Entropy: a counterpart in statistical energy analysis

Alain Le Bot, Alexandre Carbonelli, J. Perret-Liaudet

18TH INTERNATIONAL CONGRESS ON SOUND & VIBRATION
ICSV18
RIO DE JANEIRO
BRAZIL
10–14 JULY 2011

ENTROPY: A COUNTERPART IN STATISTICAL ENERGY ANALYSIS

A. Le Bot, A. Carbonelli, J. Perret-Liaudet

Laboratoire de tribologie et dynamique des systèmes UMR 5513 CNRS
École centrale de Lyon, Ecully, France.

Statistical energy analysis (SEA) is the most famous method intended to prediction of sound and vibration in high frequency range with random excitations. Although SEA is largely inspired from statistical mechanics and thermodynamics, it is up to now limited to the application of the first principle of thermodynamics. In this study, we introduced the related entropy concept and it is shown that an entropy balance is possible, as a counterpart of the energy balance. An illustration of the approach is proposed.

1. Introduction

Statistical energy analysis is an attempt to model the vibro-acoustical behaviour of structures in the frequency domain where statistics on modes begin to operate efficiently. However, the equations of motion are still valid in this domain and we could raise the question of utility of SEA. The simplicity of SEA compared with the huge amount of calculation required to solve the equations of motion by finite element method in the high frequency domain is a strong argument in favour of the statistical approach. This situation is quite similar to the problem of turbulence in fluid mechanics. Navier-Stoke's equation applies for both laminar and turbulent fluids but its direct resolution, the so-called direct numerical simulation, highlights the limitation of the brute force approach.

The statistical approach applied to small wavelengths are however conceptually interesting. The employed method is the same as in statistical physics but at a different scale. So, like in statistical physics, the fact that in SEA we *deliberately* neglect a part of information raises the question of existence of entropy and its meaning. This is what is done in this paper.

2. Energy and SEA

Statistical energy analysis is based on application of the energy balance to sub-systems. Basically, the primary variables of sub-systems are the vibrational energy E_i and mode count N_i for $i = 1 \ldots n$ contained within a frequency band of width $\Delta\omega$ and central frequency ω (typically an octave band). The sources assumed to be white noise mutually uncorrelated, are characterized by their injected power P_i.

The dissipation of vibration occurring in sub-systems is summarized in a single damping law,

$$P_i^{\text{diss}} = \eta_i \omega E_i \tag{1}$$

which states that the dissipated power is proportional to the energy level.

The net power exchanged between sub-systems i and j follows the law,

$$P_{ij} = \omega \left(\eta_{ij} E_i - \eta_{ji} E_j \right) \tag{2}$$

where the coupling loss factors verify the reciprocity relationship $\eta_{ij} N_i = \eta_{ji} N_j$. This law of proportionality of power and difference of modal energies constitutes the main result of SEA. It has been first derived from a modal approach in Refs. [1, 2, 3]. In Refs. [4, 5], its derivation is based on the geometrical approach with rays. The notion of statistical ensemble is of a great importance [6] particularly with regard to the question of variability of responses [7, 8]. Finally, in Ref. [9], some interesting arguments based the H-theorem are given in favour of this law.

The energy balance of sub-system i in steady-state condition reads,

$$\omega \eta_i E_i + \sum_{j \neq i} \omega \left(\eta_{ij} E_i - \eta_{ji} E_j \right) = P_i \tag{3}$$

This is a set of linear equations on unknowns E_i. It can be used for instance to calculate E_i from the knowledge of P_i.

3. Entropy and SEA

In this section are summarized all equations related to usage of entropy in SEA. The proofs and comments may be found in Refs. [10, 11, 12].

Entropy of a sub-system with vibrational energy E and number of modes N is,

$$S = kN \left[1 + \log \left(\frac{2\pi E}{h\omega N} \right) \right] \tag{4}$$

where $k = 1.38\ 10^{-23}$ J K^{-1} is Boltzmann's constant, $h = 6.63\ 10^{-34}$ m^2 kg/s is Planck's constant and ω the central frequency of bandwidth in rad/s. The physical unit of entropy is J/K.

The vibrational temperature of sub-systems is defined as,

$$T = \frac{E}{kN} \tag{5}$$

with physical unit K.

In a driven sub-system, excitation forces tend to warm up it by injecting energy. The rate of entropy being injected in sub-system i is,

$$\frac{dS_i^{\text{inj}}}{dt} = \frac{P_i}{T_i} \tag{6}$$

where T_i is the vibrational temperature and P_i (W) the power being injected by driven forces.

Dissipation of vibration by natural damping processes tend to cool down the sub-system since damping is responsible of reduction of energy. The rate of entropy being extracted is,

$$\frac{dS_i^{\text{diss}}}{dt} = \frac{\eta_i \omega E_i}{T_i} = k \eta_i \omega N_i \tag{7}$$

Finally, during exchange of power between sub-systems, a mixing entropy is created. This is a irreversible process. The rate of entropy created at interfaces is,

$$\frac{dS_{ij}^{\text{irr}}}{dt} = k \eta_{ij} \omega N_i \frac{N_i N_j}{E_i E_j} \left(\frac{E_i}{N_i} - \frac{E_j}{N_j} \right)^2 \tag{8}$$

It can readily be checked that a global entropy balance holds,

$$\sum_{i=1}^{n} \frac{dS_i^{\text{inj}}}{dt} + \frac{dS_i^{\text{diss}}}{dt} + \sum_{i>j} \frac{dS_{ij}^{\text{irr}}}{dt} = 0 \tag{9}$$

meaning that in steady-state condition, no entropy is supplied to the system.

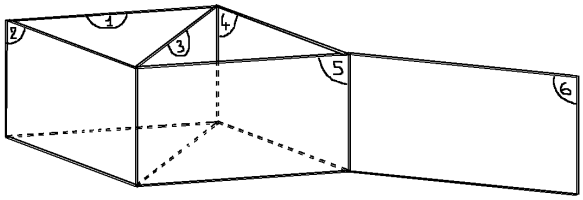

Figure 1. System composed of six coupled plates.

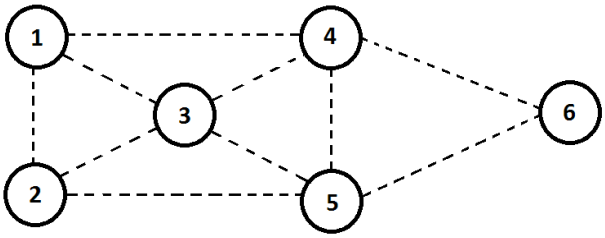

Figure 2. Sub-system connection graph.

4. Numerical simulation

The studied system is composed of six plates coupled by their edges as shown in Fig. 1. Each plate is considered as being one sub-system. The power is injected in plate 1 and the receiver is plate 6. The sub-system connection graph is shown in Fig. 2. It can be seen that from plate 1 to plate 6, a large variety of transmission paths is allowed.

SEA calculations are performed by using a home built code named CeReS which provides all usual information, i.e. coupling loss factors, modal energies, modal densities, and modal overlaps for all plates. The material characteristics of plates need to be provided as input data of the software as well as their exact geometry. Plates 1, 2, 4 and 5 have area equal to 7.2 m^2. Plates 3 and 6 have area 8 m^2. All of them have same width of 2 m and same thickness of 1 mm. The material characteristics are listed in Tab. 1.

When using SEA, the classical coupling loss factors of plate-plate junction issued from the wave-based approach is adopted *i.e.* $\eta_{ij} = Lc\bar{\tau}/\pi\omega S_i$ where L is the coupling length, c the group speed of flexural waves, $\bar{\tau}$ the mean transmission efficiency and S_i the plate area. To determine the transmission efficiency of the coupling, we consider the continuity of deflection, rotation and the balance of forces and moments. Other approaches to estimate coupling loss factors are possible [13].

The analysis is performed in the octave band centred on 1000 Hz.

5. Comments

The plates are characterized by four dimensionless parameters: The number N of modes in the octave band, the modal overlap M and the normalized attenuation factor (attenuation per mean free path) $\bar{m}$ [14] are listed in Tab. 2; Coupling strengths between connected plates γ_{ij} according to the Gamma criterion are listed in Tab. 3 [15].

It can be observed that the mode count and modal overlap are both high enough to justify the statistical approach. The normalized attenuation factor is low compared to unity, to ensure the field

Table 1. Plates material characteristics.

Young Modulus E	Density ρ	Poisson's ratio ν	Damping loss factor η
71 10^9 Pa	2400 kg/m^3	0.3	0.1 %

Table 2. Plate modal characteristics.

	N	M	$\bar{m}$
Plates 1,2,4,5	1550	2.191	0.0624
Plates 3,6	1720	2.430	0.0647

Table 3. Plate coupling strenghs.

γ_{ij}	$j=1$	2	3	4	5	6
$i=1$	-	0.2724	0.1116	0.1219	0	0
2	0.2724	-	0.1091	0	0.1220	0
3	0.1117	0.1092	-	0.1117	0.1090	0
4	0.1219	0	0.1116	-	0.1199	0.1073
5	0	0.1220	0.1091	0.1199	-	0.1141
6	0	0	0	0.1073	0.1141	-

to be diffuse [16]. And eventually, the coupling strength values are low.

Energy and entropy repartition over the sub-systems are presented respectively in Figs. 3 and 4. For both figures, the thickness of arrows and size of circles standing for each sub-systems are proportional to the related quantity. Comparison of these two figures leads to the following observations:

- The maximum of entropy created by mixing roughly corresponds to the maximum of energy transfer and both are located close to the driven plate,
- sub-system entropy levels are quite different from sub-system energy levels,
- for all sub-systems, the dissipated power is proportional to the energy level (see Eq. (1)),
- for all sub-systems, the dissipated entropy rate is proportional to the entropy level (this was not obvious from the above equations).

The energy paths are also investigated [17]. Figure 5 shows the energetic contribution of transmission paths from plate 1 to plate 6 respectively for paths having 3, 4 and 5 nodes. We consider all the energy paths possible between those two plates and order them according to their energy contribution. The most contributive paths (146 and 1256) are those for which the entropy created by mixing is maximum.

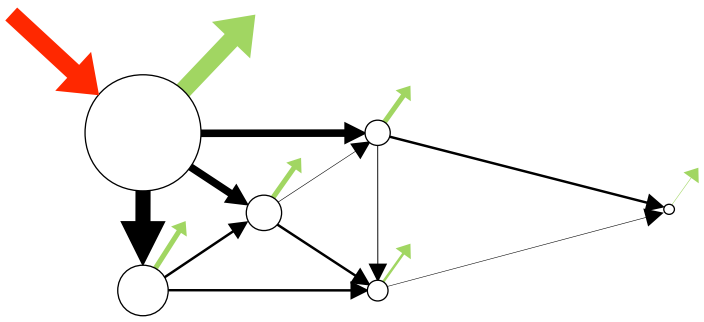

Figure 3. Energy repartition over the sub-systems. Red arrow, injected power; Black arrow, exchanged power; Green arrow, dissipated power; Black circle, sub-system energy.

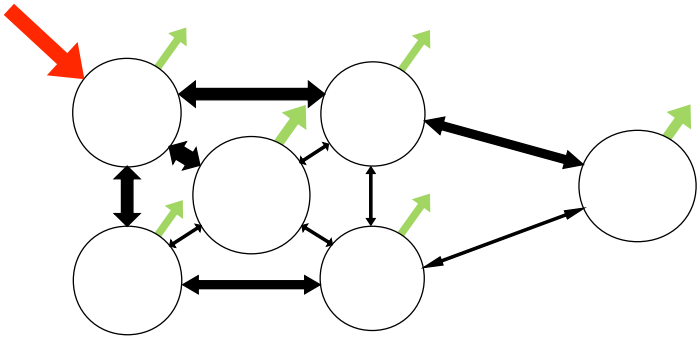

Figure 4. Entropy repartition over the sub-systems. Red arrow, injected entropy rate; Black arrow, mixing entropy rate; Green arrow, dissipated entropy rate; Black circle, sub-system entropy

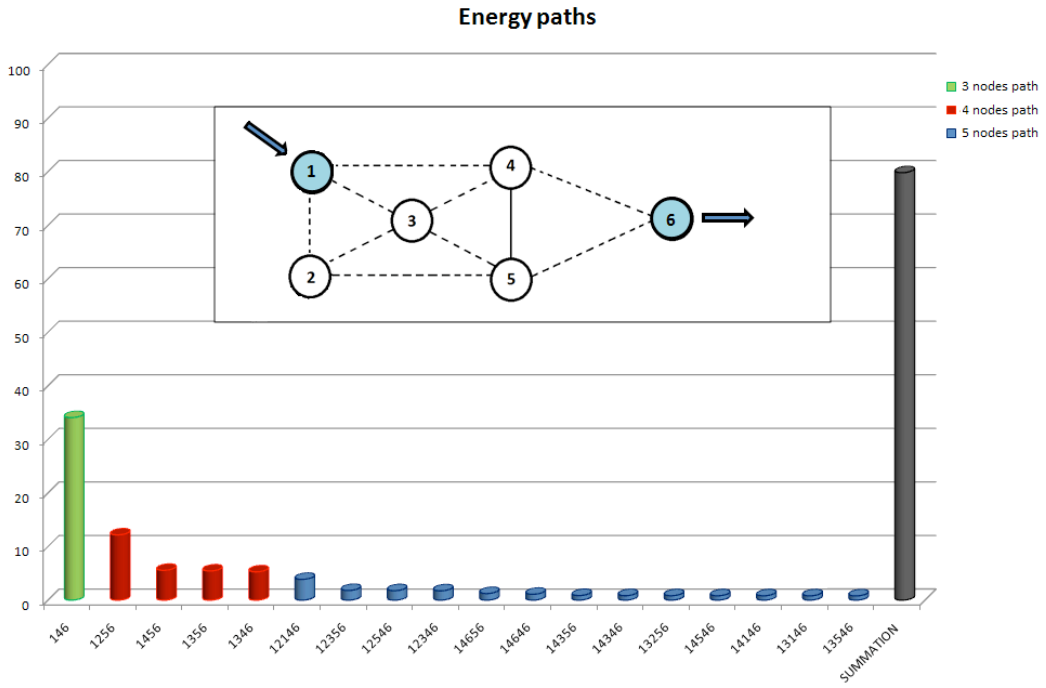

Figure 5. Classification of energy paths by order of importance.

6. Acknowledgments

This work has been supported by ANR (National Research Agency, contract number ANR-08-VTT-007-02), ADEME (French Environment and Energy Management Agency) and Lyon Urban Trucks & Bus competitivness cluster. The authors acknowledge gratefully this support.

REFERENCES

[1] T.D. Scharton and R.H. Lyon, Power flow and energy sharing in random vibration, *Journal of the Acoustical Society of America* **43** 1333–1343 (1968).

[2] D.E. Newland, Power flow between a class of coupled oscillators, *Journal of the Acoustical Society of America* **43** 553–559 (1968).

[3] B. Mace, L. Ji, The statistical energy analysis of coupled sets of oscillators, *Proc. R. Soc. A* **463** 1359–1377 (2007).

[4] A. Le Bot, Derivation of statistical energy analysis from radiative exchanges, *Journal of Sound and Vibration* **300**, 763–779 (2007).

[5] G. Tanner, Dynamical energy analysis-Determining wave energy distribution in vibro-acoustical structures in the high frequency regime, *Journal of Sound and Vibration* **320**, 1023–1038 (2009).

[6] B. Mace, Statistical energy analysis, energy distribution models and system modes, *Journal of Sound and Vibration* **264** 391–409 (2003).

[7] N. Driot, E. Rigaud, J. Sabot and J. Perret-Liaudet, Allocation of gear tolerances to minimize gearbox noise variability, *Acustica united with Acta Acustica* **87**, 67–76 (2001).

[8] R.S. Langley, V. Cotoni, Response variance prediction in the statistical energy analysis of built-up systems, *Journal of the Acoustical Society of America* **115**, 706–718 (2004).

[9] A. Carcaterra, An entropy formulation for the analysis of energy flow in mechanical resonators, *Mechanical systems and signal processing* **16**, 905–920 (2002).

[10] A. Le Bot, Entropy in statistical energy analysis, *Journal of the Acoustical Society of America* **125**, 1473–1478 (2009).

[11] A. Le Bot, A. Carcaterra and D. Mazuyer, Statistical vibroacoustics and entropy concept, *Entropy* **12**, 2418–2435 (2010).

[12] A. Le Bot, Statistical energy analysis and the second principle of thermodynamics, in *IUTAM symposium on the vibration analysis of structures with uncertainties*, Edited by A.K. Belyaev and R.S. Langley, Springer, (2011).

[13] N. Totaro, C. Dodard and J.L. Guyader, SEA coupling loss factors of complex vibro-acoustics systems, *J. Vib. Acoust. Trans. ASME* **131**, 041009-1 (2009).

[14] A. Le Bot and V. Cotoni, Validity diagram of statistical energy analysis, *Journal of Sound and Vibration* **329**, 221–235 (2010).

[15] S. Finnveden, A quantitative criterion validating coupling power proportionality in statistical energy analysis, *Journal of Sound and Vibration* **330**, 87–109 (2011).

[16] R.L. Weaver, Equipartition and mean-square responses in large undamped structures, *Journal of the Acoustical Society of America* **110**, 894–903 (2001).

[17] O. Guash and A. Aragonès, Finding the dominant energy transmission paths in statistical energy analysis, *Journal of Sound and Vibration* **330**, 2325–2338 (2011).