Streamwise rotating Poiseuille flow : modal and non-modal stability analyses

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STREAMWISE ROTATING POISEUILLE FLOW: MODAL AND NON-MODAL STABILITY ANALYSES

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Summary A better understanding of transition to turbulence in a Poiseuille flow rotating about the streamwise axis is sought by investigating the stability of the flow. Using the classical linear modal analysis, we define the instability envelop and find that the rotation increases the exponential growth of the most unstable mode and, for high levels of rotation, we observe a re-stabilization of the flow. The influence of rotation on transient energy growth is also investigated as the set of linear equations is non-normal. We show that the energetic growth can be of the order of $O\left(10^3\right)$ in the sub-critical region but for high Ro the non-normality of the equation set is reduced, resulting in a decrease of the maximal energetic growth. The maximal growth is achieved by the purely spanwise disturbances until a certain point where the maximal energy growth is caused by oblique ones.

1 Governing equations and geometry of the flow

The application of system rotation to canonical wall-bounded flows is of great academic and engineering interest. From an academic perspective, these flows provide unique insight into the influence of body forces on the flow. From an engineering point of view, the presence of rotating flow is ubiquitous in most turbo-machinery applications making the study of these flows of great practical relevance. To the knowledge of the authors, [1] were the first to study the streamwise rotating Poisseuille flow using Lie group analysis and direct numerical simulations (DNS). Althought the flow setup share some similarities with the classical spanwise rotating channel flow, the streamwise rotation, in the present case, does not break the centerline symmetry. More recently, an experimental investigation was conducted at the RWTH-Aachen ([2]) and a DNS study was performed to verify the results by [3]. A study conduced by [4] shed light on the non-modal stability characteristics of this flow. However, their study overlooked the importance of non-modal transient growth in the flow.

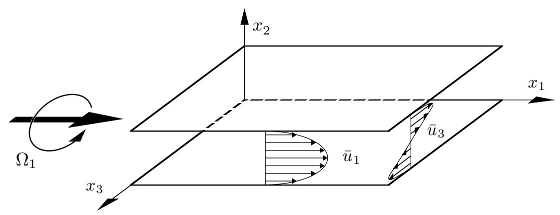

Figure 1: Streamwise rotating Poiseuille flow.

In the present work, we investigate the modal and non-modal stability characteristics of a streamwise rotating Poiseuille flow (figure 1). The non-dimensionalised Navier–Stokes and the continuity equations for the incompressible rotating channel flow are:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\,\boldsymbol{u} = -\boldsymbol{\nabla} P + \frac{1}{Re}\nabla^2 \boldsymbol{u} - Ro \times \boldsymbol{u}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

Reynolds and Rossby numbers are defined in the following form: $Re = U_{\max} h/\nu$, $Ro = 2\Omega_1 h/U_{\max}$. ν is the kinematic viscosity, $U_{\max}$ is the maximal base flow velocity and h is the half-height of the channel. We consider only a streamwise rotation (see figure 1), hence $\boldsymbol{\Omega} = (\Omega_1, 0, 0)$.

2 Exponential and transient growths

The numerical calculations were performed in MATLAB using a spectral collocation method based on Chebyshev polynomials modified from the code developed by S. Reddy ([5]). The original code was extended to include the additional system rotation.

By setting $Re = 100$ and evaluating the effective perturbations for different rates of rotation in the figures 2 we observe the appearance of spectral instabilities from a Rossby number of $Ro = 0.246$ at $\alpha \approx 0.55$ and $\beta \approx 2.0$. By increasing the Rossby number to $Ro = 0.5$ (right plot in the figure 2), one can see a great range of perturbations (represented by the shaded area) that are able to destabilise the flow.

In modal analysis, a conclusion on the stability must be made without regards to the evolution of the problem at the near initial state. We present the results of non-modal analysis of the flow and determine the

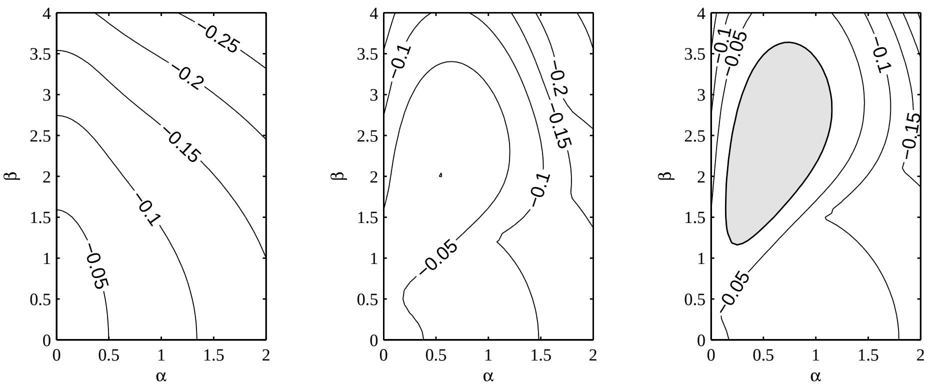

Figure 2: $Re = 100$. Results corresponding to Rossby numbers $Ro = 0,\ 0.246,\ 0.5$ are presented on left, middle and right plots correspondingly. The contour lines represent the value of the maximal imaginary eigenvalue for every perturbation couple. The first spectral instability for $Re = 100$ is seen at a rotation of $Ro = 0.246$.

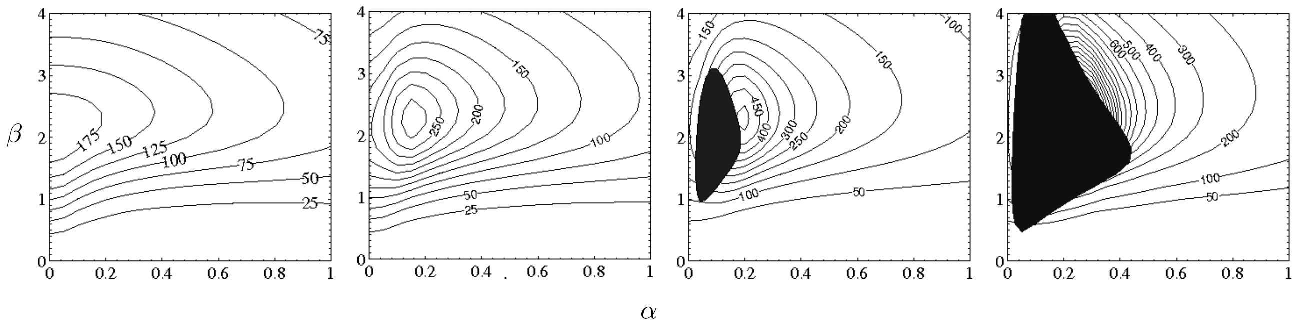

Figure 3: The energetic transient growth is shown for a large spectrum of perturbations at $Re = 1000$ with $Ro = 0.001$, 0.015, 0.025 and 0.05. The shaded area represents spectral instability calculated with the help of modal analysis. Contour lines represent the ratio of the maximum turbulent kinetic energy growth defined as $sup\frac{|u^2(\cdot,t)|}{|u^2(\cdot,0)|}$.

possible energetic growth the the perturbations. Non-modal analysis reveals that streamwise rotation allows for a significant level of transient energy growth to occur (up to $O(10^3)$) in exponentially stable flow. At high *Ro*, the transient growth is significantly reduced as the linearised governing equation set reaches an (nearly) anti-symmetric form for which the non-normality of the operator set is greatly reduced. For a given *Ro*, the initial conditions leading to the maximal energy growth change from a 2D (streamwise independant) to a $3D$ perturbation at a given Reynolds number. As the rotational terms are streamwise dependant, the shift represents the minimal Reynolds number at which the transient growth is caused primarily by the rotational body force.

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