PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2011 – 30

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Measuring Poverty Without The Mortality Paradox

Mathieu Lefebvre*, Pierre Pestieau† and Gregory Ponthiere‡

August 17, 2011

Abstract

Under income-differentiated mortality, poverty measures reflect not only the "true" poverty, but, also, the interferences or noise caused by the survival process at work. Such interferences lead to the Mortality Paradox: the worse the survival conditions of the poor are, the lower the measured poverty is. We examine several solutions to avoid that paradox. We identify conditions under which the extension, by means of a fictitious income, of lifetime income profiles of the prematurely dead neutralizes the noise due to differential mortality. Then, to account not only for the "missing" poor, but, also, for the "hidden" poverty (premature death), we use, as a fictitious income, the welfare-neutral income, making indifferent between life continuation and death. The robustness of poverty measures to the extension technique is illustrated with regional Belgian data.

Keywords: premature mortality, income-differentiated mortality, poverty measurement, censored income profile.

JEL classification code: I32.

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1 Introduction

In *An Essay on the Principle of Population* (1798), Malthus emphasized that the population is a social product, whose size adjusts to the prevalence of poverty through two kinds of population "checks". On the birth side, poverty reduces fertility through parental anticipations about future difficulties to raise children (i.e. "preventive checks"). Moreover, poverty leads to premature deaths within the low income classes (i.e. "positive checks").

Given the underdeveloped state of social statistics at Malthus's time, the existence of population checks was more a conjecture than a scientific result. However, in the recent years, empirical studies confirmed the existence of positive population checks, under the form of a relationship between income and life expectancy.[1] On average, individuals with higher incomes have, *ceteris paribus*, a longer life than individuals with lower incomes.[2]

Income-differentiated mortality raises a twofold challenge for the measurement of poverty. Actually, the two aspects of poverty measurement underlined by Sen (1976) are affected: on the one hand, the *identification* of the poor within the population; on the other hand, the construction of an *index* aggregating and weighting the information available on the identified poor.

Regarding the identification of the poor, income-differentiated mortality leads to a paradox. Poor persons tend to die, on average, earlier than non-poor persons. Hence, usual poverty measures, which focus on living individuals, do not count the "missing" poor, and, thus, reflect not only the "true" poverty, but, also, the interferences or noise due to income-differentiated mortality.[3] Those interferences push towards a lower poverty estimate. As a consequence, poverty measures tend to underestimate the "true" poverty.[4] That problem can be called the Mortality Paradox: the worse the survival conditions faced by the poor are, the lower the measured poverty is. That result is paradoxical, since poverty measures should be increasing - or, at least, non-decreasing - in the premature mortality faced by the poor due to their low income.

Income-differentiated mortality raises also an important challenge regarding the treatment of the informational basis relevant for the measurement of poverty. Undoubtedly, a shorter life is a major source of deprivation. Hence, if individuals with lower incomes face also a higher mortality, it is hard to ignore this when measuring poverty. But if one takes premature death as a part of the poverty phenomenon to be measured, then a major issue concerns the *weighting* of the two dimensions under study: income and longevity.

This paper focuses on the challenges raised by income-differentiated mortality for the measurement of poverty, by re-examining the Mortality Paradox

[1] The same cannot be said on the birth side, where the observed fertility transition has infirmed the existence of preventive population checks. Indeed, to explain the fertility decline, the substitution effect related to the rise in productivity must have overcome the income effect.

[2] See, among others, Duleep (1986), Deaton and Paxson (1998), Deaton (2003), Jusiot (2003) and Salm (2007). One exception is Snyder and Evans (2006), who show that high income groups face, *ceteris paribus*, a higher mortality than low-income groups.

[3] The existence of "missing poor" due to income-differentiated mortality is quite similar to the existence of "missing women" because of gender discriminations (see Sen 1998).

[4] This problem is general, and concerns all poverty measures, including the recent multi-dimensional poverty indexes. For instance, the Human Poverty Index (UNDP 1997), which includes, as dimensions, the probability of not surviving to ages 40 or 60, faces the paradox, since poor persons who survive until 40 or 60 years and then die will not be counted as poor after their death, so that a poverty-related death reduces the poverty measure.

and its solutions. For that purpose, we first develop a 2-period model with income mobility and income-differentiated mortality, and study the conditions under which the Mortality Paradox occurs. We propose also a solution to it: the extension, by means of a fictitious income, of lifetime income profiles of the prematurely dead. Then, in a second stage, we argue that a natural candidate for the fictitious income is the welfare-neutral income, i.e. the income making an agent indifferent between further life with that income and death. Finally, we use Belgian data to estimate the bias induced by the Mortality Paradox, and to evaluate the robustness of adjusted poverty measures to the extension of income profiles. This allows us to decompose the adjustment into counting the "missing" persons and valuing premature death as a part of poverty.

At this stage, it is important to relate our paper to the existing literature on poverty measurement. As far as we know, there exists only one paper, by Kanbur and Mukherjee (2007), which proposed a solution to the Mortality Paradox. They recommend, when computing poverty measures, to count the prematurely dead poor persons *as if* they were still alive, and to truncate their lifetime income profiles by means of a fictitious income depending on past incomes. Our paper complements that contribution on three grounds. Firstly, whereas Kanbur and Mukherjee propose general rules for the selection of a fictitious income, we argue, on the contrary, that the fictitious income should be equal to a particular level: the welfare-neutral income. Secondly, while Kanbur and Mukherjee only truncate the income profiles among the poor population, we propose to do it for *all* "missing" persons. Thirdly, whereas Kanbur and Mukherjee's paper is purely theoretical, we provide empirical estimates of the size of the bias induced by the Mortality Paradox, as well as an empirical study of the robustness of adjusted old-age poverty measures to the extension technique used.

Anticipating our results, we first show how, under income-differentiated mortality, standard old-age poverty rates are subject to the Mortality Paradox. We show that, when a fictitious income lower than the poverty line is assigned to prematurely dead poor individuals only, the adjusted poverty rate is robust to variations in survival conditions, and, thus, avoids the Mortality Paradox. Then, we consider the construction of alternative adjusted poverty measures, which count a premature death as an aspect of deprivation and poverty. Such measures, instead of being invariant to a worsening of survival conditions, are *increasing* with the strength of positive population checks. That alternative solution to the Mortality Paradox consists of assigning, to all prematurely dead persons, a fictitious income equal to the welfare-neutral income. Finally, we show, on the basis of Belgian regional data, that, while the addition of the "missing" persons with fictitious incomes equal to the incomes when being alive only raises the poverty rates by about one point, the assignment of a fictitious income equal to the welfare-neutral income raises poverty rates by 6-7 points. This suggests that taking the "hidden" burden of premature death into account is a much bigger correction than merely counting the "missing" poor.

The paper is organized as follows. Section 2 studies the measurement of poverty in a model with income mobility and income-differentiated mortality, and identifies conditions under which truncating income profiles of the prematurely dead prevents the Mortality Paradox. Section 3 explores a particular extension, which relies on the welfare-neutral income. Section 4 illustrates, on the basis of regional Belgian data, the Mortality Paradox and the robustness of poverty measures to the extension method. Conclusions are drawn in Section 5.

2 Poverty measure and income-based mortality

2.1 The framework

Let us consider a two-period model, where a cohort, of size $N \in \mathbb{N}$, lives the young age (first period) for sure, whereas only some fraction of the population will enjoy the old age (second period).[5]

There exists a finite number $K \in \mathbb{N}$ of possible income levels ($K > 1$). The set of possible income levels is: $Y = \{y_1, ..., y_K\}$. For the ease of presentation, we assume that income levels are indexed in an increasing order, so that:

$$y_1 < ... < y_K \tag{1}$$

The number of young individuals with income $y_i \in Y$ is denoted by n_i^1.[6] We denote by $\mathbf{n^1}$ the vector of size K, whose entries are n_k^1 for $k = 1, ..., K$.

The probability of survival to the old age, denoted by π, depends on the income when being young. Following the literature, we assume that a higher income when being young leads to higher survival chances.[7] Hence income-specific survival probabilities, which take K distinct values, are ranked as follows:

$$\pi_1 < ... < \pi_K \tag{2}$$

We denote by $\boldsymbol{\pi}$ the vector of size K whose entries are the income-specific survival probabilities π_k, for $k = 1, ..., K$. The number of surviving old individuals with income $y_i \in Y$ is denoted by n_i^2.[8] We denote by $\mathbf{n^2}$ the vector of size K, whose entries are n_k^2 for $k = 1, ..., K$.

Denoting by λ_{ij} the probability that a young agent with income y_i enjoys, in case of survival, an income y_j at the old age, the income mobility can be described, conditionally on survival, by the right stochastic matrix $\mathbf{\Lambda}$:

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_{11} & \lambda_{12} & ... & \lambda_{1K} \\ \lambda_{21} & \lambda_{22} & ... & \lambda_{2K} \\ ... & ... & ... & ... \\ \lambda_{K1} & \lambda_{K2} & ... & \lambda_{KK} \end{pmatrix} \tag{3}$$

The income mobility matrix $\mathbf{\Lambda}$ concerns individuals who live the two periods. As such, this does not take premature death into account, and, thus, leads to an incomplete representation of the dynamics of income distribution.

Actually, the dynamics of income distribution can be represented by means of the transition matrix $\mathbf{M}$, of size $K \times K$, which describes how the income distribution at the young age determines the income distribution at the old age:

$$\mathbf{n^2} = \mathbf{M'n^1} \tag{4}$$

The transition matrix $\mathbf{M}$ is:

$$\mathbf{M} = \begin{pmatrix} \pi_1\lambda_{11} & \pi_1\lambda_{12} & ... & \pi_1\lambda_{1K} \\ \pi_2\lambda_{21} & \pi_2\lambda_{22} & ... & \pi_2\lambda_{2K} \\ ... & ... & ... & ... \\ \pi_K\lambda_{K1} & \pi_K\lambda_{K2} & ... & \pi_K\lambda_{KK} \end{pmatrix} \tag{5}$$

[5] Our argument is robust to the number of life-periods. We focus here on a two-period framework to simplify the presentation without unnecessary material.

[6] By construction, we have: $\sum_{k=1}^{K} n_k^1 = N$.

[7] See Duleep (1986), Deaton and Paxson (1998), Jusiot (2003) and Salm (2007).

[8] By construction, we have: $\sum_{k=1}^{K} n_k^2 = \sum_{k=1}^{K} \pi_k n_k^1$.

The $\mathbf{M}$ matrix fully describes the trajectories of individuals in our economy. The lifecycle trajectory depends on survival probabilities and on income transition probabilities, which are correlated in terms of rank. We can easily decompose the matrix $\mathbf{M}$ into its two components: the income mobility component and the survival process component:

$$\mathbf{M} = \mathbf{\Lambda} \circ \mathbf{\Pi} \tag{6}$$

where $\mathbf{\Pi} \equiv \boldsymbol{\pi} \times \mathbf{1}'_{\mathbf{K}}$, $\mathbf{1}_{\mathbf{K}}$ being the identity vector of size K, while the symbol $\circ$ refers to the Hadamard product, that is, the entrywise product of two matrices.

The $\mathbf{M}$ matrix includes, as a special case, the situation where there is no premature death (i.e. $\pi_i = 1$ for all i). In that case, the matrix $\mathbf{M}$ vanishes to the income mobility matrix $\mathbf{\Lambda}$. Alternatively, if there is no mobility over the lifecycle (i.e. $\lambda_{ii} = 1$ for all i), the matrix $\mathbf{M}$ is a diagonal matrix with survival probabilities π_i as entries.

2.2 The Mortality Paradox

The Mortality Paradox is a general problem faced by various types of poverty measures. It refers to an undesirable sensitivity of poverty measures to income-differentiated mortality. That paradox can be stated as follows: the worse the survival conditions faced by the poor are, the lower the measured poverty is.

The origin of that paradox has to do with the *selection* mechanism that is at work under income-differentiated mortality. Survival laws act as a selection process: poor individuals die, on average, earlier than non-poor persons. This implies that poor persons become, with the mere passage of time, relatively less numerous than non-poor persons, yielding a lower measured poverty. That result is paradoxical, since the measured poverty should not decrease because of the mere existence of positive population checks. Actually, income-differentiated mortality can be regarded as creating some interferences or a noise preventing the measurement of the "true" poverty.

To illustrate the Mortality Paradox, let us focus on the simplest measures of poverty, i.e. head-count ratios, which measure poverty by counting the number of individuals whose incomes are below a (fixed) poverty threshold $y_P \in Y$.

Definition 1 *Assume an economy with income distribution* $\mathbf{n^i}$ *at age* $i = 1, 2$. *If* $y_P \in Y$ *is the poverty threshold, the poverty rate at age* i *is:*

$$P^i = \frac{\sum_{j=1}^{P-1} n_j^i}{\sum_{k=1}^{K} n_k^i}$$

The old-age poverty rate P^2 is subject to the Mortality Paradox. To see this, let us consider the following example. In a first situation $\left(\mathbf{n^1}, \boldsymbol{\pi}, \mathbf{\Lambda}\right)$, individuals who are poor at the young age survive to the old age with positive probabilities $0 < \pi_k < 1$ for $k < P$, and there is no income mobility (i.e. $\lambda_{jj} = 1$ for all j). In the second situation $\left(\mathbf{n^{1\prime}}, \boldsymbol{\pi}', \mathbf{\Lambda}\right)$, individuals who are poor at the young age do not survive to the old age: $\pi'_k = 0$ for $k < P$, and there is no income mobility (i.e. $\lambda_{jj} = 1$ for all j). Writing the old-age poverty rate as:

$$P^2 = \frac{\sum_{j=1}^{P-1} n_j^2}{\sum_{k=1}^{K} n_k^2} = \frac{\sum_{j=1}^{K} \pi_j n_j^1 \left(\sum_{l=1}^{P-1} \lambda_{jl}\right)}{\sum_{k=1}^{K} \pi_k n_k^1} \tag{7}$$

we obtain the following two measures of poverty, denoted by P^2 for the first situation and by $P^{2\prime}$ for the second one:

$$\frac{\sum_{j=1}^{P-1} \pi_j n_j^1}{\sum_{k=1}^{K} \pi_k n_k^1} > \frac{\sum_{j=1}^{P-1} \pi_j' n_j^{1\prime}}{\sum_{k=1}^{K} \pi_k' n_k^{1\prime}} = 0$$

The old-age poverty rate is larger in the first situation than in the second one. The reason has nothing to do with the level of poverty at the young age, which could take any possible value; nor does it have anything to do with mobility (which is absent in the two situations); nor does it have anything to do with a change in the poverty threshold y_P, which is supposed to be the same in the two situations under study.[9] Actually, the lower level of measured poverty at the old age in the second situation is the mere outcome of income-differentiated mortality. The strong positive population checks at work in the second situation have made old-age poverty vanish to - apparently - nothing.

The fact that a more severe income-based mortality *reduces* the measured poverty is paradoxical. Poverty indexes should show us the extent of poverty, and not be disturbed by the noise due to income-based mortality. Hence the Mortality Paradox invites a refinement of poverty measures.

To avoid that paradox, one solution consists of imposing that poverty measures exhibit some *independence* with respect to survival conditions. The following property, entitled Robustness to Mortality Changes, captures that intuition.

Condition 1 (Robustness to Mortality Changes) *A poverty measure P^i satisfies Robustness to Mortality Changes if and only if a deterioration of the survival conditions of the poor leaves the measured poverty unchanged:*

$$\textit{If} \begin{cases} \pi_k > \pi_k' \textit{ for some } k < P \\ \pi_k = \pi_k' \textit{ for other } k \leq K \end{cases}, \textit{ then } P^i = P^{i\prime}.$$

Robustness to Mortality Changes requires poverty measures to be *invariant* to a deterioration of the survival conditions faced by the poor. Whereas that property requires to observe the impact of a change in survival conditions on the poverty measure, a simpler way to avoid the Mortality Paradox consists of imposing that a poverty measure depends only on two things: (1) the level of poverty at younger ages; (2) the matrix of income mobility over the lifecycle. That simpler requirement is captured by the following property.

Condition 2 (No Mobility Same Poverty) *A poverty measure P^i satisfies No Mobility Same Poverty if and only if, in the absence of income mobility, the measured poverty is constant across the lifecycle:*

$$\textit{If } \lambda_{ii} = 1 \textit{ for all } i, \textit{ then } P^1 = P^2.$$

The No Mobility Same Poverty condition states that, if we consider an economy without mobility, the poverty rate should be *constant* across the lifecycle. That condition rules out the Mortality Paradox. Actually, this is also equivalent to the Robustness to Mortality Changes condition.

Lemma 1 *Robustness to Mortality Changes and No Mobility Same Poverty are equivalent conditions.*

[9] See Section 4 on the impact of varying the poverty line on the measurement of poverty.

Proof. See the Appendix. ■

That equivalence result is useful, since this will allow us, when studying whether a poverty measure satisfies Robustness to Mortality Changes or not, to focus on the hypothetical case where there is no income mobility, and to check whether the poverty rate is constant or not along the lifecycle.

Let us now examine whether poverty rates satisfy those conditions or not. As it is shown below, the old-age poverty rate P^2 does not satisfy the Robustness to Mortality Changes condition. The old-age poverty rate is actually subject to the interferences induced by income-differentiated mortality.

Proposition 1 P^2 *does not satisfy Robustness to Mortality Changes. Moreover, we have, in the absence of income mobility, that:* $P^1 > P^2$*.*

Proof. See the Appendix. ■

The old-age poverty rate is not robust to a deterioration of the survival conditions faced by the poor. The old-age poverty rate P^2 is thus subject to the Mortality Paradox. Moreover, by the above lemma, we know that P^2 does not satisfy No Mobility Same Poverty: P^2 is inferior to the P^1 under no income mobility. The reason is that poor individuals tend to die earlier than non-poor persons, pushing the old-age poverty rate below the young age poverty rate.

2.3 A general solution to the Mortality Paradox

The reason why poverty measures suffer from the Mortality Paradox has to do with the fact that, once dead, poor persons disappear from the population. Therefore, as suggested by Kanbur and Mukherjee (2007), a solution to the Mortality Paradox comes from the extension of lifetime income profiles, to take into account the persons subject to premature mortality.

The underlying idea is the following. Instead of computing old-age poverty measures on the basis of the surviving persons *only*, one should do *as if* all individuals are still alive at the old age, and benefit from some income. For that purpose, lifetime income profiles must be extended, to assign some income to prematurely dead persons.

The assignment of a fictitious income to the premature dead implies that we have now *two*, instead of one, income transition matrices: one for individuals who survived to the old age, i.e. $\mathbf{\Lambda}$, and one for those who did not survive. We will denote that latter income transition matrix by $\mathbf{\Sigma}$, of size $K \times K$:

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & ... & ... & \sigma_{1K} \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ \sigma_{K1} & ... & ... & \sigma_{KK} \end{pmatrix} \tag{8}$$

where σ_{ij} is the probability, for an individual with income y_i when being young, to have a fictitious income $e_i = y_j$ assigned to him when he is dead.

The adjusted old-age poverty rate, denoted by $\hat{P}^2$, can be written as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{K} (1-\pi_i) n_i^1 \left(\sum_{j=1}^{P-1} \sigma_{ij} \right)}{\sum_{k=1}^{K} n_k^1} \tag{9}$$

The first term of the numerator is standard: it counts the poor individuals among the old (surviving) population. But the second term is less standard: it counts the poor individuals among those who did *not* survive, their fictitious incomes being assigned to them through the matrix $\mathbf{\Sigma}$.

The adjusted poverty rate $\hat{P}^2$ can take distinct forms, depending on: (1) whether the assignment of fictitious incomes concerns all individuals or only the initially poor; (2) whether fictitious incomes exceed or are below the poverty line y_P. Those two features of the extension are captured by the matrix $\mathbf{\Sigma}$.

The next proposition examines the conditions on $\mathbf{\Sigma}$ under which $\hat{P}^2$ avoids the Mortality Paradox. As above, we rely here on the No Mobility Same Poverty (i.e. NMSP), since we know, by our lemma, that a violation of that property leads to a violation of Robustness to Mortality Changes.[10]

Proposition 2 *I: A fictitious income e_i is assigned to the dead poor only.*

- Ia: If $e_i < y_P$ for all i, $\hat{P}^2$ satisfies NMSP: $\hat{P}^2 = P^1$.

- Ib: If $e_i < y_P$ for $i < R \leq P$ and $e_i \geq y_P$ for $i \geq R$, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 < P^1$.

- Ic: If $e_i \geq y_P$ for all i, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 < P^1$.

II: A fictitious income e_i is assigned to all dead individuals.

- IIa: If $e_i < y_P$ for all i, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 > P^1$.

- IIb: If $e_i < y_P$ for $i < R \leq P$ and $e_i \geq y_P$ for $i \geq R$, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 \gtreqless P^1$.

- IIc: If $e_i \geq y_P$ for all i, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 < P^1$.

Proof. See the Appendix. ■

When fictitious incomes are assigned only to the prematurely dead poor persons, and when all fictitious incomes are lower than the poverty line, $\hat{P}^2$ satisfies the No Mobility Same Poverty, and, by our lemma, exhibits Robustness to Mortality Changes, and, thus, avoids the Mortality Paradox.

That case - case Ia - is quite specific, and - even slight - departures from this will generally imply a lack of robustness of the old-age poverty rate to changes in survival conditions. Two kinds of sensitivity can arise, and these do not have the same relationship with the Mortality Paradox.

On the one hand, if the fictitious income *exceeds* the poverty threshold (i.e. cases Ib, Ic, IIc), the old-age poverty rate does not satisfy Robustness to Mortality Changes, and is subject to the Mortality Paradox. In that case, $\hat{P}^2$ is, despite the adjustment, lower than P^1, since only a subgroup of the prematurely dead poor persons are counted as poor in the adjusted measure.

On the other hand, if fictitious incomes are assigned to *all* prematurely dead persons (i.e. cases IIa), $\hat{P}^2$ is not invariant to mortality changes. $\hat{P}^2$ is then higher than the poverty rate at the young age, because we count all prematurely dead persons as poor. Hence the Mortality Paradox does not hold here. It is quite the opposite, since $\hat{P}^2$ does not only count the "missing" poor, but counts also premature death as a part of the poverty phenomenon to be measured.[11]

[10] We assume, for the simplicity of presentation, that the poverty line y_P is invariant to the adjustment. Alternatively, if one considers a relativistic view of poverty, it could be argued that the adjustment affects also the poverty line. Given that this second-order effect depends on the precise way in which the poverty line is computed, we leave that discussion to the empirical example of Section 4.

[11] We will come back on that extension - and its justifications - below.

Finally, let us notice a special case where the two reasons why $\hat{P}^2$ violates No Mobility Same Poverty go in opposite directions. It can be shown that, under particular circumstances, those reasons cancel each other, making $\hat{P}^2$ satisfy No Mobility Same Poverty.

Corollary 1 *When (1) a fictitious income is assigned to all prematurely dead persons; (2) the fictitious income is inferior to y_P for all short-lived poor individuals, and is superior to y_P for all short-lived non-poor individuals, $\hat{P}^2$ satisfies No Mobility Same Poverty. That case coincides with case IIb with $R = P$.*

Proof. See the Appendix. ■

In that case, we know also, by the equivalence lemma, that $\hat{P}^2$ is robust to a deterioration of the survival conditions faced by the poor, so that $\hat{P}^2$ avoids the Mortality Paradox.[12]

In sum, this section shows that it is possible to escape from the Mortality Paradox by truncating the lifetime income profiles of the prematurely dead. That extension can be done in various ways, but, if one wants poverty measures to be invariant to changes in mortality, the extension should concern either only the prematurely dead poor individuals, with fictitious incomes below the poverty threshold, or every prematurely dead, but with fictitious incomes below the poverty line only for prematurely dead poor individuals.

3 The extension of income profiles revisited

The previous section identified conditions under which truncating income profiles of the dead makes poverty measures avoid the Mortality Paradox. That solution, although appealing, faces some criticisms, which concern both the selection and meaningfulness of the fictitious incomes used in the extension.

A first criticism is that, even if one sticks to the extension described above, there exist not a single, but *numerous* ways to truncate the lifetime income profiles of the prematurely dead. The problem is that the resulting poverty estimates are likely to be strongly sensitive to the chosen fictitious income e_i. The closer e_i is to y_P, the lower the measured poverty is. Hence, the measurement of poverty in real environments requires more precise information.[13]

A second criticism concerns the extent to which premature mortality *per se* matters for poverty measurement. Once lifetime income profiles are extended, the fictitious income enters the poverty measure *as if* it was an income enjoyed by a living person. This amounts to regard as equivalent two situations that

[12] That special case includes the situation where the matrix assigning fictitious incomes, i.e. $\mathbf{\Sigma}$, coincides with the income mobility matrix $\mathbf{\Lambda}$. Indeed, in that case, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij}\right)}{\sum_{k=1}^{K} n_k^1}$$

It is easy to check that this adjusted poverty rate satisfies No Mobility Same Poverty.

[13] That critique applies not only to the extension discussed above, but, also, to what was proposed by Kanbur and Mukherjee (2007). According to them, the fictitious income has to satisfy three properties: (1) it is increasing in the past income (i.e. $y_i > y_{i-1} \implies e_i > e_{i-1}$); (2) it cannot exceed the past income (i.e. $e_i \leq y_i$); (3) agents who are not poor when being alive should not be counted as poor after the extension (i.e. $e_i \geq y_P$ when $y_i > y_P$). Those conditions imply that the poverty measure falls under case Ia, and escapes from the Mortality Paradox. However, those conditions are too general for measurement exercises.

are quite different: on the one hand, being alive with some income y_i, and, on the other hand, being dead with a fictitious income e_i equal to y_i. Hence the extension of lifetime income profiles has a double effect. This does not only allow to count some - otherwise missing - poor in the measure of poverty. The extension assigns also some *weights* to two dimensions of poverty (income and longevity). Such a weighting exercise cannot remain implicit, but should have some explicit (welfare) foundations.

Those two criticisms invite a method to select a particular fictitious income, and to solve the trade-offs, in terms of poverty measurement, between low incomes and short lives. This is the task of the present section.

3.1 A welfare-neutral fictitious income

To overcome those two criticisms, we propose here to carry out the computation of fictitious incomes on the basis of individual *preferences* on lifetime income profiles. More precisely, we propose to solve those problems by selecting a particular fictitious income: the *welfare-neutral* income. This is defined as the hypothetical income that would make an individual indifferent between, on the one hand, further life with that income, and, on the other hand, death.

The reliance on individual preferences requires some justifications. Actually, the Mortality Paradox is a paradox only to the extent that premature death is a part of the poverty phenomenon to be measured. If premature death had nothing to do with poverty, then there would be nothing paradoxical in having poverty measures decreasing once survival conditions deteriorate. But the will to overcome the Mortality Paradox pushes us *de facto* in the field of multidimensional poverty measurement. Hence, it is hard to ignore individual preferences as providing an adequate informational basis for weighting the two dimensions of poverty (income and longevity). Indeed, if the level of the fictitious income contributes to assign a specific "weight" reflecting the contribution of premature death to poverty, then using individual preferences is the natural way to solve that weighting exercise.

To define that welfare-neutral income, let us assume that individuals have well-defined preferences over all possible lifetime income profiles, and that those preferences can be represented by a non-decreasing function $U(\cdot)$:[14]

$$U(u_i^1, u_j^2) \tag{10}$$

where u_i^t is the value assigned by a state-dependent temporal utility function at period t under temporal income y_i:

$$u_i^t = \begin{cases} u(y_i) \text{ if the individual is alive at that period} \\ \Omega \text{ if the individual is not alive at that period} \end{cases} \tag{11}$$

where $u(\cdot)$ is increasing, while Ω is the utility of being dead.[15]

On the basis of that, one can define the "welfare-neutral" income as follows.

Definition 2 *For an individual with income $y_i \in Y$ with premature death, the welfare-neutral income $\bar{y}_i$ is the hypothetical income that makes him indifferent between life continuation and death:*

$$U\left(u(y_i), \Omega\right) = U\left(u(y_i), u(\bar{y}_i)\right)$$

[14] For the sake of simplicity, we assume that preferences are uniform.
[15] That number is, in most applications, set to zero (see below).

Note that whether the welfare-neutral income $\bar{y}_i$ is higher or lower than the income when alive y_i is an open issue. The answer depends on the individual's preferences (i.e. the shape of the functions $U(\cdot)$ and $u(\cdot)$), and on how poor he was when alive (i.e. y_i enjoyed at the young age).

Having defined the welfare-neutral income $\bar{y}_i$, let us now explain why it is a plausible candidate for the extension of lifetime income profiles of the prematurely dead. For that purpose, we will show how the welfare-neutral income provides a solution to the two criticisms formulated above, by considering the welfare consequences of the extension of lifetime income profiles.

Actually, when one truncates an agent's income profile $(y_i, 0)$ with a fictitious income e_i, this amounts to do *as if* that person was still alive during that period, and enjoyed an income e_i. Thus, the hypothetical situation after extension can be better or worse than the actual situation depending on whether:

$$U(u(y_i), \Omega) \lesseqgtr U(u(y_i), u(e_i)) \tag{12}$$

If the RHS exceeds the LHS, the person would have preferred living one more period with the fictitious income rather than dying after the young age. If the LHS exceeds the RHS, the person thinks that his actual life is better than the same life with the addition of one period with income e_i. In the former case, the extension of the income profile amounts to do *as if* the person had enjoyed a better life than the one he actually enjoyed. In the latter case, it is the opposite. But in any case, the extension disconnects the measurement of poverty from the measurement of welfare, which is problematic.

To illustrate this, let us compare three cases.

- Case A: an individual with income $y_i < y_P$ dies after period 1, and no extension is made in the poverty measure.
- Case B: an individual with income $y_i < y_P$ dies after period 1, but his income profile is extended by means of the income $\tilde{e}_i < y_P$ such that:

$$U(u(y_i), \Omega) < U(u(y_i), u(\tilde{e}_i))$$

- Case C: an individual with income $y_i < y_P$ dies at the end of period 2, and enjoys the income $y_j = \tilde{e}_i < y_P$ at the old age.

The measured poverty is lower in Case A than in Case B. The measured poverty is also lower in Case A than in Case C, since Case C is equivalent to Case B for the measurement of poverty. Hence we have: $P^{2A} < P^{2B} = P^{2C}$. In welfare terms, Cases A and B are equivalent, since these differ only in how poverty is measured, and are exactly the same otherwise. Case C dominates the other cases in welfare terms (because of the above inequality). Thus we have: $U^A = U^B < U^C$. Hence, the death of the individual, i.e. the passage from C to B, reduces his welfare, but does not change the measured poverty. This is quite problematic. Clearly, if a society undergoes an epidemy, so that many individuals shift from Case C to Case B, it is hard to claim, despite the fall in social welfare, that poverty is, at the end, the same as if the epidemic had not occurred. Such a claim is surely subject to the Mortality Paradox. The constancy of the measured poverty despite a change in living conditions is acceptable *only if* that change is welfare-neutral.

Actually, there exists only *one* level of the fictitious income e_i that avoids that problem. It is the "welfare-neutral" income $\bar{y}_i$. It is easy to see that, when $e_i = \bar{y}_i$, there is no discrepancy between poverty measurement and welfare measurement. Indeed, if we now assume $e_i = \bar{y}_i$, we still get that poverty is larger under Cases B and C than under Case A, i.e. $P^A < P^B = P^C$. In welfare terms, we have that Cases A and B are still equivalent (since the only change concerns how poverty is measured), but that welfare in Case C is also equal to what it is in Case B, since, by definition, survival with the fictitious income does not constitute a welfare improvement. Hence we have: $U^A = U^B = U^C$.

Thus, poverty is the same in B and C, and welfare too. Here there is nothing shocking in having a constant poverty measure despite the occurrence of an event such as an epidemic, since that event is here welfare-neutral, unlike what prevailed above. Here the comparison of Cases B and C consists of comparing the emergence of an epidemic with its avoidance at the cost of extreme misery (leading individuals to indifference between life and death). Those two situations are equivalent in terms of poverty, and the Mortality Paradox does not arise.

In sum, the use of the welfare-neutral income as a fictitious income allows us to base the extension on a specific fictitious income, as well as to avoid a discrepancy between the measurement of poverty and the measurement of welfare. As such, this brings an appealing solution to the Mortality Paradox.

3.2 A specific case: time-additive welfare

The "welfare-neutral" fictitious income $\bar{y}_i$ depends on the postulated preferences, which can take various forms. However, under standard time-additive lifetime welfare, the welfare-neutral income $\bar{y}_i$ has two convenient properties. On the one hand, it is *unique*; on the other hand, it is *independent* from past income.

To see this, let us assume that lifetime welfare takes a time-additive form:

$$U(u_i^1, u_j^2) = u_i^1 + \beta u_j^2 \tag{13}$$

where β is a time preference factor ($0 < \beta < 1$). Then, if the utility of death is normalized to zero (i.e. $\Omega = 0$), we have

$$U(u_i^1, 0) = u_i^1 + 0 = u(y_i) \tag{14}$$

Hence, the welfare-neutral income $\bar{y}_i$, is implicitly defined by:

$$u(\bar{y}_i) = 0 \tag{15}$$

An additional life-period with income larger than $\bar{y}_i$ is worth being lived, whereas a life-period with income lower than $\bar{y}_i$ is not. Under that specification, the fictitious income $\bar{y}_i$ is such that temporal welfare is zero, that is, equivalent to the temporal welfare associated to death. Thus, in that case, the welfare-neutral income $\bar{y}_i$ is the same for all past income levels. In the rest of this section, we will denote it by y_N. As a consequence, it appears that, under standard time-additive lifetime utility, the welfare-neutral income level is unique and independent from past incomes. Note that those properties way not hold under alternative, less standard, preferences.[16]

[16] To see this, take the following forms, which only regard either the worst or the best period

3.3 Properties of the new adjusted poverty measure

Let us now study whether adjusted poverty measures based on welfare-neutral fictitious incomes are robust to changes in survival conditions. Remind that the adjusted old-age poverty rate $\hat{P}^2$ is now computed by assigning a single value - i.e. the welfare-neutral fictitious income - for the second-period income for all prematurely dead individuals.

As explained above, whether the welfare-neutral fictitious income lies above or below the past income when alive depends on individual preferences, and on the past income level. Therefore the transition matrix $\mathbf{\Sigma}$ can take various forms. Note, however, that, in the case of time-additive lifetime welfare, the welfare-neutral income takes a single value, denoted by y_N, which is independent from past incomes. Hence $\mathbf{\Sigma}$ takes a simple form, where $\sigma_{ij} = 0$ for $j \neq N$ and $\sigma_{iN} = 1$ for all i, and the adjusted poverty rate $\hat{P}^2$ can be written as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{K} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} \qquad \text{if } y_N < y_P$$

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right)}{\sum_{k=1}^{K} n_k^1} \qquad \text{if } y_N \geq y_P$$

In the former case, the prematurely dead persons are all counted as poor. In the latter case, they all disappear from the poverty measure. The interpretation of those two cases is as follows. When $y_N < y_P$, an individual enjoying an income equal to the poverty line still prefers that life to death, whereas, when $y_N \geq y_P$, the poverty threshold is lower than the welfare-neutral income level, revealing that the misery makes life not worth being lived.

To identify the conditions under which the so-constructed adjusted poverty measure $\hat{P}^2$ is subject to the Mortality Paradox, we will, as above, examine whether $\hat{P}^2$ satisfies No Mobility Same Poverty (NMSP), since it is a simple way to see whether $\hat{P}^2$ is robust to changes in survival conditions.[17]

Proposition 3 *Consider an economy with time-additive lifetime welfare.*

- *Under $y_N < y_P$, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 > P^1$.*
- *Under $y_N \geq y_P$, $\hat{P}^2$ does not satisfy NMSP: $\hat{P}^2 < P^1$.*

Proof. See the Appendix. ■

Thus, once the fictitious income takes its welfare-neutral level and is assigned to all prematurely dead individuals, the adjusted poverty rate does not

lived (see Broome 2004, p. 228):

$$U(u_i^1, u_j^2) = min\,\{u_i^1, u_j^2\}$$
$$U(u_i^1, u_j^2) = max\,\{u_i^1, u_j^2\}$$

Under the min specification, $\bar{y}_i$ is such that $u(\bar{y}_i) \geq u_i^1$, which implies $\bar{y}_i \geq y_i$, but does not allow us to say more. On the contrary, under the max specification, we have $\bar{y}_i \leq y_i$. Thus the uniqueness and the independence of the welfare-neutral income from past income are not general properties, but only nice corollaries of standard time-additive lifetime welfare.

[17] As above, we assume here that the poverty line y_P is invariant to the adjustment made (see Section 4 on this).

satisfy No Mobility Same Poverty, and, by the equivalence lemma, also violates Robustness to Mortality Changes. That lack of robustness occurs, since the extension based on the welfare-neutral fictitious income amounts to count early deaths as a source of the poverty phenomenon to be measured. The sensitivity of adjusted poverty measure to survival conditions has two meanings.

When y_N lies below the poverty threshold y_P, all premature deaths are regarded as a source of poverty, and this explains why $\hat{P}^2$ is sensitive to a deterioration of survival conditions. Thus, adjusted poverty measures, instead of being invariant to changes in income-based mortality, take differential mortality into account, in the opposite way as standard poverty measures do. This explains why $\hat{P}^2$ violates the Robustness to Mortality Changes in that case.

If, on the contrary, the welfare-neutral income level exceeds the poverty threshold, the adjusted poverty measure at the old age is lower than the poverty measure at the young age. The intuition is that, in that case, life is not worth being lived for all poor persons. As a consequence, in that context, premature death cannot be counted as something causing poverty, and the Mortality Paradox is hardly relevant under those circumstances.

In sum, this Section proposed to overcome the Mortality Paradox by truncating lifetime income profiles of the prematurely dead by means of the welfare-neutral income. The use of the welfare-neutral income as a fictitious income can be defended on two grounds. First, the computation of an income that yields indifference between life and death leads, under standard preferences, to a *unique* value for the fictitious income. Such a uniqueness is most welcome when we consider the empirical measurement of poverty, which is strongly sensitive to the fictitious income (see below). Second, the so-computed fictitious income can also, thanks to its welfarist foundations, sort out trade-offs between income and longevity. As such, this adjustment of poverty measures does more than counting the "missing" individuals; it also takes into account a "hidden" - but central - part of the poverty phenomenon to be measured: premature death.

4 Old-age poverty in Belgian regions

Let us now illustrate the downward bias due to the Mortality Paradox, and the sensitivity of adjusted old-age poverty measures to the extension of income profiles. For those purposes, we will use data from Belgium and its regions.

4.1 The data

We use raw poverty measures coming from the European household survey EU-SILC for the year 2006 (EU, 2006). Regarding longevity data, the empirical study of the Mortality Paradox ideally requires lifetables differentiated according to income groups. Such lifetables are not available, but these are derived from education-specific lifetables from Deboosere *et al.* (2009).[18]

Since the Mortality Paradox is about the effect of differentiated survival conditions on poverty measurement, one can expect that measurement biases induced by income-differentiated mortality are more negligible at younger ages. Therefore, to estimate the biases due to differentiated mortality, we will focus

[18] See the Appendix.

on the measurement of poverty in the population aged 60 or more. Table 1 presents the head-count poverty rates by region and by age groups in 2006.

Poverty is higher among those of age 60 and more than among the total population. While the total poverty rate is 14.2 %, the proportion of poor elderly is 20.8 %. There exist also large differences between men and women. Whatever the region and the age group are, poverty rates are larger among women than among men. That gender poverty gap is particularly high above the age of 60. Table 1 highlights also a big difference between Flanders and Wallonia. That gap is important among the younger generations, but tends to vanish at older age, thanks to the (nationwide) pension system.

Table 1 also shows life expectancy differentials between men and women, and between Flanders and Wallonia. Whereas the gender gap in life expectancy is well documented, the geographical gap is more surprising. Indeed, although both regions are geographically close to each other, life expectancy at birth in Wallonia is shorter than in Flanders, by about 2 years and a half.

Table 1 : Poverty and life expectancy in Belgium[19]

	Belgium	Flanders	Wallonia
Poverty rate			
Total population	14.2 %	11.5 %	16.0 %
Male	12.9 %	10.1 %	15.0 %
Female	15.4 %	12.9 %	16.9 %
60+	20.8 %	20.2 %	20.4 %
Male	18.7 %	18.3 %	18.9 %
Female	22.4 %	21.9 %	21.6 %
Life expectancy			
Total population	79.4	80.2	77.9
Male	76.5	77.6	74.6
Female	82.2	82.8	81.1

Besides gender and geographic location, another source of longevity inequality is the income. However, the impact of income on mortality is more difficult to observe, since there exist no income-specific lifetable. Hence, in order to derive a relation between income and mortality, we use lifetables by educational level, which are regularly published, and the correlations between education and income, to extrapolate lifetables by income levels, for each region and gender.

While our calculations are presented in the Appendix, Figures 1 and 2 below summarize our results by showing life expectancy at age 55-59 by income class, for males and females in Flanders and in Wallonia.

Those figures invite several comments. First, there exists an increasing monotonic relationship between income and life expectancy at age 55-59. That relationship is robust to all genders and regions, and is significant. For instance, a Walloon man in the lowest income class has a life expectancy that is 4 years less than the one of a Walloon man of the highest income group. Second, the income / longevity relationship is non-linear: it is between the second and the sixth deciles that the slope is the largest. But at the two extremes of the income distribution, the income / longevity relationship is less strong. Thirdly, the com-

[19]Poverty rate is the percentage of the population below the poverty threshold fixed at 60% of the median income (threshold=10236€).

parison of Figures 1 and 2 suggests that the income / longevity relationship is significantly stronger for men than for women.

In the light of Figures 1 and 2, one can expect that standard poverty measures at high ages are biased downwards. The reasons are twofold.

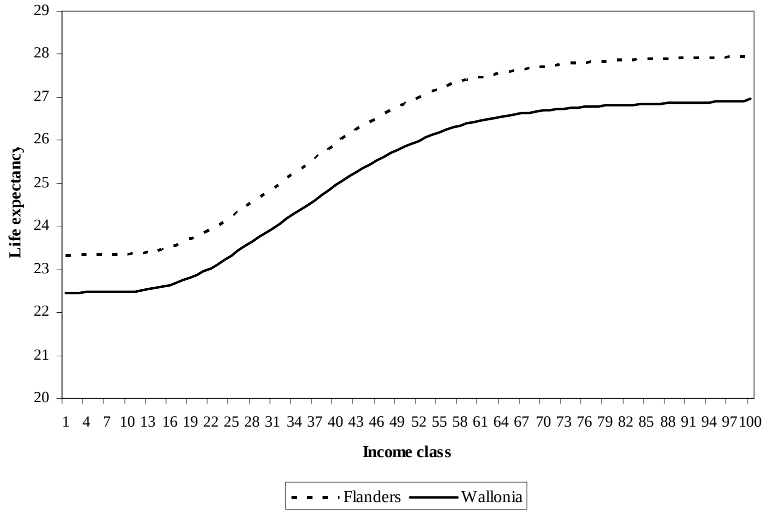

Figure 1 : Life expectancy at 55-59 by income class - Male

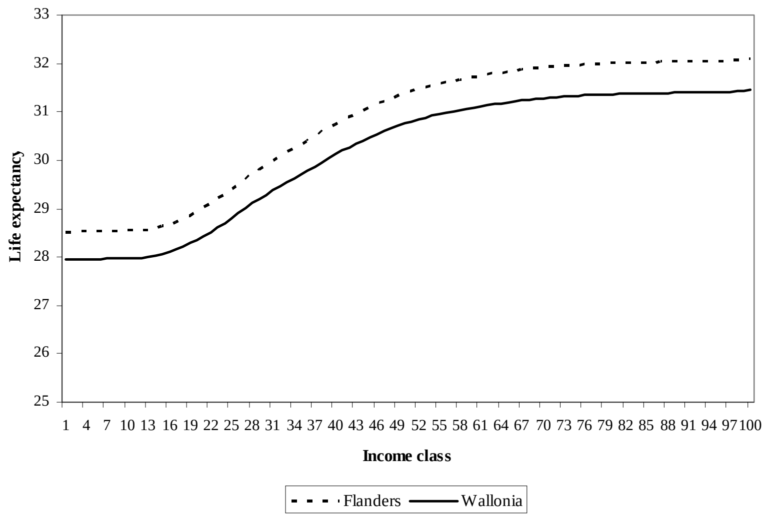

Figure 2 : Life expectancy at 55-59 by income class - Female

A first reason has to do with the selection mechanism induced by income-differentiated mortality. Given that poor persons tend to live less long than non-poor persons, the poverty rate among the surviving population at age 60 and more reflects not only the "true" poverty, but, also, the interferences associated with the differentiated survival process. That noise tends to reduce the apparent poverty, by the mere absence of the "missing" poor. Hence the poverty rate among Walloon males, equal to 20.4 %, tends, by being based on the population surviving to age 60, to forget the "missing" poor, who faced worse survival conditions than the average because of their poverty.

Besides that measurement problem - i.e. the Mortality Paradox -, one may also argue that a premature death is a part of the poverty phenomenon to be measured. Once it is acknowledged that a Walloon male of the lowest income class lives, on average, 4 years less than one of the highest income class, why should we restrict the measurement of poverty to the income dimension?

Those problems invite distinct adjustments of poverty measures. This section compares adjusted old-age poverty measures obtained by truncating the income profiles of the prematurely dead, under various extension techniques.

4.2 General methodology

The adjustment of poverty measures includes two parts. First, the addition of the "missing" poor; second, the imputation of a particular fictitious income.

Regarding the first step, we use the following method. For each income class i and region $r = F, W$, we have increased the population group N_{ir} on the basis of the largest life expectancy observed (i.e. the one of top income levels in Flanders). After correction, the adjusted population group is:

$$\hat{N}_{ir} = N_{ir} \frac{L_{100F}}{L_{ir}}$$

where $\hat{N}_{ir}$ is the adjusted population group, N_{ir} is the raw population group, L_{100F} is the life expectancy of the top income group in Flanders, and L_{ir} is the life expectancy for income group i in region r.

The above computation gives us a new distribution of the population in terms of income, which is the income distribution in the hypothetical case where all individuals had faced the same survival conditions as the ones of a group of reference (top earnings in Flanders). That computation allows us to reintegrate, in our calculations, the missing poor, equal, for each group, to $\hat{N}_{ir} - N_{ir}$.

Under such a computation, the fictitious income assigned to a prematurely dead individual coincides with his past income. As we discussed above, that extension technique, although attractive, is not the unique possible one. Hence, in the following, we present adjusted poverty measures on the basis of that extension, and contrast these with the ones under alternative extension techniques, including the one relying on the welfare-neutral fictitious income.[20]

4.3 Results

Let us first consider the simple case where the fictitious income e_i used for the extension is the one enjoyed when being alive, i.e. y_i. For that purpose, we will proceed in two stages. We will first compute the poverty rate for age 60 and more for each gender and region, for the new population computed above,

[20] Throughout this section, the poverty rate is the percentage of the population below the poverty line, fixed to 60 % of the median income.

while assuming that the poverty threshold takes the same level as before the adjustment. Then, we will compute adjusted poverty rates under a new poverty line (taking the modification of the income distribution into account).

Table 2 shows that, if one keeps the poverty threshold of Table 1, adjusted poverty rates are larger than standard poverty rates. That result is not surprising: our correction, by adding the "missing" persons, consists in adding relatively more poor individuals than non-poor individuals. Hence, under a fixed poverty threshold, there must be a rise in the poverty rate. That adjustment is relatively constant across genders and regions, and equal to about 1 point. Such an adjustment, which can be interpreted as the downward bias due to the Mortality Paradox, may be regarded as either low or high. On the one hand, when one considers poverty rates of about 20%, the addition of one point is a minor adjustment. On the other hand, that adjustment looks significant once we think that 1 percent of the population under study consists of thousands of persons and families.

Whereas the first part of Table 2 is based on the pre-adjustment poverty threshold, the modification of the population groups in such a way as to neutralize the impact of differential mortality has also the effect of changing the income distribution as a whole. Hence, if one adheres to a relativist - rather than absolutist - view of poverty, the addition of "missing" individuals may also affect the level of the poverty threshold. If one computes that new threshold, we obtain a poverty line that is 125 euros lower than the initial one. Under that new threshold, poverty rates tend to fall to levels that are close (if not inferior) to unadjusted poverty rates (second part of Table 2). Thus, if one adheres to a relativist view of poverty, taking the "missing" individuals into account may reduce - rather than raise - poverty.

Table 2: Adjusted poverty rates 60+: fictitious income = past income

	Belgium	Flanders	Wallonia
Pre-adjustment poverty threshold (10236 euros)			
Total population	21.7 %	21.1 %	21.3 %
Men	19.8 %	19.3 %	20.1 %
Women	23.3 %	22.7 %	22.3 %
New poverty threshold (10109 euros)			
Total population	20.5 %	20.2 %	20.0 %
Men	18.7 %	18.3 %	18.4 %
Women	22.1 %	21.8 %	21.2 %

Therefore, whether counting the "missing" persons affects the measured poverty or not depends on whether we adhere to an absolutist or a relativistic view of poverty. In the former case, adding the prematurely dead raises poverty. In the latter case, the fall in the poverty threshold is such that the poverty rate is close - if not lower - than before the adjustment. Those results raise the question of the "right" poverty threshold. We will not address that general issue here, and we will propose poverty measures under the two kinds of threshold.

Whereas Table 2 presupposed that the fictitious income equals the income when being alive, one can consider other values for that fictitious income. As discussed above, a natural candidate is the welfare-neutral income. That welfare-neutral income is not easy to estimate. As a starting point, we will consider the case where the welfare-neutral income equals zero, implying that death is, from a welfare perspective, equivalent to a life with zero income.

As shown in Table 3, setting the fictitious income to zero leads to much larger poverty rates, whatever the gender and the region under study. That result is robust to whether we keep a given poverty threshold, or whether we adjust it as a result of the modification of the income distribution. When comparing Table 3 with Table 2, it appears that the adjusted poverty measures are sensitive to the fictitious income assigned to prematurely dead individuals.[21]

Table 3: Adjusted poverty rates 60+: fictitious income = zero

	Belgium	Flanders	Wallonia
Pre-adjustment poverty threshold (10236 euros)			
Total population	29.0 %	27.5 %	30.6 %
Men	28.8 %	26.9 %	31.9 %
Women	29.1 %	27.9 %	29.4 %
New poverty threshold (10031 euros)			
Total population	27.3 %	25.9 %	28.5 %
Men	27.2 %	25.5 %	29.6 %
Women	27.3 %	26.3 %	27.6 %

Assuming that the welfare-neutral income is equal to zero should only be regarded as a first approximation. Actually, the recent literature on the measurement of welfare losses induced by premature death allows us to derive more precise estimates of the welfare-neutral income. For that purpose, let us assume, like Becker *et al.* (2005), that agents have the temporal utility function:

$$u(y_i) = \frac{(y_i)^{1-1/\gamma}}{1-1/\gamma} + \alpha$$

Following Becker *et al.* (2005), we fix $\gamma = 1.25$. We estimate the intercept α on the basis of the average income in our database, and we obtain: $\alpha = -15.50$. On the basis of those estimates, we obtain a welfare-neutral income equal to 284 euros. Table 4 shows adjusted poverty rates under that fictitious income.

Adjusted poverty rates under the welfare-neutral fictitious income are larger than under fictitious incomes equal to the income when being alive, and, also, larger than unadjusted poverty measures. It is also important to decompose the adjustment into (1) counting the "missing poor"; (2) counting premature death as a part of poverty. The first adjustment explains the gap between poverty rates in Tables 1 and 2. That change is small - about 1 point - and not robust to the chosen poverty threshold. The second adjustment explains the poverty differentials between Tables 2 and 4. That differential is large - about 6-7 points - and quite robust to the chosen poverty threshold.

Another important observation to be made concerns the gender poverty gap. In unadjusted terms, Walloon women are poorer than Walloon men (21.6 % against 18.9 %). In adjusted terms, and taking past incomes as a basis for the fictitious income, women are still more poor than men (22.3 % against 20.1 %). However, once we count premature death as a part of poverty, we obtain

[21] For instance, if one assumes that the fictitious income equals the past income when being alive, the poverty rate at age 60 and above lies between 20.5 % and 21.7 %, whereas it lies between 27.3 % and 29.0 % when the fictitious income is set to zero.

the opposite ranking: Walloon men, because of their worse survival conditions, turn out to be poorer than Walloon women (31.9 % against 29.4 %). Hence, the choice of fictitious incomes is relevant not only for the description of aggregate outcomes, but, also, for the description of poverty differentials between groups.

Table 4: Adjusted poverty rates 60+: welfare-neutral fictitious income

	Belgium	Flanders	Wallonia
Pre-adjustment poverty threshold (10236 euros)			
Total population	29.0 %	27.5 %	30.6 %
Men	28.8 %	26.9 %	31.9 %
Women	29.1 %	27.9 %	29.4 %
New poverty threshold (10031 euros)			
Total population	27.3 %	25.9 %	28.5 %
Men	27.2 %	25.5 %	29.6 %
Women	27.3 %	26.3 %	27.6 %

5 Conclusions

Under income-differentiated mortality, poverty measures reflect not only the "true" poverty, but, also, the interferences due to the survival process. That dependency on survival laws leads to the Mortality Paradox: the worse the survival conditions of the poor are, the lower the measured poverty is.

We proposed to re-examine a solution to that paradox, which consists of truncating lifetime income profiles, to take the "missing poor" into account. For that purpose, we developed a two-period model with income mobility and income-differentiated mortality. We identified two conditions under which the extension of income profiles neutralizes the interferences of differential mortality: (1) the fictitious income is assigned only to the prematurely dead poor; (2) that fictitious income does not exceed the income when being alive.

Although those conditions are intuitive, these suffer from two major drawbacks. First, condition (1) is not compatible with the idea that a premature death is a source of poverty for *all* individuals who face it. Second, condition (2) does not help us a lot regarding the choice of a particular fictitious income, which is problematic for empirical applications. Therefore, we proposed to extend the adjustment to all prematurely dead persons, and to use, as a fictitious income, the welfare-neutral income, i.e. the income making an individual indifferent between life continuation and death.

Finally, we used regional Belgian data to estimate the size of the Mortality Paradox, as well as the robustness of adjusted poverty measures to the fictitious incomes used. We showed that the extension of income profiles by means of fictitious incomes equal to the incomes when being alive leads to a rise of about 1 point of poverty rate at age 60 and more. But once the poverty threshold is modified to fit the adjusted income distribution, the adjusted poverty rate becomes close to the unadjusted one. We also compute adjusted poverty rates under welfare-neutral fictitious incomes, and showed that such an alternative adjustment raises poverty rates by about 6 to 7 points. Hence, while the mere addition of the "missing" poor under a constant income leads to a minor variation in the magnitude of poverty, the monetization of premature death by means of the welfare-neutral fictitious income raises the magnitude of poverty.

In sum, the comparison of standard poverty rates with adjusted ones reveals that the impact of income-differentiated mortality on the measurement of poverty is far from benign. One should thus be careful when interpreting the levels and variations of usual old-age poverty measures. Those measures hide not only a large number of "missing" poor, but, also, a strong form of deprivation: premature death. Thus, two centuries after Malthus' treatise, a particular attention should still be paid to the positive population checks at work in our economies. Otherwise, if we do *as if* positive checks do not exist, social statistics - including the ones on poverty - will be hardly useful for policy-makers.

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7 Appendix

7.1 Proof of Lemma 1

Let us first show that No Mobility Same Poverty implies Robustness to Mortality Changes. For that purpose, take a situation with poverty rate at the young age equal to P^1. By the No Mobility Same Poverty condition, we know that, in the absence of mobility, we have $P^1 = P^2$. Take now another situation, with the same poverty rate at the young age, equal to $P^{1\prime} = P^1$, but with a worsening of the survival probability for some income level below the poverty line. By the No Mobility Same Poverty condition, we know that, in the absence of mobility, we have $P^{1\prime} = P^{2\prime}$. But as $P^{1\prime} = P^1$, it follows, by transitivity of equality, that $P^2 = P^{2\prime}$ in conformity with Robustness to Mortality Changes.

Let us now prove that Robustness to Mortality Changes implies No Mobility Same Poverty. Let us start from a situation where all individuals reach the old age. Assuming the absence of mobility, we get: $P^2 = P^1$. Consider now a deterioration of survival conditons for some income group below y_P. As poverty rates at the young age do not depend on survival, we get: $P^1 = P^{1\prime}$ (as everything else except the deterioration is left unchanged). Moreover, we have, by Robustness to Mortality Changes, that $P^{2\prime} = P^2$. Hence, by transitivity, we have $P^{2\prime} = P^2 = P^1 = P^{1\prime}$. Hence, it follows that, in the absence of income mobility, we have $P^{1\prime} = P^{2\prime}$, in conformity with No Mobility Same Poverty.

7.2 Proof of Proposition 1

By Lemma 1, we can prove that proposition by merely showing that the old-age poverty rate violates No Mobility Same Poverty. No Mobility Same Poverty requires that, if no mobility, i.e. $\lambda_{ii} = 1$ for all i, poverty at the young age and at the old age should be the same. In the absence of mobility, P^2 is:

$$P^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right)}{\sum_{k=1}^{K} \pi_k n_k^1} = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1}{\sum_{k=1}^{K} \pi_k n_k^1}$$

Given $\pi_1 < ... < \pi_K$, this cannot be equal to P^1, which is given by:

$$P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Actually, in P^2, low income group numbers receive lower weights than under P^1 (where the weights are unitary). Hence, it is easy to see that: $P^2 < P^1$, which goes against No Mobility Same Poverty. By Lemma 1, we also know that P^2 does not satisfy Robustness to Mortality Changes.

7.3 Proof of Proposition 2

Consider first the case where only the initially poor who died prematurely are assigned a fictitious income. In that case, we have:

- If $e_i < y_P$ for all i:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{P-1} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

In the absence of income mobility among those who are alive, this can be rewritten as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{P-1} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} = P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Thus No Mobility Same Poverty is satisfied.

- If $e_i < y_P$ for all $i < R \leq P$ and $e_i \geq y_P$ for all $i \geq R$:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{R-1} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

In the absence of income mobility, this can be rewritten as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{R-1} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} < P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Hence No Mobility Same Poverty is not satisfied here.

- If $e_i \geq y_P$ for all i:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right)}{\sum_{k=1}^{K} n_k^1}$$

In the absence of income mobility, this can be rewritten as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1}{\sum_{k=1}^{K} n_k^1} < P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Thus the adjusted poverty measure does not satisfy No Mobility Same Poverty.

Let us now consider the case where a fictitious income level is assigned to *all* premature dead persons, whatever their past income was. We have the following three cases:

- If $e_i < y_P$ for all i:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{K} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

In the absence of income mobility, this can be rewritten as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{K} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} > P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Thus $\hat{P}^2 > P^1$, because the premature deaths who used to be rich are now counted as poor. However, when $\pi_i \to 1$ for $e_i > y_P$, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} n_i^1 +}{\sum_{k=1}^{K} n_k^1} + \underbrace{\frac{\sum_{i=P}^{K} (1-\pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}}_{\approx 0} \approx P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Hence under a low mortality of the non-poor, the adjusted poverty rate is close to satisfy Non Mobility Same Poverty.

- If $e_i < y_P$ for all $i < R \leq P$ and $e_i \geq y_P$ for all $i \geq R$:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{R-1} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Hence without income mobility, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{R-1} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} \lesseqgtr P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

- If $e_i \geq y_P$ for all i:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right)}{\sum_{k=1}^{K} n_k^1}$$

Hence without income mobility, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1}{\sum_{k=1}^{K} n_k^1} < P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Thus No Mobility Same Poverty is not satisfied.

7.4 Proof of Corollary 1

Indeed, in that case, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{P-1} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Hence, in the absence of income mobility, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{P-1} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1} = P^1$$

in conformity with No Mobility Same Poverty.

7.5 Proof of Proposition 3

Under $y_N < y_P$, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right) + \sum_{i=1}^{K} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1}$$

In the absence of mobility, this can be rewritten as:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1 + \sum_{i=1}^{K} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} = \frac{\sum_{i=1}^{P-1} n_i^1 + \sum_{i=P}^{K} (1 - \pi_i) n_i^1}{\sum_{k=1}^{K} n_k^1} > P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

Thus $\hat{P}^2 > P^1$, because the premature deaths who used to be rich are now counted as poor.

Under $y_N \geq y_P$, we have:

$$\hat{P}^2 = \frac{\sum_{i=1}^{K} \pi_i n_i^1 \left(\sum_{j=1}^{P-1} \lambda_{ij} \right)}{\sum_{k=1}^{K} n_k^1}$$

Without income mobility, this becomes:

$$\hat{P}^2 = \frac{\sum_{i=1}^{P-1} \pi_i n_i^1}{\sum_{k=1}^{K} n_k^1} < P^1 = \frac{\sum_{i=1}^{P-1} n_i^1}{\sum_{k=1}^{K} n_k^1}$$

7.6 Life tables by income class

There are no lifetable by income in Belgium. However, there exist lifetables by education levels (Deboosere *et al*, 2009). From these tables, it is possible to estimate lifetables by income class using a weighted ordinary least square regression, as in Bossuyt *et al* (2004) and Van Oyen *et al* (2005) studies on health expectancy. Indeed, the position in the social hierarchy is mainly determined by the dimensions: occupation, income and education. Given that the income and education are highly related to one another, we can extrapolate mortality by income class on the basis of the mortality by education. The social position is determined by the educational attainment. A five-category classification is used: (1) no formal education; (2) primary education; (3) lower secondary education; (4) higher secondary education; (5) tertiary education. We assume that the position of a socio-economic group is determined by its relative position, defined as the mid-point of the proportion of group represents on an ordered scale of 100% (Pamuk, 1985, 1988).

The mortality rates of the educational groups in terms of their relative socio-economic position is estimated using a weighted ordinary least square region of each region and sex and (5-year) age group using aggregate data. The weights are defined as the relative sizes of the educational levels in each age group. The slope of the regression line represents the difference in mortality between the bottom and the top of the socio-economic hierarchy. Once estimated, the coefficient is used to compute lifetable according to income by assuming that the social hierarchy is similar to education.

In our case, we used lifetables by age groups of five years in order to obtain sufficient subsample of each income class. Indeed, we consider one hundred different groups. Each income class is of 500€ except for the highest class which comprehend all income above 50000€.