



Coordination and partial decoupling in tracking control for wheeled mobile manipulators

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Abstract

We consider tracking control for mobile manipulators built from a wheeled mobile platform and a serial chain robotic arm. Two approaches are presented: the first one is based on global instantaneous kinematics for the whole system whereas the second one follows a decomposition approach that requires trajectory planning for the platform. Since the platform is nonholonomic, different issues have to be carefully studied.

Keywords: mobile manipulation, kinematics, instantaneous kinematics, nonholonomy.

1 Introduction

In this paper, we consider mobile manipulators built from a wheeled mobile platform and a serial chain robotic arm. These systems combine manipulation and mobility capabilities. So the majority of the tasks they are dedicated to require a certain level of coordination of the robotic arm and the platform. This is a specificity of mobile manipulation. In addition, in most tasks of manipulation, the user has to control the location (position and orientation) of the tool or the grip of his robot – named as the end-effector and denoted by EE from now on. This is also the case in mobile manipulation.

Recently some contributions concerning modelling and control of generic nonholonomic mobile manipulators ([12],[2, 1]) have been proposed. Based on these proposals, it is now possible to consider modelling and control of mobile manipulators on a unified basis and a comparison can be made with classical manipulation. As for any compound system, a mobile manipulator can be seen as two subsystems cooperating or as a whole. In the first case, setpoints are defined for each subsystem from the tasks at hand whereas, in the second one, the setpoint is given directly to the whole system. Of course, redundancy plays an important role here and needs to be carefully defined.

We propose to compare both approaches at kinematic level on a tracking task for which the end-effector trajectory is

imposed. This is illustrated on a planar example. In fact, we have developed:

- on one side, a generic formulation that is quite similar to the one used for holonomic arms
- on the other, a method based on decoupling the motion of the nonholonomic platform from that of the arm.

Manipulation and mobile robotics literature both provide modelling tools to solve this problem. On one hand, kinematics and instantaneous kinematics of robotic arms with a fixed base are now a very classical material along with the associated notions of redundancy, singularity and manipulability [10]. On the other hand, wheeled mobile platforms were properly described and modelled by [3]. Though less classical and less used in the robotics community, these notions are of great interest in the case of wheeled mobile manipulators.

In section 2 we first recall the main notions attached to task description. In section 3, a description of robotic arms and mobile platforms is given together with their respective kinematic models. Then, kinematics and instantaneous kinematics modelling of wheeled mobile manipulators is developed. Section 4 presents both approaches for tracking control of end-effector location. Simulation are given for a simple planar mobile manipulator.

2 Manipulation tasks

A task is defined by the user in the so-called *operational space*. A point in this space is a *location*. It is characterized by a set of *operational coordinates* that correspond to the value of the position and the orientation of a frame attached to the EE at a particular point of this EE. Note that the location of the EE can be defined in different ways, according to the task. For instance, for a planar problem, we will consider only the EE position and orientation in the plane. Both those values are measured with respect to a fixed reference frame. Let $\mathcal{R} = (O, \vec{x}, \vec{y}, \vec{z})$ be this reference frame with \vec{z} vertical. Hereafter, the location of the EE

is denoted by the $m \times 1$ vector ξ . The tasks are mainly of two types : *regulation* or *tracking*. In a task of regulation, the goal is to reach a desired value of the EE location. In a task of tracking, one needs to realize a given velocity of this location, *i. e.* a given operational velocity, to follow a prescribed operational motion.

3 Kinematic modelling

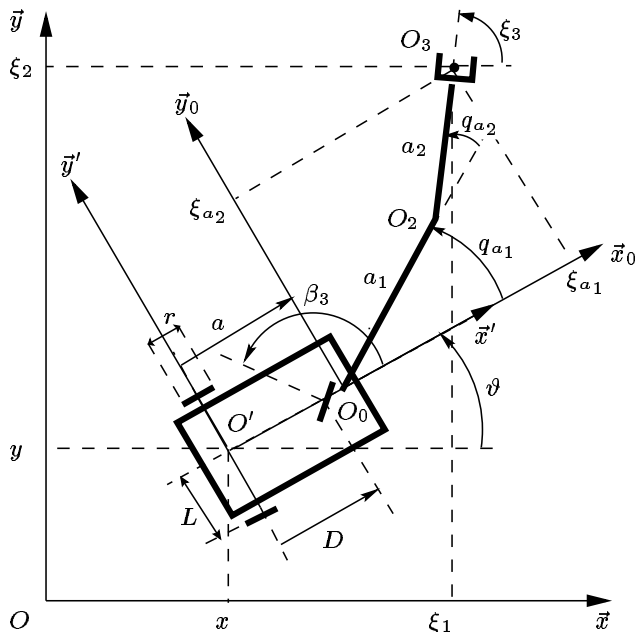


Figure 1: a planar mobile manipulator

3.1 Modelling of robotic arms

In classical manipulation, we consider a robotic arm built from n_a mobile bodies, supposed perfectly rigid and articulated by n_a revolute and/or prismatic joints. The most usual way to model it consists in using the Denavit-Hartenberg modified parameters proposed by [6]. These parameters define the location of all the bodies of the robotic arm, *i. e.* its whole geometry. This parameterization associates a frame, denoted here by $\mathcal{R}_i = (O_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$, with $i = 0, 1, \dots, n_a$, to the i -th body of the robotic arm. So, the frame \mathcal{R}_0 is linked to the base. The center of the EE is denoted by O_{n_a+1} . Hence, both points O_{n_a} and O_{n_a+1} are linked to the EE.

The *robotic arm configuration* is known when the position of all its points in \mathcal{R}_0 are known (see for instance [8]). It is defined by a vector q_a of n_a independent coordinates.

These coordinates, called *generalized coordinates of the robotic arm*, characterize the values associated to the different joints: rotation angles for the revolute joints, translations for the prismatic ones and form the configuration $q_a = [q_{a1} \ q_{a2} \ \dots \ q_{a_{n_a}}]^T$ of the arm.

In this article, we choose to define the *location of the EE* relative to \mathcal{R}_0 by a m_a -dimensional vector of *independent coordinates*, denoted by $\xi_a = [\xi_{a1} \ \xi_{a2} \ \dots \ \xi_{a_{m_a}}]^T$. This vector defines the position and the orientation of the EE in \mathcal{R}_0 .

The *kinematic model (KM) of the robotic arm* sets the location of its EE as a function of its configuration:

$$f_a : q_a \mapsto \xi_a = f_a(q_a).$$

The *instantaneous kinematic model (IKM) of the robotic arm* sets the derivative of the EE location – or operational velocity – as a function of the derivative of the configuration – or generalized velocity:

$$J_a(q_a) : \dot{q}_a \mapsto \dot{\xi}_a = J_a(q_a)\dot{q}_a,$$

where $J_a(q_a) = \frac{\partial f_a}{\partial q_a}(q_a)$ is the $m_a \times n_a$ Jacobian matrix of f_a .

3.2 Wheeled mobile platforms

Let $\mathcal{R}' = (O', \vec{x}', \vec{y}', \vec{z}')$ be a mobile frame linked to the platform (see Figure 1). The origin of \mathcal{R}' is usually chosen as a remarkable point of this platform (e.g. the midpoint of the rear axle). The *location*¹ of the platform is given by a vector ξ_p of 3 *operational coordinates*, which define its position and orientation in \mathcal{R} . We write $\xi_p = [x \ y \ \vartheta]^T$, where x and y are respectively the abscissa and the ordinate of O' in \mathcal{R} and ϑ the angle (\vec{x}, \vec{x}') .

Let $R(\vartheta)$ be the 3-order rotation matrix expressing the orientation of \mathcal{R}' with respect to \mathcal{R} . Rolling without slipping conditions give two kinds of instantaneous kinematics constraints. The first one defines allowable velocities for the platform location. It can be seen as the fact that there exists a δ_m -dimensional vector η_p of independent components (η_p is the *control of mobility of the platform*) such that:

$$\dot{\xi}_p = R(\vartheta)\Sigma(\beta_s)\eta_p. \quad (1)$$

where β_s is the δ_s -dimensional vector of the steering angles of the wheels when the platform has such wheels and $\Sigma(\beta_s)$ a $3 \times \delta_m$ matrix, constant or not, depending on the

¹This quantity is not distinguished from the *posture* in [3]. Nonetheless, it seems important to us to isolate the part of the posture that is related to the EE location, on one side, and that will appear as the range of a linear map defined over the control of mobility (see hereafter).

existence of steering wheels. Equation (1) forms the *instantaneous location kinematic model* (ILKM) of the platform. It sets the derivative of the platform location as a function of the control of mobility, for a given configuration.

When there are steering wheels, instantaneous kinematics are not completely described by the ILKM in the sense that this model does not fix the whole generalized velocity. We assume that we control the velocity of the δ_s (δ_s is the *degree of steerability of the platform*) steering wheels around their orientation axis. As a consequence, the δ_s -dimensional vector ζ_p is termed as the *control of steerability of the platform*:

$$\dot{\beta}_s = \zeta_p, \quad (2)$$

With [3], let the $(3 + \delta_s)$ - dimensional vector $z_p = [\xi_p^T \beta_s^T]^T$ be the *posture of the platform*². If we define the *control of manoeuvrability of the platform* by the δ_M -dimensional vector $u_p = [\eta_p^T \zeta_p^T]^T$ ($\delta_M = \delta_m + \delta_s$), we get:

$$\dot{z}_p = B_p(\vartheta, \beta_s)u_p,$$

with:

$$B_p(\vartheta, \beta_s) = \begin{bmatrix} R(\vartheta)\Sigma(\beta_s) & 0 \\ 0 & I_{\delta_s} \end{bmatrix},$$

where I_{δ_s} is the δ_s -order identity matrix. This is the *instantaneous posture kinematic model* (IPKM). It can be shown that this model is irreducible (see [4]). It constitutes the minimal model that allows to obtain the whole generalized velocity in the form:

$$\dot{q}_p = S_p(\vartheta, \beta_s)u_p, \quad (3)$$

This equation relates the derivative of the platform configuration, for a given configuration, to its control of manoeuvrability. It is termed as the *instantaneous kinematic configuration model* (ICKM) of the platform.

Remark: for simplicity, we have assumed that the platform always has a number of actuators equal to its degree of manoeuvrability.

3.3 Wheeled mobile manipulators

The usual models of manipulation systems have been modified to take into account the nature of the platform. It results that the models of mobile manipulators are compromises between the models of platforms and those of robotic arms. Particularly, as mobile manipulators are manipulation systems, we write models to describe the location ξ of the EE, which is important from an operational point of view. The *kinematic model* (KM) of a mobile manipulator

²Posture and location need to be distinguished only when the platform has steering wheels. When there is no steering wheels, posture and location are the same notion.

sets the location of its EE as a function of the robotic arm configuration q_a and of the platform location ξ_p :

$$\xi = f(q_a, \xi_p)$$

At the velocity level the *instantaneous location kinematic model* (ILKM) of a mobile manipulator sets the derivative of its location as a function of a set of Δ_m parameters of control, which form the *mobile manipulator control vector of mobility* η . These parameters are the control inputs of the system which have an influence on the EE velocity:

$$\dot{\xi} = \bar{J}\eta. \quad (4)$$

The *mobile manipulator degree of mobility* Δ_m is the dimension of vector η .

The rest of the control parameters of a mobile manipulator consist of the δ_s velocities $\dot{\beta}_s(t)$ of the steering wheels – when they exist – of the platform about their orientation axis. The *mobile manipulator control vector of manoeuvrability* is given by the set of all the control parameters, *i. e.* by $u = [\eta^T \zeta_p^T]^T$. Finally the *mobile manipulator degree of manoeuvrability* Δ_M is the dimension of vector u .

To know the evolution of the mobile manipulator configuration we also define the *mobile manipulator instantaneous configuration kinematic model* (ICKM):

$$\dot{q} = Su. \quad (5)$$

It is fundamental to notice that, in general, the dimension m of operational space is less than the degree of mobility Δ_m of the mobile manipulator. In this case the problem, mobile manipulator and task, is redundant. From now on, we assume that it is the case.

4 Global or decomposed approach

From a kinematic point of view the problem of motion generation has been studied by various authors [11, 5]. The difficulty consists in the necessary coordination of the two kinematically different subsystems: the platform and the arm. Whereas the tasks of a mobile manipulator are often described by the EE evolution – thus concerning manipulation aspects – it is compulsory to control the mobile platform adequately.

In fact, there are different factors acting for the definition of a planning and control strategy. First of all, we can think that the sole specification of the end-effector motion is not sufficient since the platform has to move without collision with the environment (and the arm). In the same way, since a kinematic scheme is incremental by nature, it does not take explicitly into account the proximity of the bounds.

Finally, different criteria may be considered. So, the task does not come down only to the end-effector location. Of course, these questions arise when there is more than one way to realize the imposed end-effector location.

Another point of view comes from the control problems. First of all, a kinematic scheme will be ill-conditioned near singularity of the linear map when the robot is not redundant w.r.t. the task. In the redundant case, convergence will be dependent on the chosen generalized inversion scheme. That is why different methods aim at defining additional tasks that must lead to robust invertible kinematic schemes [9].

From the control viewpoint, nonholonomy must be studied carefully. It is now well-known that kinematic models of wheeled mobile platforms are quite difficult to control [4]. In particular, point stabilisability is not ensured by continuous static feedback. Thus, this difficulty has to be taken into account when we are thinking in decomposing the task into a particular motion of the platform and an adapted motion of the arm. On the other side, when looking at the global mobile manipulator, the kinematic constraints of the platform may be compensated by an adequate generalized velocity of the arm to realize a prescribed end-effector task. In that case, nonholonomy of the platform is somewhat hidden in the global instantaneous kinematics.

So, it is interesting to compare both viewpoints.

Two methods are possible: i) to get a solution for the robotic arm joint values, an adequate platform position and orientation, and then to compute a control law to stabilize the system around the corresponding configuration [13]; ii) to compute directly the control inputs from the EE specified motion by an inverse of the adequate kinematic model [11, 5].

We consider the operational motion tracking problem. In order to simplify the presentation, we suppose, from now on, that the mobile manipulators have no steering wheel. The manoeuvrability control of the mobile manipulator reduces to its mobility control, and so $\mathbf{u} = \boldsymbol{\eta}$.

4.1 A global approach

We synthesize the control inputs of the mobile manipulator to obtain the specified motion of the EE. The proposed scheme is as general as possible and consistent with on-line computation. Our control laws are based on the generic modelling proposed in [1] and recalled in the previous section.

Here, for a given operational motion $\boldsymbol{\xi}^*(t)$, the problem is to find the mobility control $\boldsymbol{\eta}(t)$ such that

$$\boldsymbol{\xi}^*(t) = \bar{J}(t)\boldsymbol{\eta}(t)$$

(see equation 4), in order to asymptotically stabilize the

operational error $\mathbf{e}(t) = \boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t)$. The matrix $\bar{J}(t)$ is $m \times \Delta_m$. We suppose (this is the case in practice), that $m \leq \Delta_m$ and that the rank of this matrix is m . Then the previous linear system is *consistent* and all the *exact* solutions are given by :

$$\boldsymbol{\eta} = \bar{J}^+(t)\dot{\boldsymbol{\xi}}^*(t) + (I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))\mathbf{g}(t),$$

in which $\bar{J}^+(t)$ is the pseudo-inverse of $\bar{J}(t)$ and $\mathbf{g}(t)$ any Δ_m -dimensional vector. The solution obtained is the one that minimizes the Euclidean norm $\|\boldsymbol{\eta} - \mathbf{g}\|$

In fact, in order to asymptotically stabilize the error $\mathbf{e}(t)$, one can choose:

$$\boldsymbol{\eta}(t) = \bar{J}^+(t)(\dot{\boldsymbol{\xi}}^*(t) + W(\boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t))) + (I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))\mathbf{g}(t), \quad (6)$$

in which W is a m -order definite positive matrix.

Actually the previous control leads to the asymptotic stability of the transient error $\mathbf{e}(t)$, due to the equation:

$$\dot{\mathbf{e}}(t) + W\mathbf{e}(t) = 0,$$

because $\bar{J}^+(t)$ is a right-inverse of $\bar{J}(t)$.

On one hand the mobile manipulator is redundant, because the dimension Δ_m of its control of mobility $\boldsymbol{\eta}$ is greater than the dimension m of its location $\boldsymbol{\xi}$. On the other hand, we recall that $\dot{\mathbf{q}}(t) = S(t)\boldsymbol{\eta}(t)$ (see equation (5)). Then (6) writes:

$$\dot{\mathbf{q}}(t) = S(t)\bar{J}^+(t)(\dot{\boldsymbol{\xi}}^*(t) + W(\boldsymbol{\xi}^*(t) - \boldsymbol{\xi}(t))) + S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))\mathbf{g}(t).$$

In this equation the first term is due to the input and the second one is the *internal motion*. We can use the previous redundancy to propose a coordination strategy for the internal motion. For instance, it is interesting to avoid great variations of the components q_i of \mathbf{q} and constrain these coordinates around their mean values. This can be done by a gradient descent method in which the potential function has its minimum value for the previous means. In general, let \mathcal{P} be a scalar function depending on the mobile manipulator configuration $\mathbf{q}(t)$. We can write:

$$\begin{aligned} \dot{\mathcal{P}}(t) &= \nabla^T \mathcal{P}(\mathbf{q}(t))\dot{\mathbf{q}} \\ &= \nabla^T \mathcal{P}(\mathbf{q}(t))S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))\mathbf{g}(t), \end{aligned}$$

for the internal motion ($\nabla \mathcal{P}(\mathbf{q}(t))$ is the gradient of the function $\mathcal{P}(\mathbf{q}(t))$). In order to decrease $\mathcal{P}(\mathbf{q}(t))$, that is $(\dot{\mathcal{P}}(t)) \leq 0$, we propose the choice:

$$\mathbf{g}(t) = -k (S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t)))^T,$$

where k is a positive scalar. Indeed, with this choice:

$$\begin{aligned} \dot{\mathcal{P}}(t) &= -k (\nabla^T \mathcal{P}(\mathbf{q}(t))S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))) \\ &\quad (S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t)))^T, \end{aligned}$$

and then $(\dot{\mathcal{P}}(t)) \leq 0$.

Finally, the mobility control is:

$$\eta(t) = \bar{J}^+(t)(\dot{\xi}^*(t) + W(\xi^*(t) - \xi(t))) - kS(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t))(S(t)(I_{\Delta_m} - \bar{J}^+(t)\bar{J}(t)))^T.$$

This approach can be applied with various choices for \mathcal{P} . The first figure (Fig. 2) shows an example with criteria based on maximum distance to obstacle and the second one (Fig. 3) illustrates how manipulability measure can be maximized (the ellipse shown is the ellipse of manipulability of the whole mobile manipulator) [2]. In both case, only the position of the end-effector in the plane is imposed.

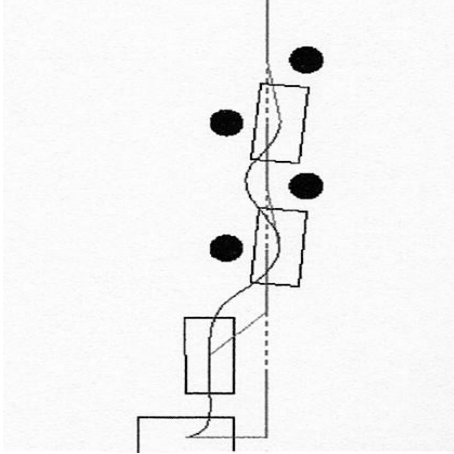


Figure 2: Obstacle avoidance

4.2 A decomposition approach

In that part, we consider a way to decompose the task. First of all, we have to verify that the platform is always near enough to the end-effector. So, a trajectory is planned for the platform from the end-effector trajectory. Then, it is necessary to have a control scheme that ensures that the actual motion is close to the planned one. Specifically, we must characterize the distance between the planned platform location and the actual one. A way to do that is to plan a feasible trajectory for the platform i.e. a trajectory that respects the nonholonomic constraints. Different techniques exist but they do not solve the online control problem. We follow another direction through the use of *practical stabilisability of the platform by transverse functions* [7]. This technique allows to plan a trajectory, feasible or not, and to follow it actually with a location error that can be tuned quite easily. For simplicity, let us take the instantaneous kinematic model for a unicycle-like platform (i.e.

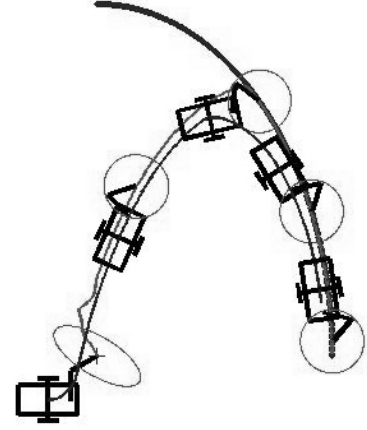


Figure 3: Maximization of manipulability

a platform with two independent driven wheels on the same axle and no steering wheels). The ILKM writes:

$$\dot{\xi}_p = B(\xi_p)\eta_p$$

where η_p is a 2×1 control vector and $B(\xi_p)$ a 3×2 matrix. The method by Samson et al. [7] allows by a dynamic extension to write a system :

$$\dot{y} = \mathcal{A}(y)U$$

with $U = [\eta_p^t \ \dot{\alpha}]^t$ the new 3-dimensional control vector with the external dynamic in α , \mathcal{A} a full rank 3×3 matrix, and y a new variable such that when y goes to zero, ξ_p is arbitrarily small. Then, by inverting the above system, arbitrary dynamics can be given to y and, as a result, ξ_p can be *practically* stabilized i.e. can be steered into a arbitrary small ball around the target.

Then, the arm adapts its motion from the end-effector set-point and the current value of the platform location through a simple linear controller.

Two results of simulation are presented for the same end-effector task that consists in following a line. In the first one, the induced platform trajectory is feasible since the platform can roll along the line parallel to the blackboard. In the second one, the induced trajectory of the platform is such that the position (x, y) of the platform must move along the same parallel line but with the orientation normal to this line. This trajectory is clearly non feasible since it needs a slipping of the platform. The transverse function

method allows to follow this trajectory with a bounded error and then the arm can adapt itself to this error. Typically, the first simulation (Fig. 4) could be obtained easily with a global scheme whereas the second one (Fig. 5) shows how the dynamic feedback introduced by transverse functions method allows to produce efficient motions of the platform in the direction its instantaneous kinematics prevent.

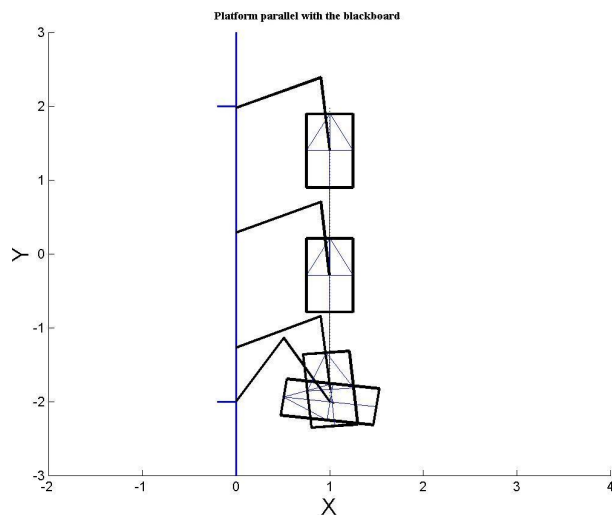


Figure 4: trajectory of the platform parallel to the EE trajectory

Finally, both approaches have respective qualities and drawbacks and do not produce the same kind of solutions. Additional features appear in the case with steering wheels.

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References

- [1] B. Bayle, J.-Y. Fourquet, and M. Renaud. From manipulation to wheeled mobile manipulation: analogies and differences. In *Proceedings of 7th Symposium on Robot Control (SYROCO'03)*, Wroclaw, Poland, September 2003.
- [2] B. Bayle, J.-Y. Fourquet, and M. Renaud. Manipulability of wheeled mobile manipulation: application to motion generation. *The International Journal of Robotics Research*, vol. 22(7-8):565–581, July 2003.
- [3] G. Campion, G. Bastin, and B. D’Andréa-Novél. Structural properties and classification of kinematic and dynamic models of wheeled mobile robots. *IEEE Transactions on Robotics and Automation*, 12(1):47–62, February 1996.
- [4] C. Canudas de Wit, B. Siciliano, and G. Bastin, editors. *Theory of Robot Control*. Springer, 1996.

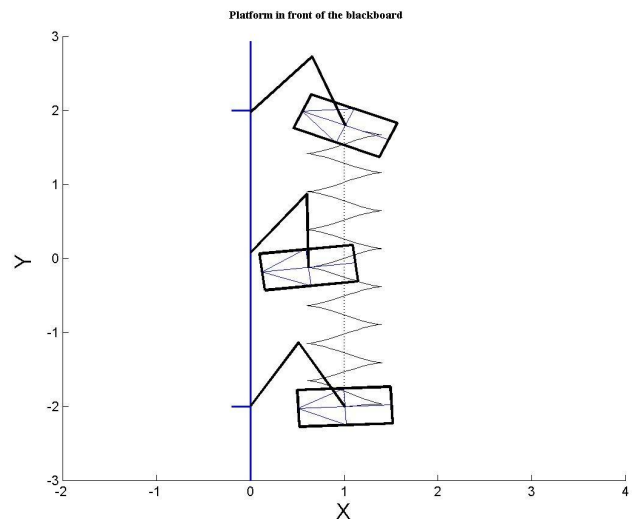


Figure 5: the platform must be turned to the blackboard

- [5] G. Foulon, J.-Y. Fourquet, and M. Renaud. Control of a rover-mounted manipulator. In *ISER'97*, pages 96–107, Barcelona, Spain, June 1997.
- [6] W. Khalil and J. Kleininger. A new geometric notation for open and closed loop robots. *ICRA'86*, pages 75–79, 1986.
- [7] P. Morin and C. Samson. Practical stabilization of a class of nonlinear systems. application to chain systems and mobile robots. In *IEEE Conf. on Decision and Control (CDC)*, pages 2989–2994, 2000.
- [8] J. Neimark and N. Fufaev. *Dynamics of Nonholonomic Systems*, volume 33. Translations of Mathematical Monographs, 1972.
- [9] C. Samson, M. Leborgne, and B. Espiau. *Robot Control. The Task Function Approach*, volume 22 of *Oxford Engineering Series*. Oxford University Press, 1991.
- [10] L. Sciacivico and B. Siciliano. *Modelling and Control of Robot Manipulators*. Springer-Verlag, 2000.
- [11] H. Seraji. An on-line approach to coordinated mobility and manipulation. In *ICRA'93*, pages 28–35, Atlanta, USA, May 1993.
- [12] K. Tchon, J. Jakubiak, and R. Muszynski. Kinematics of mobile manipulators: a control theoretic perspective. *Arch Control Sci*, 11:195–221, 2001.
- [13] Y. Yamamoto and X. Yun. Coordinating locomotion and manipulation of a mobile manipulator. *IEEE Transactions on Robotics and Automation*, 39(6):1326–1332, June 1994.