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1 Introduction

Vortex breakdown occurs in some swirling flows, such as those in gas turbine combustion chambers. Ruith *et al* [1] and Gallaire *et al* [2] established that the initial breakdown is steady and axisymmetric but that an unsteady spiralling breakdown mode develops on top of this, due to a region of absolute instability.

In this paper, we investigate the linear stability of the steady axisymmetric solutions obtained by Vyazmina *et al* [3] for vortex breakdown in incompressible flows at Re = 200. We compare local and global stability analyses and relate the global behaviour of the flow to its local stability properties. Using results from the global stability analysis, we map the regions of the flow that are most sensitive to external forcing and internal feedback.

2 Methodology

We use a Low Mach Number (LMN) formulation of the Navier–Stokes (N–S) equations to obtain steady axisymmetric solutions in a cylindrical domain with open lateral and convective outlet boundaries. The Grabowski velocity profile [1] is imposed at the inlet. We study the linear stability of these flows for a range of values of the swirl parameter.

In the global stability analysis, we superpose small perturbations of the form $\hat{\mathbf{u}}(x,r)\exp(\mathrm{i}m\theta+\sigma t)$, where m is the azimuthal wavenumber and σ is the complex eigenvalue. The direct and adjoint Linearized N–S (LN–S) equations are discretized with a compact finite difference scheme in space and a 4^{th} order Runge Kutta scheme in time. The resultant matrix eigenvalue problems are solved with the implicitly restarted Arnoldi algorithm. The regions that are most sensitive to external forcing and internal feedback are found from the direct and adjoint eigenmodes [4].

In the local stability analysis we apply the WKBJ approximation (*i.e.* we assume a locally parallel base flow) and superpose small perturbations of the form $\hat{\mathbf{u}}(r)\exp(\mathrm{i}(kx+m\theta-\omega t))$, where k is the local complex wavenumber and ω is the local complex angular frequency. The absolute frequency, ω_0, is calculated at each axial location. This distribution of ω_0 is interpolated with a Padé polynomial and then continued analytically into the complex x-plane. The linear global frequency, ω_g (which is equivalent to $\mathrm{i}\sigma$), and the wavemaker position are estimated from the position of the relevant saddle point of ω_0 in the complex x-plane. The response of each slice to the estimated global frequency, ω_g, is then calculated and the WKBJ approximation is inverted in order to obtain the mode shape $\tilde{\mathbf{u}}(x,r)$.

3 Results

Figure 1 shows the base flow at a swirl parameter of $Sw = 1.0$, together with the absolute growth rate, the most unstable direct and adjoint global modes and the structural sensitivity maps for $m = 1$ helical perturbations. There is an axisymmetric breakdown bubble around $x = 1$, which causes a wake downstream. The global stability analysis predicts one unstable eigenmode, which has $\mathrm{i}\sigma = 1.1655 + 0.0352\mathrm{i}$. This matches the results from nonlinear DNS [1], suggesting that linear wavefront theory is valid in this case. The direct global mode, which represents the flow structure that dominates in the long time limit, has maximum amplitude at $x = 9$, in the wake downstream of the bubble. The adjoint global mode, which represents the most unstable initial condition or the receptivity of the direct mode to external forcing, is localized between the inlet and the upstream edge of the bubble.

The direct and adjoint global modes are combined to give the structural sensitivity maps. These show where a modification in the linearized equations produces the greatest drift of the eigenvalue [4]. The first map shows the regions that have most influence on the growth rate of the direct global mode. The second map shows the regions where there is maximum coupling between the direct and adjoint velocity components, which indicates the core of the instability (Fig. 17 of [4]). We find that the instability core (as defined by [4]) is located in the upstream end of the breakdown bubble. By considering the global stability of the breakdown bubble and the wake separately, we find that the global frequency of the flow matches that of the bubble rather than that of the wake. We conclude that the wavemaker region is in the recirculation bubble, contrary to expectation.

The local stability analysis predicts two regions of absolute instability: a small region corresponding to the bubble and a large region corresponding to the wake. We find that the instability core obtained from the global analysis corresponds to the region of absolute instability in the bubble. This supports the conclusion that the linear global mode is being driven by a wavemaker situated in the breakdown bubble.

For a given azimuthal wavenumber, m, more eigenmodes become unstable as the swirl increases. Furthermore, eigenmodes at higher m also become unstable as the swirl increases. The local analysis indicates that there are one or two valid saddle points (*i.e.* k^+/k^- pinch points) at each streamwise location. Broadly, these group into one saddle point that dominates in the bubble and another saddle point that dominates in the wake. As the swirl increases, the saddle point in the wake becomes more absolutely unstable.

We find that the local analysis is valid only for slowly developing, weakly non-parallel flows. Furthermore, it cannot conclusively identify the location of the wavemaker region in flows which have more than one region of absolute instability.

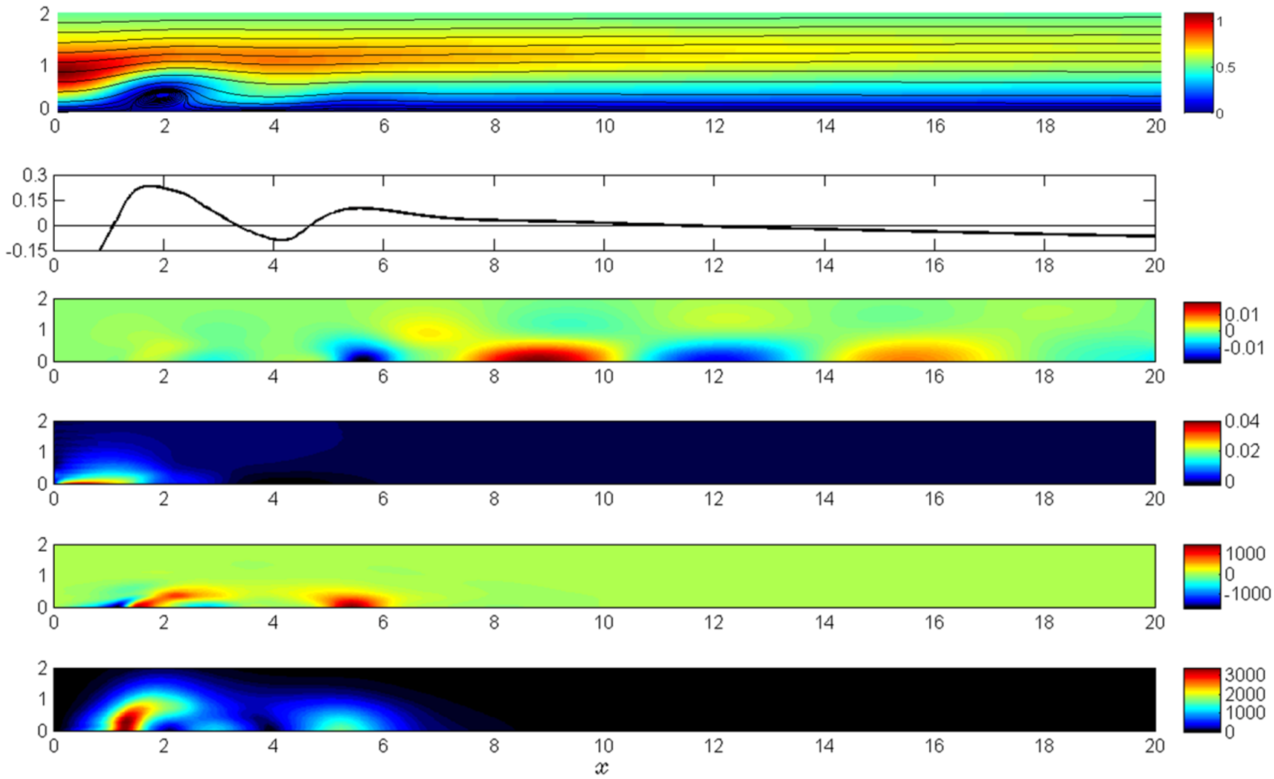

Figure 1: (a) streamlines and azimuthal velocity (colour) of the base flow for $Sw = 1.0$; (b) local absolute growth rate, $\omega_{0,i}$; (c) real component of the direct global mode for radial velocity; (d) real component of the adjoint global mode for radial velocity; (e) sensitivity of growth rate; (f) structural sensitivity as defined by [4]

References

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