarly $\ddot{w}_t$ corresponds to a vector of indices of the form $(w_{3,t}, w_{1,t}, w_{2,t})$. Finally, $\dddot{w}_t$ corresponds to w_t,$\dot{w}_t$ or $\ddot{w}_t$ respectively, depending on the way the last integral is decomposed. The computation of the last term is not trivial. However, this integral can be decomposed in a non-unique way as follows:

$$
\begin{aligned}
&\int_0^{\rho_{12}}\int_0^{\rho_{13}}\int_0^{\bar{\rho}_{23}} \frac{\partial^3 \phi_3(\dddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t} \partial w_{2,t} \partial w_{3,t}} d\lambda_{12} d\lambda_{13} d\lambda_{23} \\
&\quad = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} - \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t; 0, \lambda_{13}, \lambda_{23})}{\partial w_{3,t}} d\lambda_{13} d\lambda_{23} \\
&\quad = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\bar{\rho}_{23} - \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t; \lambda_{12}, 0, \lambda_{23})}{\partial w_{2,t}} d\lambda_{12} d\lambda_{23} \\
&\quad = \int_0^{\lambda_{12}}\int_0^{\lambda_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, \lambda_{23})}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13} - \int_0^{\rho_{12}}\int_0^{\rho_{13}} \frac{\partial \phi_3(w_t; \lambda_{12}, \lambda_{13}, 0)}{\partial w_{1,t}} d\lambda_{12} d\lambda_{13}.
\end{aligned}
\tag{16}
$$

These finite-range multiple integrals are numerically evaluated by using a Gauss-Legendre Quadrature rule [6] over bounded intervals. In such a context, two possibilities can be considered: whether the likelihood function is directly maximized, or the first order conditions [7] are derived so as to obtain an exact score vector. As stressed by Huguenin, Pelgrin and Holly (2009), the two methods may not lead to the same results if the objective function is not sufficiently smooth.

4 Empirical Application

This section aims at implementing the multivariate dynamic probit methodology presented above to a system composed by three types of crises, *i.e.* currency, banking and sovereign debt crises. We thus evaluate the probability of mutation of one type of crisis into another one. After a short data description and the presentation of the criteria implemented to detect the three types of crises, we estimate bivariate models by excluding sovereign debt crises. This constitutes a benchmark for the second part where the sovereign debt crises are included in the system.

4.1 Dating the crises

4.1.1 The Database

Monthly macroeconomic indicators expressed in US dollars covering the period from January 1985 to June 2010 have been extracted for 17 emerging countries [8] from the IMF-IFS

6. Details about this quadrature are available in Appendix 2.

7. The score vector of the trivariate probit model is presented in Appendix 3.

8. Argentina, Brazil, Chile, Colombia, Ecuador, Egypt, El Salvador, Indonesia, Lebanon, Malaysia, Mexico, Panama, Peru, Philippines, South Africa, Turkey and Venezuela.

database as well as from the national banks data of the countries under analysis via Datastream.[9] The government bond returns are obtained via the JPMorgan EMDB database. More exactly, we have selected the main leading indicators used in the literature for the three types of crises that we analyze (see Candelon et al., 2009, Jacobs et al., 2003, Glick and Hutchison, 1999, Hagen and Ho, 2004, Pescatori and Sy, 2007), namely, the one-year growth rate of international reserves, the growth rate of M2 to reserves ratio, one-year growth of domestic credit over GDP ratio, one-year growth of domestic credit, one-year growth of GDP, government deficit, debt service ratio and external debt ratio.

4.1.2 Dating the Crisis Periods

1. **The Currency Crises**

   Currency crises are generally identified using the market pressure index (MPI), which is a linear combination between exchange rate and foreign reserves changes. Hence if the pressure index exceeds a predetermined threshold[10] a crisis period is identified.

   As in Lestano and Jacobs (2004) and Candelon et al. (2009), a modified version of the pressure index proposed by Kaminski et al.(1998), which also incorporates the interest rate is used. It is denoted by (KLRm) and takes the following form of a weighted average of the percentage of the exchange rate and of the foreign reserves and of the change in the domestic interest rate:

$$\mathrm{KLRm}_{i,t} = \frac{\Delta e_{i,t}}{e_{i,t}} - \frac{\sigma_e}{\sigma_{res}} \frac{\Delta res_{i,t}}{res_{i,t}} + \frac{\sigma_e}{\sigma_r} \Delta r_{i,t}, \tag{17}$$

   where $e_{i,t}$ denotes the exchange rate (*i.e.*, units of country i's currency per US dollar in period t), $res_{i,t}$ represents the foreign reserves of country i in period t (expressed in $US\$$), while $r_{i,t}$ is the interest rate in country i at time t. σ_x denotes the standard deviation of the relative changes in the variable $\Delta x_{i,t}/x_{i,t}$, where x denotes each variable separately, including the exchange rate, foreign reserves, and the interest rate, with $\Delta x_{i,t} = x_{i,t} - x_{i,t-6}$.[11] For both subsamples, the currency crisis ($CC_{i,t}$) threshold equals 1.5 standard deviations above the mean:

$$CC_{i,t} = \begin{cases} 1, & \text{if } \mathrm{KLRm}_{i,t} > 1.5\sigma_{\mathrm{KLRm}_{i,t}} + \mu_{\mathrm{KLRm}_{i,t}} \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

9. We choose not to include any European country, as 1) only few of them have suffered from the three types of crises and 2) if this is the case it corresponds to a single episode: the recent turmoil.

10. Usually fixed to 2 or 3 times the sample's standard deviation as in Kaminski et al.(1998).

11. Additionally, we take into account the existence of higher volatility in periods of high inflation, and consequently the sample is split into high and low inflation periods. The cut-off corresponds to a six months inflation rate being higher than 50%.

2. **The Banking Crises**

Banking crises are most commonly identified using the banking sector balance sheet, policy responses to bank runs and bank failures on a yearly basis (see the recent dating of Leaven and Valencia (2008). Nevertheless, our crisis dating requires a monthly frequency. Moreover, Eichengreen (1995, 1996) notices that banking crises are not always associated with a visible policy intervention. Indeed, some interventions may take place in the absence of a crisis in order to solve structural economic problems and perhaps to prevent a crisis. Besides, some measures can be taken only when the crisis has spread to the whole economy. Thus, Hagen and Ho (2004) propose a money market pressure index, accounting for the increasing demand for central bank reserves, to identify banking crises. Thus, it resembles a banking pressure index (BPI), available at monthly frequency:

$$BPI_{i,t} = \frac{\Delta\gamma_{i,t}}{\sigma_{\Delta\gamma}} + \frac{\Delta r_{i,t}}{\sigma_{\Delta r}}, \tag{19}$$

where $\gamma_{i,t}$ is the ratio of reserves to bank deposits in country i at time t, r is the real interest rate, Δ operator represents the six-months difference operator, and $\sigma_{\Delta\gamma}$ and $\sigma_{\Delta r}$ are the standard deviations of the two components. Sharp increases in the indicator (greater than the 90^{th} percentile denoted as $P_{BPI,90}$) signal a banking crisis:

$$BC_{i,t} = \begin{cases} 1, & \text{if } \mathrm{IMP}_{i,t} > P_{BPI,90,i} \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

3. **The Sovereign Debt Crises**

Countries' 'default' does not constitute an adequate measure to characterize a sovereign debt crisis. Indeed a country may face debt-servicing difficulties or problems to refinance its debt on the international capital markets, without being in default. In order to overcome this problem, Pescatori and Sy (2007), consider a market-oriented measure of debt-servicing difficulties based on sovereign bond spreads.

In the line of this study, we consider that a sovereign debt crisis ($SC_{n,t}$) occurs if the CDS spreads exceed a critical threshold estimated by using kernel density estimation. More precisely, the existence of a mode around high spread values can be used to define crisis and calm periods, since whenever spreads are close to a limit that cannot be passed smoothly, the observations will concentrate around it until the limit is finally broken or the increasing pressure is reduced. Additionally, as expected, this estimated threshold corresponds to a percentile between the 90^{th} and the 99^{th} percentiles, depending on the country (the number of crisis periods varies from one country to another), since crises are extreme events:

$$SC_{i,t} = \begin{cases} 1, & \text{if CDSspread}_{i,t} > Kernel\ Threshold_i \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

It is worth noting that most of the crisis periods we have identified by using the three aforementioned methods correspond to the ones reported in the literature on financial crises, e.g Reinhart and Rogoff (2008).

4.1.3 Remarks

1. As in Kumar (2003), we dampen the magnitude of every variable using the formula : $f(x_t) = sign(x_t)\log(1 + |x_t|)$, so as to reduce the impact of extreme values. [12]
2. It should also be noted that the entire sample is used for the identification of currency and banking crises, while the identification of debt crises is realized by using data from December 1997 (See Table 1) since the CDS spread series used for the identification of sovereign debt crises are not available before 1997 in the JPMorgan EMDB database. Consequently our empirical analysis will consist of two parts, the first one analyzing the case of twin crises (currency and banking) for which the entire database can be used, while the second part focuses on the interactions between the three types of crises and is thus based on data from 1997 onwards. The data sample actually used for each of the 17 countries and the two types of analyses is available in Table 1.
3. We only retain the countries for which the percentage of crisis periods is superior to 5% (See Table 2). [13]
4. As mentioned in section 2, there are three dynamic multivariate specifications that can be used. However, as shown by Candelon et al. (2010), the dynamic model including the lagged binary variable seems to be the best choice according to model selection using the Akaike information criterion. However, since we cannot expect a crisis to have a certain impact on the probability of emergence of another type of crisis from one month to another, which would justify the notation $y_{m,t-1}$ from the theoretical part, in the empirical application we consider a response lag l of 3, 6 and respectively 12 months for the bivariate models and one of 3 or 6 months for the trivariate models [14]. Therefore, for each type of crisis we build a lagged variable $y_{m,t-k}$ which takes the value of one if there was crisis in the past k periods or at time t, and the value of 0 otherwise:

12. Missing values of the series are replaced by cubic spline interpolation.
13. Argentina, Chile, Ecuador, Egypt, Indonesia, Lebanon, Mexico, South Africa and Venezuela are included in the bivariate analysis, whereas a trivariate model is specified for Ecuador and South Africa. Since the threshold has been arbitrarily set to 5%, we have also checked the borderline countries, like Colombia or Turkey in the bivariate analysis and Egypt in the trivariate analysis respectively, and similar results have been obtained.
14. A 12 months lag is not used in the case of trivariate models since it would significantly reduce the already small number of observations we have at our disposal.

$$y_{m,t-k} = \begin{cases} 1, & \text{if } \sum_{j=0}^{k} y_{m,t-j} > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

5. The significance of the parameters of each model is tested by using simple t-statistics based on robust estimates of standard-errors (which rely on a Gallant kernel, as in Kauppi and Saikkonen, 2008). A special attention is given to the interpretation of cross-effects which stand for the transmission channels of the shocks/crisis. Besides, the joint nullity of the contemporaneous correlations between shocks is tested using a log-likelihood ratio test for the trivariate models.

4.2 Bivariate Analysis

Along the lines of Kaminsky et al. (1998) it is possible to find a large number of explanatory variables that may signal the occurrence of a crisis. Nevertheless, Candelon et al. (2010) showed that a univariate dynamic probit model presents the advantage of yielding plausible results while being fairly parsimoniously parametrized. Indeed, a large part of the information is integrated either in the past state variable or in the lagged index and thus, only a few explanatory variables turn out to be significant. In this context, we expect their multivariate (bivariate or trivariate) extension to be even more parsimonious. Therefore, we consider the four explanatory variables which are significant in Candelon et al. (2010), i.e. one-year growth of international reserves, one-year growth of M2 to reserves for currency crises as well as one-year growth of domestic credit over GDP and one-year growth of domestic credit for banking crises, resulting in four different specifications including one explanatory variable for each type of crisis. Moreover, three different lags (3 months, 6 months and 12 months) are considered for the lagged binary variable $y_{m,t-k}$. The dynamic probit model is estimated country-by-country using the Exact maximum likelihood. [15] It is indeed a simplification as contagion (or spill-overs) from one country to another are not taken account. A panel version of the model would lead to several problems. First, as shown by Berg et al. (2008) heterogeneity due to country specificities would have to be accounted for. Second, the estimation of a fixed effect panel would be biased without a correction on the score vector. [16] Third, in a country by country analysis contagion has to be ignored. For all these reasons, we consider this extension to be beyond the scope of this paper and leave it for future research.

Each model is estimated via maximum-likelihood, the bivariate normal cumulative distribution function being approximated using the Gauss-Legendre quadrature, as proposed by Huguenin, Pelgrin and Holly, (2009). However, the quadrature specified in Matlab by

15. Initial conditions are introduced as given by the univariate static probit.
16. See Candelon et al., (2010) for a discussion about this point.

default, *i.e.* the adaptive Simpson quadrature, has been considered as a benchmark.

Information criteria, namely AIC and SBC, are used to identify the best model for each country. Specification (14), i.e. with the lagged binary variable turns out to be preferred. Optimal lag lengths are determined similarly. It is nevertheless worth stressing that the results are generally robust to the choice of explanatory variables and even to the choice of lags.

A summary of the results for the selected models is given in Table 3.

insert Table 3

First of all, it seems that most of the models exhibit dynamics, whatever the lag used to construct the 'past crisis' variable is. This result confirms the findings of Candelon et al. (2010) and Bussière (2007), showing that crises exhibit a regime dependence: if the country is proven to be more vulnerable than investors had initially thought, investors will start withdrawing their investments, thus increasing the probability of a new crisis. More precisely, most of the countries are found to have experienced banking and currency crises, with a significant autoregressive coefficient, i.e the crisis variable depends on its own past, *e.g.* Argentina, Egypt, Lebanon, Mexico, South Africa, Venezuela. Besides, only for a small number of cases, only one of the two types of crises is best reproduced by a dynamic model (currency crises in Chile (3 and 12 months), Mexico (6 and 12 months) ; banking crises in Argentina (6 and 12 months), Ecuador, Lebanon (6 months), South Africa (12 months) and Venezuela (12 months)). Actually, in Chile a past currency crisis has only a short term positive impact on the emergence of another currency crisis, whereas a banking crisis has just a long term effect on the probability of occurrence of another banking crisis. Mexico, however, seems to be more prone to recurring currency crises than banking crises as the former type of crisis has a long-term impact on the probability of experiencing a new crisis, whereas the latter has a positive effect only in the short run. On the contrary, for Argentina, South Africa and Venezuela the impact of past banking crises on currency crises is longer (up to one year) as opposed to that of past currency crises on banking ones (up to three and six months, respectively).

Second, for the majority of the countries (Argentina, Chile, Lebanon, Mexico and Venezuela), currency and banking crises are interconnected. This link between crises can take two forms. On the one hand, a certain type of crisis increases (or diminishes) the probability of occurrence of the other type of crisis. This strong link between banking to currency crisis was emphasized by Glick and Hutchinson (1999) within a panel framework. Nevertheless, there is no reason for the transmission of shocks to be symmetric. Indeed, our country per country analysis reveals that for some countries like Argentina (3 and 6 months) a banking crisis in the past months increased the probability of a currency crisis at time t. At the same time, a banking crisis in Chile in the last 12 months reduced the probability of experiencing a currency crisis. Conversely, a currency crisis in Egypt and in Lebanon (3 months) diminished the probability of a banking crisis.

On the other hand, crisis shocks can be contemporaneously positively correlated. This feature seems to be very stable across models (independent of the lag used). The only exceptions are Egypt and Lebanon, for which there is no instantaneous correlation in the model with 3-months lagged binary variables and Mexico, for which such a correlation appears only for the 12-months lag. To sum up, but for Egypt, all countries are characterized by a positive instantaneous correlation between shocks of currency and banking crises variables, corroborating the previous findings of Glick and Hutchinson (1999).

Third, the macroeconomic variables are rarely significant.[17] These results corroborate our previous findings (see Candelon et al. 2010) that the dynamics of crises captures most of the information explaining the emergence of such phenomena. Furthermore, when these coefficients are significant, they have the expected sign (an increase in the growth of international reserves diminishes the probability of a crisis, while a surprise in the rest of indicators soars the probability of a crisis).

To summarize, these results confirm the presence of interaction between the banking and currency crisis. The twin crisis phenomenon is thus confirmed empirically. Besides, our findings are robust to the quadrature choice and the lags considered when constructing the dynamic binary variables.

4.3 Trivariate Analysis

But is it really enough to look at two crises only ? This subsection extends the previous analysis to the trivariate case by modeling simultaneously the occurrence of a currency, banking and debt crises. However, only two countries experienced these three events during a sufficiently long period. Ecuador presents for our sample an *ex-post* probability larger than 5% for whatever the type of crisis. Such a result is not surprising if one remembers that Ecuador faced a strong financial turmoil in the late 1990, affecting first the banking sector,[18] then the Sucre[19], and the government budget. Jacone (2004) showed that institutional weaknesses, rigidities in public finances, and high financial dollarization have amplified this crisis. South Africa constitutes a borderline case as the sovereign debt crisis probability is slightly below 5%.

Each of the models is estimated for these countries using both the methodology proposed by Huguenin et al. (2009) based on the Gauss-Legendre quadrature and the direct approxi-

17. These results are available upon request.
18. 16 out of the 40 banks existing in 1997 faced liquidity problems.
19. The Ecuadorian currency has been replaced by the U.S. dollar on March, 13, 2000.

mation of a triple integral based on the adaptive Simpson quadrature that Matlab uses by default. Similar results are obtained for the two methods.[20] However, the latter implies a significant gain in time without any loss in accuracy proving that recently developed quadrature methods are good approximations of the normal cumulative distribution function. Besides, 6 and 12 month-lags of the dynamic crisis variable are considered.

insert Table 4

In the case of Ecuador, the results corroborate our bivariate findings: the banking crises are persistent, while currency crises are not. Nevertheless, it is clear that the bivariate model is misspecified, since it cannot capture the impact of a banking crisis on the occurrence of a currency crisis when using the 6-months lagged binary variables to account for the dynamics of these phenomena (see Table 4).

Moreover, the trivariate model turns out to be more parsimonious in terms of parameters to be estimated since the index of past debt crisis has a positive effect on the probability of occurrence of both currency and debt crises. Therefore it supports the implementation of a trivariate crisis model whenever when it is feasible. We also observe that the contemporaneous correlation matrix is diagonal, ruling out common shocks. Crises in Ecuador turn out to be exclusively driven by transmission channels, as in the late 1990, when the banking distress was diffused to the currency and the government budget.

In the case of South-Africa, both currency and debt crises are dynamic. There is no evidence of causality between the different types of crises, but significant contemporaneous correlation. It highlights the fact that contrary to Ecuador, South African crises did not mutate but they originated from a common shock. It is worth noting that the results are found to be robust in the sensitivity analyses performed, namely to the choice of macroeconomic variables and the use of different lags for the past crisis variables.

4.4 Further results

To grasp better the properties of the models estimated and selected, a conditional probability as well as an Impulse Response Functions (IRF) analysis are provided. For sake of space, we only report the results obtained for Ecuador.[21]

First, Figure 1 reports the conditional probabilities for each type of crisis obtained from both the bi- and trivariate models considering a forecast horizon of 3 and 6-months. To allow a fair comparison, both models are estimated from the same sample, *i.e.* from 1997

20. The results for Ecuador when considering a 6-months lag have been obtained with Matlab's quadrature since the model based on the Gauss-Legendre Quadrature did not converge.

21. For South Africa, crisis mutation is exclusively driven by the contemporaneous correlation matrix as indicated in Table 4. Otherwise we can see that currency and sovereign debt crises are more persistent than banking ones. All figures are available from the authors upon request.

onwards. It goes without saying that the bivariate model does not provide any conditional probabilities for sovereign debt crisis.

It turns out that the trivariate model outperforms the bivariate one whatever the forecast horizon is, *i.e.* the conditional probabilities issued from the trivariate model are higher than those obtained from the bivariate model during observed crisis periods, while they appear to be similar for calm periods. Such results corroborate hence our previous findings, stressing that a crisis model should take into account the whole sequence of crises to be accurate. Besides, the conditional probabilities obtained from the trivariate model do not immediately collapse after the occurrence of the crisis, which is the case for the bivariate model. It stresses hence the vulnerability of the economy after the exit from a turmoil in particular if it affects the foreign exchange market.

Second, to evaluate the effect of a crisis, considered here as a shock, an IRF analysis is performed for the trivariate model. As the order of the variables has been shown to be crucial, we consider the historical sequence of crises observed in Ecuador, *i.e.* banking crises (the most exogenous ones), debt crises and currency crises (the most endogenous ones). Orthogonal impulse response functions are considered on the latent variable for a 3 month-horizon. The exogenous variables are fixed to the unconditional mean ($\bar{x}_{m,t}$). Departing from eq. 1, we express the IRF in terms of the latent model, i.e. the probability of being in a crisis state at time t and the binary crisis/calm variable as follows:

$$\begin{aligned} y^*_{m,t} &= \hat{\alpha}_m + \bar{x}_{m,t-1}\hat{\beta}_m + \Delta_{m,m'}\tilde{y}_{m',t-1} + \hat{\varepsilon}_{m,t}, \\ \Pr(\hat{y}_{m,t} = 1) &= \Phi_3(y^*_{m,t}), \\ \hat{y}_{m,t} &= \mathbb{1}(y^*_{m,t} > 0) = \mathbb{1}(\Pr(\hat{y}_{m,t} = 1) > 0.5), \end{aligned} \tag{23}$$

where $\hat{\alpha}, \hat{\beta}, \ \hat{\Delta}$ are obtained from the estimated trivariate model for Ecuador and the correlated residuals $\hat{\epsilon}_{m,t}$ are transformed into orthogonal ones via a Choleski decomposition of the covariance matrix $\hat{\Omega}$. Therefore, a crisis is to arise at time t, if $\hat{y}_{m,t} = 1$.

Additionally, as in any non-linear model, the IRFs are calculated for two initial states: a tranquil one, $\tilde{y}_{t-1} = 0$, *i.e.* *"no type of crisis is observed at time* $t = 0$ *or in the previous 3 months"* and a turmoil regime, $\tilde{y}_{t-1} = 1$, *i.e.* *"all types of crisis are observed in* $t = 0$*"*. Confidence intervals are built taking the 2.5% and 97.5% percentiles of IRF's distribution obtained from 10,000 simulations of the model. The magnitude of the shock is fixed to 5 [22], allowing for a potential mutation of the crisis.

It is important to distinguish between a significant IRF and a significant shift from a calm to a crisis period. First, IRFs are demeaned, so that they are significant if the corresponding confidence interval does not include the value of 0. Second, the shift probability from calm

22. Results for shocks of magnitude 10 are available upon request.

to crisis or the probability of remaining in a crisis period is significantly different from zero at time t if the confidence interval associated to the IRF_t remains in the grey area, *i.e.* the centered IRF_t is significantly lying above its unconditional mean $(\hat{\alpha} + \bar{x}_{m,t-1}\hat{\beta}_m)$. While the first analysis is common to all vector autoregressive (VAR) models, the second one is specific to non-linear (threshold) time series models.

Figures 2 to 4 report the diffusion of banking, currency and debt crises respectively through the system.

First, it appears in figure 2 that banking crisis shock has almost no persistence in a calm initial state, as the IRF function reverses to mean after a single period. On the contrary the persistence jumps to 5 months for an initial crisis state. Similarly, the diffusion of a banking crisis shock to another type of turmoil is exclusively observed in a crisis initial state. Besides, the shift probability from calm to crisis period is significant only for the banking crisis and up to the second period (see the left part of figure 2), whereas the probability of remaining in a crisis period is significant for all three types of crises until $t = 2$ (see the right part of figure 2). This underlines the uncertainty surrounding the duration of a crisis beyond one month after the shock. Overall, these first results clearly correspond to the path exhibited by the crisis sequence faced by Ecuador in the late 90's. Figure 3 reports the response of the three latent variables to a debt crisis shock. In such a case, the impact of the shock on the banking and currency crises vanishes almost instantaneously in the case of a calm initial state, while it disappears after 4 or 5 months, if the economy is facing initially a joint crisis. As for the debt crisis, the impact of the shock lasts at least 5 months even though we are certain of being in a crisis period during the first two periods (the confidence interval is in the grey area at that time). Finally, Figure 4 presents the IRF after currency crisis shock. As in the previous cases, the impact on the banking crisis is not important if we depart from a calm situation, while it becomes significant during 4 periods for an initial crisis period. At the same time, the response of the debt crisis is slowly dampened towards the baseline for a calm initial state, whereas it is significant during the first 4 periods if the shock occurs while being in a crisis state. It seems that the persistence of the effect of this shock is around two months for a calm initial period while it dies away only after 5 months in the alternative situation.

Overall, the conditional probability and the IRF analyses stress the superiority of the trivariate model to scrutinize the diffusion mechanisms that occurred in Ecuador after the banking crisis at the end of the 1990. Strong interactions between the three types of crises are clearly present in particular between banking and other crises.

5 Conclusion

This paper is the first attempt to model simultaneously the three types of crises (currency, banking and sovereign debt), thus allowing to investigate the potential mutations between not only the currency and the banking crises but also the sovereign debt one. It is actually an extension of previous papers which investigate the twin crises phenomenon (in particular Glick and Hutchinson, 1999). To achieve this objective, a methodological novelty has been introduced consisting of an exact maximum likelihood approach to estimate the multivariate dynamic probit model, extending hence the Huguenin, Pelgrin and Holly (2009)'s method to dynamic models. Applied to a large sample of emerging countries, we find that in the bivariate case causality from banking to currency (and vice-versa) are quite common. More importantly, for the two countries, Ecuador and South Africa, which suffered from the 3 types of crises, the trivariate model turns out to be the best performing in term of conditional probabilities and comprehension of the reasons why a specific crisis mutates to another one: this can be due to common shocks (as in South Africa) or to a strong causal structure (as in Ecuador). More generally, this paper advocates the use of trivariate probit crisis models whenever it is possible, so as to have a better insight on the financial turmoils. Finally, the work in this paper can be extended in several directions. First, a panel data approach could be adopted to jointly estimate the models for a set of countries, thereby appropriately accounting for heterogeneity across countries and imposing parameter heterogeneity wherever it is supported by the data. Provided the specifications across countries are sufficiently homogenous, it might be possible to also include countries which have experienced some but not all types of crises into the joint analysis and to make probability statements on the occurrence in the future of as yet unobserved crises types for these countries.

Second, a comparison of the multivariate probit models with alternative models such as multivariate extensions of the Markov switching model of Hamilton (1989) might give further insights into the dynamics of the generation and transmission of crises.

Third, it would be interesting to investigate the interaction between the method of construction of the crises indicator variables and the nature of the DGP for the indicator variables from which they have been derived, along lines proposed by Harding and Pagan (2011). Fourth, another extension could be to extend the model by specifying the process for the explanatory variables, for instance along the lines of Dueker (2005) who considers a univariate probit whereby the underlying latent variable and the set of its explanatory variables are generated by a VAR-model. Fifth, an extension should deal with the forecasting properties of the proposed models to find out whether they provide an accurate early warning against an imminent crisis.

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Appendix 1: Proof of lemma 1

By definition, the likelihood of observation t is given by:

$$\begin{aligned} L_t(y_t|z_{t-1},\theta;\Omega) &= \Pr((-q_{1,t}y^*_{1,t} \leq 0),...,(-q_{M,t}y^*_{M,t} \leq 0)) \\ &= \Pr(-q_{1,t}\varepsilon_{1,t} \leq q_{1,t}\pi_{1,t},...,-q_{M,t}\pi_{M,t} \leq q_{M,t}\pi_{M,t}) \\ &= \Phi_{M,-Q_t\varepsilon_t}(w_t|0_M;\Omega) \\ &= \int_{-\infty}^{w_{M,t}} ... \int_{-\infty}^{w_{1,t}} \phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t,\Omega) \prod_{m=1}^{M} d\varepsilon_{m,t}. \end{aligned}$$

Since each $q_{m,t}$ takes only the values $\{-1,1\}$, it is straightforward to show that $Q_t = Q_t^{-1}$ and $|Q_t\Omega Q_t| = |\Omega|$. Moreover, the density of an M-variate standardized normal vector $-Q_t\varepsilon_t$ with covariance matrix Ω may be re-written as the density of an M-variate standardized normal vector ε_t with variance-covariance matrix $Q_t\Omega Q_t$:

$$\begin{aligned} \phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t;\Omega) &= |2\pi\Omega|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}(-Q_t\varepsilon_t)'\Omega^{-1}(-Q_t\varepsilon_t)\right\} \\ &= |2\pi(Q_t\Omega Q_t)|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}\varepsilon_t'(Q_t\Omega Q_t)^{-1}\varepsilon_t\right\} \\ &= \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t). \end{aligned}$$

Therefore, the likelihood of observation t is given by:

$$\begin{aligned} L_t(y_t|Z_{t-1},\theta;\Omega) &= \int_{-\infty}^{q_{M,t}\pi_{M,t}} ... \int_{-\infty}^{q_{1,t}\pi_{1,t}} \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t) \prod_{m=1}^{M} d\varepsilon_{m,t} \\ &= \Phi_{M,\varepsilon_t}(Q_t\pi_t;Q_t\Omega Q_t). \end{aligned}$$

Appendix 2: The Gauss-Legendre Quadrature rule

The goal of the Gauss-Legendre Quadrature rule is to provide an approximation of the following integral:

$$\int_a^b f(x)dx. \tag{24}$$

In a first step, the bounds of the integral must be changed from $[a, b]$ to [-1,1] before applying the Gaussian Quadrature rule:

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^1 f(z)dz, \tag{25}$$

where $z_i = \frac{b-a}{2}abs_i + \frac{b+a}{2}$ and the nodes abs_i, $i \in \{1, 2, ..., p\}$ are zeros of the Legendre polynomial $P_p(abs)$.

Definition 1. *Then, the standard p-point Gauss-Legendre quadrature rule over a bounded arbitrary interval [a,b] is given by the following approximation*:

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\sum_{i=1}^p v_i f(z_i) + R_p, \tag{26}$$

where v_i *are the corresponding weights,* $v_i = \frac{2}{(1-abs_i^2)\left(\frac{\partial P_p(abs)}{\partial abs}|_{abs_i}\right)^2}$, $\sum_{i=1}^p v_i = 2$, *and* R_p *is the error term,* $R_p = Q_p f^{(2p)}(\xi) = \frac{(b-a)^{2p+1}(p!)^4}{(2p+1)(2p!)^3} f^{2p}(\xi)$, *with* $\xi \in (a, b)$.

Appendix 3: The EML score vector for a trivariate dynamic probit model

For ease of notation, let us denote by $\rho_{i,j}$, $i, j = \{1, 2, 3\}$, $i \neq j$ the correlation coefficients associated to the Ω matrix. The likelihood of observation t may be written as:

$$\begin{aligned} P_t &= \Phi_3(q_1\pi_{1,t}, q_2\pi_{2,t}, q_3\pi_{3,t}, q_1q_2\rho_{12}, q_1q_3\rho_{13}, q_2q_3\rho_{23}) \\ &= \Phi(q_1\pi_{1,t})\Phi(q_2\pi_{2,t})\Phi(q_3\pi_{3,t}) \\ &+ q_1q_2\Phi(q_3\pi_{3,t})\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ &+ q_1q_3\Phi(q_2\pi_{2,t})\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ &+ q_2q_3\Phi(q_1\pi_{1,t})\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ &+ q_1q_2q_3\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ &+ q_1q_2q_3\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ &+ q_1q_2q_3\Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}), \end{aligned} \tag{27}$$

where

$$\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) = \int_0^{\rho_{12}} \psi_2(\pi_{1,t}, \pi_{2,t}, \lambda_{12}) d\lambda_{12}$$

$$\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) = \int_0^{\rho_{13}} \psi_2(\pi_{1,t}, \pi_{3,t}, \lambda_{13}) d\lambda_{13}$$

$$\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) = \int_0^{\rho_{23}} \psi_2(\pi_{2,t}, \pi_{3,t}, \lambda_{23}) d\lambda_{23},$$

and

$$\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) = \int_0^{\rho_{13}} \int_0^{\rho_{23}} \frac{-\pi_{3,t} + \lambda_{13}\pi_{1,t} + \lambda_{23}\pi_{2,t}}{1 - \lambda_{13}^2 - \lambda_{23}^2} \psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \lambda_{13}, \lambda_{23}, 0) d\lambda_{13} d\lambda_{23}$$

$$\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) = \int_0^{\rho_{23}} \int_0^{\rho_{12}} \frac{-\pi_{2,t} + \lambda_{23}\pi_{3,t} + \lambda_{12}\pi_{1,t}}{1 - \lambda_{23}^2 - \lambda_{12}^2} \psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \lambda_{23}, \lambda_{12}, 0) d\lambda_{23} d\lambda_{12}$$

$$\begin{aligned} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = &\int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{-(1-\rho_{23}^2)\pi_{1,t} + (\lambda_{12} - \lambda_{13}\rho_{23})\pi_{2,t} + (\lambda_{13} - \lambda_{12}\rho_{23})\pi_{3,t}}{1 - \lambda_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\lambda_{13}\rho_{23}} \\ &\times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{23}. \end{aligned}$$

Therefore, the first order partial derivatives can be obtained as follows:

$$\begin{aligned}\frac{\partial}{\partial \pi_1} P_t = {} & q_1 \psi(\pi_{1,t}) \Phi(q_2 \pi_{2,t}) \Phi(q_3 \pi_{3,t}) \\ & + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ & + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ & + q_1 q_2 q_3 \psi(\pi_{1,t}) \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),\end{aligned} \tag{28}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_2} P_t = {} & q_2 \psi(\pi_{2,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_3 \pi_{3,t}) \\ & + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ & + q_1 q_2 q_3 \psi(\pi_{2,t}) \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ & + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ & + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),\end{aligned} \tag{29}$$

$$
\begin{aligned}
\frac{\partial}{\partial \pi_3} P_t = {} & q_1 \psi(\pi_{3,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_2 \pi_{2,t}) \\
& + q_1 q_2 q_3 \psi(\pi_{3,t}) \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{12}} P_t = {} & q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \rho_{12}} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{13}} P_t = {} & q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \rho_{13}} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} P_t = {} & q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \rho_{23}} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned}
\tag{33}
$$

where

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}}\int_0^{\rho_{13}} \frac{\partial}{\partial \lambda_{13}}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{23}\lambda_{13}$$
$$= \int_0^{\rho_{13}} \psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} [(\pi_{3,t}-\lambda_{13}\pi_{1,t}-\lambda_{23}\pi_{2,t})^2-(1-\lambda_{13}^2-\lambda_{23}^2)]$$
$$\times \frac{1}{(1-\lambda_{13}^2-\lambda_{23}^2)^2}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{13}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}} \frac{-\pi_{3,t}+\rho_{13}\pi_{3,t}+\lambda_{23}\pi_{2,t}}{1-\rho_{13}^2-\lambda_{23}^2}\psi_3(\pi_3,\pi_1,\pi_2,\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}} \frac{-\pi_{3,t}+\lambda_{13}\pi_{3,t}+\rho_{23}\pi_{2,t}}{1-\lambda_{13}^2-\rho_{23}^2}\psi_3(\pi_3,\pi_1,\pi_2,\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} \frac{\partial}{\partial \lambda_{12}}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{12}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t,},\lambda_{23},\rho_{12},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} [(\pi_{2,t}-\lambda_{23}\pi_{3,t}-\lambda_{12}\pi_{1,t})^2-(1-\lambda_{23}^2-\lambda_{12}^2)]$$
$$\times \frac{1}{(1-\lambda_{23}^2-\lambda_{12}^2)^2}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}d\lambda_{12},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\psi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}\lambda_{12}$$
$$= \int_0^{\rho_{12}} \psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\lambda_{12},0)d\lambda_{12},$$

$$\frac{\partial}{\partial \rho_{12}}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}} \frac{-\pi_{2,t}+\lambda_{23}\pi_{3,t}+\rho_{12}\pi_{1,t}}{1-\lambda_{23}^2-\rho_{12}^2}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\rho_{12},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}}\Psi_3(\pi_{2,t},,\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{12}} \frac{-\pi_{2,t}+\rho_{23}\pi_{3,t}+\lambda_{12}\pi_{1,t}}{1-\rho_{23}^2-\lambda_{12}^2}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\lambda_{12},0)d\lambda_{12},$$

$$\begin{aligned}\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\int_0^{\rho_{13}}\{[(1-\rho_{23}^2)\pi_{1,t}-(\lambda_{12}-\lambda_{13}\rho_{23})\pi_{2,t}-(\lambda_{13}-\lambda_{12}\lambda_{23})\pi_{3,t}]^2 \\ &\quad -(1-\rho_{23}^2)(1-\lambda_{12}^2-\lambda 13^3-\rho_{23}^2+2\lambda_{12}\lambda_{13}\rho_{23})\}\times \\ &\quad \frac{1}{(1-\lambda_{12}^2-\lambda_{13}^2-\rho_{23}^2+2\lambda_{12}\lambda_{13}\rho_{23})^2}\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{12}d\lambda_{13},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{13}}\int_0^{\rho_{12}}\frac{\partial}{\partial\lambda_{12}}\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},,\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{12}\lambda_{13} \\ &= \int_0^{\rho_{13}}\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\lambda_{13},\rho_{23})d\lambda_{13},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\int_0^{\rho_{13}}\frac{\partial}{\partial\lambda_{13}}\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\lambda_{13},\rho_{23})d\lambda_{13}\lambda_{12} \\ &= \int_0^{\rho_{12}}\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\rho_{13},\rho_{23})d\lambda_{12},\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \rho_{12}}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{13}}\frac{(1-\rho_{23}^2)\pi_{1,t}+(\rho_{12}-\lambda_{13}\rho_{23})\pi_{2,t}+(\lambda_{13}-\rho_{12}\rho_{23})\pi_{3,t}}{1-\rho_{12}^2-\lambda_{13}^2-\rho_{23}^2+2\rho_{12}\lambda_{13}\rho_{23}} \\ &\quad\times\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\lambda_{13},\rho_{23}d\lambda_{13}),\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \rho_{13}}\Psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\rho_{12},\rho_{13},\rho_{23}) &= \int_0^{\rho_{12}}\frac{(1-\rho_{23}^2)\pi_{1,t}+(\lambda_{12}-\rho_{13}\rho_{23})\pi_{2,t}+(\rho_{13}-\lambda_{12}\rho_{23})\pi_{3,t}}{1-\lambda_{12}^2-\rho_{13}^2-\rho_{23}^2+2\lambda_{12}\rho_{13}\rho_{23}} \\ &\quad\times\psi_3(\pi_{1,t},\pi_{2,t},\pi_{3,t},\lambda_{12},\rho_{13},\rho_{23}d\lambda_{12}),\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) &= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{2,t} \partial \lambda_{13}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{12}} \frac{-(1-\rho_{13}^2)\pi_{2,t} + (\lambda_{12} - \rho_{13}\rho_{23})\pi_{1,t} + (\rho_{23} - \lambda_{12}\rho_{13})\pi_{3,t}}{1 - \lambda_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\rho_{13}\rho_{23}} \\
&\times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \rho_{13}, \rho_{23}) d\lambda_{12} \\
&= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{3,t} \partial \lambda_{12}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{13}} \frac{-(1-\rho_{12}^2)\pi_{3,t} + (\lambda_{13} - \rho_{12}\rho_{23})\pi_{1,t} + (\rho_{23} - \rho_{12}\lambda_{13})\pi_{2,t}}{1 - \rho_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\rho_{12}\lambda_{13}\rho_{23}} \\
&\times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \lambda_{13}, \rho_{23}) d\lambda_{13}.
\end{aligned}
$$

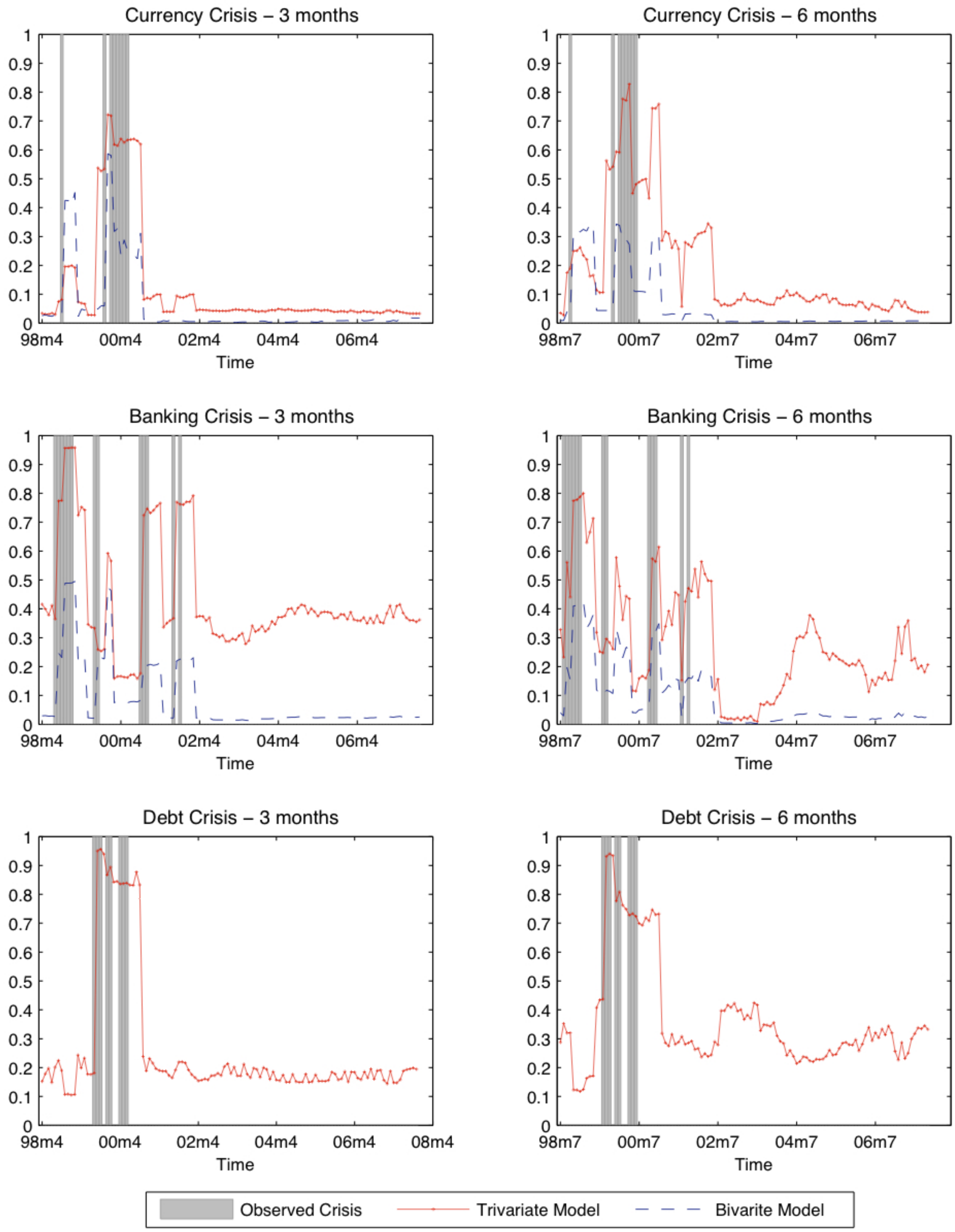

FIGURE 1 – Conditional crisis probabilities - Ecuador

Note : Probabilities at time t are calculated including observed information prior 3 or 6 months.

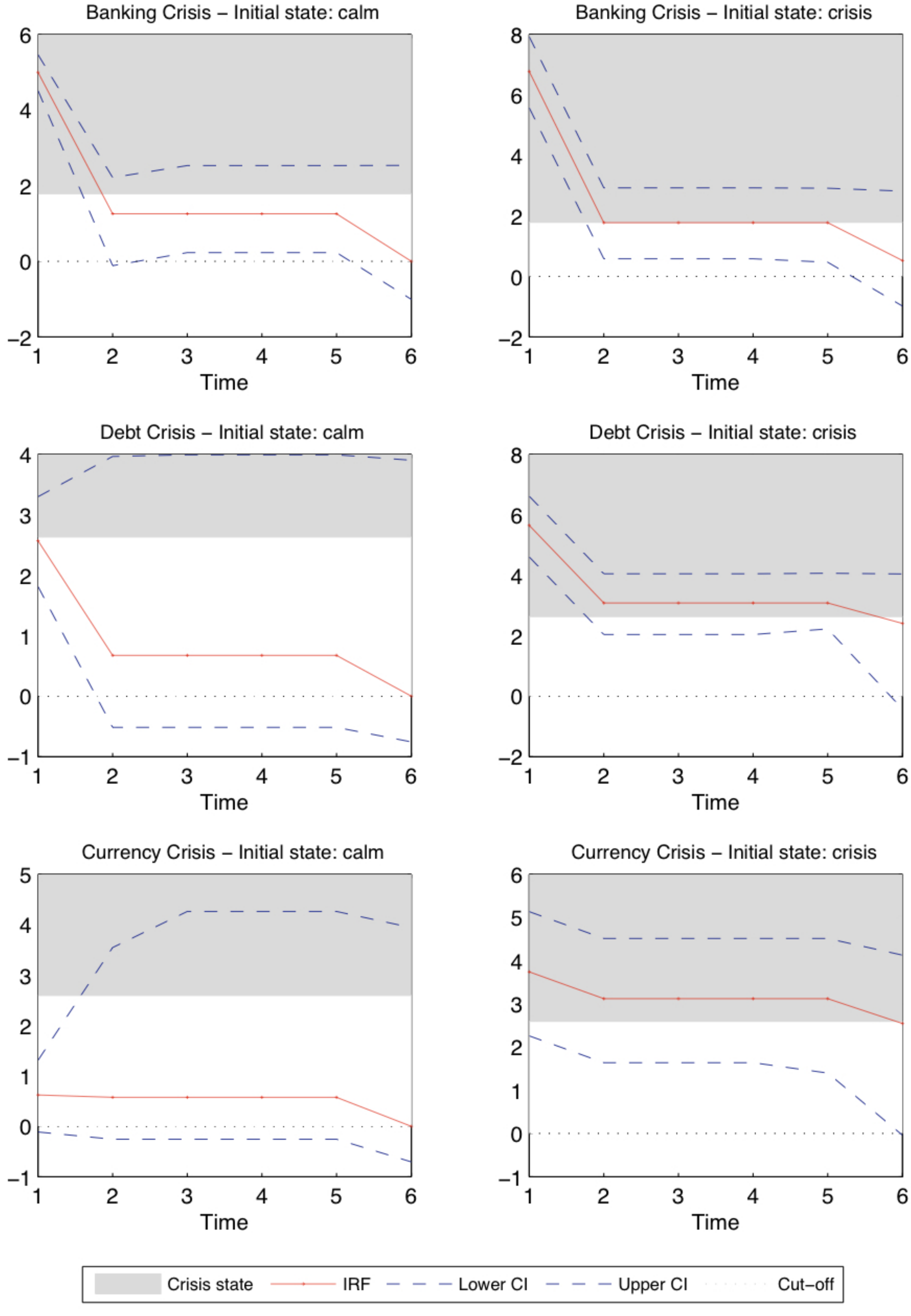

FIGURE 2 – IRF after a banking crisis shock - Ecuador 3 months

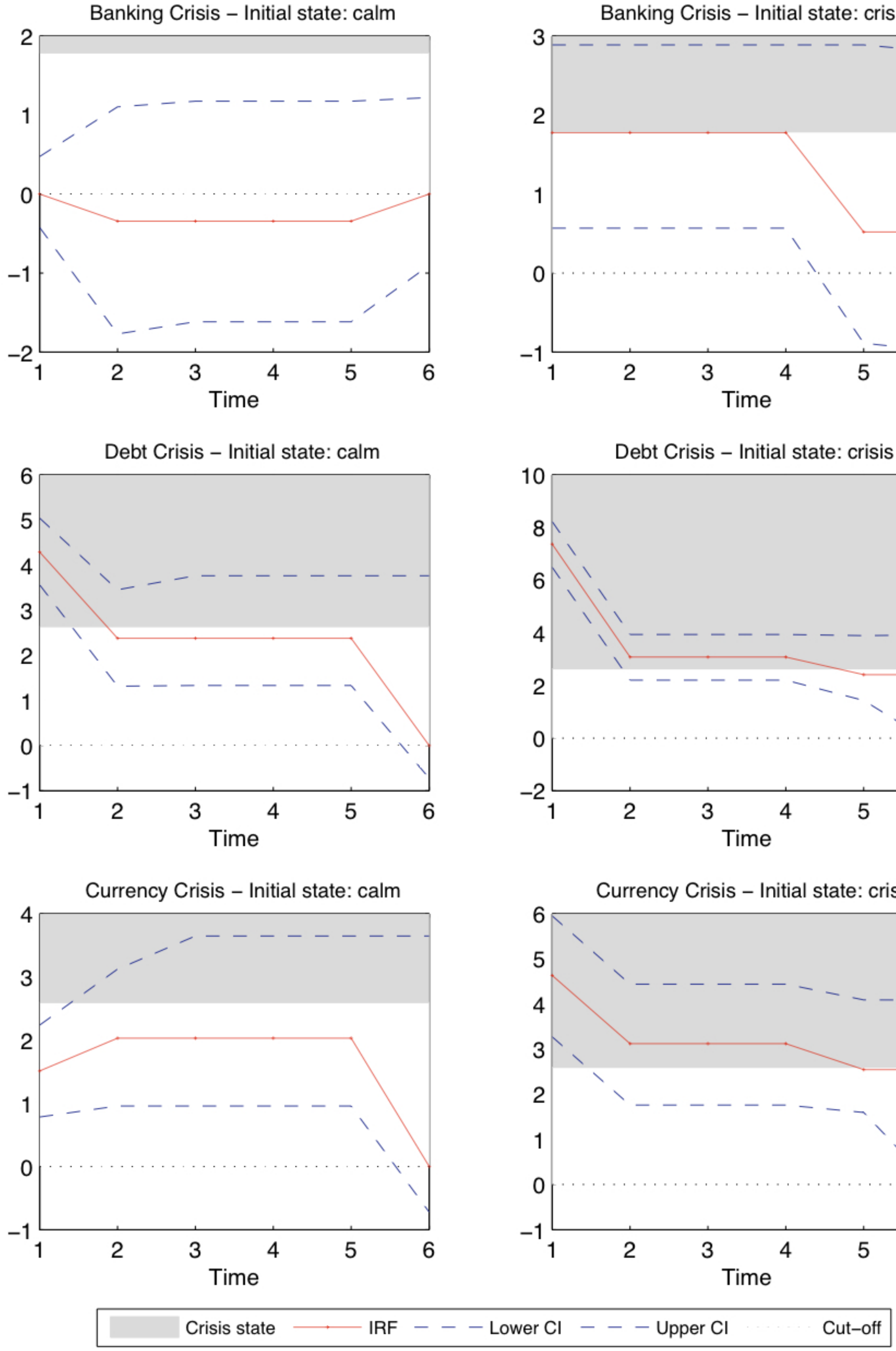

FIGURE 3 – IRF after a debt crisis shock - Ecuador 3 months

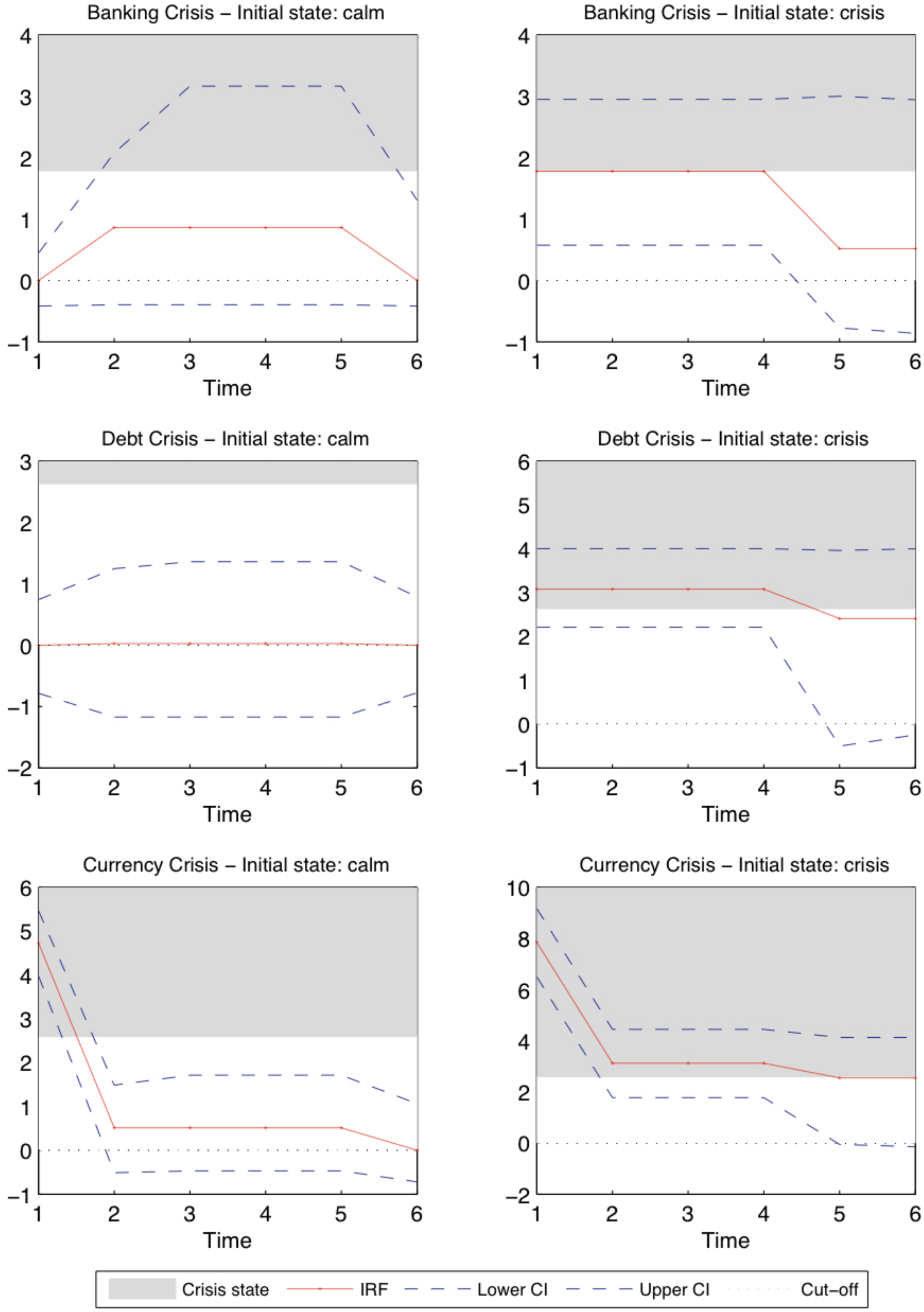

Figure 4 – IRF after a currency crisis shock - Ecuador 3 months

Table 1 – Database

Country	Bivariate model	Trivariate model
Argentina	February 1988 - May 2010	December 1997 - May 2010
Brazil	September 1990 - May 2010	December 1997 - May 2010
Chile	January 1989 - May 2009	May 1999 - May 2010
Colombia	February 1986 - August 2009	December 1997 - August 2009
Ecuador	January 1994 - November 2007	December 1997 - November 2007
Egypt	February 1986 - June 2009	July 2001 - June 2009
El Salvador	January 1991 - November 2008	April 2002 - November 2008
Indonesia	January 1989 - August 2009	May 2004 - August 2009
Lebanon	January 1989 - April 2010	April 1998 - April 2010
Malaysia	January 1988 - March 2010	December 1997 - March 2010
Mexico	January 1988 - May 2010	December 1997 - May 2010
Peru	January 1990 - May 2010	December 1997 - May 2010
Philippines	January 1995 - February 2008	December 1997 - February 2008
South Africa	January 1988 - August 2009	December 1997 - August 2009
Turkey	January 1988 - May 2010	December 1997 - May 2010
Venezuela	February 1986 - November 2009	December 1997 - November 2009

Note: Data sample.

Table 2 – Percentage of crisis periods

	Bivariate model		Trivariate model		
	Currency crisis	Banking crisis	Currency crisis	Banking crisis	Debt crisis
Argentina	**5.13**	**8.90**	4.00	**6.67**	**10.0**
Brazil	3.77	**7.19**	0.00	3.33	2.67
Chile	**6.07**	**10.0**	**5.79**	**5.79**	3.31
Colombia	4.95	**9.90**	**9.22**	**12.8**	0.00
Ecuador	**5.73**	**9.93**	**6.67**	**10.8**	**6.67**
Egypt	**6.76**	**9.96**	4.17	**7.30**	**7.30**
El Salvador	3.65	**9.85**	0.00	0.00	2.50
Indonesia	5.30	**9.90**	0.00	**14.0**	**6.25**
Lebanon	**9.62**	**9.96**	1.38	**8.97**	2.76
Malaysia	3.10	**10.0**	4.05	**6.08**	4.73
Mexico	**6.50**	**9.93**	0.00	**9.33**	0.00
Panama	0.00	**9.89**	0.00	**6.38**	0.00
Peru	4.45	**8.22**	0.00	**10.7**	0.00
Phillipines	4.90	**9.80**	**5.69**	**6.50**	3.25
South Africa	**6.71**	**9.89**	**7.09**	**7.80**	4.26
Turkey	4.80	**8.56**	4.00	**6.67**	0.00
Venezuela	**7.33**	**10.1**	4.17	**7.64**	2.78

Note: The entries represent the proportion of crises period over the whole sample. It is indicated in bold as it exceeds 5%.

TABLE 3 – Bivariate Analysis

Country		3 months Δ	3 months Ω	6 months Δ	6 months Ω	12 months Δ	12 months Ω
Argentina	currency	+ +	1 +	. +	1 +	. .	1 .
	banking	. +	+ 1	. +	+ 1	. +	. 1
Chile	currency	+ .	1 +	. .	1 +	. −	1 +
	banking	. .	+ 1	. .	+ 1	. +	+ 1
Ecuador	currency	. .	1 .	. .	1 .	. .	1 .
	banking	. +	. 1	. +	. 1	. +	. 1
Egypt	currency	+ .	1 .	+ .	1 −	+ .	1 −
	banking	− +	. 1	− +	− 1	. +	− 1
Lebanon	currency	+ .	1 .	. .	1 +	+ .	1 +
	banking	− +	. 1	. +	+ 1	. +	+ 1
Mexico	currency	+ .	1 .	+ .	1 .	+ .	1 +
	banking	. +	. 1	. .	. 1	. .	+ 1
South Africa	currency	+ .	1 .	+ .	1 .	. .	1 .
	banking	. +	. 1	. +	. 1	. +	. 1
Venezuela	currency	+ .	1 +	+ .	1 +	. .	1 +
	banking	. +	+ 1	. +	+ 1	. +	+ 1

Note: Three different lags of the dependent variable are used, namely 3, 6 and 12 months. 'Δ' stands for the parameters of the lagged crisis variables, while Ω represents the covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Argentina, 3 months, all the parameters are positive and significative except for the impact of a currency crisis on the probability of occurrence of banking crises. Similarly, the correlation coefficient between currency and banking crises is significative.

TABLE 4 – Trivariate Analysis

Country		3 months Δ	3 months Ω	6 months Δ	6 months Ω
Ecuador	currency	. . +	1 . .	. + +	1 . .
	banking	. + .	. 1 .	. + .	. 1 .
	sovereign	. . +	. . 1	. . +	. . 1
South Africa	currency	+ . .	1 . +	+ . .	1 . +
	banking	. . .	. 1 .	. . .	. 1 .
	sovereign	. . +	+ . 1	. . +	+ . 1

Note: Two different lags of the dependent variable are used, namely 3 and 6 months. 'Δ' stands for the parameters of the lagged crisis variables, while Ω represents the variance-covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Ecuador, 3 months, sovereign debt crises have a positive and significative impact on the probability of occurrence of currency crises.