Quality of Service in Wireless Cellular Networks Subject to Log-Normal Shadowing

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Abstract

Shadowing is believed to degrade the quality of service (QoS) in wireless cellular networks. Assuming log-normal shadowing, and studying mobile's path-loss with respect to the strongest (serving) base station (BS) and the corresponding interference factor (the ratio of the sum of the path-gains form interfering BS's to the path-gain from the serving BS), which are two key ingredients of the analysis and design of the cellular networks, we discovered a more subtle reality. We observe, as commonly expected, that a strong variance of the shadowing increases the mean path-loss with respect to the serving BS, which in consequence, may compromise QoS. However, in some cases, an increase of the variance of the shadowing can significantly reduce the mean interference factor and, in consequence, improve some QoS metrics in interference limited systems, provided the handover policy selects the strongest BS as the serving one. We confirm this phenomenon, similar to stochastic resonance, studying the blocking probability in regular, hexagonal networks both by simulation and in a semi-analytic manner, using a spatial version of the Erlang's loss formula combined with Kaufman-Roberts algorithm. More detailed probabilistic analysis explains that increasing variance of the log-normal shadowing amplifies the ratio between the strongest signal and all other signals thus reducing the interference. The above observations might shed new light in particular on the design of indoor communication scenario.

Index Terms

Wireless cellular networks, blocking probability, path-loss, shadowing, indoor, interference factor, stochastic resonance, geometry, honeycomb, Poisson.

I. INTRODUCTION

Modeling of the attenuation of an electromagnetic wave as it propagates in space is a major component in the analysis and design of wireless systems. This phenomenon, also called propagation loss, is caused by the decay of the signal power with the distance from the emitter (existing even in the free space propagation models) and due to various obstacles between emitters and receivers (trees, buildings, hills, etc.) present in real network profiles. Complexity and haphazard character of actual network profiles makes pertinent the statistical modeling of the propagation loss. In this approach, the propagation loss is typically modeled by the product of a deterministic function of the distance, which represents average path-loss on the given distance in the network, and a random variable, called *shadowing*, that takes into account in a statistical manner the deviation from this average, observed for each particular pair of emitter and receiver. The deterministic path loss function is commonly assumed to be some power of the distance, with the exponent called *path-loss exponent*. The random variable of the shadowing is often assumed to have *log-normal distribution*, normalized to have mean one and parametrized by its variance or standard deviation.

Various QoS metrics in cellular networks, as *blocking probability* for constant bit-rate (CBR) connections and *spectral efficiency* for variable bit-rate (VBR) connections, depend on the strength (i.e., variance) of the shadowing. It is commonly believed that an increase of the variance of shadowing penalizes the network performance. The results presented in this paper shed some new light on this problem. Namely, studying the blocking probability (defined as the fraction of the CBR connections that cannot be established due to insufficient transmission resources, in the long run of the system) we have discovered that it is not always increasing with the variance of the shadowing. For example, in the (CDMA or OFDMA)

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hexagonal network consisting of 36 BS, with cell radius 0.525km and the path-loss exponent equal to 2.5, the blocking probability evaluated at the presence of the log-normal shadowing with the standard deviation of 25dB is four times smaller than in the scenario with no-shadowing. Even if this spectacular example regards a strong shadowing (adequate for some indoor communications), we obtain a smaller, but still very significant, decrease of the blocking probability for the shadowing with quite moderate standard deviation from 7 to 15dB. In all cases, a very strong shadowing ultimately makes the blocking probability tend to 1 and this dependence indeed becomes (as expected) monotone for higher path-loss exponent (larger than 4 in the considered examples).

To explain the above, somewhat surprising, observations and extend them to other QoS metrics we study the impact of the shadowing and the path-loss exponent on the following two key characteristics of any given mobile in the network:

- its *path-loss to the serving BS*, which is the one received with the strongest signal (and not necessarily the closest one),
- the so called mobile's *interference factor*, defined as the ratio of the sum of the path-gains form interfering BS to the path-gain from the serving BS.

These are two key ingredients in the analysis of wireless cellular networks and thus their mean values can be considered as some QoS "pre-metrics". In particular, they are explicitly present in the call blocking condition — the one used to control the admission of streaming users, and hence intrinsically related to the blocking probability. Thy are, even more straightforwardly, related to the spectral efficiency of the data networks.

We have studied the mean values of the above two basic QoS ingredients, with the averaging taken over all possible locations of users in the network and over the distribution of the shadowing. Our main findings are as follows:

- The mean path-loss (with respect to the serving BS) is always increasing in the variance of the shadowing. The ultimate degradation of the QoS for large shadowing variance is due to this increasing path-loss. (When possible, this may be however remedied by increasing the power of the emitted signals).
- The mean interference factor is not monotonic in the variance of the shadowing. It first increases and then decreases (asymptotically to zero!), when the shadowing variance goes to infinity.
- The above two facts lead to the phenomenon that we may call a *stochastic resonance for QoS in path-loss-and-interference limited systems*: when QoS is not yet compromised by path-loss conditions, a moderate increase of the shadowing variance may make it profit from the reduction of the interference.

We confirm the above findings by a mathematical analysis of the respective stochastic models. We also compare in this matter the performance of the perfect (hexagonal) and irregular (Poisson) networks and find that both architectures exhibit very similar QoS "pre-metrics" for the standard deviation of the shadowing larger than 20dB. Moreover, we prove an interesting invariance of the QoS metrics of the infinite Poisson cellular networks with respect to the distribution of the shadowing. As a consequence we also obtain fully explicit, analytical results for the mean path-loss and interference factors in the case of the infinite Poisson network.

The remaining part of this paper is organized as follows. In the next section we briefly present related works. In Section III we describe our models. The main results obtained by simulations of these models are presented in Section IV. Next, in Section V we present mathematical analysis of the models, which supports and completes our numerical findings. Finally, in Section VI we provide some concluding remarks. In order to make this paper more self-contained, in the Appendix we recall the relations between the path-loss and interference factors and the blocking probability, and how this latter can be evaluated by means of the (spatial) Erlang's formula and the Kaufman-Roberts algorithm in the context of a cellular network with shadowing.

II. Related works

The propagation loss model considered in this paper is commonly accepted in the literature; see e.g. [2] where log-normal shadowing of mean 1 is considered. A possible extension of this model consists in assuming shadowing distribution (say, its variance) that depends on the distance, cf. [3].

The impact of the shadowing on the distribution of the interference factor is studied numerically in [4] and analytically in [5]. However, the above two articles do not take into account the modification of the network geometry induced by the shadowing; i.e., assume that mobiles are served by their geographically closest BS. This is not a realistic assumption and, as we will show in this paper, leads to misleading conclusions that the shadowing dramatically increases the mean interference factor.

The paper [6] focuses on the interference factor averaged over a given cell, and in particular the effect of shadowing on this average. It is shown there that the cell shape modification induced by the shadowing affects significantly the mean interference factor. More precisely, that this mean decreases substantially if mobiles are served by the BS offering the smallest path-loss. We adopt this assumption throughout the present paper in the context of regular (hexagonal) and irregular (Poisson) geometry of BS, as proposed in [7].

Some papers (see e.g. [8, 9]) propose more explicit approximations of the interference factor and its moments (mean and variance) assuming only deterministic propagation loss models (without random shadowing). [10] studies the distribution of the interference factor in such a case.

The interference factor was recognized very early as a key element in the performance evaluation of cellular networks; cf. [11, 12]. Fundamental to our approach to the evaluation of the blocking probability are papers [13, 14]. They show how the power allocation problem without power limitations can be reduced to an algebraic system of linear inequalities. Moreover, they recognize that the spectral radius of the (non-negative) matrix corresponding to this system not greater than 1 is the necessary and sufficient condition of the feasibility of power allocation without power limitations. This approach lead to the development of a comprehensive framework of the evaluation of the blocking probability in CDMA, HSDPA and OFDMA, via a spatial version of the famous Erlang's formula in [7, 15–17]. QoS in data networks are studied using this approach in [18].

III. Model description

A. Location of base stations

In this paper we will consider two particular models for the location of BS, hexagonal and Poisson one. The former is commonly considered as an "ideal" model for the cellular networks, while the latter one can be seen as an extremal case of very irregular network.

1) Infinite Models:

- *Hexagonal network.* Consider BS located on a regular hexagonal grid on $\mathbb{R}^2$ with the distance Δ between two adjacent vertexes of this grid [1]; cf. Figure 1. Note that the surface area of a given cell (hexagon; i.e., subset of the plane whose points are closer to a given point of the grid than to any other) of this model is equal to $\sqrt{3}\Delta^2/2$. Thus the intensity of the BS in this model is equal to $\lambda = 2/(\sqrt{3}\Delta^2)$ BS/km^2. In what follows it will be customary to consider a stationary version Φ_H of this grid, which can be obtained by randomly shifting it through a vector uniformly distributed in one given hexagon (cf. [19, Example 4.2.5]). In this model a given location, say the origin of the plane, corresponds to an "arbitrary" location of a mobile, "randomly chosen" in the network.
- *Poisson network.* Assume that BS are located at the points of a stationary, homogeneous Poisson point process (p.p.) Φ_P of intensity λ BS/km^2 on the plane $\mathbb{R}^2$. When comparing performance of Poisson and hexagonal model we will always take them with the same intensity $\lambda = 2/(\sqrt{3}\Delta^2)$.

Considering infinite models is often a convenient way of studying phenomena arising in very large networks. A particular property of these models is lack of (geographic) boundary effects, which in real, large but finite, networks, have often a negligible impact on performance characteristics measured in the "middle" of the network. However, as we will see in this paper, sometimes mathematical assumption of an infinite network may create some artifacts, which are not observed in more realistic, large but finite, networks.

[1] The set of vertexes of this grid can be described on the complex plane by $\{\Delta(u_1 + u_2 e^{i\pi/3}),\ u = (u_1, u_2) \in \{0, \pm 1, \dots\}^2\}$.

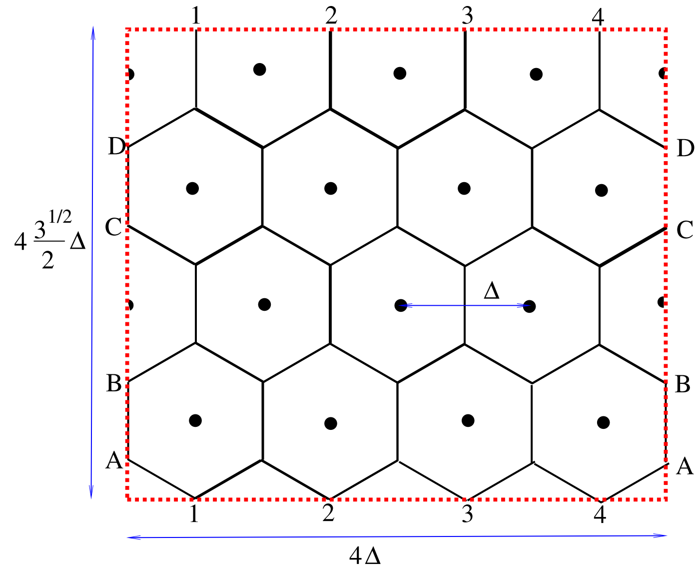


Fig. 1. hexagonal pattern of 4×4 BS on rectangular torus $\mathbb{T}_4$. Identified points are denoted by the same digits or characters.

2) Bounded Models: In order to have finite network models, and still neglect the boundary effects (which might be reasonable for large networks) one often considers *toroidal model*. Recall that, roughly speaking, rectangular torus is a rectangle whose opposite sites are "identified". For $N = 2, 4, 6, \ldots$, we will denote by $\mathbb{T}_N$ the rectangle $[-N\Delta/2, N\Delta/2) \times [-N\sqrt{3}\Delta/4, N\sqrt{3}\Delta/4)$ with toroidal metric. Restricting Φ_H to $\mathbb{T}_N$, i.e. taking $\Phi_H^{\mathbb{T}_N} = \Phi_H \cap \mathbb{T}_N$ one obtains the model whose distribution is invariant with respect to translations on the torus. Thus we obtain a hexagonal network model that consists of N^2 cells (cf. Figure 1) and which does not exhibit any border effects. Similarly we will consider the restriction $\Phi_P^{\mathbb{T}}$ of the Poisson p.p. Φ_P to $\mathbb{T}_N$.

B. Path-loss model with shadowing

For a given BS $X \in \Phi$ ($\Phi = \Phi_P$ or Φ_H) and a given location $y \in \mathbb{R}^2$ on the plane we denote by $L_X(y)$ the (time-average, i.e., averaged out over the fading) propagation-loss between BS X and location y. In what follows we will always assume that

$$L_X(y) = \frac{L\left(|X - y|\right)}{S_X(y)}, \tag{1}$$

where $L(\cdot)$ is a non-decreasing, *deterministic* function of the distance between an emitter and a receiver, and $S_X(\cdot)$ is a *random shadowing field* related to the BS X. In what follows we will always assume that given locations of BS $\{X_i \in \Phi\}$ their shadowing fields $\{S_{X_i}(\cdot)\}$ are *independent* non-negative stochastic processes, each being indexed by locations $y \in \mathbb{R}^2$. More formally speaking, the locations of BS X and their respective shadowing fields $S_X(\cdot)$ form an *independently marked* version $\tilde{\Phi} = \{(X, S_X(\cdot))\}_{X \in \Phi}$ of the point process Φ.

Regarding the distribution of the marks (shadowing fields) of this process, they are assumed to have *the same marginal distributions*; i.e., given X, $S_X(y)$ has the same distribution for all $y \in \mathbb{R}^2$, of normalized mean $\mathbf{E}[S_X(y)] = 1$, with the following two cases being of particular interest

- $S_X(y) \equiv 1$, which corresponds to a case with negligible shadowing (we will say also "no shadowing"),
- for all y, $S_X(y)$ is log-nornal random variable with mean 1. Recall that such a mean-1 log-normal variable S can be expressed as $S = e^{\mu + \sigma N}$ where N is standard Gaussian random variable (with mean 0 and variance 1) with $\mu = -\sigma^2/2$. Indeed, in this case $\mathbf{E}[S] = e^{\mu + \sigma^2/2} = 1$. Note that if the shadowing is log-normal random variable then the path-loss (at a given distance) expressed in dB is Gaussian random variable. Furthermore, in this context it is common to parametrize the log-normal shadowing by the standard deviation (SD) of S expressed in dB; i.e., the SD of $10\log_{10} S$. We will denote it by v. With respect to the previous parametrization we have $v = \sigma 10/\log 10$. Throughout the paper we will call v the *logarithmic standard deviation (log-SD) of the shadowing*.

If not otherwise specified, we do not make any particular assumption on the correlation of the shadowing field $S_X(y)$ for given X and different locations y. Throughout the paper we will implicitly assume also that mean *path-gain* is finite, i.e., $\mathbf{E}[1/S] < \infty$. Note that this condition is satisfied for log-normal variable, indeed, in our case of mean-1 variable $\mathbf{E}[1/S] = e^{\sigma^2} = e^{v^2 \log^2 10/100}$.

For the deterministic path-loss function $L(\cdot)$ the following particular model is often used and will be our default assumption in this paper:

$$L(r) = (Kr)^{\beta} \tag{2}$$

where $K > 0$ and $\beta > 2$ are some constants.

C. Handover policy and path-loss factor

In what follows we will assume that each given location $y \in \mathbb{R}^2$ is served by the BS $X_y^* \in \Phi$ with respect to which it has the weakest path-loss $L_{X_y^*}(y)$ (so, in other words, the strongest received signal, given all BS emit with the same power), i.e, such that

$$L_{X_y^*}(y) \le L_X(y) \qquad \text{for all } X \in \Phi\,, \tag{3}$$

with any tie-breaking rule. Note that in the case of negligible shadowing ($S_X(y) \equiv 1$) and strictly increasing function $L(\cdot)$ the above policy corresponds to the geographically closest BS. Note also that for our infinite network models with random shadowing, one has to prove that the minimum of the path-loss is achieved for some BS, i.e.; that X_y^* is well defined.

Note that $L_{X_y^*}(y)$ is the path-loss experienced by a user located at y with respect to its serving BS. Obviously it determines the QoS of this user (we will be more specific on this in Section III-E). In this context we will call it *path-loss factor*[2] of user y and denote by $l(y) = L_{X_y^*}(y)$. Note that it depends on the location y but also on the path-loss conditions of this location with respect to all BS in the network $l(y) = l(y, \tilde{\Phi})$. Path-loss factor $l(y)$ is typically not enough to determine the QoS of a given user.

D. Interference factor

For a given location $y \in \mathbb{R}^2$ we define the interference factor $f(y)$ as

$$f(y) = f(y, \tilde{\Phi}) = \sum_{X \in \Phi, X \neq X_y^*} \frac{L_{X_y^*}(y)}{L_X(y)} = \sum_{X \in \Phi} \frac{l(y)}{L_X(y)} - 1 \tag{4}$$

provided X_y^* is well defined.

Study of the path-loss and interference factors, which are relatively simple objects, can give an important insight into more involved QoS metrics, such as blocking probability in streaming traffic and mean throughput in data traffic. In what follows we recall how $l(y)$ and $f(y)$ appear naturally in the evaluation of the blocking probabilities.

E. Blocking probability

Consider a spatio-temporal Poisson arrival process of calls (for more details we refer to Section B in the Appendix), which require from the network some predefined transmission rates for some exponential transmission times. These rates can be maintained at the price of blocking of some call arrivals when a network congestion occurs. The fractions of blocked arrivals in the long run of the system is called the *blocking probability*. By the famous *Erlang's loss formula*, it is equal to the conditional probability that in the stationary configuration of the (non-blocked) arrival process the system cannot admit a new user, given all users in the current configuration can be served (cf. Section C in the Appendix). Moreover, if the decision whether to block a given call (or admit it) is based on the verification of some *feasibility condition* that has the so called *multi-Erlang form* (cf. Section D in the Appendix), then the Erlang's loss formula can be relatively easily evaluated, e.g. using *Kaufman-Roberts algorithm*. In what follows we will

[2]not to be confused with the path-loss exponent β

denote by $\mathbf{E}[b] = \mathbf{E}[b(\tilde{\Phi})]$ the blocking probability averaged over possible scenarios regarding locations of BS and their shadowing conditions; for more details see Sections A, C and E in the Appendix.

A canonical form of the multi-Erlang feasibility condition involves verification by each BS X of the following condition

$$\sum_{y: X^*_y = X} \varphi\Big(l(y), f(y)\Big) \le 1\,, \tag{5}$$

where the summation is taken over all users (including a new arrival) to be served by the BS X and $\varphi(\cdot,\cdot)$ is some function of the path-loss and interference factors of user y. This condition guarantees sufficient wireless resources to maintain the predefined transmissions rates for all served mobiles. Specific form of the function $\varphi(\cdot,\cdot)$ needs to be developed for each particular cellular technology (taking into account the performance of the coding schemes, type of the multiplexing, etc.). Below we show two examples borrowed from our previous studies. They give some insight into how the feasibility condition (5) depends on the user transmissions rates, it is supposed to guarantee.

- For the down-link in CDMA network

$$\varphi(l,f) = \frac{\xi}{1+\alpha\xi}\frac{1}{1-\epsilon}\Big(\frac{Nl}{\tilde{P}} + \alpha + f\Big) \tag{6}$$

  (cf. [16]), where $\tilde{P}$ is the maximal BS power, ϵ is the fraction of this maximal power used in common (pilot) channels, α is the intra-cell orthogonality factor, N external noise power, and $\xi = \psi^{-1}(r/W)$ is the SINR threshold corresponding to the required bit-rate r of user given the link performance function ψ and the system bandwidth W [3]. Note that in the absence of the maximal power constraint ($\tilde{P} = \infty$, the so called *pole-capacity regime*) φ depends only on the interference factor f (and not on the path-loss factor l) through the simple relation $\varphi(l,f) = (\alpha+f)\xi/((1-\epsilon)(1+\alpha\xi))$.
- For the down-link in the OFDMA network

$$\varphi(l,f) = \frac{r}{W\psi\Big((1-\epsilon)/((Nl/\tilde{P}) + \alpha + f)\Big)}\,, \tag{7}$$

  with the notation as above; cf. [17].

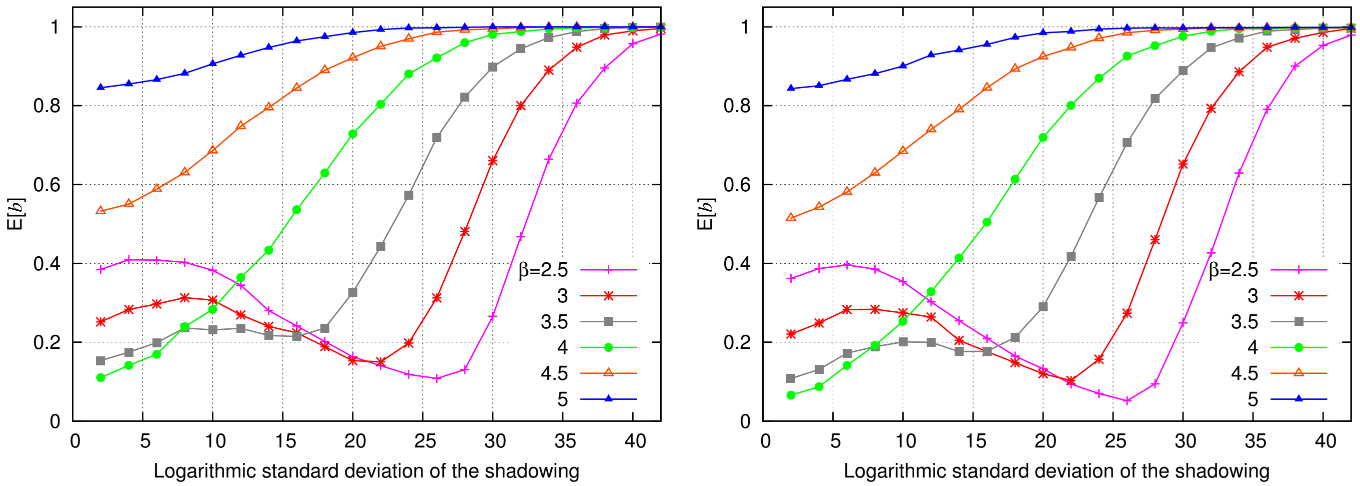
F. Our methodology in the study of the network QoS

In section III-E we will show some numerical examples, which show the typical dependence of the blocking probability $\mathbf{E}[b]$ on the parameters of the path-loss model. These examples are not supposed to be exhaustive. The goal is to show the main tendencies and, in particular, to draw the reader's attention to some astonishing (or at least not so intuitive) non-monotonicity of $\mathbf{E}[b]$ with respect to the standard deviation of the shadowing.

In order to explain these tendencies, in Sections IV-B and IV-C we will study more thoroughly the mean values of the interference and path-loss factor $\mathbf{E}[f(y)] = \mathbf{E}[f(y,\tilde{\Phi})]$, $\mathbf{E}[l(y)] = \mathbf{E}[l(y,\tilde{\Phi})]$ (where the expectation $\mathbf{E}[\ldots]$ corresponds to the distribution of $\tilde{\Phi}$; i.e., this of the shadowing field and of the random location of the user). By the translation invariance of the distribution of our infinite and toroidal models, these expectations do not depend on the user location and thus, for these models, $\mathbf{E}[l(y)] = \mathbf{E}[l(0)]$ and $\mathbf{E}[f(y)] = \mathbf{E}[f(0)]$ [4].

[3] e.g., assuming additive white Gaussian noise (AWGN) channel and the link performance closed to the optimal one, ψ is given by the famous Shannon's formula $\psi(\xi) = \log_2(1+\xi)$. Taking $\psi(\xi) = b\log_2(1+\xi/a)$ with some constants $a \ge 1, b \le 1$ permits to account for a degradation of the link performance in practical systems compared to the ideal AWGN case; cf. [20]. Further extensions consider the Single-Input-Single-Output (SISO) AWGN channel with fading, for which the known formula for the ergodic capacity is $\psi(\xi) = \mathbf{E}[\log_2(1+\xi|F|^2)]$, where the expectation is with respect to the distribution of the channel fading F, and the Multiple-Input-Multiple-Output (MIMO) AWGN channel, whose ergodic capacity is $\psi(\xi) = \mathbf{E}[\log_2 \det(I + \xi \mathbf{F}\mathbf{F}^T)]$, where $\mathbf{F}$ is the vector of channel fading; cf. [21].

[4] Often the mathematical expectation $\mathbf{E}[f(0,\tilde{\Phi})]$ (and similarly for $\mathbf{E}[l(0,\tilde{\Phi}))$ corresponds to the *empirical mean value* $\lim_{n\to\infty} 1/n \sum f(y_i,\tilde{\Phi})$ of the interference factor measured at many locations "uniformly" sampled in one given realization of the network and shadowing. A precise statement and rigorous proof of such an ergodic result is beyond the scope of this paper. We remark only that for the hexagon network on the torus, this result follows simply form the Law of Large Numbers, when y_i are independently and uniformly distributed and provided the shadowing variables $S_X(y_i)$ are independent across different values of y_i. However, recall that the latter assumption, corresponding to spatially uncorrelated shadowing, is *not* our default assumption, since it is not needed for other results regarding $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$.



(a) CDMA, 46.2 Erlang per km^2 (b) OFDMA, 46.2 Erlang per km^2

Fig. 2. Blocking probability in CDMA and OFDMA hexagonal network on the torus $\mathbb{T}_6$ with log-normal shadowing with log-SD v and path-loss exponent β, evaluated using the Kaufman-Roberts algorithm for the traffic 46.2 Erlang per km^2.

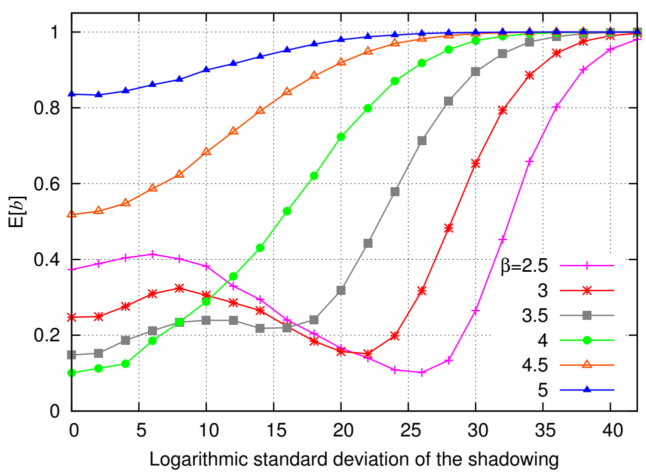


Fig. 3. Blocking probability in CDMA hexagonal network as on Figure 2(a) evaluated by the crude simulation of the network.

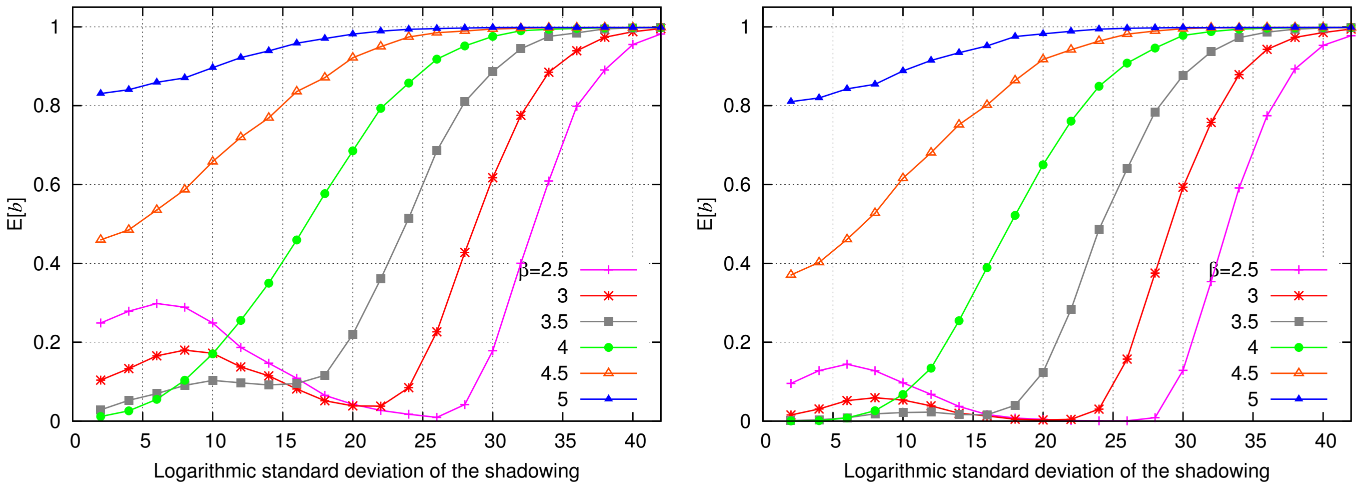
Our methodological conjecture is as follows. We believe that the *mean path-loss and interference factors* $\mathbf{E}[l(0)]$, $\mathbf{E}[f(0)]$ *can be considered as primitive (basic) metrics of the QoS and their behavior can (at least qualitatively) explain the main tendencies observed for more involved QoS metrics*. This methodological conjecture is motivated by the observation that the function φ in the feasibility condition (5) is an increasing function of some linear combination of $l(y)$ and $f(y)$ (at least for the examples of CDMA and OFDMA given above). Indeed, we will show that the study of $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$ can explain the aforementioned non-monotonicity of the blocking probability $\mathbf{E}[b]$ with respect to the standard deviation of the shadowing.

IV. Numerical results

Following the methodology described in Section III-F, in this section we will first numerically study the blocking probability and then the mean path-loss and interference factors for the hexagonal and Poisson network models with log-normal shadowing.

A. Blocking probability

In this section we consider only the hexagonal network on the torus $\mathbb{T}_6$. We evaluate the blocking probability $\mathbf{E}[b]$ in CDMA and OFDMA network using the Kaufman-Roberts algorithm. For a validation



(a) OFDMA, 34.6 Erlang per km^2 (b) OFDMA, 23.1 Erlang per km^2

Fig. 4. Blocking probability for OFDMA network as on Figure 2(b) with traffic 34.6 Erlang per km^2 and 23.1 Erlang per km^2.

of the method, the results obtained by a crude simulation (implemented in MATLAB) of the CDMA network will be given as well.

We assume the following parameter values for CDMA and OFDMA systems: System bandwidth $W =$ 5MHz, BS are equipped with antennas having a gain 9dBi and transmit with the maximal power 43dBm; thus $\tilde{P} = 43 + 9 = 52$dBm when we account for antenna gain. The common channel power is the fraction $\epsilon = 0.12$ of $\tilde{P}$. The intra-cell orthogonality factor is $\alpha = 0.4$ for CDMA and 0 for OFDMA. We assume the ambient noise power $N = -103$dBm and the Shanon's link performance formula $r/W = \psi(\xi) = \log_2(1 + \xi)$.

We assume a traffic demand of 46.2 Erlang par km^2 consisting of streaming calls at the bit-rate $r =$ 180Kbits/s that is served by the hexagonal network consisting of 36 BS (on the tours $\mathbb{T}_6$) with the distance between adjacent BS $\Delta = 1$km.

The (deterministic) path-loss function is $L(x) = (Kx)^\beta$ with $K = 8667\text{km}^{-1}$. Moreover, we assume that the values of the shadowing $S_X(y)$ for given X and different locations y are independent. Figures 2(a) and 2(b) show the dependence of the blocking probability $\mathbf{E}[b]$ on the path-loss exponent β and the logarithmic standard deviation v of the log-normal shadowing in CDMA and OFDMA network, respectively.

Note that both technologies have very similar profiles of the blocking probabilities, which exhibit the following two kinds non-monotonicity.

Remark 4.1:

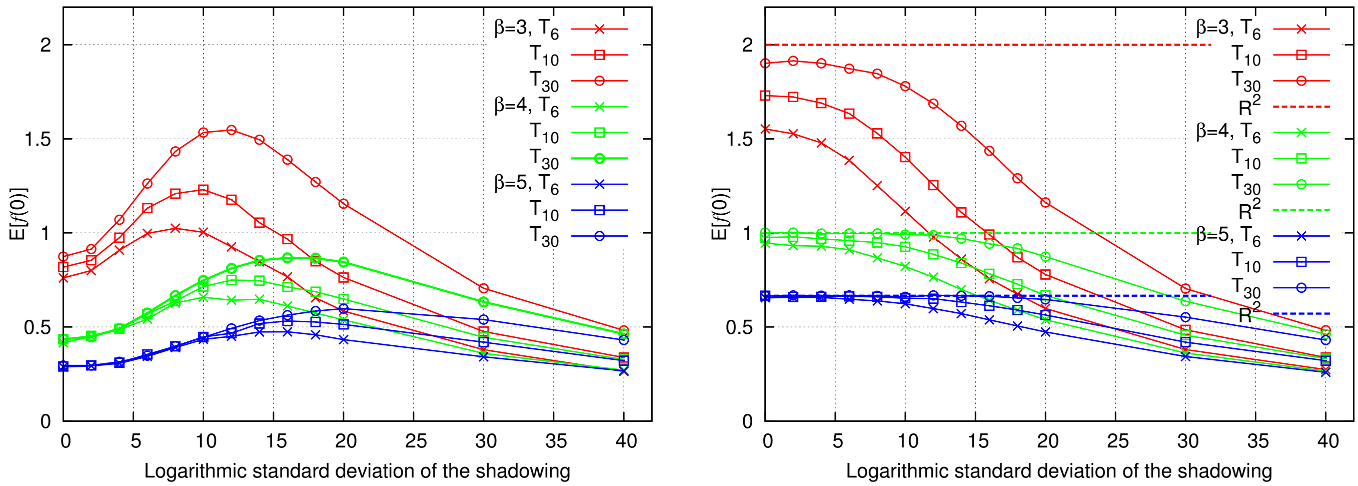
- For negligible shadowing (logarithmic standard deviation close to 0) the blocking probability $\mathbf{E}[b]$ first decreases in the path-loss exponent β (on our figures for β from 2.5 to 4) and then increases in β.
- The blocking probability is not always increasing in the standard deviation of the shadowing. Indeed, on our figures with $\beta \geq 4$ it is monotone increasing. However, for $\beta \leq 3.5$ the blocking probability $\mathbf{E}[b]$ first increases, then decreases, and ultimately increases to 1.

Note the decrease of the blocking probability in the standard deviation of the shadowing can be quite significant even between 7 and 15 dB, depending on the path-loss exponent.

The lack of monotonicity observed in Remark 4.1 is not specific for our choice of the traffic of 46.2 Erlang par km^2 as can be remarked on Figure 4, where we have assumed two different smaller values of the traffic. It is also not an artifact of our semi-analytical evaluation of the blocking probability. Indeed, it is confirmed by the results presented on Figure 3, which are obtained by calculating the fraction of call-blockings in the crude simulation of the network with the arrivals and departures of users.

In the next section we will explain this behavior and argue that it my be expected for other QoS metrics which depend on some combination of the path-loss factor and the interference factor.

Finally, we remark that for a scientific completeness we study the problem for a large range of the



(a) hexagonal network (b) Poisson network

Fig. 5. Mean interference factor in hexagonal and Poisson network on the torus $\mathbb{T}_N$ with log-normal shadowing with log-SD v and path-loss exponent β. Note that $\mathbf{E}[f(0)]$ increases with the size of the network. The straight lines correspond to the infinite (on $\mathbb{R}^2$) Poisson model; cf. Proposition 5.5.

values of the logarithmic standard deviation of the shadowing, even if the practically used values seem not to exceed the value of 20dB.

B. Analysis of the interference factor

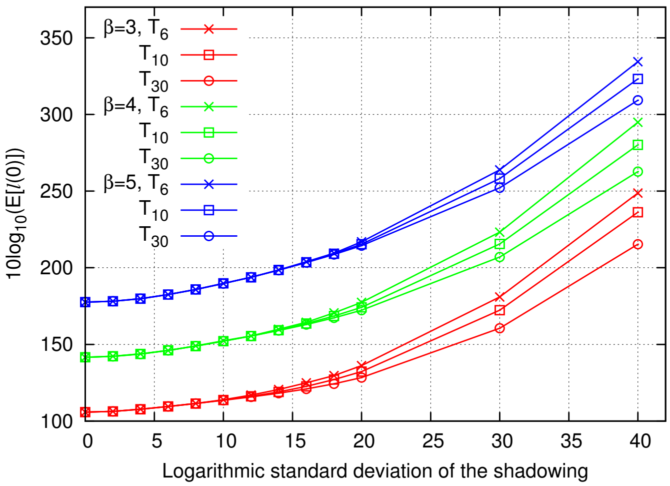
Now, we will study the impact of the shadowing and also the geometry and size of the network on the interference factor $\mathbf{E}[f(0)]$ that is a key to the understanding of the strange non-monotonicity of the blocking probability shown above. Recall that, contrarily to the blocking probability, the expectation $\mathbf{E}[f(0)]$ (as well as $\mathbf{E}[l(0)]$) does not depend on any particular correlation of the values of the shadowing $S_X(y)$ for given X and different locations y.

We begin with an important observation made directly from our model.

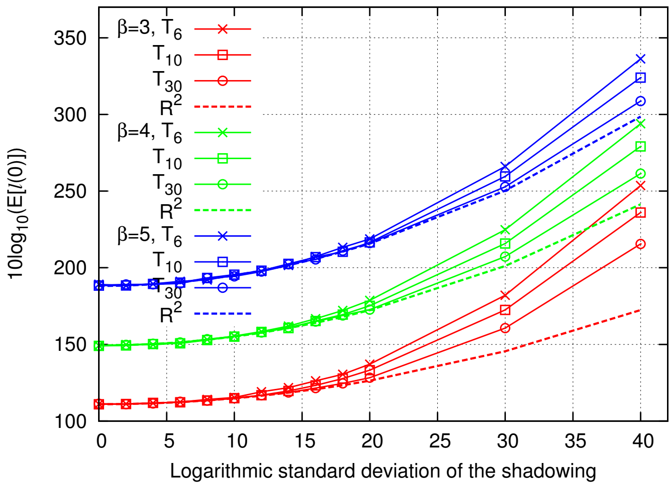
Remark 4.2: By the homothetic invariance of our hexagonal and Poisson models on the torus, or in the infinite models, with the path-loss function (2), the *mean interference factor does not depend on the intensity* λ of BS but only on the size N of the network.

Figures 5(a) and 5(b) show the impact of the path-loss exponent, shadowing and the size of the network in the case of the hexagonal and Poisson network architecture, respectively. Here are our main observations.

Remark 4.3:
1) Observe on Figure 5(a) *for hexagonal network* of a given size N^2 BS, with $N = 6, 10, 30$, and a given path-loss exponent $\beta = 3, 4, 5$, that *the mean interference factor* $\mathbf{E}[f(0)]$ *first increases and then decreases to 0 when the value* v *of logarithmic standard deviation (log-DS) of the shadowing increases*.
2) *For the Poisson network* (see Figure 5(b)) $\mathbf{E}[f(0)]$ *decreases in log-SD starting already from very small values of* v.
3) The *actual size of the network consisting of* N^2 *BS, when* $N \geq 100$*, has negligible impact on* $\mathbf{E}[f(0)]$ *when* $\beta = 4$ *and* $v \leq 10$ *or* $\beta = 5$ *and* $v \leq 15$ both in hexagonal and Poisson case (in this latter case N^2 is the expected number of BS). In this regime the *value of* $\mathbf{E}[f(0)]$ *corresponds to this in the respective infinite model*. In particular, for Poisson network it is equal to $2/(\beta - 2)$ and does not depend on log-SD v (cf. Proposition 5.5 below).
4) When $\beta = 4$ and $v \geq 10$ or $\beta = 5$ and $v \geq 15$ the mean interference factor $\mathbf{E}[f(0)]$ non-negligibly increases with the network size.
5) Comparing Figures 5(a) and 5(b) for $v \geq 20$ we observe that *for large log-SD of the shadowing the mean interference factor evaluated for the Poisson network is almost exactly the same as for the hexagonal network of the same size*.



(a) hexagonal network



(b) Poisson network

Fig. 6. Mean path-loss factor expressed in dB, in hexagonal and Poisson network on the torus $\mathbb{T}_N$ with log-normal shadowing with log-SD v and path-loss exponent β. Note that $\mathbf{E}[l(0)]$ decreases with the size of the network. The analytical expression for the infinite (on $\mathbb{R}^2$) Poisson model is given in Proposition 5.5.

Remark 4.4: The seminal paper [6] considers only the hexagonal network architecture, however, the beneficial impact of the shadowing is not observed there. The reason is that the model considered in [6] assumes that the smallest-path-loss BS (the serving one) is selected among the N_C closest BS. In particular, $N_C = 1$ ignores the shadowing in the hand-over policy as it corresponds to the situation where the serving BS is always the closest one. On the other hand the model considered in our paper corresponds to N_C equal to the total number of BS in the network. In consequence, for a higher path-loss exponent (say $\beta = 4$) and small and moderate log-SD of the shadowing ($0 \le v \le 12$) our numerical results are close to those of [6] with $N_C = 4$; cf. our Figure 5(a) and the last column in Table 1 in [6]. The fact that the average interference factor decreases in some cases with log-SD of the shadowing has not been observed in [6] due to the set of parameters considered there. Indeed, for a smaller path-loss exponent, $\beta = 3$, our Figure 5(a) shows the mean interference factor decreasing in v starting from $v \approx 8$. This range of parameters is also considered in [6, Table 2] however, with the $N_C = 2$. Apparently the beneficial impact of the shadowing cannot be observed in this case, when the BS can be chosen only among two closest BS. A general remark is of the following order: strong shadowing requires larger geographical domain in which the serving BS is searched, as the optimal one may be located far from the mobile.

C. Analysis of the path-loss factor

We begin with an important remark regarding the scaling of $\mathbf{E}[l(0)]$ with respect to the density of the BS.

Remark 4.5: Unlike the mean interference factor $\mathbf{E}[f(0)]$ (cf. Remark 4.2), *the mean path-loss factor* $\mathbf{E}[l(0)]$ *depends on the intensity* λ *of BS*. By the homothetic invariance of our hexagonal and Poisson models, it is easy to see in the case of the path-loss function (2) that this dependence has the following form $\mathbf{E}[l(0)]|_{@\lambda} = \lambda^{-\beta/2}\mathbf{E}[l(0)]|_{@\lambda=1}$. Consequently, in particular, the path-loss factor becomes preponderant in the case of sparse networks (small λ) and negligible for dense networks (large λ). We will see in Section V-B that $\mathbf{E}[l(0)]$ can be evaluated explicitly in the case of the infinite Poisson network with an arbitrary distribution of the shadowing.
Figures 6(a) and 6(b) show the mean path-loss factor $\mathbf{E}[l(0)]$ evaluated for the intensity of BS $\lambda = 1.155$BS/km^2 (equivalent to $\Delta = 1$km). The main observations are presented in the next section.

D. Conclusions on numerical results

For the hexagonal network we have observed the following facts regarding our two QoS “pre-metrics”.

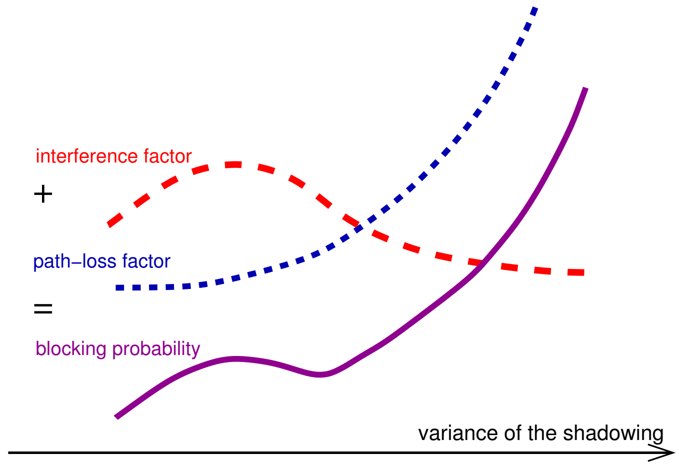


Fig. 7. Graphical explanation of a possible shape of the dependence of the blocking probability on the variance of the shadowing.

- The mean path-loss factor increases to infinity in the standard deviation of the shadowing, increases in the path-loss exponent, increases in the cell radius, but (slightly) decreases in the number of base stations.
- The mean interference factor is not monotone in the standard deviation of the shadowing: first increases and then decreases to 0. It decreases in the path-loss exponent, is invariant with respect to the cell radius and increases in the number of base stations.

Knowing that the blocking probability depends on some combination of the path-loss and interference factors of users (cf. formulas (6) and (7)), and having observed that the mean values of these two factors have opposite monotonicities in the path-loss exponent, it is not surprising that the blocking probability is not monotone in the path-loss exponent; cf. the first observation of Remark 4.1. A similar argument explains a possible non-monotonicity of the blocking probability in the standard deviation of the shadowing; cf. the second observation of Remark 4.1 and the scheme on Figure 7.

For the Poisson network we have observed the same tendencies of QoS "pre-metrics" as for hexagonal network mentioned above, except that the mean interference factor is monotone decreasing in the shadowing. Moreover, for large standard deviation of the shadowing, the "pre-metrics" of the Poisson network are very close to those of the hexagonal network.

In the next section we will prove also that for the infinite Poisson network the distributions of our QoS "pre-metrics" do not depend on the shadowing and admit explicit formulas for their means.

V. Mathematical results

In this section we will state and prove some mathematical results regarding $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$, which support and extend the numerical findings of Section IV.

A. Toroidal models

We begin by a simple observation regrading the log-normal distribution of the shadowing S with mean 1. Recall, it can be represented as $S = e^{-\sigma^2/2+\sigma N}$ where N is the standard Gaussian random variable. Thus, for any fixed $\epsilon > 0$ we have

$$\Pr\{S \geq \epsilon\} \;=\; \Pr\{N \geq \sigma/2 + (\log \epsilon)/\sigma\} \stackrel{\sigma\to\infty}{\longrightarrow} 0\,,$$

which shows that the random variable S *converges in probability to* 0 when σ (and hence $v = \sigma 10/\log 10$) tends to infinity (and this even if $\mathbf{E}[S] \equiv 1$!). From this, we have that the *path-loss between any location y and any BS X, $L_X(y) = L(|X-y|)/S_X(y)$, converges in probability to infinity when the variance of the shadowing increases*. Consequently, for any *finite* network $\tilde{\Phi}$ of base stations, the *path-loss factor $l(y) = \min_{X\in\Phi} L_X(y)$ converges in probability and in expectation to infinity*. This explains the asymptotics of $\mathbf{E}[l(0)]$ for large v observed on Figures 6(a) and 6(b).

The somewhat surprising observation on Figures 5(a) and 5(b) regarding the beneficial impact of the strong log-SD v of the shadowing on the mean interference factor can be also confirmed mathematically.

Proposition 5.1: *Assume an arbitrary, fixed, finite pattern $\{X_1, X_2, \ldots, X_n\}$ of BS locations. Consider any deterministic path-loss function $0 < L(r) < \infty$ and (independent) log-normal shadowing $S_{X_i}(\cdot)$ with the log-SD v. Then for any location y we have $\lim_{v\to\infty} f(y) = 0$ in probability.*

Proof: We will show that $\lim_{v\to\infty} \Pr\{ f(y) \geq \epsilon \} = 0$ for any $\epsilon > 0$. Denote by $G_i = S_{X_i}(y)/L(|X_i - y|)$ the path-gain from X_i to y. Consider ordered vector $(G_{(1)}, \ldots, G_{(n)})$ of these path gains, where $\min_i G_i = G_{(1)} \leq \ldots \leq G_{(n)} = \max_i G_i$. Note that $f(y) = 1/G_{(n)} \sum_{i=1}^{n} G_{(i)} - 1 \leq (n-1)G_{(n-1)}/G_{(n)}$. In order to prove our claim it is enough to show that $\Pr\{ G_{(n-1)}/G_{(n)} \geq \epsilon \} \to 0$ when $v \to \infty$. To this regard denote $L(|X_i - y|) = l_i$, and recall from the definition of our path-loss model that we can represent $G_i(y) = e^{\tilde{N}_i}$, where $\{\tilde{N}_i\}_{i=1,\ldots,n}$ are independent Gaussian random variables, with mean $\mathbf{E}[\tilde{N}_i] = -\log l_i - \sigma^2/2$ and the same SD $\sigma = v \log 10/10$. Since G_i is monotone increasing in $\tilde{N}_i$ we have $G_{(i)} = e^{\tilde{N}_{(i)}}$, where $\min_i \tilde{N}_i = \tilde{N}_{(1)} \leq \ldots \leq \tilde{N}_{(n)} = \max_i \tilde{N}_i$. Moreover, $A := \{ G_{(n-1)}/G_{(n)} \geq \epsilon \} = \{ \tilde{N}_{(n)} - \tilde{N}_{(n-1)} \leq M \}$, where $M = -\log \epsilon$. Denote by $A_{ij} = \{ 0 \leq \tilde{N}_i - \tilde{N}_j \leq M \}$. Note that $A \subset \bigcup_{i,j=1,\ldots,n, i\neq j} A_{ij}$ and the result follows from the fact that for any $i \neq j$ $\Pr\{A_{ij}\} \to 0$ when $v \to \infty$. Indeed, for $i \neq j$, $\tilde{N}_i - \tilde{N}_j = \bar{N}$ is Gaussian random variable with mean $\log(l_j/l_i)$ and variance σ^2 and thus $\Pr\{A_{ij}\} = \Pr\{ 0 \leq \bar{N} \leq M\} \to 0$ for any given finite M when $\sigma^2 = v^2 \log^2 10/100 \to \infty$. This completes the proof. ■

Corollary 5.2: *The mean interference factor $f(0)$ in the Poisson and hexagonal network on the torus $\mathbb{T}_N$, with log-normal shadowing converges in distribution and in expectation to 0 when log-SD of the shadowing goes to infinity.*

Proof: For any $\epsilon > 0$ by Proposition 5.1 and Lebesgue dominated convergence theorem we have $\Pr\{ f(0, \tilde{\Phi}) > \epsilon \} = \mathbf{E}[\Pr\{ f(0, \tilde{\Phi}) > \epsilon | \Phi\}] \to 0$, when $v \to \infty$. This proves that $f(0)$ converges in distribution to 0. Convergence of $\mathbf{E}[f(0)]$ to 0 follows again from the Lebesgue dominated convergence theorem by the observation $f(y, \tilde{\Phi}) \leq \Phi(\mathbb{T}_N) - 1$ and $\mathbf{E}[\Phi(\mathbb{T}_N)] < \infty$. ■

B. *Infinite models*

In this section we will consider infinite hexagonal and Poisson models. We will show first that serving BS X_0^*, and hence the path-loss and interference factors, are well defined. Then we will argue that values of these factors in the infinite models can be seen as limits of respective toroidal models on $\mathbb{T}_N$ when $n \to \infty$. Finally we will prove a (surprising ?) invariance of $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$ in the infinite Poisson model with respect to the distribution of the shadowing. In this case the values $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$ can be evaluated explicitly.

Proposition 5.3: *Consider infinite Poisson $\Phi = \Phi_P$ or hexagonal $\Phi = \Phi_H$ model of BS, with shadowing whose marginal distribution has finite moment of order $2/\beta$ ([5]). Then there exist $X_0^* \in \Phi$ satisfying (3). Moreover, the path-loss factor and the interference factor calculated with respect to the restriction of Φ to $\mathbb{T}_N$, i.e., $l(0, \tilde{\Phi}^{\mathbb{T}_N})$ and $f(0, \tilde{\Phi}^{\mathbb{T}_N})$, converge almost surely and in expectation to $l(0, \tilde{\Phi})$ and $f(0, \tilde{\Phi})$, respectively.*

Proof: To prove the first statement it is enough to show that the expected number of BS X_i such that $S_{X_i}(0)/L(|X_i|) > M$ is finite for any $M < \infty$. In the case of the Poisson p.p. this will be shown in the proof of Proposition 5.5 below. Here we consider only hexagonal case $\Phi = \Phi_H$. Denote by $\overline{G}(x) = \Pr\{ S > x \}$. We have

$$
\begin{aligned}
&\mathbf{E}[\#\{X_i \in \Phi_H : S_{X_i(0)}/L(|X_i|) > M\}] \\
&\quad= \mathbf{E}\Big[\sum_{X_i \in \Phi_H} \mathbb{1}\Big(S_{X_i(0)} > ML(|X_i|)\Big)\Big] \\
&\quad= \mathbf{E}\Big[\sum_{X_i \in \Phi_H} \overline{G}\Big(ML(|X_i|)\Big)\Big]
\end{aligned}
$$

[5] i.e., $\mathbf{E}[S^{2/\beta}] < \infty$. Note that $2/\beta < 1$ and thus the above assumption follows from our default assumption $\mathbf{E}[S] = 1 < \infty$.

$$\leq \sum_{i=1}^{\infty} 6n\overline{G}\Big((n\Delta K/2)^{\beta}/M\Big) < \infty,$$

where the last inequality follows from the assumption $\mathbf{E}[S^{2/\beta}] = 2/\beta\int_0^\infty s^{2/\beta-1}\overline{G}(s)\,ds < \infty$. This completes the proof of the first statement.

In order to prove the second statement, note that for any realization the network $\tilde{\Phi}$, for N large enough $X_0^* \in \mathbb{T}_N$. Consequently, $l(0,\tilde{\Phi}^{\mathbb{T}_N})$ is eventually constant in N while $f(0,\tilde{\Phi}^{\mathbb{T}_N})$ eventually increases in N (the serving BS is not changing any more and only interference is added). The convergence of the expectation of the path-loss factor follows from the monotone convergence theorem, noting that $l(0,\tilde{\Phi}^{\mathbb{T}_N})$ is decreasing in N. The convergence of the expectation of the interference factor follows form the dominated convergence theorem knowing that $f(0,\tilde{\Phi}) \leq f'(0,\tilde{\Phi})$, where $f'(0,\tilde{\Phi})$ is the interference factor calculated under assumption that the handover policy selects the geographically closest BS as the serving one. By the independence of the shadowing fields given the locations of BS and the assumption that the mean shadowing is equal to 1

$$E[f'(0,\tilde{\Phi})] = E\Big[\frac{1}{S}\Big]\mathbf{E}\Big[\sum_{X\in\Phi}\frac{L(|X_0'^*|)}{L(|X|)}\Big] - 1\,, \tag{8}$$

where $X_0'^*$ is a point of Φ closest to the origin 0. By our assumption on the mean path-gain $\mathbf{E}[1/S] < \infty$. The second expectation (8) is equal to the mean interference factor in the infinite model with constant shadowing $S \equiv 1$, and it is known to be finite in the infinite hexagonal and Poisson model; cf. respectively Remark 5.4 and Proposition 5.5 below. ■

Remark 5.4: It was shown in [7] that in the case of $S \equiv 1$ and the deterministic path-loss function (2) $\mathbf{E}[l(0)]$ and $\mathbf{E}[f(0)]$ in the hexagonal model can be approximated by the following expressions

$$\begin{aligned}\mathbf{E}[f(0,\Phi_H)] &\approx \frac{0.9365}{\beta-2},\\ \mathbf{E}[l(0,\Phi_H)] &\approx \frac{K^\beta}{(\pi\lambda)^{\beta/2}(1+\beta/2)}\,.\end{aligned}$$

To the best of our knowledge, analytical expressions (approximations) in the case of the infinite hexagonal network with random shadowing are not known. We consider now infinite Poisson model.

Proposition 5.5: *Assume infinite Poisson network, deterministic path-loss function (2) and a general distribution of the shadowing S satisfying $\mathbf{E}[S^{2/\beta}] < \infty$. Then the distribution of the interference factor $f(0) = f(0,\tilde{\Phi})$ does not depend on the distribution S and the distribution of $l(0)$ depends on S only through the product $\lambda\mathbf{E}[S^{2/\beta}]$. Moreover*

$$\begin{aligned}\mathbf{E}[f(0)] &= \frac{2}{\beta-2}\,,\\ \mathbf{E}[l(0)] &= \frac{K^\beta\Gamma(1+\beta/2)}{(\pi\lambda\mathbf{E}[S^{2/\beta}])^{\beta/2}}\,,\end{aligned}$$

where $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$. In particular, for the log-normal shadowing

$$\mathbf{E}[l(0)] = \frac{K^\beta\Gamma(1+\beta/2)\exp[(1-2/\beta)\sigma^2/2]}{(\pi\lambda)^{\beta/2}}\,.$$

Remark 5.6: The above result says that in the infinite Poisson notwork the existence of shadowing has no impact on the mean interference factor. The impact of the shadowing on the mean path-loss factor in this model consists in a "fictitious" scaling of the intensity of the BS by the factor $(\mathbf{E}[S^{2/\beta}])^{\beta/2} \leq 1$. The respective expressions in the case of $S \equiv 1$ has been found for the first time (to the best of our knowledge) in [15]. Note however, that the above observation is valid only if the handover policy selects the strongest BS as described in Section III-C. Indeed, assume that, despite non-constant shadowing, the handover policy selects the geographically closest BS as the serving one. Then, the mean interference factor $\mathbf{E}[f'(0)]$ can be expressed as in (8). Recall that the second expectation in this expression is equal to the mean interference factor in the same model without shadowing (i.e., $S \equiv 1$). By the Jensen's inequality

$\mathbf{E}[1/S] \geq 1/\mathbf{E}[S] = 1$ and consequently we observe the increase of the mean interference factor compared to the "shadowing-dependent" handover policy. In particular, for log-normal S with mean 1 and log-SD v we have $\mathbf{E}[1/S] = e^{\sigma^2} = e^{v^2 \log^2 10/100}$, which means that the *log-normal shadowing in any geometric model of BS in which it is not taken into account in the handover policy increases the mean interference factor by* $v^2 \log 10/10\,\mathrm{dB}$, where v is log-SD of the shadowing.

Proof of Proposition 5.5: Note that the values of $l(0)$ and $f(0)$ are entirely defined by the collection of random variables $\{L_X(0) = L(|X|)/S_X(0) : X \in \Phi\}$. Given Φ these random variables are independent. Thus by the displacement theorem for Poisson p.p. (cf. [19, Theorem 1.3.9]) $\{L_X(0)\} = \Psi$ constitutes a (non-homogeneous) Poisson p.p. on $\mathbb{R}^+ = [0, \infty)$ of intensity measure Λ' given by

$$\begin{aligned}
\Lambda'([0,s]) &= \mathbf{E}[\Psi([0,s])] \\
&= \lambda \int_{\mathbb{R}^2} \Pr\{\, L(|z|)/S \leq s \,\}\, dz \\
&= 2\pi\lambda \int_0^\infty r \Pr\{\, L(r)/S \leq s \,\}\, dr \\
&= 2\pi\lambda \int_0^\infty r \mathbf{E}\left[\mathbb{1}\left(L(r)/S \leq s\right)\right] dr \\
&= 2\pi\lambda \mathbf{E}\left[\int_0^{(sS)^{1/\beta}/K} r\, dr\right] \\
&= \frac{\lambda s^{2/\beta}\pi}{K^2} \mathbf{E}\left[S^{\frac{2}{\beta}}\right] .
\end{aligned}$$

Note that the latter expression is finite, which proves that the serving BS X_0^* is well defined (cf. proof of Proposition 5.3). Note also that it depends on the shadowing only through its moment $\mathbf{E}[S^{2/\beta}]$. Moreover one obtains the same expression in the model without shadowing and the density of BS multiplied by $\mathbf{E}[S^{2/\beta}]$. By the homothetic invariance of the Poisson model with the path-loss function (2) the distribution of $f(0)$ does not depend on the intensity of the BS. Thus the invariance of the distribution of $f(0)$ on the distribution of the shadowing. In particular, we can conclude that $\mathbf{E}[f(0)] = 2/(\beta - 2)$ — the value obtained in the model without shadowing; see [15], cf. also [19, Example 4.5.1]. The formula for the mean path-loss factor follows from its dependence on the intensity of the base stations via the function $\lambda^{-\beta/2}$. This completes the proof. ■

VI. Concluding remarks

We show that the QoS in path-loss-and-interference limited cellular networks is not always decreasing in the strength (variance) of the log-normal shadowing, provided the handover policy selects the strongest BS as the serving one. Under strong shadowing it principally suffers from the poor path-loss conditions with respect to the serving BS. For moderate shadowing however, when the QoS is not yet compromised by the path-loss conditions, it may profit from the reduction of the interference. This is because increasing variance of the shadowing tends to "separate" the strongest (serving BS) signal from all other signals. This latter mathematical result seems to be in line with a recent real-network observation [22] that mobiles in indoor communications (typically subject to strong shadowing) report fewer BSs for potential hand-over. The results presented in this paper regard the network-average of the QoS metrics. More study is needed, to analyze the impact of the shadowing on the distribution of these metrics in the network.

Appendix

From the interference and path-loss factors to blocking probability via Erlang's loss formula

In order to make this paper more self-contained, in this Appendix we briefly recall a simple and intrinsic relation between the path-loss and interference factors and the blocking probability in the network serving streaming traffic. This relation, whose very essence can be explained by the famous Erlang's loss formula, was observed in the current geometric context (however without shadowing) for the first time in [15]. It explains why the (mean) path-loss and interference factors can be considered as some primitive (basic) QoS metrics in cellular networks serving streaming traffic. For (more involved) relations between interference factor and the QoS in networks serving data traffic see e.g. [18].

In order to evaluate the blocking probability it is necessary to specify the dynamics of call arrivals and their durations, as well as to identify the set of feasible configurations of users (which can be served simultaneously at their requested bit-rates).

A. Space-time scenario

Consider a *bounded* (only finite models will be considered in this section) subset $\mathbb{D}$ of the plane $\mathbb{R}^2$ that models a geographical region in which call arrivals take place, and in which some network of BS's Φ is deployed as described in Section III-A2; for example $\mathbb{D} = \mathbb{T}_N$ with hexagonal $\Phi = \Phi_H^{\mathbb{T}_N}$ or Poisson $\Phi = \Phi_P^{\mathbb{T}_N}$ network of BS's. As described in Section III-B, we assume that the set $\mathbb{D}$ is endowed with the random path-loss fields $\{S_X(y)\}_{X\in\Phi}$ describing the shadowing from all BS to any point $y \in \mathbb{D}$. Recall from Section III-B that this network model was considered as a marked pp $\tilde{\Phi}$. The randomness of this pp (of the shadowing fields, as well as the locations of BS) should be considered as sampled "once for ever"; i.e., not varying in time. Given a realization of $\tilde{\Phi}$ in what follows we will consider a random process (in time) of user arrivals and departures to $\mathbb{D}$. The stationary distribution of this process will allow us to evaluate the *blocking rates* $b(y)$ of users arriving at $y \in \mathbb{D}$ *given a realization of the network* $\tilde{\Phi}$. By averaging over all possible locations of users y and then configurations of $\tilde{\Phi}$ we will evaluate a one-value QoS metric of the given network with shadowing, that we call the *blocking probability*.

B. Traffic demand

Regarding the traffic of users (calls), we assume that their inter-arrival times are independent and identically, exponentially, distributed, with rate α (mean $1/\alpha$). The position of each arriving user is picked at random in $\mathbb{D}$ according to some distribution $Q(dy)$ (which we take uniform for the numerical evaluations). We assume that users don't move during their calls.

Each call requires to be served by the network at a given bit-rate during some service time. This service is demanded from the BS with the strongest signal; i.e., the one chosen according to the handover policy described in Section III-C.

The durations of the different calls are assumed to be i.i.d. exponentially distributed with mean $1/\mu$. (This assumption may be relaxed due to the so-called insensitivity property, but this is not in the scope of the present paper.) The quantity $\rho = \alpha/\mu$ is called the *traffic demand* (expressed in Erlangs) in the whole network.

The set of positions of all users served at a given time is called *configuration of users*. In accordance with the point process formalism it is customary to identify configurations of users with finite counting measures on $\mathbb{D}$, assuming that each user corresponds to a unit (Dirac) measure concentrated at its location. Let $\mathbb{M}$ be the set of such measures (all possible configurations ν of users, not necessarily all served by the network).

We denote by $\{N_t\}_{t\geq 0}$ the process describing the evolution in time of the user configurations in $\mathbb{D}$ (due to arrivals and departures) in the absence of any admission control. It is often called the *free process*. Due to our convention it takes its values in $\mathbb{M}$.

By our previous assumptions, the free process $\{N_t\}_{t\geq 0}$ is a Markov process, which is ergodic with stationary invariant distribution Π corresponding to the Poisson point process on $\mathbb{D}$ with mean measure $\rho Q(dy)$. In other words: the stationary free process (offered traffic) of positions of users is Poisson with mean measure equal to $\rho Q(dy)$. Moreover $\{N_t\}_{t\geq 0}$ is reversible with respect to Π. Note that the free process (and hence its stationary distribution) does not depend on $\tilde{\Phi}$; i.e., on the configuration of the BS and shadowing-fields.

C. Erlang's loss formula

We assume some call-admission condition consisting in verifying whether a given configuration of users (with a new arrival) belongs to some set of *feasible configurations* $\mathbb{M}^{\mathrm{f}}$. Users of these configurations can be served simultaneously by the network at the requested bit-rates. Contrary to the configurations of the free process, feasible configurations depend on $\tilde{\Phi}$; i.e., on the locations of BS and their shadowing fields (one needs in particular to know which user is served by which BS). Particular forms of this dependence will be specified latter on. In general, we tacitly assume that a user departure from a feasible configuration cannot make it unfeasible.

Denote the evolution of the free process modified (controlled) by the given admission condition by $\left\{N_t^{\mathrm{f}}\right\}_{t\geq 0}$. Due to the form of the admission condition, this process is also Markov. More precisely, it has the same dynamics as the marked free process except that the transitions (i.e. arrivals) that would lead outside $\mathbb{M}^{\mathrm{f}}$ are blocked. Such a modification of the Markov process is called *truncation* to $\mathbb{M}^{\mathrm{f}}$. The crucial observation, made by the reversibility of the free process, is that the truncated process $\left\{N_t^{\mathrm{f}}\right\}_{t\geq 0}$ admits as its invariant distribution the truncation of Π to $\mathbb{M}^{\mathrm{f}}$; that is, $\Pi^{\mathrm{f}}\left(\Gamma\right) = \Pi\left(\Gamma \cap \mathbb{M}^{\mathrm{f}}\right)/\Pi\left(\mathbb{M}^{\mathrm{f}}\right)$ for $\Gamma \subset \mathbb{M}$.

The blocking probability (in some region of the network) is usually defined as the proportion of the blocked calls to the total number of arrivals in the long run of the system. The celebrated Erlang's loss formula allows to express this ergodic time-average via the invariant measure of the free process, and in our space-time scenario takes the form

$$b(y) = \frac{\Pi(\{\nu \in \mathbb{M}^{\mathrm{f}} : \nu + \epsilon_y \notin \mathbb{M}^{\mathrm{f}}\})}{\Pi(\mathbb{M}^{\mathrm{f}})}\,, \tag{9}$$

where $b(y) = b(y, \tilde{\Phi})$ is the blocking rate of users arriving at the location $y \in \mathbb{D}$ (given some particular repartition $\tilde{\Phi}$ of BS and their shadowing conditions; cf. [16]). Note that, in analogy to the classical version of the Erlang's loss formula, the blocking rate $b(y)$ of users arriving at y is equal to the conditional probability that the stationary configuration of users in the free process, given it is feasible, cannot admit a new user at y.

Integrating $b(y)$ over $\mathbb{D}$ against the distribution of the mobile location $Q(dy)$ one obtains the *blocking probability in the whole network, given the realization of the network* $\tilde{\Phi}$

$$b = b(\tilde{\Phi}) = \int_{\mathbb{D}} b(y, \tilde{\Phi})Q(dy)\,. \tag{10}$$

Finally, since we are not interested in the performance of one particular realization of the network (of its shadowing field), we will *average* $b(\tilde{\Phi})$ *over all possible realizations of* $\tilde{\Phi}$ and call

$$\mathbf{E}[b] = \mathbf{E}[b(\tilde{\Phi})] \tag{11}$$

the blocking probability (strictly speaking we should call it "mean" blocking probability). In the case of the uniform distribution of $Q(dy)$, which is our default assumption, we have by changing the order of integration and the translation invariance of the distribution of our toroidal network models

$$\begin{aligned}\mathbf{E}[b] &= \int_{\mathbb{D}} \mathbf{E}[b(y, \tilde{\Phi})]Q(dy) \\ &= \int_{\mathbb{D}} \mathbf{E}[b(0, \tilde{\Phi})]Q(dy) \\ &= \mathbf{E}[b(0, \tilde{\Phi})] = \mathbf{E}[b(0)].\end{aligned}$$

D. Multi-Erlang admission condition and Kaufman-Roberts algorithm

For the above formula for blocking probability to be of any use in the dimensioning process one needs an efficient way of the evaluation of the Poisson probabilities in the numerator and the denominator in (9), in particular $\Pi(\mathbb{M}^{\mathrm{f}})$. Such efficient method exists for some particular form of the admission condition defining $\mathbb{M}^{\mathrm{f}}$, as we explain in what follows.

Recall that Φ denotes the locations of BS's. We say that the admission condition has the *multi-Erlang* form if the corresponding set of feasible configurations $\mathbb{M}^{\mathrm{f}}$ has the following form

$$\mathbb{M}^{\mathrm{f}} = \bigcap_{X \in \Phi} \left\{\nu \in \mathbb{M} : \sum_{y \in \nu} \mathbb{1}(y \text{ served by } X)\varphi\left(y\right) \leq 1\right\} \tag{12}$$

where $\varphi(y) = \varphi(y; \tilde{\Phi})$, is some function of user location y and the realization of the network $\tilde{\Phi}$ (and possibly other characteristics of the user, as e.g. its bit-rate, and the BS X, as e.g. its maximal power, the ambient noise power etc.), whose value, for each $BS\ X$ is summed over all users y served by this BS and compared to the constant 1. When the admission condition has the above multi-Erlang, then one may easily evaluate the blocking probability $b = b(\tilde{\Phi})$ given by (10) using the Kaufman-Roberts algorithm; cf. [23, 24].

Examples of the function φ for the down-link in CDMA and OFDMA networks are given in Section III-E. Note that in these cases $\varphi(y) = \varphi(l(y), f(y))$ depends in an explicit way on the path-loss and interference factors of the user y.

E. Spatial ergodicity

Recall that in order to obtain the final blocking probability $\mathbf{E}[b(\tilde{\Phi})]$ in (11), it remains to do the final averaging over the distribution of $\tilde{\Phi}$. It can be done by the simulation of several realizations of $\tilde{\Phi}$, evaluation of $b(\tilde{\Phi})$ for each of them by the Kaufman-Roberts algorithm, and then taking the empirical average. However, in practice *one realization of* $\tilde{\Phi}$ *is enough*, provided the shadowing fields $S_X(y)$ do not exhibit high spatial correlation across y (recall that we have assumed them to be independent across X). Indeed, we have noticed in our experiments, that for large enough networks (in the case of the hexagonal network $\mathbb{T}_6$ is enough!) with spatially uncorrelated shadowing, the value of $b(\tilde{\Phi})$ is almost invariant with respect to $\tilde{\Phi}$ and hence very close to $\mathbf{E}[b(\tilde{\Phi})]$. This is due to the integration over $\mathbb{D}$ in (10) and spatial ergodic properties of the process $\tilde{\Phi}$. We have also observed that this invariance (with respect to $\tilde{\Phi}$) in general is not present for spatially correlated shadowing.

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