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Source localisation in deep water using waveguide invariant distribution

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When recorded on an horizontal array, the acoustic intensity of a broadband source presents a striation pattern due to interferences between modes. This pattern is a robust feature of waveguide propagation and is described by a scalar parameter called waveguide invariant. In classical shallow water configurations, this invariant is close to 1 and has been used to perform source localization or to study the environment. For deep water waveguides, the invariant varies and no straightforward methods exist to take benefit of it. Indeed, in deep water many modes contributes to the acoustic intensity and the invariant depends on different modal contributions at different frequencies. This paper proposes a study of the invariant in deep water in a propagation channel. It is viewed as a three dimensional distribution depending on frequency, on central mode and on number of modes considered. Then, it is shown that the invariant distribution can be used to perform range estimation of the source in deep water. To compute it, a priori knowledge of the environment is required. However, the proposed localization method is robust to realistic errors on the environment knowledge as it uses mean of the invariant distribution.

1 Introduction

In oceanic waveguides, an acoustic field can be viewed as a coherent sum of modes. It is described by the modal propagation theory [1]. When one considers a broadband source and a field recorded on an horizontal array, interferences will naturally appear in the domain $r - f$ (where r represents the radial distance between source and frequency and f the frequency). The formation of the interference structure and their localization in the $r - f$ plane depends on the waveguide properties : the sound speed profile in the water and sediments and the source localization.

This interference structure is summarized by a quantity β called waveguide invariant and introduced by Chuprov [2]. It is a single scalar parameter that takes into account the dispersive nature of the oceanic waveguide. Indeed, when plotted versus range r and pulsation f, the sound field intensity $I(r,f)$ exhibits a striation patterns. The slope of this striations $\frac{df}{dr}$ is related to the frequency and the source/receiver range through the waveguide invariant by [3]

$$\beta = \frac{r}{f}\frac{df}{dr}. \tag{1}$$

In an isovelocity waveguide with a perfectly reflecting bottom, the waveguide invariant is actually invariant and is equal to 1. For more complicated waveguides with arbitrary sound speed profile, β can vary when distance and/or frequency vary. In shallow water waveguides, the variations of β are often minor, and the waveguide invariant represents an attractive simple feature of the propagation which can even be used in an environment that varies in range and azimuth [4]. It has been applied for various purposes, such as moving the focal range in a time-reversal experiment [5], source range estimation [6], geoacoustic characterization [7] or source motion compensation [8]. However, it has rarely been considered in deep water waveguides, were only very low frequencies are dispersive.

In this paper, we consider the waveguide invariant for very low frequency propagation (less than 150 Hz) in deep water (more than 1500 m). We first show that it is no longer invariant and has to be considered as a distribution. Then, we use that distribution to device an estimation of the source/receiver range. Finally, we apply this estimation on simulated deep water data.

2 The waveguide invariant in a range independent deep water waveguide

In this section, we will first provide a quick reminder on how the waveguide invariant is computed. Then, we will use it to show that it can be problematic when applied to deep water propagation.

2.1 Waveguide invariant calculation

In any range independent waveguide, the acoustic intensity described by modal theory is given by [1]:

$$I(r,f) = \left| Q \sum_n \psi_m(z_s,f)\psi_m(z_r,f)\frac{e^{irk_{rm}(f)}}{\sqrt{rk_{rm}(f)}}\right|^2, \tag{2}$$

where Q is a constant depending on the environment, $\psi_m(z,f)$ is depth function of mode m, r is the source-receiver range and k_{rm} the horizontal wavenumber. The waveguide invariant characterizes the interference pattern for which acoustical intensity is constant. Thus, it is computed by differentiating the acoustical intensity

$$I(r,f) = cst \Rightarrow \frac{\delta f}{\delta r} = -\frac{\partial I}{\partial r}\Big/\frac{\partial I}{\partial f}. \tag{3}$$

To approximate the the quantity $\frac{\partial I}{\partial r}$ and $\frac{\partial I}{\partial f}$, one has to assume a functional relationship between the group slowness and the phase slowness. It allows to expand S_g^n, the group slowness of mode n, in Taylor series at order 1 in the neighborhood of S_g, the mean group slowness of a group of modes:

$$S_g^n = S_g + \frac{dS_g}{dS_p}(S_p^n - S_p) \tag{4}$$

where S_p^n is the phase slowness of mode n and S_p the mean phase slowness of a group of modes.

Some straightforward algebra allows to obtain the final result [2]:

$$\frac{\delta r}{\delta f} = -\frac{r}{f}\frac{dS_g}{dS_p}, \tag{5}$$

with waveguide invariant β defined as:

$$\frac{1}{\beta} \equiv -\frac{dS_g}{dS_p}. \tag{6}$$

2.2 Analysis of the waveguide invariant using simulated deep water waveguide

Equation (4) is the most critical step of the waveguide invariant definition. As we are doing an order 1 Taylor series, we obtain a local result (in term of frequency). Moreover, the mean group and phase slowness of a group of mode have been defined. This brings questions about that group of modes. What is its size? Which modes are considered? For shallow water propagation, these are not critical question as the functional relationship between phase and group slownesses does not change much between different propagating modes. It is not the case for deep water propagation where there is a big number of propagating modes, which have changing properties in term of phase and group slownesses.

To illustrate it, we consider a typical Mediterranean sound speed profile represented in figure 1. Water depth is 2500 m, and there is a propagation channel at $D = 100\ m$. We consider an impulsive source with a flat spectrum between 0 and 130 Hz, and a 238 element horizontal array for reception. Both source and receivers are in the channel, and are separated by a radial distance $R = 30$ km. Simulation was computed using the simulator MOCTESUMA [9, 10]. Figure 2 presents the simulated recorded signal on the array in the distance-frequency $r - f$ domain. Note that the first hydrophone of the array has been take as the distance origin $r = 0$. One can clearly see the striation pattern of the field. Each striation seems to have a constant slope. We will see that it is not strictly true.

To show the variations of β, phase and group slownesses of modes 30 and 35 are computed in this environment for frequencies from 35 to 130 Hz. Figure 3 presents the functional relationship between these slownesses. One can see that the relationship depends on the frequency and on the mode number. It confirms the idea that, in deep water, waveguide invariant is a local notion ; and that as pointed in [3], equation (5) is valid only for a group of modes.

As in [11], we will now consider the waveguide invariant as a distribution. The idea of our distribution is to take into account the approximations brought by the Taylor expansion of the group slowness in equation (4). As mean slownesses are computed over a group of modes, we consider β as a 3 dimensional distribution depending on :

- the number of modes over which the mean slownesses are computed,
- the number of the central mode of this group,
- the frequency at which the invariant is considered.

Next section will show how to compute the invariant as a 3 dimensional distribution, and that it can easily be applied to perform range estimation using an horizontal array of receivers in deep water.

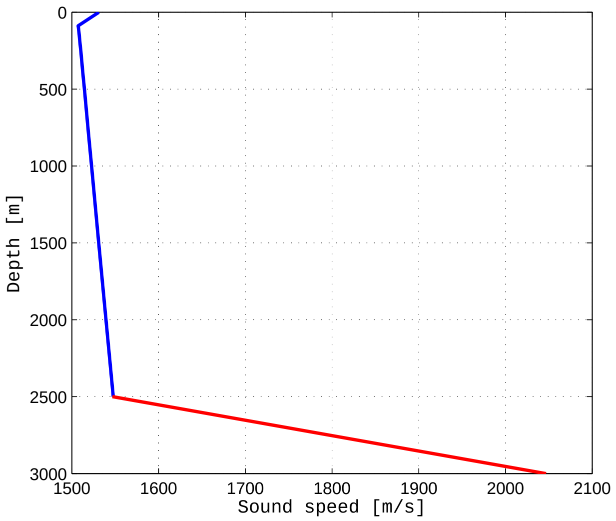

Figure 1: Sound speed profile used for Mediterranean deep water simulation

3 The source localization scheme

3.1 Computation of the invariant as a 3 dimensional distribution

To compute the invariant distribution, we apply the following method:

1. Follow a ridge line along a striation (local maximum or minimum).
2. Interpolate this curve with a second order polynomial. This interpolation smooth the curve to get ride of the sampling stair effect and of the eventual noise.

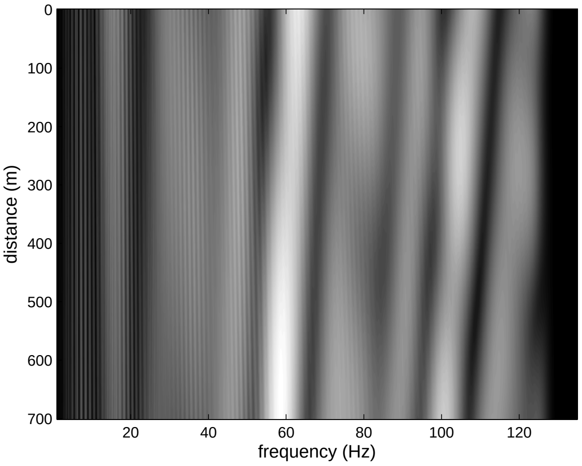

Figure 2: Simulated signal in the Mediterranean environment for a 30 km range

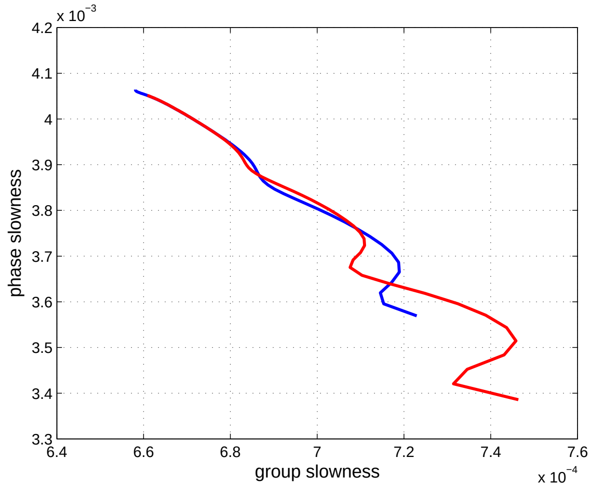

Figure 3: Functional relationship between phase and group slowness for mode 30 (blue) and 35 (red) for a simulated Mediterranean environment

3. Compute the invariant as a 3 dimensional distribution along the interpolated line using equation (6) and *a priori* knowledge of the environment.

The invariant is now noted $\beta(N,m,f)$. The quantity N is the number of modes in the group that we use to compute $\beta(N,m,f)$, m is the central mode of this group, and f is the frequency at which it is computed. Note that thanks to interpolation, a single distance r corresponds to each frequency f.

3.2 Range estimation

By combining equations (5) and (6), an estimated range $\hat{R}(N,m,f)$ can be computed for each value of $\beta(N,m,f)$:

$$\hat{R}(N,m,f) \;=\; \beta(N,m,f)\; f\; \frac{\delta r}{\delta f}, \tag{7}$$

where $\frac{\delta r}{\delta f}$ is the slope of the striation computed for every frequency/distance pair thanks to the interpolation.

The final estimated range $\hat{R}$ is obtained by averaging the distribution $\hat{R}(N,m,f)$

$$\hat{R} \;=\; \text{mean}\ \hat{R}(N,m,f), \tag{8}$$

for every N, m, f verifying

- $f_{min} < f < f_{max}$, with f_{min} and f_{max} the minimum and maximum frequencies of the striation,
- N is odd (so that the considered group of mode has a central mode) and $3 < N < M_{max}$, where N_{max} is the highest mode at frequency f_{min},
- $\frac{N-1}{2} < m < N_{max} - \frac{N-1}{2}$.

4 Application to simulated data

The source localization scheme is applied on the simulated data presented in section 2.2. We follow the ridge between 89 and 109 Hz. Figure 4 presents the result. We see that even for that simple simulation, the slope of the striation -and thus the invariant- is not constant. We compute the invariant distribution for each frequency of the green curve. Figure 5 presents a 2 dimensional slice of this distribution, with parameter N constrained to 7. Figure 6 presents the corresponding estimated range distribution. As the slope of the considered striation is not strictly constant, the link between that two distributions is not linear. The range is estimated by averaging the full 3 dimensional distribution, and the result is $\hat{R} = 32$ km. This is a satisfying result as

$$\hat{R} = R \pm 6.6\%. \tag{9}$$

The same treatment has been applied for several striations, and also for another dataset presented in figure 7. This dataset was computed in the same Mediterranean environment but with a 40 km range.

Final range estimations are :

- Dataset with $r = 40\ km$:
  - striation around 105 Hz : $\hat{r} = 38.0$ km,
  - striation around 95 Hz : $\hat{r} = 42.2$ km,
  - striation around 95 Hz : $\hat{r} = 41.2$ km.

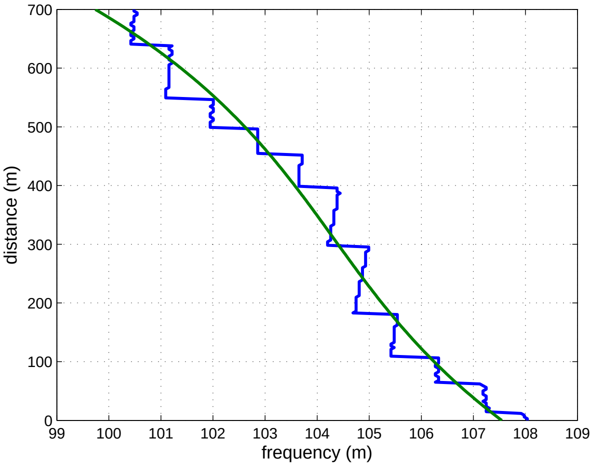

Figure 4: Ridge line along a striation from the Mediterranean data : raw data (blue) and interpolation (green)

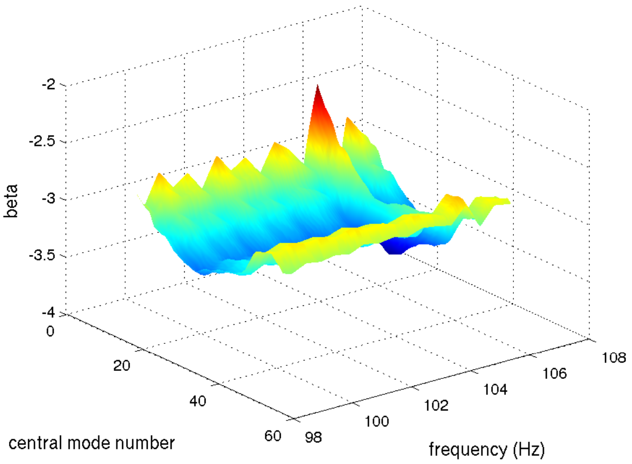

Figure 5: Invariant distribution slice $N = 7$ for the Mediterranean data and the striation around 105 Hz

- Dataset with $r = 30\ km$:
    - striation around 110 Hz : $\hat{r} = 31.5$ km,
    - striation around 105 Hz : $\hat{r} = 32.0$ km,
    - striation around 60 Hz : $\hat{r} = 26.9$ km.

These are good results as each time the estimation error is less than 10%. The last estimation (striation around 60 Hz in the 30 km dataset) is voluntary not as good as the others. Indeed, invariant computation based on equation 3 assumes to follow a constant line striation. That is impossible in a realistic case because of noise and hydrophone calibration. To overcome this, the method follow a maximum (or minimum line) which is assumed to be locally constant. This condition is less respected for this last estimation than for the others, which explains that the result is less accurate than the others.

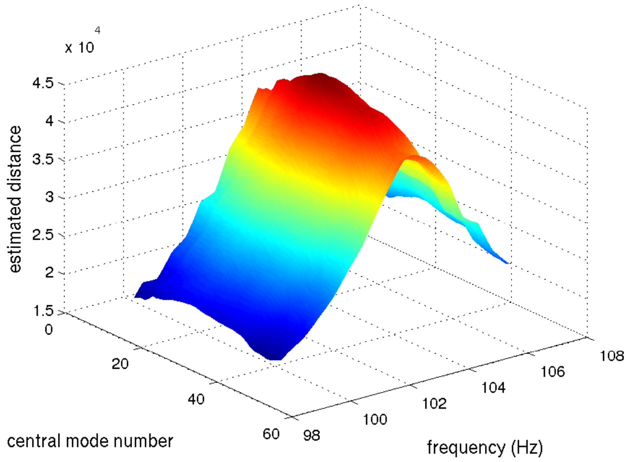

Figure 6: Estimated range distribution slice $N = 7$ for the Mediterranean data and the striation around 105 Hz

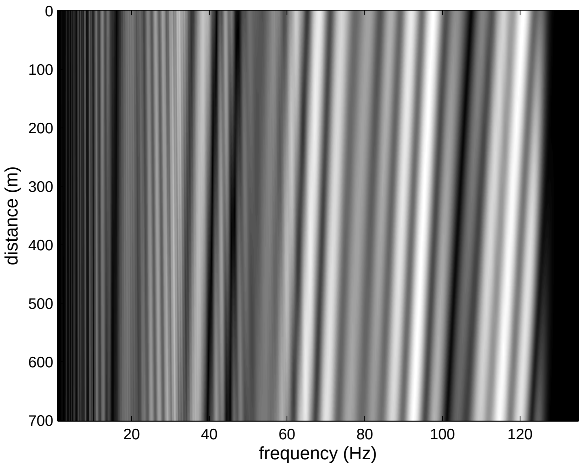

Figure 7: Simulated signal in the Mediterranean environment for a 40 km range

As the proposed method is based on a 3 dimensional mean of the invariant distribution it should be robust to environmental mismatch. To test it, the method is applied on simulated data coming from two new datasets, but the invariant distribution is computed using the celerity profile presented in figure 1. This corresponds to two realistic scenario where there is an error on the environment:

- the propagation channel is 10 m shallower than what we think.
- there is an undetected surface duct profile, resulting in a first propagation channel at depth 50 m.

Each time, the range estimation is good and the estimation error is about 5%. The error of the environment have been filtered out by the 3 dimensional mean of the invariant.

5 Conclusion

This article proposes to consider the waveguide invariant as a 3 dimensional distribution, depending on the frequency and the modes that are used to compute it. It is mainly useful for deep water environment. Indeed, when recorded on an horizontal array, the acoustical intensity presents an interference pattern, and the slope of the striation is not constant for deep water propagation. Then, this article proposes a simple method to estimate the source distance from an horizontal array based on the invariant. This method is validated on simulations. It requires knowledge of the environment. However, as it is based on a 3 dimensional mean of the waveguide invariant, it is robust to realistic errors on environmental knowledge.

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