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Excitations of Bose-Einstein condensates in a one-dimensional periodic potential

N. Fabbri,* D. Clément, L. Fallani, C. Fort, M. Modugno, K. M. R. van der Stam, and M. Inguscio
European Laboratory for Non-linear Spectroscopy (LENS), Dipartimento di Fisica, Università di Firenze and INFM-CNR, via N. Carrara 1, 50019 Sesto Fiorentino (FI), Italy

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We report on the experimental investigation of the response of a three-dimensional Bose-Einstein condensate (BEC) in the presence of a one-dimensional (1D) optical lattice. By means of Bragg spectroscopy we probe the band structure of the excitation spectrum in the presence of the periodic potential. We selectively induce elementary excitations of the BEC choosing the transferred momentum and we observe different resonances in the energy transfer, corresponding to the transitions to different bands. The frequency, the width, and the strength of these resonances are investigated as functions of the amplitude of the 1D optical lattice.

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The knowledge of the linear response of a complex system gives crucial information about its many-body behavior. For example, the superfluid properties of a three-dimensional (3D) Bose-Einstein condensate (BEC) are related to the linear part of the phonon-dispersion relation at low momenta [1]. The presence of optical lattices enriches the excitation spectrum of a BEC in a remarkable way. For deep three-dimensional lattices, the gas enters the strongly correlated Mott insulator phase and the spectrum exhibits a gap at low energies [2]. The response of a BEC in the superfluid phase is also drastically modified by the presence of a one-dimensional (1D) optical lattice [3–7]. Indeed, as in any periodic system, energy gaps open in the spectrum at the multiples of the lattice momentum and it is possible to excite several states corresponding to different energy bands at a given value of the momentum transfer [8,9]. In addition, the linear dispersion relation of the superfluid, and thus its sound velocity, is changed. In the mean-field regime of interactions these peculiar features of the excitations of a superfluid BEC in the presence of an optical lattice are captured by the Bogoliubov theory [1].

Bragg spectroscopy represents an excellent experimental tool for investigating the linear response of gaseous BECs [10]. It has allowed measurement of the dispersion relation of interacting BECs in mean-field regime [11–13], characterization of the presence of phase fluctuations in elongated BECs [14], study of signatures of vortices [15], and more recently study of strongly interacting 3D Bose [16] and Fermi [17] gases close to Feshbach resonances as well as 1D Bose gases across the superfluid–to–Mott insulator transition [18].

In this work we use Bragg spectroscopy to probe the excitation spectrum of a 3D BEC loaded in a 1D optical lattice. Previous experimental studies have so far investigated the excitations of superfluid BECs within the lowest energy band of a 3D optical lattice by means of lattice modulation [19] and Bragg spectroscopy [18,20]. This paper presents a detailed experimental study of the different bands in the excitation spectrum of an interacting 3D BEC in the presence of a 1D optical lattice. We measure the resonance frequencies, the strengths, and the widths of the transitions to different bands of the 1D optical lattice. The measurements are quantitatively compared with Bogoliubov mean-field calculations for our experimental system [7].

We produce a 3D cigar-shaped BEC of $N \simeq 3\times10^5$ ^{87}Rb atoms in a Ioffe-Pritchard magnetic trap whose axial and radial frequencies are $\omega_y = 2\pi\times 8.9$ Hz and $\omega_x=\omega_z=2\pi\times 90$ Hz, respectively, corresponding to a chemical potential $\mu \simeq h\times 1$ kHz, with h being the Planck constant. The condensate is loaded in an optical lattice along the longitudinal

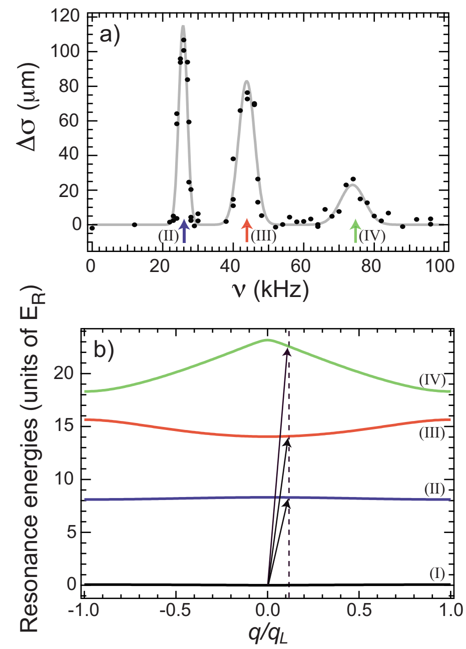

FIG. 1. (Color online) (a) Measured BEC excitation spectrum in the presence of a lattice with height $s=(22\pm2)$. The increase $\Delta\sigma$ in the width of the atomic density distribution is monitored as a function of the relative detuning ν between the two counterpropagating Bragg beams. The data are fitted with Gaussian functions (gray line). The arrows below the resonances indicate the corresponding bands, represented in (b) with the same colors (numbers). (b) Band structure of the excitation spectrum of a BEC in a 1D optical lattice with $s=22$: first, second, third, and fourth bands are represented [black (I), blue (II), red (III), and green (IV) lines]. The arrows indicate the processes starting from a BEC at $q=0$ and inducing the creation of excitations in the different bands at a quasimomentum transfer $0.12q_L$.

*fabbri@lens.unifi.it

direction ($\hat{y}$ axis). Two counterpropagating laser beams with wavelength $\lambda_L=830$ nm create the lattice potential $V(y)=sE_R\sin^2(q_L y)$, where $q_L=2\pi/\lambda_L=7.57\ \mu\text{m}^{-1}$ is the wave number, s measures the height of the lattice in units of the recoil energy $E_R=h^2/2m\lambda_L^2\simeq h\times 3.3$ kHz, and m is the mass of a ^{87}Rb atom. The loading of the BEC in the lattice is performed by slowly increasing the laser intensity up to a height s with a 140-ms-long exponential ramp with time constant $\tau=30$ ms.

After a holding time of typically 20 ms in the lattice at the height s, we excite the gas by shining two off-resonant laser beams (Bragg beams) for a time $\Delta t_B=3$ ms. The Bragg beams induce a two-photon transition transferring momentum and energy to the atomic sample. Their wavelength is $\lambda_B=780$ nm, corresponding to a wave number $q_B=2\pi/\lambda_B=8.05\ \mu\text{m}^{-1}$, and they are typically detuned by 350 GHz with respect to the D_2 transition of ^{87}Rb. To change the transferred momentum we use two different geometries of the Bragg beams. In the first configuration the two beams are counterpropagating along the $\hat{y}$ direction and the transferred momentum is $q=2q_B=2.12q_L$, which corresponds to a quasimomentum $0.12q_L$ in the first Brillouin zone. In the second configuration the angle between the Bragg beams is smaller and the measured value of the transferred momentum (and quasimomentum, in this case) along the $\hat{y}$ direction is $q=0.96q_L$. In both the cases, the two beams are detuned from each other by a frequency difference ν using two phase-locked acousto-optic modulators. We quantify the response to the excitation by measuring the energy transferred to the gaseous BEC. The measurement of the energy transfer $E(\nu,q)$ is connected with the dynamical structure factor $S(\nu,q)$ (giving information on the excitation spectrum) by the relation [1]

$$E(\nu,q)\propto \nu S(\nu,q), \tag{1}$$

where ν and q are the frequency and the momentum of the excitation. In particular, this result applies for long enough Bragg pulses, namely, $\nu\Delta t_B\gg 1$, which is the case in our experiment since ν is on the order of several kHz.

In order to get an estimate of the transferred energy $E(\nu,q)$, we adopt the following procedure. We linearly ramp in 15 ms the longitudinal optical lattice from s (the lattice height at which we have applied the Bragg pulse) to the fixed value $s_f=5$. Then we let the excitation be redistributed over the entire system by means of the interatomic collisions for 5 ms. After this time interval we abruptly switch off both the optical lattice and the magnetic trap, letting the cloud expand for a time of flight $t_{\text{TOF}}=20$ ms and we then take an absorption image of the density distribution integrated along the $\hat{x}$ axis. Since the atoms are released from an optical lattice of relatively small amplitude ($s_f=5$), the density distribution exhibits an interference pattern [21]. We extract the rms width σ of the central peak of this density distribution by fitting it with a Gaussian function. The increase $\Delta\sigma$ in this quantity is used as a measurement of the energy transfer [19]. For a given value of the transferred momentum q and amplitude s of the lattice, this procedure is repeated varying the energy $h\nu$ of the excitation in order to obtain the spectrum. In our regime of weak interatomic interactions, the

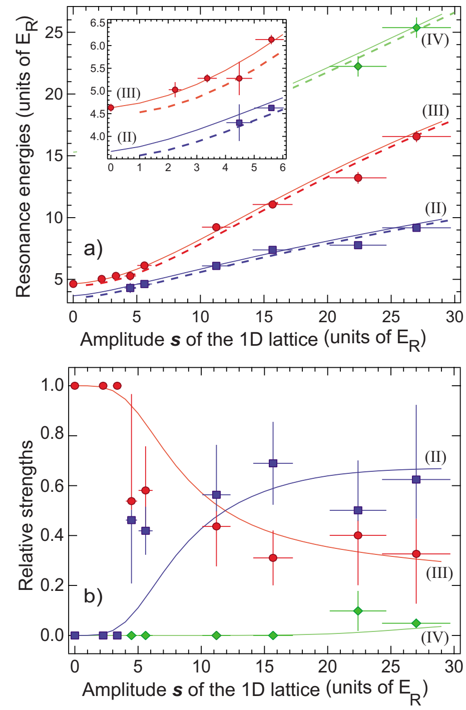

FIG. 2. (Color online) (a) Band spectroscopy of a BEC in the presence of a 1D optical lattice: the energy of the resonances is reported as a function of the height s of the lattice. The experimental points (blue squares, red circles, and green diamonds) are compared with the numerical calculation of the Bogoliubov spectrum in the presence of a 1D lattice (solid lines) and the single-particle Bloch spectrum (dashed lines). The lines correspond to the energies of an excitation in the second [blue (II) line], third [red (III) line], and fourth [green (IV) line] Bogoliubov bands. Inset of (a): zoom of graph (a) for low values of s. (b) Relative strengths of the excitations in the second (II), third (III), and fourth (IV) bands. Symbols and colors (numbers) are the same as in (a).

excitation spectrum of the BEC in the presence of a 1D optical lattice can be described by the mean-field Bogoliubov approach [3,5], by which we calculate resonance frequencies ν_j and transition strengths Z_j to create an excitation in the Bogoliubov band j.

We first discuss the results obtained with the configuration of counterpropagating beams, i.e., for a transferred momentum $q=2.12q_L$. The induced two-photon transition is characterized by a measured Rabi frequency for the BEC in the absence of the optical lattice $\Omega_R\simeq 2\pi\times 1$ kHz for the typical power and detuning of the beams used in the experiment.

A typical Bragg spectrum is presented in Fig. 1(a) corresponding to a lattice height $s=(22\pm 2)$. The spectrum exhibits multiple resonances corresponding to the creation of excitations in the different Bogoliubov bands as shown in Fig. 1(b). From Gaussian fit of each resonance we extract central frequency, width, and relative strength of the transition toward the corresponding band. In Fig. 2(a), we plot the energy values corresponding to the measured central frequencies as a function of s. Vertical error bars come from the result of the fitting procedure, while horizontal error bars correspond to possible systematic errors in the lattice calibration (estimated within 10%). For large enough amplitude s of the periodic

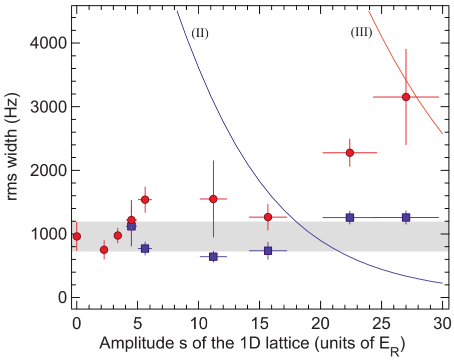

FIG. 3. (Color online) rms width of the resonances to the second (blue squares) and third (red circles) bands as a function of s. The gray region corresponds to the experimental rms width (with its uncertainty) for the BEC in the absence of the lattice ($s=0$). The blue (II) and red (III) lines are, respectively, the bandwidths of the second and the third bands, calculated in the mean-field Bogoliubov approach.

potential we observe up to three different bands. We find good agreement between the experimental data and the numerical results of the Bogoliubov calculation [solid lines in Fig. 2(a)]. In particular, for low amplitudes of the 1D lattice ($s<6$) the agreement of the resonance energies with the Bogoliubov bands (full lines) is better than with the single-particle (dashed lines) Bloch bands [see inset of Fig. 2(a)]. For larger amplitude of the 1D lattice, we cannot explicitly distinguish between the Bogoliubov and Bloch results. This comes from the experimental uncertainty on the calibration of the lattice amplitude.

In the entire range of s values used in this work, we observe a resonance corresponding to an excitation created in the third band (red circles in Fig. 2, labelled III). For larger lattice amplitudes two other resonances appear, respectively, for $s>4$ and $s>20$, corresponding to excitations in the second band [blue squares in Fig. 2(a)] and in the fourth band [green diamonds in Fig. 2(a)]. This demonstrates the possibility to excite, in a periodic system, several states for a given momentum transfer [8]. For weak optical lattices creating an excitation in the third band is the most efficient process since the excitation energy of this band is continuously connected as $s\rightarrow 0$ to that of the BEC in the absence of the 1D optical lattice at the transferred momentum $q=2.12q_L$. On the contrary, the possibility to excite states in the second and fourth bands of the optical lattice requires a large enough amplitude s. These observations can be quantified in terms of the strength Z_j of the different excitations, which can be extracted from the energy spectrum. The strengths Z_j are proportional to the integral $\int d\nu S_j(q,\nu)$, with $S_j(q,\nu)$ being the structure factor corresponding to the creation of an excitation in the Bogoliubov band j [5]. From Eq. (1) and assuming that ν_j is much larger than the width of the resonances of $S_j(q,\nu)$, we obtain

$$Z_j(q) \propto \int d\nu S_j(q,\nu) \propto \frac{1}{\nu_j}\int d\nu E_j(q,\nu) \equiv g_j. \qquad (2)$$

In the experiment, we extract the quantity g_j from a Gaussian fit of the different resonances. Normalizing the sum

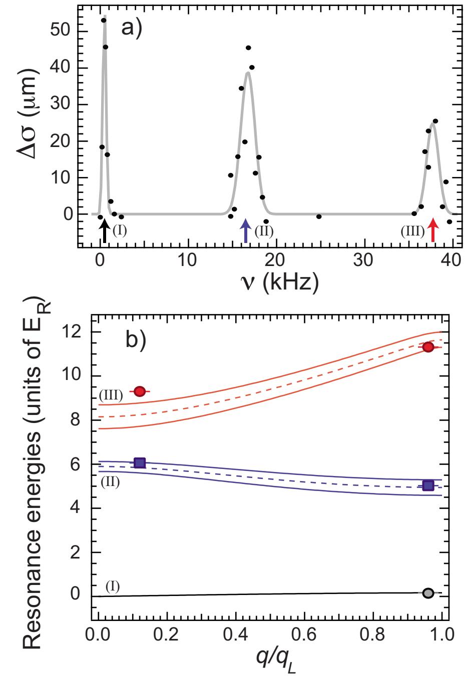

FIG. 4. (Color online) (a) Excitation spectrum of a BEC in the presence of a 1D lattice with height $s=11$ at a transferred momentum $q=0.96q_L$ along the $\hat{y}$ direction. The arrows below the resonances indicate the corresponding bands, represented in (b) with the same colors (numbers). (b) Energies of the resonances corresponding to excitations in bands I (black circle), II (blue squares), and III (red circle) as functions of the transferred quasimomentum q for a fixed value of the lattice height $s=(11\pm 1)$. The experimental points are compared with the numerical calculation of the energy bands in the Bogoliubov approach for $s=11$ [black (I), blue (II), and red (III) dashed lines]. The solid lines correspond to the bands for $s=10$ and $s=12$ to take into account the 10% uncertainty of s.

of these quantities to one for the first three observed resonances ($g_{\rm II}+g_{\rm III}+g_{\rm IV}=1$) allows direct comparison with the relative strengths $Z_j/(Z_{\rm II}+Z_{\rm III}+Z_{\rm IV})$ for $j={\rm II},{\rm III},{\rm IV}$. The comparison between the experimental data and the calculation reveals reasonable agreement [see Fig. 2(b)].

From the Gaussian fit of the experimental spectra [see Fig. 1(a)], we also extract the rms width of the resonances toward the second and third bands with the results plotted in Fig. 3. Different sources contribute to broaden the observed resonances. The inhomogeneous density of the trapped BEC is a first source [11]. From the measured spectrum of the BEC in the absence of the optical lattice ($s=0$), we extract this contribution as being (0.36 ± 0.11) kHz, consistent with the expected value of $\simeq 0.26$ kHz [11]. The other contributions to the width are related to the Bragg spectroscopic scheme. For our experimental parameters the largest contribution comes from the power broadening ($\Delta\nu_P\simeq 1$ kHz), whereas the atom-light interaction time broadening ($\Delta\nu_t \simeq 167$ Hz) is much smaller. The total resonance width can be obtained by quadratically adding up all these rms contributions. In the presence of the optical lattice we observe that the widths of the resonances corresponding to the excitations in bands II and III lie within the experimental range of the resolution as expected for a coherent system, except in the case of the third band (III) for large amplitudes of the lattice

($s>20$) where the width is much larger. We attribute these larger widths at high amplitude of the 1D lattice to the long tunneling times (0.11 s for $s=20$), implying that the system is not fully coherent along the $\hat{y}$ direction on the time scale of the experiment. Indeed, the loss of coherence spreads the population of quasimomenta across a larger fraction of the Brillouin zone. This results in a wider range of resonance energies in the system and, for large amplitude s, one expects the width of the resonances in the energy spectrum to be equal to the bandwidths. In Fig. 3 we have plotted the bandwidths of the second and third bands [blue (II) and red (III) lines]. When the system becomes incoherent the width of the resonance toward the third band is equal to the bandwidth. This effect is not observable for the second band where the bandwidth [blue (II) line in Fig. 3] is smaller than the experimental resolution.

We also perform the experiment with a different configuration of the Bragg beams corresponding to a transferred momentum along $\hat{y}$, $q=0.96q_L$. In Fig. 4(a) an excitation spectrum in the presence of an optical lattice of height $s=11$ is depicted. Note that a first resonance at low frequency is visible corresponding to an excitation with nonzero momentum within the lowest energy band (I). Such a resonance is not observed using counterpropagating Bragg beams because the strength of this transition is negligible for $q=2.12q_L$. Due to the variation in the transferred momentum q with respect to the previous case, the frequencies of the resonances are shifted according to the dispersion relation of the different energy bands of the system. In Fig. 4(b) we report the frequencies of the resonances to bands I–III for the two values of quasimomentum used in the experiment ($0.12q_L$ and $0.96q_L$). We use the region comprised between the calculated bands for $s=10$ and $s=12$ (solid lines) to take into account the 10% error in the lattice calibration. The experimental points are in good agreement with the numerical calculation of the Bogoliubov bands for $s=11$ [dashed lines in Fig. 4(b)].

In conclusion, Bragg spectroscopy has been used to probe the response of a Bose-Einstein condensate in the presence of a 1D optical lattice. Changing the angle of the Bragg beams allowed us to investigate excitations for a transferred quasimomentum close to the center and to the edge of the reduced Brillouin zone. We have observed different resonances in the response function of the system corresponding to the different bands of the periodic potential. The system being in a weakly interacting regime, the experimental results are in quantitative agreement with the Bogoliubov band approach. This work opens the way to investigation of the excitation spectrum in the presence of an additional lattice with different wavelength (bichromatic potential) [22] and eventually to study of the localization of the excitations in a true disordered potential [23].

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