**LOCAL LINEAR STABILITY ANALYSIS OF A TURBULENT SWIRLING JET UNDERGOING VORTEX BREAKDOWN**

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# 1 Introduction

Swirling jets are known to exhibit a variety of instability phenomena. A number of different stability theories are available today that can partly explain these phenomena. Of the many theories the linear theory applied to a laminar base flow has probably been studied the most. Although this theory is in a strict theoretical sense only valid for a base flow that is a steady solution of the Navier-Stokes-equations, it can be successfully applied to time-averaged turbulent flows [2]. The results of such studies are often in very good agreement with experiments. Oberleithner et al. [4] found the wave-number and frequency of a global mode derived from the local linear theory to be in good agreement with the experimental observations. The goal of this study is to extend the results of [4] by using the linear theory to examine the properties of a global mode in a swirling turbulent jet for different stages of the vortex breakdown. Properties of the global mode are obtained from the results of a local analysis, as shown in Gallaire et al. [1]. Therefore, it is necessary to locate regions of absolute and convective instability in the flow domain. The analysis will be based on the time-averaged turbulent flow derived from PIV-measurements.

# 2 Experimental setup and data processing

The experimental apparatus used to generate the swirling jet is shown in fig. 1. The three-dimensional velocity-

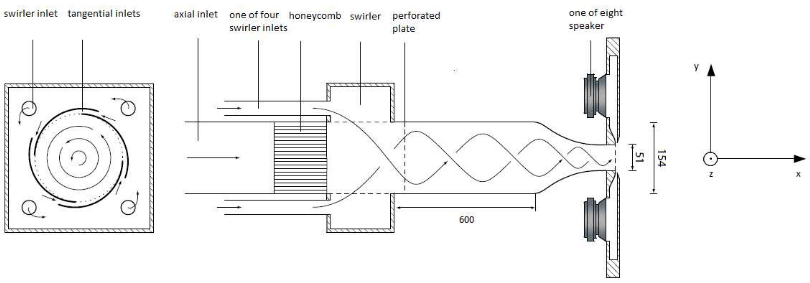


Figure 1: Experimental apparatus (all dimensions are in millimeters) [5]. The used coordinate system is indicated. The measurements presented here were obtained at a Reynolds number of around 21000.

field was measured using a Stereo-PIV system. The laser sheet was introduced perpendicular to the nozzle in the x-z-plane, extending up to seven nozzle diameters in the x- and 3.5 nozzle diameters in the z-direction (positive and negative). Measurements were taken for swirl numbers between 0 and 1.2, covering flow configurations from a non-swirling-jet to a fully developed breakdown state. Especially in the swirl number range between 0.7 and 0.9 measurements were concentrated, because in this range the vortex-breakdown was observed to develop.

# 3 Numerical method

The numerical discretization method employed is based on a Chebyshev collocation approach [3]. The stability of the flow is analyzed by solving the resulting matrix eigenvalue problem with MATLABs eig-, respectively eigs-routine. In order to find the saddle-points determining the type of the instability a method adapted from [6] is used. This method is based on a truncated Taylor-series expansion around the saddle point. Because the truncated series can only be expected to converge in the vicinity of the saddle-point, good starting values are needed. These are obtained by constructing a global picture and the corresponding cusp map at two axial positions and identifying the relevant saddle points by human eye. From these two known saddle points the wave-number and frequency of the relevant saddle point at the next axial location can be extrapolated and the truncated Taylor series in conjunction with the eigs-routine is used to converge to the exact wave-number and frequency of the saddle-point.

# 4 Results

An exemplary result of the analysis is given in fig. 2. The streamline-plot illustrates the analyzed flow and the bottom right plot provides a magnified view of the absolutely unstable region. The computational code used to produce this graph was used in [4] as well, thus demonstrating that it produces correct predictions of the global modes frequency and wave-number. Furthermore, the existence of two regions of absolute instability seems plausible, since Gallaire et al. [1] found a similar behavior in their study. The results of the stability analysis are expected to provide a better understanding of the mode selection of the naturally occurring global mode and its properties. Additionally, the results are supposed to shed light on the global modes interaction with the dynamics of the vortex breakdown.

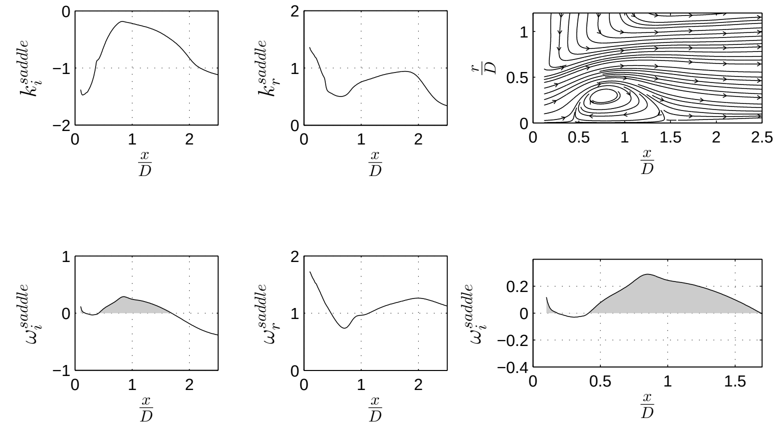


Figure 2: Wave-number $k$ and frequency $\omega$ at the saddle point location for one measurement, plotted over the nondimensionalized axial coordinate ($D$ denotes the nozzle diameter). The swirling jet exhibits vortex-breakdown in this case. The azimuthal wave-number was set to one, which is the azimuthal wave-number of the naturally occurring mode, as it is observed from experimental data. Two regions of absolute instability are present (indicated by gray shading in the graph). One immediately downstream of the nozzle exit and the other one in the region of the breakdown bubble.

# References

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