Two-Hop Relay Channels with Limited FeedBack

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***Abstract*— In this paper, we consider the layered two-hop relay channel, operating under the linear rotate-and-forward (RF) scheme [1], where the relays rotate the received signal before its retransmission. In this work, a limited feedback channel is considered between the relays and the destination. The destination finds the optimal rotation vector and feeds it back to the relays. We propose an iterative algorithm to find this optimal rotation vector. The proposed algorithm is shown to have optimal performance with low complexity.**

I. Introduction

Cooperative Diversity techniques have recently received a great interest as a way to improve both the throughput and the reliability in wireless networks.

Cooperative diversity was first studied in [2] for the single antenna case and then extended in [3] to the multi-antenna case, with distributed space-time coding. The proposed cooperative protocols were compared in terms of diversity-multiplexing-tradeoff (DMT). The DMT was originally introduced by Zheng and Tse in [4] as a mean of evaluating the point-to-point multi-antenna (MIMO) schemes in slow fading scenarios at high SNR. The cut-set bound is an upper bound on the achievable DMT for a given network. An essential question is whether a cooperative protocol can achieve the cut-set bound for all the range of multiplexing gain.

In this paper, we are interested in the multi-hop relay networks where the direct source-destination link is absent. In [5], Yang and Belfiore have considered the MIMO multi-hop networks operating under the Amplify-and-Forward (AF) protocol. They have proved that the AF scheme is equivalent, in the DMT sense, to the rayleigh product (RP) channel and give an exact characterization of the DMT of the AF scheme in multi-hop channel of arbitrary size. While the AF scheme achieves the maximum multiplexing gain, it fails in achieving the maximum diversity gain in all the multi-hop networks. The Flip-and-Forward (FF) protocol, also proposed in [5], allows to achieve both the maximum diversity and multiplexing gains in these multi-hop networks.

Gharan, Bayesteh, and Khandani in [6] have proposed the multiplication by a random unitary matrix at the relay nodes. This multiplication used in a Random-Sequential (RS) scheme, proposed in [7], allows to achieve the optimal DMT in multi-antenna multi-hop networks consisting of a single-source, a single-destination, and full-duplex relays with exactly one relay in each hop.

Avestimehr, Diggavi and Tse have proposed a non-linear scheme called Quantize-and-Forward in [8]. This scheme is shown to achieve any rate within a constant gap to the capacity of the channel and thus attains the optimal DMT of the considered channel.

Recently, Yang and Belfiore have proposed a linear relaying scheme called Rotate-and-Forward in [1]. The idea is to use time-varying distributed rotation to recover spatial diversity. This scheme is shown to achieve the optimal DMT in layered two-hop relay channel in the two-relay case. In this scheme, the relays are independent and act in a distributed fashion. On the contrary, in the RS scheme, full cooperation is needed where the restriction to one relay in each layer.

In this work, we consider the layered two-hop relay channel with multi-antenna source-destination pair and multiple full-duplex relays. We assume that a limited feedback channel exists between the destination and relays. Based on the RF scheme, we propose an algorithm with low complexity to find the optimal rotation by the destination and feed it back to the relays. This algorithm is shown to have the same performance as an optimal exhaustive algorithm. The rest of this paper is organized as follows. The system model and some basic assumptions are presented is section II. A general idea about the RF scheme is given in section III. Section IV contains the main contribution of this work where the algorithm is exposed. We give some numerical results in section V and conclude in section VI.

II. System Model and Assumptions

Regarding the notations, we use boldface lower case letters $\boldsymbol{v}$ to denote vectors and boldface capital letters $\boldsymbol{M}$ to denote matrices. $\mathcal{CN}(\mu, \sigma^2)$ represents a complex Gaussian random variable with mean μ and variance σ^2. $[\cdot]^T$ and $[\cdot]^\dagger$ respectively denote the matrix transposition and the matrix conjugated transposition operation. $\mathbb{E}[\cdot]$ stands for the expectation operator. $|\cdot|$ is the scalar norm. $\log(\cdot)$ stands for the base-2 logarithm.

We consider a two-hop relay network composed of one source, one destination, and N full-duplex single-antenna relays. The source and the destination are equipped with n_t and n_r antennas respectively. The two-hop channel is then denoted by (n_t, N, n_r). We focus on distributed relaying schemes. By this, we mean that the relays are independent and change no information between them on the message or channel state information (CSI). The CSI are only available at the receiver side, no transmitter CSI at all. The terminals are considered perfectly synchronized. Moreover, we assume that the destination knows the topology of the network, *i.e*, it knows the number of *active* relays: relays who are retransmitting their received signals.

The faded sub-channels are flat, Rayleigh-faded, and quasi-static with a coherence time much larger than N. The channel matrices are independent, and have i.i.d. zero-mean complex Gaussian entries with unit variance, *i.e.*, $h_{i,j} \sim \mathcal{CN}(0,1)$. We assume an additive white Gaussian noise (AWGN) at the relays and the destination.

In the numerical results, we use the outage probability as a comparison criterion. An *outage event* is the situation when the channel is so poor that no scheme could communicate reliably at a certain fixed data rate. The maximum rate of reliable communication is the mutual information, I, between the source signal and destination signal. This quantity is function of the random channel gain $\boldsymbol{H}$ and is therefore random. Now, suppose that the transmitter encodes data at a rate R bits per channel use (BPCU). The system is said to be in *outage* if $I < R$ and the outage probability is

$$p_{out}(R) = \mathsf{Prob}\{\underbrace{I < R}_{\mathcal{O}(R)}\}, \quad (1)$$

$\mathcal{O}(R)$ is the outage event.

III. The Rotate-And-Forward Protocol

The authors in [1] proposed a linear relaying strategy, *i.e.*, the relays perform a linear processing on the received signal and forward it, this scheme is called *rotate-and-forward* (RF) protocol. A sequence of rotation vectors $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_T$ is defined based on a distributed rotation sequence (DRS) where $\boldsymbol{\theta}_i \triangleq [\theta_{i,1}, \ldots, \theta_{i,N}]$ (see definition in Appendix B). A codeword $\boldsymbol{X} \in \mathbb{C}^{n_t \times T}$ is transmitted by the source over T symbol times. At instant i, each relay transmits a rotated version of what it received at instant $i-1$. The rotation used by the relay j is $\theta_{i,j}$, the jth element of the vector $\boldsymbol{\theta}_i$. Thus, we have, by ignoring the power normalization terms,

$$\boldsymbol{y}_D[i+1] = \boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{H}_1\boldsymbol{x}[i] + \boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{z}_R[i] + \boldsymbol{z}_D[i+1] \quad (2)$$

where $\boldsymbol{x} \in \mathbb{C}^{n_t\times 1}$, $\boldsymbol{x}_R \in \mathbb{C}^{N\times 1}$, $\boldsymbol{y}_R \in \mathbb{C}^{N\times 1}$, and $\boldsymbol{y}_D \in \mathbb{C}^{n_r\times 1}$ are the transmitted signal from the source, transmitted signal from the relays, received signal at the relays, and received signal at the destination, respectively; $\boldsymbol{z}_R \sim \mathcal{CN}(0, \boldsymbol{I}_N)$, and $\boldsymbol{z}_D \sim \mathcal{CN}(0, \boldsymbol{I}_{nr})$ are the AWGN at relays and destination respectively; $\boldsymbol{H}_1 \in \mathbb{C}^{N\times n_t}$ and $\boldsymbol{H}_2 \in \mathbb{C}^{n_r\times N}$ are the source-relays and relays-destination channel matrix, respectively.

Then, the transmitted codeword $\boldsymbol{X}$ goes through an equivalent time-varying fading channel and the equivalent channel of the RF scheme is a sequence $\boldsymbol{H}_2\boldsymbol{F}_1\boldsymbol{H}_1, \ldots, \boldsymbol{H}_2\boldsymbol{F}_T\boldsymbol{H}_1$ with $\boldsymbol{F}_i \triangleq \mathsf{diag}\big(e^{j\theta_{i,1}}, \ldots, e^{j\theta_{i,N}}\big)$.

The DMT of the proposed protocol depends uniquely on the time-variant equivalent channel matrix $\boldsymbol{H} \triangleq \boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{H}_1$,

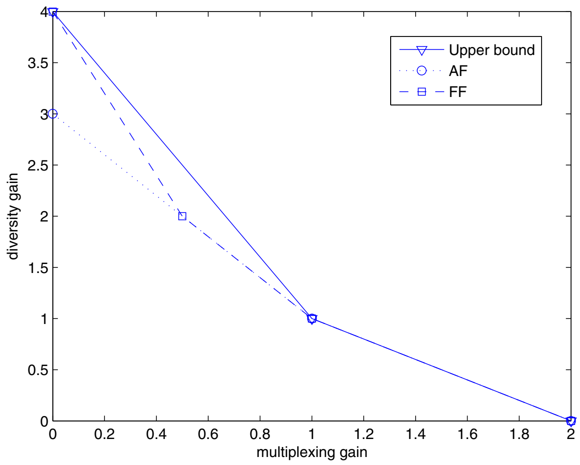

Fig. 1. Diversity-multiplexing tradeoff of (2, 2, 2) channel with different schemes.

[1]. Therefore, the mutual information for a given channel realization is

$$\frac{1}{T}\sum_{i=1}^{T} \log\det\left(\boldsymbol{I} + \mathsf{SNR}(\boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{H}_1)(\boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{H}_1)^\dagger\right) \quad (3)$$

With a large *T*, the instantaneous mutual information (3) converges to the following term

$$I_{inst} = \mathbb{E}_\theta\left\{\log\det\left(\boldsymbol{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^\dagger\right)\right\} \quad (4)$$

According to *theorem 2* in [1], the RF scheme achieves the DMT upper-bound in a N-relay two-hop layered channel with i.i.d. Rayleigh fading for $N \le 2$. This upper-bound is given by

$$d^{(RF)}(r) = (N-r)(n_{min} - r) \quad (5)$$

where $n_{min} = \min\{n_t, n_r\}$, r is the multiplexing gain and d is the diversity gain. In fig.1, we recall the DMT, $d(r)$, of the (2, 2, 2) network, (n_t=N=n_r=2), for the AF and FF schemes. The DMT upper-bound of this network is the piece-wise linear function joining the points (0,4), (1,1), and (2,0). The RF scheme achieves this upper-bound in the (2, 2, 2) network.

However, for more than two relays, $(N > 2)$, it is difficult to prove or unprove the DMT optimality of the RF scheme, [1]. In the following section, we assume a scheme with feedback channel between the destination and relays.

IV. FeedBack and Iterative Algorithm

In the RF scheme, the relays rotate their transmitted signal based on a fixed DRS, these rotation vectors are not all optimal for the channel realization. In this section, we assume that relays are provided the optimal rotation, $\boldsymbol{\theta}_{opt}$, that maximizes the mutual information. Indeed, the destination estimates the rotation vector $\boldsymbol{\theta}_{opt}$ and feeds it back to the relays using a feedback channel. The relays use the same rotation during the transmission of the codeword $\boldsymbol{X}$. Thus, the equivalent channel of the RF scheme is reduced to $\boldsymbol{H}_\theta$=$\boldsymbol{H}_2\boldsymbol{F}_{opt}\boldsymbol{H}_1$, and we omit the time index in the input-output relation (2),

$$\boldsymbol{y}_D = \boldsymbol{H}_2\boldsymbol{F}_{opt}\boldsymbol{H}_1\boldsymbol{x} + \boldsymbol{H}_2\boldsymbol{F}_{opt}\boldsymbol{z}_R + \boldsymbol{z}_D \qquad (6)$$

To this end, an exhaustive research can be done by the destination to find the optimal rotation vector $\boldsymbol{\theta}_{opt}$ by jointly searching over all possible N-tuples. The destination performs

$$\boldsymbol{\theta}_{opt} \triangleq \arg \max_{\boldsymbol{\theta}, \theta_j \in [0,2\pi)} \log\det(\boldsymbol{I} + \mathsf{SNR}\boldsymbol{H}_{\boldsymbol{\theta}}\boldsymbol{H}_{\boldsymbol{\theta}}^\dagger) \qquad (7)$$

This is an optimal exhaustive research, its cost is very heavy and grows exponentially in N. Instead, we can benefit from the separability of θ_j's in the expression of the mutual information and propose a simple iterative algorithm to find $\boldsymbol{\theta}_{opt}$ which complexity is linear in N. To see this, we must write the mutual information expression in an alternative manner.

A. Simple case: one angle

First, we will consider that only one relay is rotating its received signal by an angle θ. The remaining relays perform the AF scheme; the rotation matrix is $\boldsymbol{F} = \mathrm{diag}\big(1, \ldots, 1, e^{j\theta}\big)$. In this case, the mutual information can be written as,

$$\begin{aligned} I(\theta) = &\ \log\det(\boldsymbol{I} + \boldsymbol{W}_\theta) \\ = &\ \log\det(\boldsymbol{I}_2 + \boldsymbol{D}\boldsymbol{W}_2) + \log\{1 - \det(\boldsymbol{P}) \\ &\ + 2|P_{12}|\cos(\theta + \angle P_{12})\} \end{aligned} \qquad (8)$$

where $\boldsymbol{W}_\theta$=$\boldsymbol{H}_\theta\boldsymbol{H}_\theta^\dagger$. We omit SNR and θ's index for simplicity, the details are deferred to the Appendix A. Considering one angle, the mutual information is periodic in θ over $[0, 2\pi)$. $\boldsymbol{D}$, $\boldsymbol{W}_2$, and $\boldsymbol{P}$ are independent of θ. The destination feeds back to the relay the value of θ which maximizes (8),

$$\theta_{opt} = -\angle P_{1,2} \qquad (9)$$

B. General case: N angles

Looking at (8), we notice that we are able to separate the considered rotation angle from all the other angles. Therefore, the destination will be able to calculate each rotation angle, θ_j, like in (9). Then, for a given θ_j, $\forall j$,

$$I(\theta_j) = \log\{a_j + b_j\cos(\theta_j + \phi_j)\} \qquad (10)$$

In order to write $I(\theta_j)$ in this form, the considered angle θ_j must be at the position (N, N). Therefore, we have to permute the lines and columns of the matrices in (8).

Let us define $\boldsymbol{R}_{j,\theta}$ as a $N \times N$ diagonal matrix where $\boldsymbol{R}_{j,\theta}(j,j) = e^{j\theta}$ and $\boldsymbol{R}_{j,\theta}(k,k) = 1\ \forall\ k \neq j$.

We have the following lemma

Lemma 1

$$\det(\boldsymbol{I}+\boldsymbol{R}_{j,\theta}\boldsymbol{A}\boldsymbol{R}_{j,\theta}^\dagger\boldsymbol{B}) = f_j(\boldsymbol{A},\boldsymbol{B}) + \mathcal{R}\left\{g_j(\boldsymbol{A},\boldsymbol{B})e^{j\theta}\right\} \qquad (11)$$

where f_j and g_j are some functions of $\boldsymbol{A}$ and $\boldsymbol{B}$, $\mathcal{R}$ is the real part of a complex number.

In case $j = N$, the angle at the position (N, N), we have found that

$$f_N = (1 - \det(\boldsymbol{P}))\det(\boldsymbol{I} + \boldsymbol{D}\boldsymbol{W}_2) \qquad (12)$$

$$g_N = 2\det(\boldsymbol{I} + \boldsymbol{D}\boldsymbol{W}_2)P_{1,2} \qquad (13)$$

Now, when all relays are performing the RF scheme, the mutual information is function of the vector $\boldsymbol{\theta}$, we can write

$$\begin{aligned} I(\boldsymbol{\theta}) &= \log\det(\boldsymbol{I} + \boldsymbol{F}\boldsymbol{W}_1\boldsymbol{F}^\dagger\boldsymbol{W}_2) \\ &= \log\det(\boldsymbol{I} + \prod_{j=1}^{N}\boldsymbol{R}_{j,\theta_j}\boldsymbol{W}_1\prod_{j=1}^{N}\boldsymbol{R}_{j,\theta_j}^\dagger\boldsymbol{W}_2) \end{aligned} \qquad (14)$$

Let us define $\boldsymbol{P}_{j,N}$ the unitary matrix (*i.e.* $\boldsymbol{P}\boldsymbol{P}^\dagger = \boldsymbol{I}$) where $\boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^\dagger$ is similar to the permutation of the columns j and N of the matrix $\boldsymbol{A}$. For a given θ_j at a position (j, j) (θ_j, $j \neq i$ are considered as constant), the expression of f_j and g_j can be obtained as follows

$$\begin{aligned} \mathcal{A}(\boldsymbol{\theta}) &= \det(\boldsymbol{I} + \boldsymbol{R}_{j,\theta_j}\boldsymbol{W}_{1,j}\boldsymbol{R}_{j,\theta_j}^\dagger\boldsymbol{W}_{2,j}) \\ &= \det(\boldsymbol{I} + \boldsymbol{P}_{j,N}\boldsymbol{R}_{j,\theta_j}\boldsymbol{W}_{1,j}\boldsymbol{R}_{j,\theta_j}^\dagger\boldsymbol{W}_{2,j}\boldsymbol{P}_{j,N}^\dagger) \\ &= \det(\boldsymbol{I} + \boldsymbol{R}_{N,\theta_j}\overline{\boldsymbol{W}}_{1,j}\boldsymbol{R}_{N,\theta_j}^\dagger\overline{\boldsymbol{W}}_{2,j}) \\ &= f_N(\overline{\boldsymbol{W}}_{1,j}, \overline{\boldsymbol{W}}_{2,j}) + \mathcal{R}\left\{g_N(\overline{\boldsymbol{W}}_{1,j}, \overline{\boldsymbol{W}}_{2,j})e^{j\theta_j}\right\} \end{aligned}$$

(15) is obtained from *lemma 1*,

$$\boldsymbol{W}_{1,j} \triangleq \prod_{k=1,k\neq j}^{N}\boldsymbol{R}_{k,\theta_k}\boldsymbol{W}_1 \qquad (15)$$

$$\overline{\boldsymbol{W}}_{h,j} = \boldsymbol{P}_{j,N}\boldsymbol{W}_{h,j}\boldsymbol{P}_{j,N}^\dagger, \qquad h = 1, 2 \qquad (16)$$

$$\boldsymbol{R}_{N,\theta_j} = \boldsymbol{P}_{j,N}\boldsymbol{R}_{j,\theta_j}\boldsymbol{P}_{j,N}^\dagger, \qquad \forall j \qquad (17)$$

and,

$$f_j(\boldsymbol{W}_{1,j}, \boldsymbol{W}_{2,j}) = f_N(\overline{\boldsymbol{W}}_{1,j}, \overline{\boldsymbol{W}}_{2,j}) \qquad (18)$$

$$g_j(\boldsymbol{W}_{1,j}, \boldsymbol{W}_{2,j}) = g_N(\overline{\boldsymbol{W}}_{1,j}, \overline{\boldsymbol{W}}_{2,j}) \qquad (19)$$

For a given angle θ_j, f_j and g_j are function of all θ_k, $k \neq j$.

Thanks to this separability, the destination can calculate each θ_j and feed it back to the corresponding relay.

C. Iterative Algorithm

The destination knows the topology of the network, hence the number of angles θ_j to calculate. In order to evaluate the performance of the proposed algorithm, we compare its performance to an optimal algorithm where the destination search exhaustively the optimal vector $\boldsymbol{\theta}_{opt} \triangleq [\theta_1, \ldots, \theta_N]$ that maximizes the mutual information over all possible N-tuples.

We propose to calculate the optimal vector $\boldsymbol{\theta}_{opt}$ in an iterative manner. Starting with an initial rotation vector chosen randomly, $\boldsymbol{\theta}_{init}$, the destination calculates f_j, g_j, ϕ_j and the corresponding θ_j, $j = 1, \ldots N$. Then, the destination uses the calculated $\boldsymbol{\theta}$ as an initial vector and recalculates $\boldsymbol{\theta}_{opt}$. The destination repeats this procedure M times. The iterative algorithm consists of the following steps:

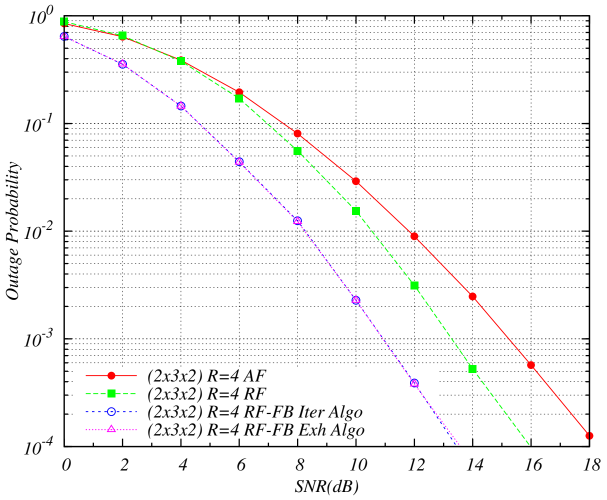

Fig. 2. Outage probability of the (2, 3, 2) channel under AF, RF, and RF with FeedBack schemes.

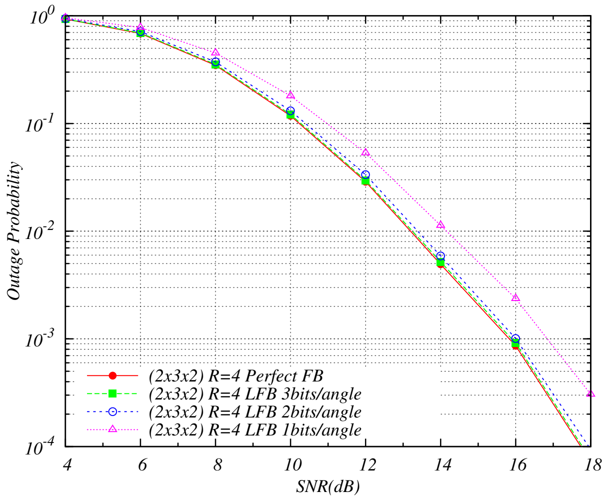

Fig. 3. Outage probability of the (2, 3, 2) channel under limited FeedBack.

step 1: choose an initial rotation vector $\boldsymbol{\theta}_{init}$ arbitrarily

step 2: calculate

$$\begin{aligned} f_1, g_1, \phi_1, \theta_1^{opt} &= -\phi_1(\theta_2, \ldots, \theta_N) \\ &\vdots \\ f_j, g_j, \phi_j, \theta_j^{opt} &= -\phi_j(\theta_1^{opt}, \ldots, \theta_{j-1}^{opt}, \theta_{j+1}, \ldots, \theta_N) \\ &\vdots \\ f_N, g_N, \phi_N, \theta_N^{opt} &= -\phi_N(\theta_2^{opt}, \ldots, \theta_{N-1}^{opt}) \end{aligned}$$

step 3: back to step 2 where $\boldsymbol{\theta}_{init} = \boldsymbol{\theta}_{opt}$

The vector $\boldsymbol{\theta}^{opt}$ found in the first iteration is used for a second iteration. Experimentally, the number of iterations needed to get an accurate $\boldsymbol{\theta}^{opt}$ is less than 5. This is interesting when we consider a perfect feedback channel. However, when the feedback is limited, which is the case in real systems, too many iterations don't have any added-value. In the next section, we present the simulation results.

V. Simulation Results

For the simulation results, we adopt the outage probability as a performance criterion. In Fig.2 and Fig.3, we show the performance of the (2, 3, 2) network under different schemes. In Fig.2, it is clear that the RF scheme outperforms the AF scheme. In feedback case, we compare the outage probability of the exhaustive algorithm and that of the proposed iterative algorithm. We can see that both algorithms have the same performance. The feedback allows us to have 2.3dB gain over the RF scheme at a target outage probability of 10^{-3} for a rate $R = 4$ BPCU. Indeed, by averaging over $\boldsymbol{\theta}$ in RF scheme, the second term in $\mathcal{A}(\boldsymbol{\theta})$ goes to zero, while in case of feedback, this term is maximized.

Now, if we consider that the feedback channel is limited, the destination is forced to use a limited number of bits to send back the optimal rotation angles to the relays, let us say B bits/angle (bpa). In this case, the interval $[0, 2\pi)$ is divided into 2^B levels. The source quantizes each angle θ_j over the 2^B levels and sends back the level index to the corresponding relay. Fig.3 shows the gain loss due to limitation on feedback for $B = 2, 1$(bpa); 0.2, 1dB respectively at an outage probability 10^{-3} and rate $R = 4$. It is worth noting that $B = 1$bpa corresponds to the case ON/OFF, *i.e.*, either rotating the signal of π before retransmission or not, the decision here depends on the channel conditions while in the FF sheme, proposed in [5], the rotation was independent of the channel which is not optimal. Based on the observed performance, we can conclude that the number of iterations in the proposed algorithm may not be so big, and this is the case in the simulations where we did three iterations for each vector $\boldsymbol{\theta}_{opt}$.

In the (2, 4, 2) network, we have the same observation, Fig.4 and Fig.5. The exhaustive algorithm and the proposed iterative algorithm have the same performance in terms of outage probability. The feedback allows us to have 3.2dB gain over the RF scheme at a target outage probability of 10^{-3} for a rate $R = 6$ BPCU. Again, when the feedback channel is limited, Fig.5, we have a gain loss of order 0.2, 1dB for $B = 2, 1$(bpa) respectively at 10^{-3}.

We notice that in the simulation result with limited feedback, we have considered the amplified noise in the mutual information expression where $\boldsymbol{F} = \boldsymbol{F}_{opt}$,

$$I = \log\det\left\{\boldsymbol{I} + \mathsf{SNR}(\boldsymbol{I} + \boldsymbol{F}^{\dagger}\boldsymbol{W}_2\boldsymbol{F})^{-1}\boldsymbol{F}^{\dagger}\boldsymbol{W}_2\boldsymbol{F}\boldsymbol{W}_1\right\}$$

VI. Conclusion

We have proposed an iterative algorithm, based on the RF scheme, to calculate the optimal rotation vector for the two-hop relay channel with limited feedback from the destination to the relays. We have showed that this algorithm offers the same performance in terms of outage probability as an optimal exhaustive algorithm. The complexity of the iterative algorithm grows linearly with the number of relays while it grows exponentially in the exhaustive case.

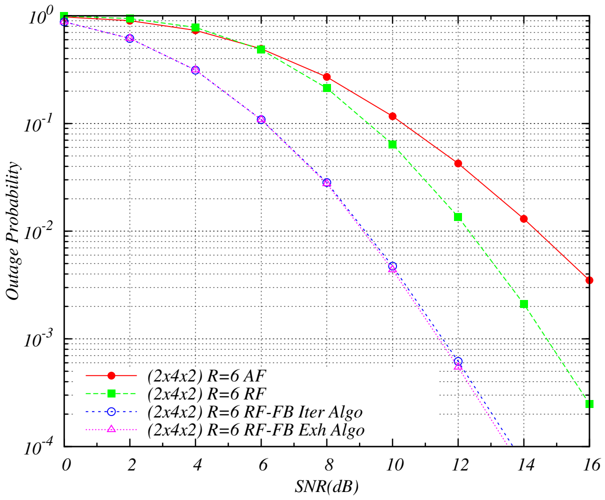

Fig. 4. Outage probability of the (2, 4, 2) channel under AF, RF, and RF with FeedBack schemes.

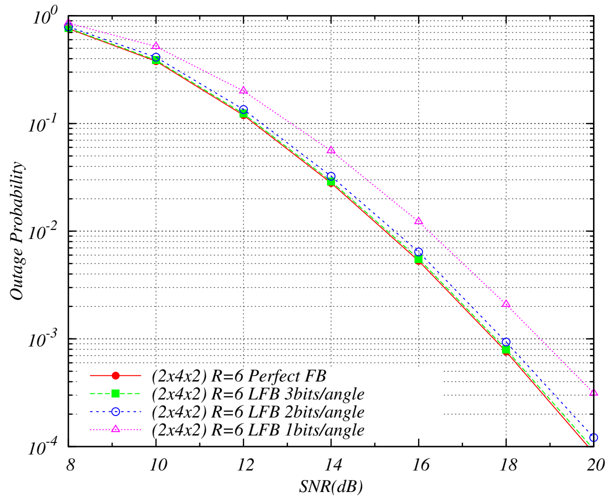

Fig. 5. Outage probability of the (2, 4, 2) channel under limited FeedBack.

APPENDIX

A. Proof of equation (8)

We detail here how we extract the equation (8) in the section IV-A. In the following, we omit θ's indices and SNR for simplicity. We define, $\boldsymbol{H}_\theta$, $\boldsymbol{W}_\theta$, $\boldsymbol{W}_1$, and $\boldsymbol{W}_2$ as,

$$\boldsymbol{H}_\theta \triangleq \boldsymbol{H}_2\boldsymbol{F}\boldsymbol{H}_1 = \boldsymbol{H}_2 \begin{bmatrix} \boldsymbol{I}_{N-1} & 0 \\ 0 & e^{j\theta} \end{bmatrix} \boldsymbol{H}_1 \qquad (20)$$

$$\boldsymbol{W}_\theta \triangleq \boldsymbol{H}_\theta\boldsymbol{H}_\theta^\dagger, \qquad \boldsymbol{W}_1 \triangleq \boldsymbol{H}_1\boldsymbol{H}_1^\dagger, \qquad \boldsymbol{W}_2 \triangleq \boldsymbol{H}_2^\dagger\boldsymbol{H}_2$$

And we decompose $\boldsymbol{W}_1$ as

$$\boldsymbol{W}_1 \triangleq \boldsymbol{D} + \boldsymbol{A} = \begin{bmatrix} \boldsymbol{W}_{11} & 0 \\ 0 & w_{12} \end{bmatrix} + \begin{bmatrix} 0 & \boldsymbol{w}_{1,c} \\ \boldsymbol{w}_{1,c}^\dagger & 0 \end{bmatrix} \qquad (21)$$

Then, we can write

$$\begin{aligned} \boldsymbol{W}_\theta &= \boldsymbol{H}_2\boldsymbol{D}\boldsymbol{H}_2^\dagger + \boldsymbol{H}_2\boldsymbol{F}\boldsymbol{A}\boldsymbol{F}^\dagger\boldsymbol{H}_2^\dagger \\ &= \boldsymbol{H}_2\boldsymbol{D}\boldsymbol{H}_2^\dagger + \boldsymbol{H}_2\boldsymbol{\Delta}\boldsymbol{R}_\theta\boldsymbol{\Delta}^\dagger\boldsymbol{H}_2^\dagger \end{aligned} \qquad (22)$$

where

$$\boldsymbol{\Delta} \triangleq \begin{bmatrix} \boldsymbol{w}_{1,c} & 0 \\ 0 & 1 \end{bmatrix}, \qquad \boldsymbol{R}_\theta \triangleq \begin{bmatrix} 0 & e^{-j\theta} \\ e^{j\theta} & 0 \end{bmatrix}$$

Let

$$\begin{aligned} \mathcal{D} &\triangleq \det(\boldsymbol{I}_2 + \boldsymbol{H}_2\boldsymbol{\Delta}\boldsymbol{R}_\theta\boldsymbol{\Delta}^\dagger\boldsymbol{H}_2^\dagger(\boldsymbol{I} + \boldsymbol{H}_2\boldsymbol{D}\boldsymbol{H}_2^\dagger)^{-1}) \\ &= \det(\boldsymbol{I}_2 + \boldsymbol{R}_\theta\boldsymbol{\Delta}^\dagger\boldsymbol{H}_2^\dagger(\boldsymbol{I} + \boldsymbol{H}_2\boldsymbol{D}\boldsymbol{H}_2^\dagger)^{-1}\boldsymbol{H}_2\boldsymbol{\Delta}) \\ &= \det(\boldsymbol{I}_2 + \boldsymbol{R}_\theta\boldsymbol{\Delta}^\dagger\boldsymbol{W}_2\left(\boldsymbol{I} + \boldsymbol{D}\boldsymbol{W}_2\right)^{-1}\boldsymbol{\Delta}) \\ &= 1 + \det\left(\boldsymbol{R}_\theta\right)\det(\boldsymbol{\Delta}^\dagger\boldsymbol{Q}\boldsymbol{\Delta}) + \mathrm{Tr}(\boldsymbol{R}_\theta\boldsymbol{\Delta}^\dagger\boldsymbol{Q}\boldsymbol{\Delta}) \\ &= 1 - \det\left(\boldsymbol{P}\right) + \mathrm{Tr}\left(\boldsymbol{R}_\theta\boldsymbol{P}\right) \\ &= 1 - \det\left(\boldsymbol{P}\right) + 2\mathcal{R}\left\{P_{12}e^{j\theta}\right\} \\ &= 1 - \det\left(\boldsymbol{P}\right) + 2|P_{12}|\cos\left(\theta + \angle P_{12}\right) \end{aligned} \qquad (23)$$

where

$$\boldsymbol{Q} \triangleq \boldsymbol{W}_2\left(\boldsymbol{I} + \boldsymbol{D}\boldsymbol{W}_2\right)^{-1} \text{ and } \boldsymbol{P} \triangleq \boldsymbol{\Delta}^\dagger\boldsymbol{Q}\boldsymbol{\Delta} \in \mathbb{C}^{2\times 2}$$

Considering one rotation, the expression of $\mathcal{D}$ is periodic in θ over $[0, 2\pi)$. The destination feeds back to the relay the value of θ which maximizes $\mathcal{D}$, *i.e.* this θ maximizes the mutual information of the channel operating under the RS protocol.

$$\theta_{opt} = -\angle P_{1,2}$$

B. Definition of a DRS

Define a set of L equally spaced angles in $[0, 2\pi)$ and the corresponding set of complex rotations as follows, [1],

$$\mathcal{A}_L \triangleq \left\{0, \frac{2\pi}{L}, \ldots, \frac{2(L-1)\pi}{L}\right\} \qquad (24)$$

$$\mathcal{R}_L \triangleq \{e^{j\theta} | \theta \in \mathcal{A}_L\} \qquad (25)$$

A sequence of diagonal matrix Δ_t, $t = 1, \ldots, L^N$ is said to be a distributed rotation sequence (DRS) if
1)$\Delta_t = \mathrm{diag}\{\xi_t\}$ with $\xi_t \in \mathcal{R}_L^{N\times 1}$,
2)$\Delta_t \neq \Delta_t'$, $\forall t \neq t'$.

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