

On the Performance of the Rotate-and-Forward Protocol in the Two-Hop Relay Channels

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Abstract—In wireless cooperative systems, the outage probability is a relevant performance measure from the information theoretic point of view. Deriving its exact value with respect to the relaying protocol parameters and for all the range of the Signal to Noise Ratio is in general a hard task. Instead, one can extract the *outage gain* in order to approximate the outage probability as the SNR goes to infinity at a fixed rate R . In this paper, we consider the layered two-hop relay channel, operating under the linear Rotate-and-Forward (RF) protocol, where each relay rotates its received signal and forwards it towards the destination. We consider two network configurations for which we derive exact outage gain values. We use the outage gain to compare the performance of the RF scheme to that of other schemes having the same diversity order. The derived outage gain values are verified through numerical simulations.

I. INTRODUCTION

In wireless communication, several techniques have been proposed to combat the channel's attenuation factors such as fading, shadowing and path loss. In particular, cooperative diversity techniques, first studied in [1] and [2], have recently received a great interest as a way to improve both the throughput and the reliability in wireless networks.

In the literature, most proposed cooperative schemes have relied on one of the following three relaying strategies. In Decode-and-Forward (DF) strategy, the relay decodes some part of the transmitted signal. The decoded bits are then re-encoded and sent to the next relay or destination. In Compress-and-Forward (CF) strategy, the signal observed at the relay is quantized and forwarded to the destination without being decoded. In Amplify-and-Forward (AF) strategy, the relay transmits a scaled version of what he has received, it simply acts as a repeater. The AF scheme is widely used due to its simplicity while the DF scheme requires more signal processing on the relay side. The CF scheme requires perfect global channel state information (CSI) and is generally excluded. The AF based cooperative diversity schemes are linear relaying schemes while those DF and CF based are non-linear. The proposed protocols are compared in terms of Diversity-Multiplexing-Tradeoff (DMT), [3].

In this paper, we are interested in the multi-hop MIMO relay networks where no direct source-destination link. For these networks, several relaying strategies have been proposed in [4] - [8]. In particular, Yang and Belfiore have recently proposed a linear relaying scheme called Rotate-and-Forward

(RF) protocol in [9]. The idea is to convert the spatial diversity into time diversity by creating an artificial fast fading channel using time-varying distributed rotations. They show that time-varying distributed rotations can recover spatial diversity. This scheme is shown to achieve the optimal DMT in layered two-hop relay channel, [10]. In this scheme, the relays are independent and act in a distributed fashion.

In this work, we consider the layered two-hop relay channel with a single source-destination pair and multiple full-duplex relays. We derive *outage gain* values for some network configurations operating under the RF protocol. The outage gain values are used to approximate the outage probability in the high signal to noise ratio, ρ , regime. The results show that those values are accurate for moderate ρ values as well. We use the outage gain values to compare the performance of the RF to that of other relaying schemes having the same diversity order. The rest of this paper is organized as follows. The system model and some basic assumptions are presented in section II. The *outage gain* criterion is detailed in section III. A general idea about the RF scheme is given in section IV. In section V, we include the main contribution of this work where we detailed the study of the performance of the RF scheme. We give some numerical results in section VI and conclude in section VII.

II. SYSTEM MODEL AND ASSUMPTIONS

In the paper, we use boldface lower case letters \mathbf{v} to denote vectors and boldface capital letters \mathbf{M} to denote matrices. $\mathcal{CN}(\mu, \sigma^2)$ represents a complex Gaussian random variable with mean μ and variance σ^2 . $[\cdot]^T$ and $[\cdot]^\dagger$ respectively denote the matrix transposition and the matrix conjugated transposition operation. $\mathbb{E}[\cdot]$ stands for the expectation operator. $|\cdot|$ is the scalar norm. $\log(\cdot)$ stands for the base-2 logarithm.

We consider a two-hop relay network composed of one source, one destination, and N full-duplex single-antenna relays. The source and the destination are equipped with n_t and n_r antennas respectively. The two-hop channel is then denoted by (n_t, N, n_r) . We focus on distributed relaying schemes. By this, we mean that the relays are independent and exchange no information between them on the message or channel state information (CSI). The CSI are only available at the receiver side, no transmitter CSI at all. The terminals are considered perfectly synchronized.

The faded sub-channels are flat, Rayleigh-faded, and quasi-static with a coherence time much larger than N . The channel matrices are independent, and have i.i.d. zero-mean complex Gaussian entries with variance σ^2 , i.e., $h_{i,j} \sim \mathcal{CN}(0, \sigma^2)$. We assume an additive white Gaussian noise (AWGN) at the relays and the destination.

III. OUTAGE GAIN CRITERION

The outage probability P_o which is the probability that the mutual information lies under a given rate, is a relevant performance measure from the information theoretic point of view. In general, the exact characterization of the outage probability P_o for every value of the signal to noise ratio (SNR), ρ , is a difficult task. However, a simpler task is to study the behavior of P_o as ρ goes to infinity, $\rho \rightarrow \infty$, at a fixed rate R . Usually, $\rho^d P_o$ converges as $\rho \rightarrow \infty$ to a positive constant in which d is the diversity order of the scheme and the rate R is fixed independently of ρ . The constant $\xi = \lim_{\rho \rightarrow \infty} \rho^d P_o$ is called the *outage gain* and provides useful information on the behavior of P_o . The outage gain was used to study the performance of the N-relay network under AF and DF protocols in [11]. It is useful to optimize the power distribution among the transmitting nodes and the durations of the slots specified by the relaying protocols. In [12], it is used to study the asynchronous two-hop two-relay scheme for the AF and DF protocols. It can also be used to compare the performance of two protocols having the same diversity gain.

In [12], the definition of the outage gain is extended as follows. Assuming there exists a real function $f > 0$ such that $\lim_{\rho \rightarrow \infty} f(\rho) P_o = \xi$, then ξ is called the outage gain associated with f .

IV. THE ROTATE-AND-FORWARD PROTOCOL

The Rotate-and-Forward relaying strategy was proposed in [9]. In this scheme, the relays perform a linear processing on the received signal and forward it. A sequence of rotation vectors $\theta_1, \dots, \theta_T$ is defined based on a distributed rotation sequence (DRS) where $\theta_i \triangleq [\theta_{i,1}, \dots, \theta_{i,N}]$. A codeword $\mathbf{X} \in \mathbb{C}^{n_t \times T}$ is transmitted by the source over T symbol times. At instant i , each relay transmits a rotated version of what it received at instant $i - 1$. The rotation used by the relay j is $\theta_{i,j}$, the j th element of the vector θ_i . Thus, we have, by ignoring the power normalization terms,

$$\mathbf{y}_D[i+1] = \mathbf{H}_2 \mathbf{F}_i \mathbf{H}_1 \mathbf{x}[i] + \mathbf{H}_2 \mathbf{F}_i \mathbf{z}_R[i] + \mathbf{z}_D[i+1] \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^{n_t \times 1}$ and $\mathbf{y}_D \in \mathbb{C}^{n_r \times 1}$ are the transmitted signal from the source and received signal at the destination, respectively; $\mathbf{z}_R \sim \mathcal{CN}(0, \mathbf{I}_N)$, and $\mathbf{z}_D \sim \mathcal{CN}(0, \mathbf{I}_{nr})$ are the AWGN at relays and destination respectively; $\mathbf{H}_1 \in \mathbb{C}^{N \times n_t}$ and $\mathbf{H}_2 \in \mathbb{C}^{n_r \times N}$ are the source-relays and relays-destination channel matrix, respectively. \mathbf{F}_i is the rotation matrix at instant i .

Then, the transmitted codeword \mathbf{X} goes through an equivalent time-varying fading channel and the equivalent channel of the RF scheme is a sequence $\mathbf{H}_2 \mathbf{F}_1 \mathbf{H}_1, \dots, \mathbf{H}_2 \mathbf{F}_T \mathbf{H}_1$ with $\mathbf{F}_i \triangleq \text{diag}(e^{j\theta_{i,1}}, \dots, e^{j\theta_{i,N}})$.

The DMT of the proposed protocol depends uniquely on the time-variant equivalent channel matrix $\mathbf{H} \triangleq \mathbf{H}_2 \mathbf{F}_i \mathbf{H}_1$, [9]. Therefore, the mutual information for a given channel realization is

$$\frac{1}{T} \sum_{i=1}^T \log \det \left(\mathbf{I} + \text{SNR}(\mathbf{H}_2 \mathbf{F}_i \mathbf{H}_1)(\mathbf{H}_2 \mathbf{F}_i \mathbf{H}_1)^\dagger \right). \quad (2)$$

With a large T , the instantaneous mutual information converges to the following term

$$I_{mean} = \mathbb{E}_\theta \left\{ \log \det \left(\mathbf{I} + \text{SNR} \mathbf{H} \mathbf{H}^\dagger \right) \right\}. \quad (3)$$

The RF scheme is shown to achieve the optimal DMT trade-off in a two-hop relay network with two antennas at the relay node in [9] and for an arbitrary number of antennas at the source, relay and destination nodes, in [10].

V. OUTAGE PROBABILITY BEHAVIOR IN RF SCHEMES

In this section, we consider two network configurations: the (1, N, 1) and the (N, 1, N) networks. As a beginning, and for sake of simplicity, we assume that the relays are noiseless. The input-output relation becomes

$$\mathbf{y}_D = \mathbf{H}_2 \mathbf{F} \mathbf{H}_1 \mathbf{x} + \mathbf{z}_D \quad (4)$$

we omit the time index for simplicity.

A. The Two-Hop (1, N, 1) Network

We consider the two-hop (1, N, 1) channel operating under the RF protocol. Let $\mathbf{H}_1 = [h_{11} \dots h_{1N}]^T$ and $\mathbf{H}_2 = [h_{21} \dots h_{2N}]$, the channel matrices corresponding to the first and the second hops respectively. $\mathbf{F} = \text{diag}\{e^{i\theta_1}, \dots, e^{i\theta_N}\}$ is the rotation matrix. $\mathbf{H} = \mathbf{H}_2 \mathbf{F} \mathbf{H}_1$ is the equivalent channel matrix. The mutual information of the channel can be written as

$$I_{mean} = \mathbb{E}_\theta \left\{ \log \left\{ a + \sum_{i=1}^{N-1} \sum_{j=i+1}^N b_{ij} \cos(\theta_i - \theta_j + \theta_{ij}) \right\} \right\} \quad (5)$$

with

$$a = 1 + \rho \sum_{l=1}^N g_{1l} g_{2l} \quad (6)$$

$$b_{ij} = 2\rho |h_{1i} h_{2i} h_{1j}^* h_{2j}^*| \quad (7)$$

where $g_{ji} = |h_{ji}|^2$ are the channels power gains. We have that [9]

$$\log(a) - (N-1) < I_{mean} < \log(a). \quad (8)$$

Hence, for a $\rho \rightarrow \infty$ and for a large rate R , the mutual information of the channel depends only on the constant term a , and we have

$$\text{Prob} \left\{ \mathbb{E}_\theta \left\{ \log \det(\mathbf{I} + \rho \mathbf{H} \mathbf{H}^\dagger) \right\} < R \right\} \doteq \text{Prob} \{ \log(a) < R \} \quad (9)$$

Using an appropriate function $f(\rho)$, we need to show that $f(\rho) P_o$ converges to a positive constant representing the outage gain ξ , as $\rho \rightarrow \infty$.

Let $g_{1l}g_{2l} = g_l$. We have

$$P_0 = \int \mathbf{1}\{(g_1, \dots, g_N) \in \mathbb{R}_+^N : \log(a) < R\} \\ \times f_{G_1}(g_1) \dots f_{G_N}(g_N) \prod_{l=1}^N dg_l. \quad (10)$$

$f_G(g)$ is the probability density function (pdf) of g . By making the change of variables $x_l = \rho g_l$, the outage gain becomes

$$\xi = \lim_{\rho \rightarrow \infty} f(\rho) \rho^{-N} \int \mathbf{1}\{(x_1, \dots, x_N) \in \mathbb{R}_+^N : \log(a) < R\} \\ \times f_{G_1}(x_1/\rho) \dots f_{G_N}(x_N/\rho) \prod_{l=1}^N dx_l. \quad (11)$$

g_l is the product of 2 independent central chi-square variates of two degrees of freedom each with respective variances σ_{1l}^2 and σ_{2l}^2 . Its density function is right continuous at zero and admits a limit, c_l , that is positive (see Appendix B). Then, the function $f_G(x/\rho)$ satisfies as $\rho \rightarrow \infty$

$$c_l \triangleq f_{G_l}(g_l \propto \frac{1}{\rho}) = \sigma_{1l}^{-2} \sigma_{2l}^{-2} \ln \rho. \quad (12)$$

we obtain

$$\xi = \lim_{\rho \rightarrow \infty} f(\rho) \rho^{-N} \prod_{l=1}^N \sigma_{1l}^{-2} \sigma_{2l}^{-2} \ln \rho \\ \times \int \mathbf{1}\{(x_1, \dots, x_N) \in \mathbb{R}_+^N : \log(a) < R\} \prod_{l=1}^N dx_l. \quad (13)$$

The integration in (14) is described by the following equation set

$$0 < x_i < 2^R - 1 - \sum_{j=1}^{i-1} x_j, \quad i = 1, \dots, N$$

Thus, after simple derivations

$$\int \mathbf{1}\{(x_1, \dots, x_N) \in \mathbb{R}_+^N : \log(a) < R\} \prod_{l=1}^N dx_l = \frac{(2^R - 1)^N}{N!} \quad (14)$$

We define $f(\rho) = \frac{\rho^N}{(\ln \rho)^N}$. With this, the outage gain of a (1, N, 1) network operating under the RF protocol is given as follows

$$\xi_{1,N,1} = \frac{(2^R - 1)^N}{N!} \prod_{l=1}^N \sigma_{1l}^{-2} \sigma_{2l}^{-2} \quad (15)$$

$f(\rho)^{-1}$ gives us the slope of the outage probability curve and ξ is a scaling factor. The outage gain factor ξ depends on the number of relays N and the channels variances. It decreases as N increases.

B. Orthogonal RF in the (1, N, 1) Network

We consider the following scenario in which the relays operate individually. At each time slot, only one relay, chosen arbitrarily, relays its received signal after rotating it. All relays use the same amount of power to retransmit their received signals. In each time slot, the destination attempts to decode the received signal. This system is in outage if each two-hop source-relay-destination link is in outage. The links are independent. Therefore, the outage probability of this scheme is the product of the outage probabilities of each link. This outage probability can be written in the following way

$$P_o(R, \sigma_1, \sigma_2, \rho) = \left[\frac{(2^R - 1)(1 - 2\gamma - \ln(2^R - 1) + \ln(\sigma_1^2 \sigma_2^2) + \ln(\rho))}{\sigma_1^2 \sigma_2^2 \rho} \right]^N \quad (16)$$

σ_1^2 and σ_2^2 are the channel variances of the first and the second hops respectively.

For this scheme, we define $f(\rho) = \frac{\rho^N}{(\ln \rho)^N}$. The outage gain is then

$$\xi_{1,N,1/orth} = \frac{(2^R - 1)^N}{\sigma_1^{2N} \sigma_2^{2N}} = (2^R - 1)^N \prod_{l=1}^N \sigma_{1l}^{-2} \sigma_{2l}^{-2} \quad (17)$$

Both the RF and the orthogonal RF schemes achieve the same diversity order in the (1, N, 1) networks since we have defined the same function $f(\rho)$ in the two cases. However, the RF scheme has a smaller outage gain and then it performs better.

As another settings, we can require the destination to combine the received messages via the N relays and then to attempt to decode the message. In this case, the scheme has the same diversity order as before, *i.e.*, we define the same function $f(\rho)$. The outage gain values are smaller than those obtained for the orthogonal RF. The RF outperforms these two schemes in the (1, N, 1) networks.

C. The Two-Hop (N, 1, N) Network

In this part, we consider the two-hop (N, 1, N) channel operating under the RF protocol. Let $\mathbf{H}_1 = [h_{11} \dots h_{1N}]$ and $\mathbf{H}_2 = [h_{21} \dots h_{2N}]^T$ denote the channel matrices of the first and the second hops respectively. $\mathbf{F} = e^{i\theta}$ is a scalar. $\mathbf{H} = \mathbf{H}_2 \mathbf{F} \mathbf{H}_1$ is the equivalent channel matrix. The mutual information of the channel can be written as

$$I = \log \det(\mathbf{I} + \rho \mathbf{H} \mathbf{H}^\dagger) = \log(1 + \rho x_1 x_2) \quad (18)$$

where $x_1 = g_{11} + \dots + g_{1N}$ and $x_2 = g_{21} + \dots + g_{2N}$, $g_{ij} = |h_{ij}|^2$. x_1 and x_2 are two central chi-square variates with $k_1 = k_2 = 2N$ degrees of freedom, *i.e.* $x_1 \sim \mathcal{X}_{2N}^2$ and $x_2 \sim \mathcal{X}_{2N}^2$. Consider $w = x_1 x_2$ and its probability density function $f_W(w)$, the outage gain becomes

$$\xi = \lim_{\rho \rightarrow \infty} f(\rho) \int \mathbf{1}\{w \in \mathbb{R}_+ : I(w) < R\} f_W(w) dw \\ = \lim_{\rho \rightarrow \infty} f(\rho) \int \mathbf{1}\left\{w < \frac{2^R - 1}{\rho}\right\} f_W(w) dw. \quad (19)$$

Resolving the integral, we get a general expression for the outage probability in a (N, 1, N) network operating under the RF protocol

$$P_o(R, \sigma_1, \sigma_2, \rho) = \frac{(2^R - 1)^N (1 - 2N\gamma - N \ln(2^R - 1) + N \ln(\sigma_1^2 \sigma_2^2) + N \ln(\rho))}{(N!)^2 \sigma_1^{2N} \sigma_2^{2N} \rho^N} \quad (20)$$

γ is the Euler's constant, σ_1^2 and σ_2^2 are the channel variances of the first and the second hops respectively (see Appendix A for more details).

We define $f(\rho) = \frac{\rho^N}{\ln \rho}$. With this, the outage gain of a (N, 1, N) network operating under the RF protocol is

$$\xi_{N,1,N} = \frac{N(2^R - 1)^N}{(N!)^2 \sigma_1^{2N} \sigma_2^{2N}}. \quad (21)$$

D. The Two-Hop (1, N, 1) Network with Noise at The Relays

In this section, $z_R \sim \mathcal{CN}(0, \mathbf{I}_N)$ and $z_D \sim \mathcal{CN}(0, 1)$ are the AWGN at the relays and the destination, respectively. $\mathbf{H} = \mathbf{H}_2 \mathbf{F} \mathbf{H}_1$ is the equivalent channel matrix, and $z_t = \mathbf{H}_2 \mathbf{F} z_R + z_D$ is the equivalent additive noise at the destination. Let $\mathbf{H}_n = \mathbf{H}_2 \mathbf{F}$. The mutual information can be written as

$$I_{mean} = \mathbb{E}_\theta \left\{ \log \det \left\{ \mathbf{I} + \rho \left(\mathbf{I} + \mathbf{H}_n \mathbf{H}_n^\dagger \right)^{-1} \mathbf{H} \mathbf{H}^\dagger \right\} \right\} \quad (22)$$

Using (8), the mutual information depends only on the constant term and we can write

$$I_{mean} = \log \left(1 + \rho \left(1 + \sum_{l=1}^N g_{2l} \right)^{-1} \sum_{l=1}^N g_{1l} g_{2l} \right) \quad (23)$$

$g_{ij} = |h_{ij}|^2$ are the channels power gains. Let $x_l = \rho g_{1l} g_{2l}$. The outage gain becomes

$$\xi = \lim_{\rho \rightarrow \infty} f(\rho) \rho^{-N} \int \mathbf{1} \{x_l, g_{2l} \in \mathbb{R}_+ : I_{mean} < R\} 24 \\ \times \prod_{l=1}^N f_{G_l}(x_l/\rho) f(g_{2l}) \prod_{l=1}^N dx_l dg_{2l} \quad (25)$$

$f_G(x/\rho)$ satisfy (12). We obtain

$$\xi = \lim_{\rho \rightarrow \infty} f(\rho) \rho^{-N} \prod_{l=1}^N \sigma_{1l}^{-2} \sigma_{2l}^{-2} \ln \rho \times I_c \quad (26)$$

I_c is the following integration

$$I_c = \int \mathbf{1} \{x_l, g_{2l} \in \mathbb{R}_+ : I_{mean} < R\} \prod_{l=1}^N f(g_{2l}) \prod_{l=1}^N dx_l dg_{2l} \quad (27)$$

We define $f(\rho) = \frac{\rho^N}{(\ln \rho)^N}$. The outage gain is given as follows

$$\xi_{1,N,1/noise} = I_c \times \prod_{l=1}^N \sigma_{1l}^{-2} \sigma_{2l}^{-2} \quad (28)$$

The function $f(\rho)$ being the same, the noise at the relays does not affect the diversity order of the RF scheme. I_c is

complicated to solve, we have to delineate the domain of variation of all the variables. Intuitively

$$\xi_{1,N,1/noise} > \xi_{1,N,1} \quad (29)$$

VI. NUMERICAL RESULTS

We proceeded to verify the outage gain expression given in (15) and (21). We plot the simulation results and the theoretical values. We can notice a very good fit between the outage probability approximation $P_o \approx \xi f(\rho)$ and the numerical results. The expression in (15) is verified in the (1, 2, 1) and (1, 4, 1) networks (Fig.1, Fig.2). The expression in (21) is also verified for $N = 2, 3, 4$ (Fig.3). Notice that the outage gain expressions, found for a signal to noise ratio $\rho \rightarrow \infty$, hold for moderate values of ρ . In Fig.4, with noise on the relays, we have 2dB and 3dB loss at 10^{-3} for $N = 2, 3$ respectively.

VII. CONCLUSION

The rotate-and-forward protocol performance is studied in terms of outage gain in some network configurations in the high SNR regime. We extract the expressions of the outage gain factors for these networks and verify them using simulation results. The numerical results show the accuracy of this criterion at moderate SNR as well. Some future work directions are to extract exact outage gain values for noisy relays.

APPENDIX

A. Distribution of the product of two central independent chi-square variates

Let $x_1 = \sum_{i=1}^{k_1} x_{1i}^2$ and $x_2 = \sum_{i=1}^{k_2} x_{2i}^2$ be two central independent chi-square variates of k_1 and k_2 degrees of freedom respectively. x_{1i}^2 and x_{2i}^2 are independent normal variates with zero means and deviations $\sigma_1^2/2$ and $\sigma_2^2/2$ respectively.

Theorem 1: Consider the product of two independent variates $w = x_1 x_2$, where x_1 is a central chi-square variate with k_1 degrees of freedom and x_2 is a central chi-square variate with k_2 degrees of freedom, then the density function of w is [13]

$$f_W(w) = \frac{4}{\sigma_1^2 \sigma_2^2} \frac{\left(\frac{4w}{\sigma_1^2 \sigma_2^2} \right)^{\frac{1}{4}k_1 + \frac{1}{4}k_2 - 1} K_{\frac{1}{2}k_1 - \frac{1}{2}k_2} \left(\sqrt{\frac{4w}{\sigma_1^2 \sigma_2^2}} \right)}{2^{\frac{1}{2}k_1 + \frac{1}{2}k_2 - 1} \Gamma(\frac{1}{2}k_1) \Gamma(\frac{1}{2}k_2)} \quad (30)$$

$K_\nu(w)$ is the modified Bessel function of the second kind. Using (30) in (19) and making the development in Taylor series as $\rho \rightarrow \infty$, we get the expression (20) of the outage probability of the (N, 1, N) network operating under the RF protocol.

B. The value of c_l

g_l is the product of two independent central χ^2 -variates of 2 degrees of freedom each with variances σ_{1l}^2 and σ_{2l}^2 . Using (30) with $k_1 = k_2 = 2$, the density function of g_l is

$$f_{G_l}(g_l) = \frac{2}{\sigma_{1l}^2 \sigma_{2l}^2} K_0 \left(\sqrt{\frac{4g_l}{\sigma_{1l}^2 \sigma_{2l}^2}} \right) \quad (31)$$

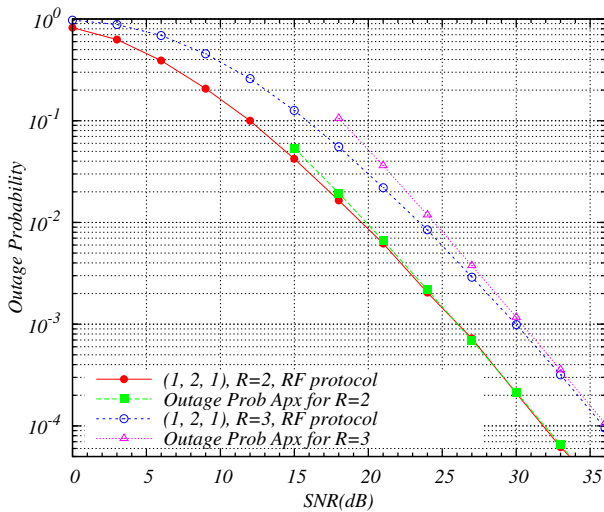


Fig. 1. Experimental and theoretical outage probability in the (1, 2, 1) network operating under the RF protocol for R=2 and R=3, $\sigma_{1l}^2 = \sigma_{2l}^2 = 1$.

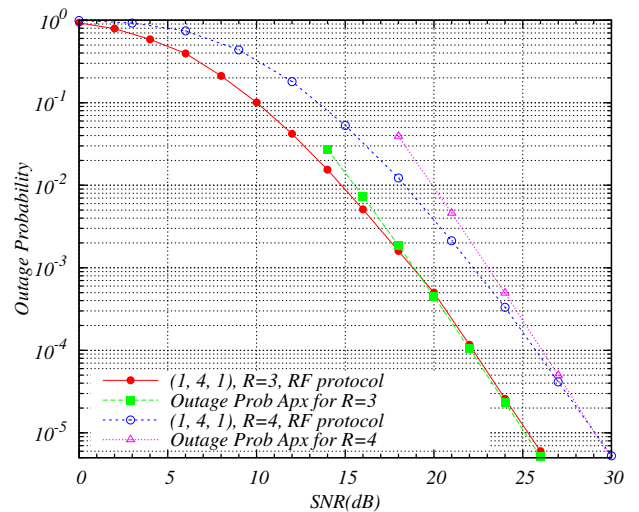


Fig. 2. Experimental and theoretical outage probability in the (1, 4, 1) network operating under the RF protocol for R=3 and R=4, $\sigma_{1l}^2 = \sigma_{2l}^2 = 1$.

K_0 is the zeroth order Bessel function of the second kind. Using the development of the function K_0 in [[14], page 909], we obtain the value of (31) as $\rho \rightarrow \infty$:

$$f_{G_l} \left(g_l \propto \frac{1}{\rho} \right) = \frac{1}{\sigma_{1l}^2 \sigma_{2l}^2} \ln \rho \quad (32)$$

REFERENCES

- [1] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. W. C.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.
- [2] —, "Cooperative diversity in wireless relay networks with multiple-antenna nodes," ISIT 2005, Adelaide, SA, p.p. 815-819, Sept 2005.
- [3] L. Zheng and D. Tse, "Diversity and Multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1073–1096, May 2003.
- [4] S. Yang and J.-C. Belfiore, "Diversity of MIMO Multihop Relay Channels," [Online]: <http://arxiv.org/abs/0708.0386>
- [5] K. Sreeram, S. Birenjith, and P. V. Kumar, "DMT of Multi-hop Cooperative Networks - Part I: Basic Results," August 2008, submitted, [Online]: <http://arxiv.org/abs/0808.0234>
- [6] K. Sreeram, S. Birenjith, and P. V. Kumar, "DMT of Multi-hop Cooperative Networks - Part II: Half-Duplex Networks with Full-Duplex Performance," August 2008, submitted, [Online]: <http://arxiv.org/abs/0808.0235>
- [7] S. O. Gharan, A. Bayesteh, and A. K. Khandani, "On the diversity-multiplexing tradeoff in multiple relay networks," *IEEE Trans. Inform. Theory*, vol. 55, pp. 5423–5444, December 2009.
- [8] S. Avestimehr, S. Diggavi and D. N. C. Tse, "Wireless network information flow: a deterministic approach," *IEEE Trans. Inf Theory*, Jun 2009, submitted, [Online]: <http://arxiv.org/abs/0906.5394>
- [9] S. Yang and J.-C. Belfiore, "Distributed Rotation Recovers Spatial Diversity," ISIT 2010, Austin, Texas, p.p. 2158-2162, June 2010
- [10] R. Pedarsani, O. Lévêque and S. Yang, "On the DMT Optimality of the Rotate-and-Forward Scheme in a Two-Hop MIMO Relay Channel," to appear in the Proceedings of Allerton Conf. on Com., October 2010
- [11] W. Hachem, P. Bianchi and Ph. Ciblat, "Outage Probability Based Power and Time Optimization for Relay Networks," *IEEE Trans. on Sig. Process.*, vol. 57, no. 2, pp. 764-782, Feb. 2009.
- [12] M. Nahas, A. Saadani and W. Hachem "Performance of Asynchronous Two-Relay Two-Hop Wireless Cooperative Networks," *IEEE Trans. on Wire. Com.*, vol. 9, no. 3, pp. 1086-1096, March 2010.
- [13] W. T. Wells, R. L. Anderson and John W. Cell "The Distribution of the Product of Two Central of Non-Central Chi-Square Variates," Jan. 1962.
- [14] L. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*, 6th ed. Alan Jeffrey and Daniel Zwillinger, July 2000.

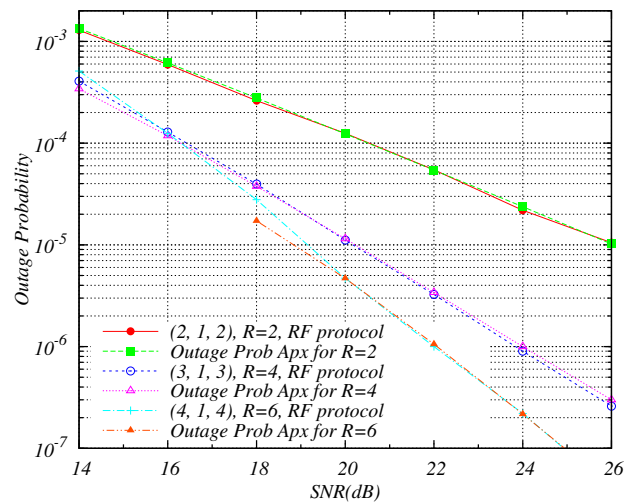


Fig. 3. Experimental and theoretical outage probability in the (N, 1, N) networks operating under the RF protocol, $\sigma_1^2 = \sigma_2^2 = 2$.

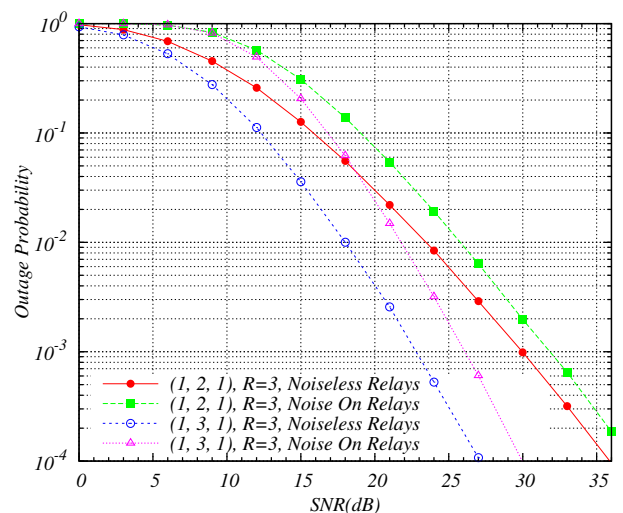


Fig. 4. Outage probability in the (1, 2, 1) and (1, 3, 1) networks with noiseless and noisy relays for R=3, $\sigma_{1l}^2 = \sigma_{2l}^2 = 1$.