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1 **Zonal shear and super-rotation in a magnetized spherical Couette**  
2 **flow experiment**

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## Abstract

We present measurements performed in a spherical shell filled with liquid sodium, where a 74 mm-radius inner sphere is rotated while a 210 mm-radius outer sphere is at rest. The inner sphere holds a dipolar magnetic field and acts as a magnetic propeller when rotated. In this experimental set-up called *DTS*, direct measurements of the velocity are performed by ultrasonic Doppler velocimetry. Differences in electric potential and the induced magnetic field are also measured to characterize the magnetohydrodynamic flow. Rotation frequencies of the inner sphere are varied between -30 Hz and +30 Hz, the magnetic Reynolds number based on measured sodium velocities and on the shell radius reaching to about 33. We have investigated the mean axisymmetric part of the flow, which consists of differential rotation. Strong super-rotation of the fluid with respect to the rotating inner sphere is directly measured. It is found that the organization of the mean flow does not change much throughout the entire range of parameters covered by our experiment. The direct measurements of zonal velocity give a nice illustration of Ferraro's law of isorotation in the vicinity of the inner sphere where magnetic forces dominate inertial ones. The transition from a Ferraro regime in the interior to a geostrophic regime, where inertial forces predominate, in the outer regions has been well documented. It takes place where the local Elsasser number is about 1. A quantitative agreement with non-linear numerical simulations is obtained when keeping the same Elsasser number. The experiments also reveal a region that violates Ferraro's law just above the inner sphere.

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## 13 I. INTRODUCTION

14 The Earth's fluid core below the solid mantle consists of a 3480 km-radius spherical cavity  
15 filled with a liquid iron alloy. A 1220 km-radius solid inner core sits in its center. It has  
16 been accepted since the 1940's [1, 2] that the flows stirring the electrically conducting liquid  
17 iron in the outer core produce the Earth's magnetic field by dynamo action. The fluid  
18 motion is thought to originate from the cooling of the Earth's core, which results both in  
19 crystallization of the inner core and in convection in the liquid outer core [3].

20 The last decade has seen enormous progress in the numerical computation of the geo-  
21 dynamo problem after the first simulation of a dynamo powered by convection [4–7]. It  
22 is however still unclear why many characteristics of the Earth's magnetic field are so-well  
23 retrieved with simulations [8] since the latter are performed with values of important di-  
24 mensionless parameters that differ much from the appropriate values for the Earth's core.  
25 The main numerical difficulty is the simultaneous computation of the velocity, the magnetic  
26 and the temperature fields with realistic diffusivities, respectively the fluid viscosity, the  
27 magnetic and the thermal diffusivities. Those differ indeed by six orders of magnitude in  
28 the outer core [9]; such a wide range is at present out of reach numerically, the simulations  
29 being performed at best with two orders of magnitude difference between the values of the  
30 diffusivities. An experimental approach of the geodynamo is, in that respect, promising since  
31 the fluid metals used in experiments have physical properties, specifically diffusivities, very  
32 close to the properties of the liquid iron alloy in the Earth's outer core. Moreover, experi-  
33 ments and simulations are complementary since they span different ranges of dimensionless  
34 parameters.

35 Magnetohydrodynamics experiments devoted to the dynamo study have started some 50  
36 years ago (see the chapter authored by Cardin and Brito in [10] for a review). To possibly  
37 induce magnetic fields, the working fluid must be liquid sodium in such experiments. Sodium  
38 is indeed the fluid that best conducts electricity in laboratory conditions. A breakthrough  
39 in these dynamo experiments occurred at the end of 1999 when amplification and saturation  
40 of an imposed magnetic field were measured for the first time in two experiments, in Riga  
41 [11] and in Karlsruhe [12]. The common property of those set-ups was to have the sodium  
42 motion very much constrained spatially, in order to closely follow fluid flows well known  
43 analytically to lead to a kinematic dynamo, respectively the Ponomarenko flow [13] and the

44 G.O. Roberts flow [14]. More recently, the first experimental dynamo in a fully turbulent  
45 flow was obtained in a configuration where two crenelated ferromagnetic rotating discs drive  
46 a von Kàrmàn swirling flow in a cylinder [15]. Earth’s like magnetic field reversals were  
47 also obtained in this experimental dynamo [16]. Other similar experiments have been run  
48 where sodium flows are driven by propellers in a spherical geometry [17, 18]. In order to  
49 emphasize the specificity of the experimental study presented in the present paper, it is  
50 worth mentioning two common features of the previously mentioned sodium experiments:  
51 the forcing of the sodium motion is always purely mechanical and the magnetic field is *weak*  
52 in the sense that Lorentz forces are small compared to the non-linear velocity terms in the  
53 equation of motion [19].

54 The experiment called *DTS* for ”Derviche Tourneur Sodium” has been designed to in-  
55 vestigate a supposedly relevant regime for the Earth’s core, the magnetostrophic regime  
56 [20–22] where the ratio of Coriolis to Lorentz forces is of the order one. The container made  
57 of weakly conducting stainless steel is spherical and can rotate about a vertical axis. An  
58 inner sphere consisting of a copper envelope enclosing permanent magnets is placed at the  
59 center of the outer sphere; the force free magnetic field produced by those magnets enables  
60 to explore dynamical regimes where Coriolis and Lorentz forces are comparable. The sodium  
61 motion in the spherical gap is driven by the differential rotation between the inner sphere  
62 and the outer sphere, unlike in the Earth’s core where the iron motion is predominantly  
63 driven by convection [23] and maybe minorly by differential rotation of the inner core [24].

64 The *DTS* experiment has not been designed to run in a dynamo regime. It has instead  
65 been conceived as a small prototype of a possible future large sodium spherical dynamo  
66 experiment which would benefit from its results. Note that meanwhile Daniel Lathrop and  
67 collaborators have built a 3m-diameter sodium spherical experiment with an inner sphere  
68 differentially rotating with respect to the outer sphere, like in *DTS*. Schaeffer, Cardin and  
69 Guervilly [25, 26] have shown numerically that a dynamo could occur in a spherical Couette  
70 flow at large  $Rm$  in a low magnetic Prandtl number fluid such as sodium ( $Pm = \nu/\lambda$ , see  
71 TABLE I).

72 Numerical simulations in a *DTS*-type configuration [27–29] of Couette spherical flows  
73 with an imposed magnetic field all show azimuthal flows stabilized by magnetic and rotation  
74 forces. Using electric potential measurements along a meridian of the outer sphere boundary,  
75 we concluded in our first report of *DTS* experimental results [30] that the amplitude of the

76 azimuthal flow may exceed the velocity of sodium in solid body rotation with the inner  
77 sphere, as predicted theoretically in the linear regime [31].

78 The *DTS* experiment offers a tool to investigate non uniform rotation of an electrically  
79 conducting fluid in the presence of rotation and magnetic forces. The differential rotation  
80 of a body permeated by a strong magnetic field and the waves driven by the non uniform  
81 rotation have received considerable attention since the work of Ferraro [32, 33]. Indeed,  
82 the absence of solid envelopes makes non uniform rotation possible in stars, where it plays  
83 an important role in the mixing of chemical elements [34], in contrast with the case of  
84 planetary fluid cores. Ferraro found that the angular rotation in an electrically conducting  
85 body permeated by a steady magnetic field symmetric about the axis of rotation tends to  
86 be constant along magnetic lines of force. MacGregor and Charbonneau [35] illustrated this  
87 result and showed, in a weakly rotating case, that Ferraro’s theorem holds for  $\text{Ha} \gg 1$  ( $\text{Ha}$ ,  
88 the Hartmann number, measures the magnetic strength, see TABLE II). An intense magnetic  
89 field, probably of primordial origin, is the key actor in the transfer of angular momentum  
90 from the solar radiative interior to the convection zone [36, 37]. Finally, in a geophysical  
91 context, Aubert recently found, investigating zonal flows in spherical shell dynamos, that  
92 Ferraro’s law of isorotation gives a good description of the geometry of the zonal flows of  
93 thermal origin [38].

94 In the second study of the *DTS* experiment [39], we investigated azimuthal flows when  
95 both the inner boundary and the outer boundary are rotating but at different speeds, using  
96 Doppler velocimetry and electric potential measurements. Specifically, we discussed the  
97 transition between the outer geostrophic region and the inner region where magnetic forces  
98 dominate. Extending the asymptotic model of Kleorin et al. [40], we could explain the  
99 shape of the measured azimuthal velocity profiles. We had to use a specific electric potential  
100 difference as a proxy of the differential rotation between the two spheres as, unfortunately,  
101 the electrical coupling between the liquid sodium and the copper casing of the interior  
102 magnets was apparently both imperfect and unreliable. Finally, we reported in on our third  
103 article [41] about the *DTS* experiment the presence of azimuthally traveling hydromagnetic  
104 waves that we inferred mainly from electric potential measurements along parallels.

105 We investigate here again the main flows when the outer sphere is at rest. Our new study  
106 benefits from a comparison with our earlier work [39] for a rotating outer sphere. There is no  
107 need any more to use an indirect measure of the global rotation of the fluid as the electrical

108 coupling between liquid sodium and copper has become unimpaired. Furthermore, the *DTS*  
109 experiment has been equipped with a host of new measurement tools. The flow amplitude  
110 is measured along 7 different beams using Doppler velocimetry. Assuming axisymmetry, we  
111 have thus been able to map the azimuthal flow in most of the fluid. It turns out that the  
112 electric potential differences evolve monotonically with the inner core rotation but cannot  
113 be interpreted directly as a measure of the velocity below the outer viscous boundary layer.  
114 We have also entered a probe inside the cavity to measure the induced magnetic field in  
115 the interior. The dense measurements in the *DTS* experiment give a nice illustration of  
116 the Ferraro law of isorotation [32] in the inner region where magnetic forces dominate. In the  
117 outer region, we retrieve axially invariant azimuthal flow as the Proudman-Taylor theorem  
118 holds there, even though the outer sphere is at rest. The variation of the geostrophic velocity  
119 with the distance to the axis differs nevertheless from the case of a rotating outer sphere as  
120 recirculation in the outer Ekman layer plays an important role in the latter case.

121 The organisation of the paper is as follows. In section II, we describe the experimental set-  
122 up and the techniques that we use to measure the magnetic, electric and velocity fields; we  
123 illustrate them with a discussion of a typical experimental run. In section III, we present the  
124 governing equations and the relevant dimensionless numbers of the experiment. We devote  
125 one section of the article to the observation of differential rotation and another one to the  
126 meridional circulation. Then, the experimental measurements are compared to numerical  
127 simulations of *DTS*. We summarize and discuss the results of our study in section VII.

128 **II. THE *DTS* EXPERIMENT**

129 **A. The experimental set-up**

130 The *DTS* experimental set-up [30, 39, 41] is shown in FIG. 1. It has been installed in a  
131 small building purpose-designed for sodium experiments.

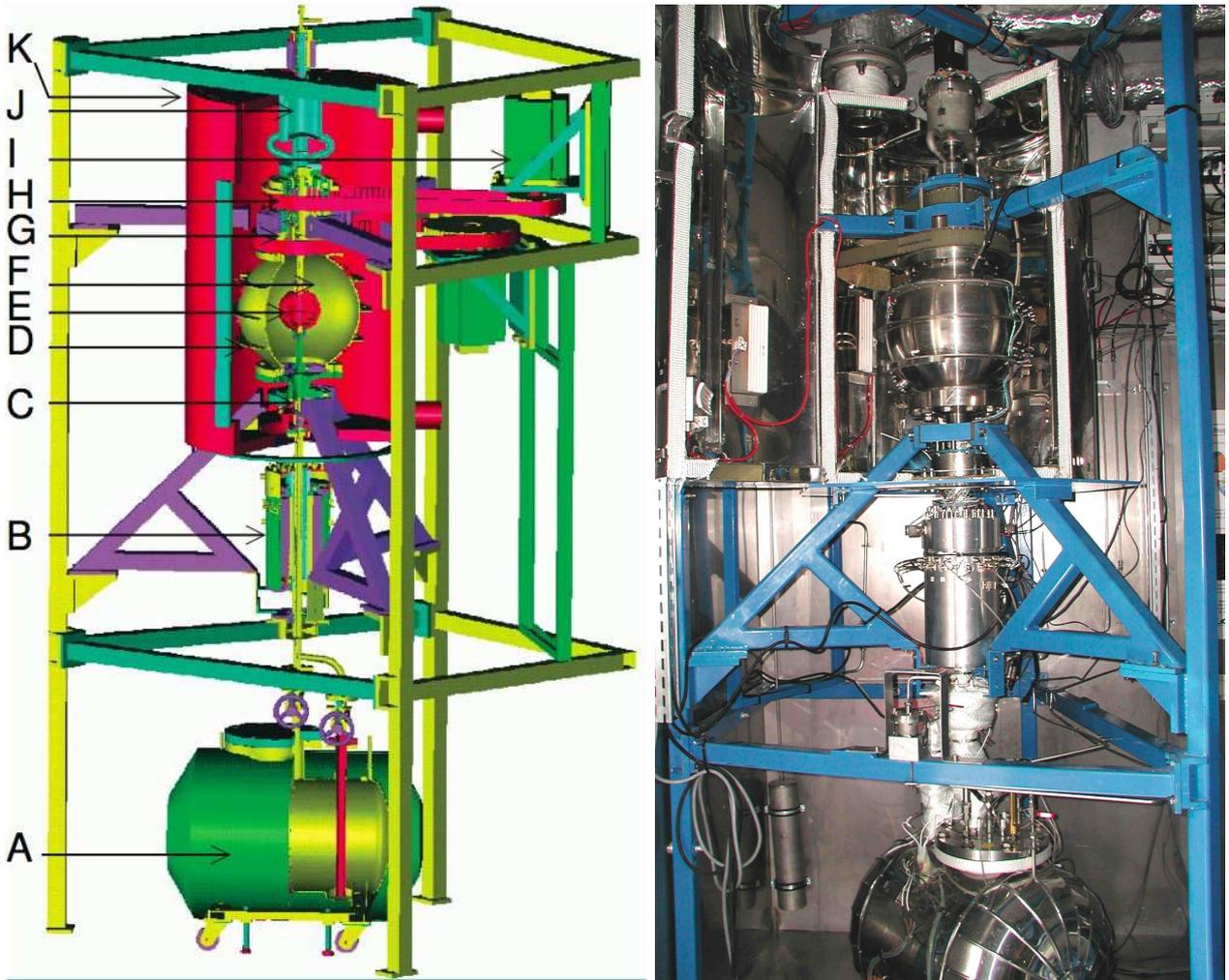


FIG. 1. (Color online) Diagram and picture of the experimental set-up. A: moveable sodium reservoir, B: shielded electric slip-ring, C: electromagnetic valve, D: outer sphere, E: magnetized rotating inner sphere, F: spherical shell containing liquid sodium, G: magnetic coupling entraining the inner sphere shaft, H: crenelated belt, I: brushless electric motor driving the inner sphere, J: expansion tank for sodium, K: thermostated chamber. The total height of the set-up is 3.9 m.

132 As shown in FIG. 1, liquid sodium is contained in a spherical shell between an outer

133 sphere and an inner sphere. The radius of the outer sphere is  $a = 210$  mm and that of the  
 134 inner sphere  $b = 74$  mm. The outer sphere is made of stainless steel and is 5 mm thick.  
 135 The copper inner sphere (FIG. 2 and FIG. 3) contains magnetized Rare-Earth cobalt bricks  
 136 assembled such that the resulting permanent magnetic field is very close to an axial dipole  
 137 of moment intensity  $700 \text{ Am}^2$ , with its axis of symmetry aligned with the axis of rotation.  
 138 The magnetic field points upward along the rotation axis and its magnitude ranges from 345  
 139 mT at the poles of the inner sphere down to 8 mT at the equator of the outer sphere.

140 Sodium is kept most of the time in the reservoir at the bottom of the set-up. When  
 141 needed to run an experiment, sodium is melted and pushed up from that reservoir into the  
 142 spherical shell by imposing an overpressure of Argon in the reservoir. When liquid sodium  
 143 reaches the expansion tank at the top of the spherical shell, an electromagnetic valve located  
 144 just below the sphere (see FIG. 1) is locked such that sodium is kept in the upper part during  
 145 experiments. In case of emergency, the valve is opened and sodium pours directly into the  
 146 reservoir.

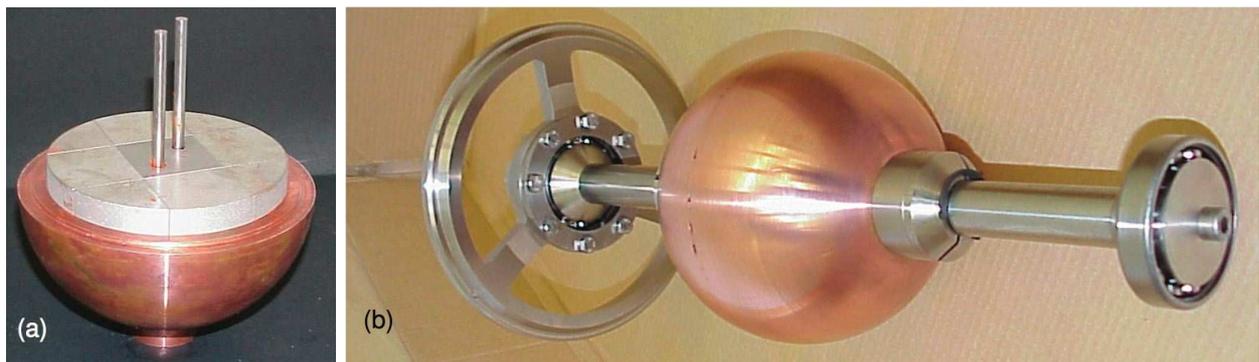


FIG. 2. (Color online) (a) Picture of one hemisphere of the inner sphere. Different pieces of magnets in gray are assembled in the bulk of the inner sphere. (b) View from the side of the inner sphere and its rotating shaft. Note that the wheels at the top and bottom (only one is shown in the picture) of the rotating shaft are attached to the outer sphere.

147 The central part of the experiment is air-conditioned in a chamber maintained at around  
 148  $130^\circ\text{C}$  during experiments: four 1 kW infrared radiants disposed around the outer sphere  
 149 heat the chamber, whereas cold air pumped from outside cools the set-up when necessary.  
 150 Liquid sodium is therefore usually kept some  $30^\circ\text{C}$  above its melting temperature during  
 151 experiments. Some physical properties of sodium relevant to our study are listed in TABLE I.

TABLE I. Physical properties of pure liquid sodium at 130°C (Documents from CEA, Commissariat à l’Energie Atomique et aux énergies alternatives). \*The sound velocity in sodium has been precisely measured in the present study using the UDV apparatus.

$\rho$	density	$9.3 \cdot 10^2 \text{ kg m}^{-3}$
$\sigma$	electric conductivity	$9 \cdot 10^6 \text{ } \Omega^{-1} \text{ m}^{-1}$
$\nu$	kinematic viscosity	$6.5 \cdot 10^{-7} \text{ m}^2 \text{ s}^{-1}$
$\eta$	magnetic diffusivity	$8.7 \cdot 10^{-2} \text{ m}^2 \text{ s}^{-1}$
$c$	sound velocity*	$2.45 \cdot 10^3 \text{ m s}^{-1}$

152 The whole volume containing sodium, from the reservoir tank up to the expansion tank is  
 153 kept under Argon pressure at all times in order to limit oxidization of sodium.

154 The rotation of the inner sphere, between  $f = -30 \text{ Hz}$  and  $f = 30 \text{ Hz}$ , is driven by  
 155 a crenelated belt attached to a 11 kW brushless motor (SGMH-1ADCA61 from Yaskawa  
 156 Electric Corporation, Tokyo, Japan). The belt entrains a home-made magnetic coupler  
 157 located around the inner sphere shaft as seen in FIG. 1. The coupler is composed of an  
 158 array of magnets located outside the sodium container, another array of magnets inside the  
 159 container being immersed in liquid sodium. The inner magnets are anchored to the rotating  
 160 shaft of the inner sphere such that when the belt is rotated outside, the inner sphere is  
 161 rotated as well. Such a coupler has the advantage of not requiring any rotating seal in liquid  
 162 sodium. Torque values up to about 70 N·m have been efficiently transmitted through this  
 163 coupler in the experiment.

## 164 B. Measurements

### 165 1. Ultrasonic Doppler velocimetry

166 We use UDV ultrasonic Doppler velocimetry [42] in order to measure liquid sodium ve-  
 167 locities in the spherical shell. This non intrusive technique has been intensively used in our  
 168 group for the last decade, in particular in rotating experiments performed either in water  
 169 or in liquid metals [43–46]. The technique consists in the emission from a piezoelectric  
 170 transducer of a succession of bursts of ultrasonic waves that propagate in the fluid. When

171 the wave encounters a particle with a different acoustic impedance, part of the ultrasonic  
 172 wave is backscattered towards the transducer. The time elapsed between the emitted and  
 173 the reflected waves and the change in that time respectively give the position of the par-  
 174 ticle with respect to the transducer and the fluid velocity along the beam direction. Data  
 175 processing is internal to the DOP2000 apparatus (<http://www.signal-processing.com>,  
 176 Signal Processing company, Lausanne, Switzerland).

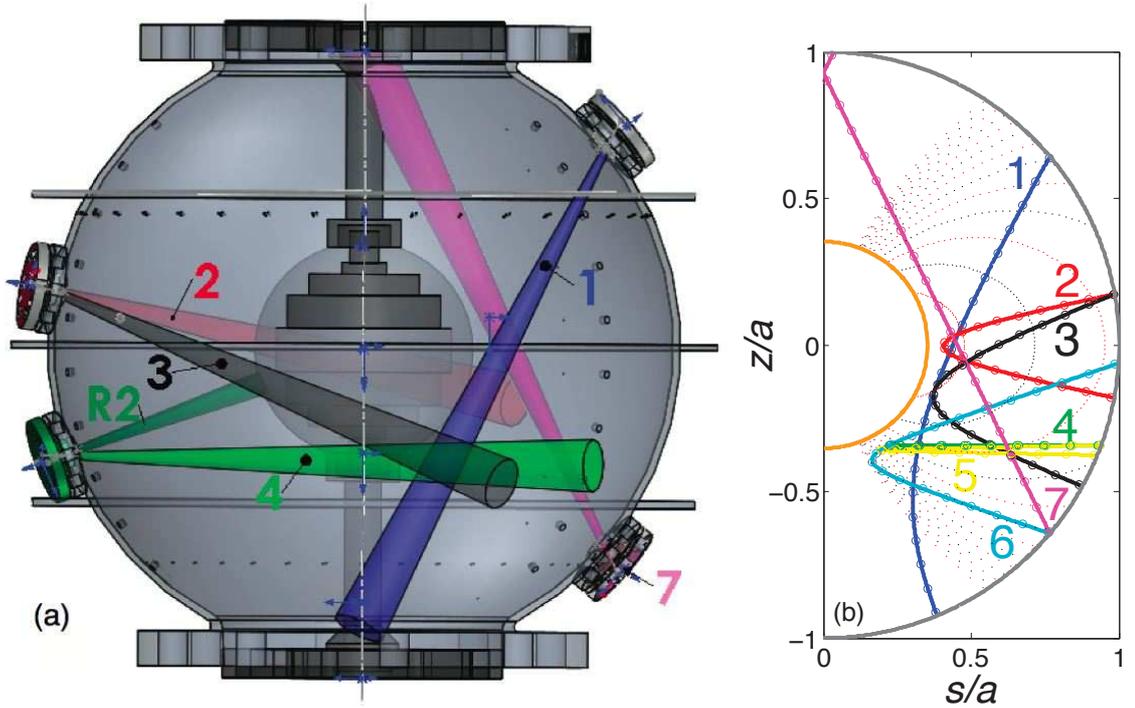


FIG. 3. (Color online) (a) 3D perspective view of the outer sphere and its interior. Caps at various latitudes hold ultrasonic velocity probes to perform UDV. The divergent ultrasonic beams emitted from each cap are shown in perspective with different colors (and numbers for the grayscale version). The five superimposed horizontal slices of magnets are assembled in the heart of the inner sphere. Differences in electric potential are measured between points from latitude  $+45^\circ$  to latitude  $-45^\circ$ , with steps of  $10^\circ$  (holes along a meridian at the right of the Figure). (b) Meridional view of the normalized coordinates  $(s/a, z/a)$  covered by the ultrasonic trajectories numbered from 1 to 7. Some of the corresponding rays are plotted in (a) with the same color code (same numbers). The distance  $d$  from the outer sphere along the ultrasonic beam is marked by small dots drawn every 20 mm. The dotted lines are field lines of the imposed dipolar magnetic field.

177 The ultrasonic probes are held in circular stainless steel caps attached to the outer sphere,

178 as shown in FIG. 3(a). There are six locations with interchangeable caps on the outer  
 179 sphere such that fluid velocities can be measured from any of these different positions. The  
 180 thickness of the stainless steel wall between the probes and liquid sodium has been very  
 181 precisely machined to 1.4 mm in order to insure the best transmission of energy from the  
 182 probe to the fluid [47]. Small sodium oxides and/or gas bubbles are present and backscatter  
 183 ultrasonic waves as in gallium experiments [46]. We keep the surface of the caps in contact  
 184 with sodium as smooth and clean as possible to perform UDV measurements.

185 We use high temperature 4 MHz ultrasonic transducers (TR0405AH from Signal Pro-  
 186 cessing) 10 mm long and 8 or 12 mm in diameter (piezoelectric diameter 5 or 7 mm). The  
 187 measurements shown throughout the paper were performed with pulse repetition frequency  
 188 (*prf*) varying from 3 kHz to 12 kHz and with a number of *prf* per profile varying from 8 to  
 189 128. A present limitation of this UDV technique is that the maximum measurable veloc-  
 190 ity obeys the following function  $u_{\max} = c^2/4f_e P_{\max}$  where  $c$  is the ultrasonic velocity of the  
 191 medium,  $f_e$  is the emitting frequency, and  $P_{\max}$  is the maximum measurable depth along  
 192 the velocity profile. Applying this relationship to the parameters used in *DTS*,  $P_{\max} \simeq 200$   
 193 mm (approximative length of the first half of the beam in Figure 3) and  $f_e = 4$  Mhz, the  
 194 maximum measurable velocity is of the order 2.2 m/s. In particular cases, it is possible to  
 195 overcome this limitation by using aliased profiles of velocity [43] as shown later in the paper.  
 196 The spatial resolution of the velocity profiles is about 1 mm, and the velocity resolution is  
 197 about 0.5%, or better for the aliased profiles.

198 We have measured both the radial and oblique components of velocity in the bulk of the  
 199 spherical shell. The radial measurements were performed from the latitudes  $+10^\circ$ ,  $-20^\circ$  and  
 200  $-40^\circ$ . The oblique measurements were performed from different locations and in different  
 201 planes, along rays that all deviate from the radial direction by the same angle ( $24^\circ$ ). Thus,  
 202 they all have the same length in the fluid cavity. At the point of closest approach, the rays  
 203 are 11 mm away from the inner sphere. The seven oblique beams used in *DTS* are sketched  
 204 in FIG. 3(b). The way to retrieve the meridional and azimuthal components of the velocity  
 205 field along the ultrasonic beam is detailed in the Appendix.

206 We use UDV measurements to confirm the strong magnetic coupling between the inner  
 207 rotating sphere and sodium. In a smaller version of *DTS* performed in water, maximum  
 208 angular velocities (normalized by that of the inner sphere) of the order 0.16 are obtained  
 209 for a hydrodynamic Reynolds number of  $10^5$  in the vicinity of the equatorial plane, close

210 to the rotating inner sphere [26]. For similar  $Re$  in *DTS*, sodium is in super-rotation close  
211 to the inner rotating sphere and maximum measured velocities are instead around 1.2 (see  
212 FIG. 11(b) for example).

## 213 2. *Magnetic field inside the sphere*

214 The measurement technique described so far does not requires probes that protrude inside  
215 the sphere. In order to measure the magnetic field inside the sphere, in the liquid, we have  
216 installed magnetometers inside a sleeve, which enters deep into the liquid. The external  
217 dimensions of the sleeve are 114 mm (length inside the sphere) and 16 mm (diameter). It  
218 contains a board equipped with high-temperature Hall magnetometers (model A1384LUA-T  
219 of Allegro Microsystems Inc). We measure the radial component of the magnetic field at  
220 radii (normalized by  $a$  the inner radius of the outer sphere) 0.93 and 0.74. The orthoradial  
221 component is measured at 0.97 and 0.78, and the azimuthal component at 0.99, 0.89, 0.79,  
222 0.69, 0.60 and 0.50. The sleeve is mounted in place of a removable port (at a latitude of  
223 either  $40^\circ$ ,  $10^\circ$  or  $-20^\circ$ ). A top view of the sleeve is shown in FIG. 6. The measured voltage  
224 is sampled at 2000 samples/second with a 16-bit 250 kHz PXI-6229 National Instruments  
225 acquisition card. The precision of the measurements (estimated from actual measurements  
226 when  $f = 0$ ) is about  $140 \mu\text{T}$ , and corresponds to about 20 unit bits of the A/D converter.  
227 Magnetic fields up to 60 mT have been measured.

## 228 3. *Differences in electric potentials on the outer sphere*

229 Differences in electric potentials are measured along several meridians and along one  
230 parallel of the outer sphere [30, 39, 41]. In the present study, we are interested in the  
231 measurements performed along meridians since they are linked to the azimuthal flow velocity  
232  $u_\varphi$  (we denote  $(r, \theta, \varphi)$  the spherical coordinates). The measurements are performed between  
233 successive electrodes located from  $-45^\circ$  to  $+45^\circ$  in latitude, with electrodes  $10^\circ$  apart as  
234 sketched in FIG. 3(a). We note  $\Delta V_{40} = V_{45} - V_{35}$  the difference between the electric potential  
235 at latitudes  $45^\circ$  and  $35^\circ$ . Electric potentials are measured by electrodes soldered to brass  
236 bolts 3 mm long, those being screwed into 1 mm-diameter, 4 mm-deep blind holes drilled in  
237 the stainless steel wall of the outer sphere. The measured voltage is filtered by an RC anti-

238 aliasing 215 Hz low-pass filter and then sampled at 1000 samples/second with a 16-bit 250  
 239 kHz PXI-6229 National Instruments acquisition card. The precision of the measurements  
 240 (estimated from actual measurements at  $f = 0$ ) is about  $80 \mu\text{V}$ , and corresponds to about  
 241 10 unit bits of the A/D converter. Electric potential differences up to 7 mV have been  
 242 measured.

243 Denoting  $\mathbf{E}$  the electric field, we introduce the electric potential  $V$  through  $\mathbf{E} = -\nabla V$ ,  
 244 which is valid in a steady state. Then, the electric potential measurements are analysed using  
 245 Ohm's law for a moving conductor,  $\mathbf{j} = \sigma (\mathbf{u} \times \mathbf{B} + \mathbf{E})$  where  $\sigma$  is the electric conductivity,  
 246  $\mathbf{j}$  the electric current density vector,  $\mathbf{u}$  the velocity field and  $\mathbf{B}$  the magnetic field. If the  
 247 meridional electric currents  $j_\theta$  are small compared to  $\sigma u_\varphi B_r$  in the fluid interior and away  
 248 from the equatorial plane where  $B_r = 0$ , and if the viscous boundary layer adjacent to the  
 249 outer sphere is thin, which ensures the continuity of  $E_\theta$  through the layer, then the measured  
 250 differences in electric potential depend on the product of the local radial magnetic field  $B_r$   
 251 by  $u_\varphi$ , the azimuthal fluid velocity:

$$\frac{\Delta V}{a\Delta\theta} = u_\varphi B_r, \quad (1)$$

252 where  $\Delta\theta = 10^\circ$  is the angle between two electrodes. However, we shall question below the  
 253 assumption on the smallness of  $j_\theta$ , referred to as the frozen flux hypothesis.

#### 254 4. *Velocity and torque measured from the motor driving the inner sphere*

255 The electronic drive of the motor entraining the inner sphere delivers an analog signal  
 256 for its angular velocity and its torque. We checked and improved the velocity measurement  
 257 by calibrating it using a rotation counter, which consists of a small magnet glued on the  
 258 entrainment pellet and passing once per turn in front of a magnetometer. The torque signal  
 259 is used to infer the power consumption in section II D.

### 260 C. **A typical experiment : a complete set of measurements**

261 A complete set of measurements performed during a typical experiment is analyzed below.  
 262 The run was chosen to illustrate the various measurements but also to depict how the  
 263 different observables evolve with  $f$ . During that run of 600 seconds, the inner sphere was

264 first accelerated from 0 to  $f = 30$  Hz in around 120 seconds, then decelerated back to 0 during  
 265 120 seconds. The inner sphere was then kept at rest for about 100 seconds and accelerated  
 266 in the opposite direction to  $f = -30$  Hz in 120 seconds. It returned to zero rotation in 120  
 267 seconds again. That cycle of rotation is shown in FIG. 4. The torque delivered by the inner  
 268 sphere motor is also shown and evolves clearly non-linearly during those cycles.

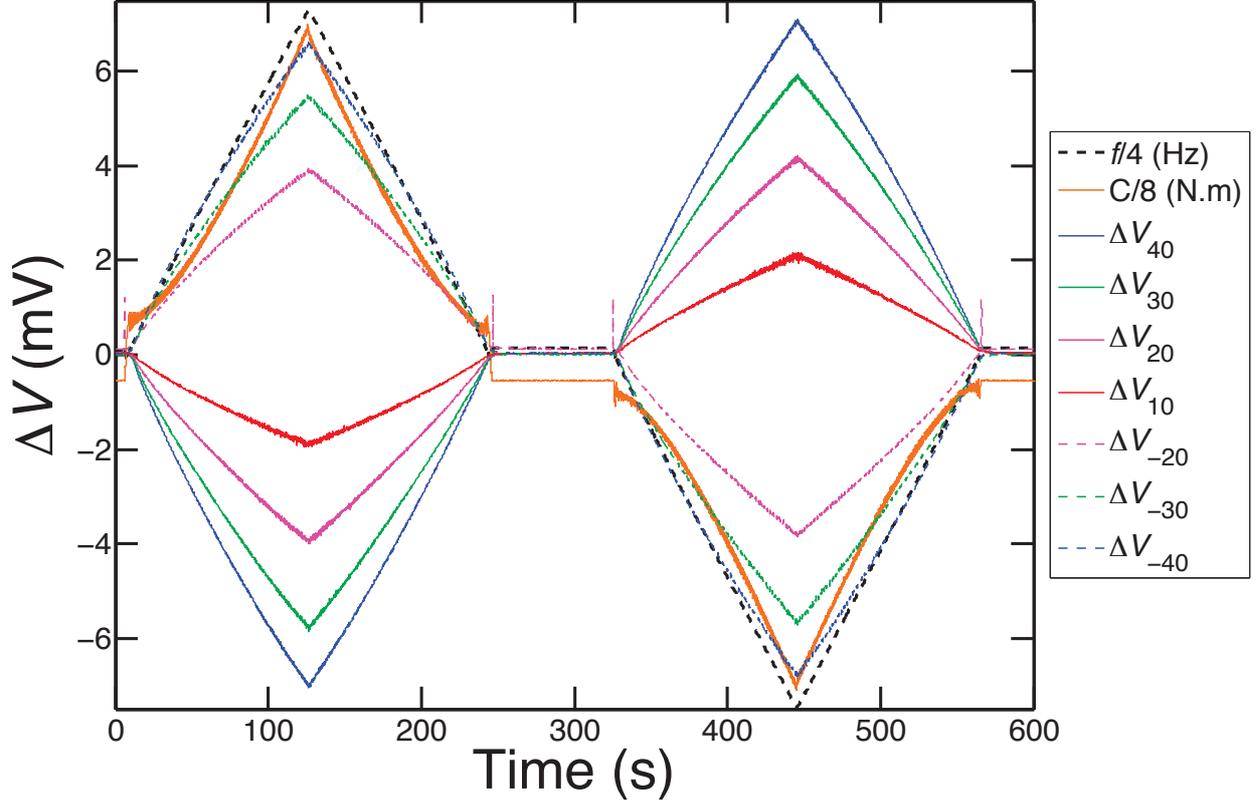


FIG. 4. (Color online) Records of the inner core rotation frequency  $f$ , torque  $C$  and differences in electric potential  $\Delta V_{40}$ ,  $\Delta V_{30}$ ,  $\Delta V_{20}$ ,  $\Delta V_{10}$ ,  $\Delta V_{-20}$ ,  $\Delta V_{-30}$ ,  $\Delta V_{-40}$  as a function of time. The subscript denotes the latitude (in degrees) of the electric potential difference.

269 FIG. 4 shows electric potential records (see part II B 3) obtained during this experiment  
 270 and time averaged over 0.1 s windows. The differences of potential vary in sync with the  
 271 inner sphere rotation frequency as expected if the various  $\Delta V$ 's measure the differential  
 272 rotation between the liquid sodium and the outer sphere to which the electrodes are affixed  
 273 (II B 3). However, it is also apparent that the fluid rotation as measured from the  $\Delta V$ 's does  
 274 not increase linearly with the inner sphere frequency. We interpret it as an indication that  
 275 braking at the outer boundary, which opposes the entrainment by the inner core rotation,

276 varies non linearly with the differential rotation. As expected, records from electrodes pairs  
277 are anti-symmetrical with respect to the equator, since the forcing is symmetrical while the  
278 radial component of the imposed magnetic field changes sign across the equator.

279 FIG. 5 shows the fluid velocity  $u(d)$  measured by UDV during the first half of the exper-  
280 iment along the ray 6 as a function of time and distance. Velocity profiles were recorded  
281 along a total distance  $d \simeq 80$  mm. As demonstrated in FIG. 5(b), the velocity is aliased  
282 since the maximum measurable velocity, for the ultrasonic frequency used during the exper-  
283 iment, is exceeded. Since the azimuthal velocity profiles are quite simple in shape, it has  
284 been straightforward to unfold those profiles and retrieve the correct amplitudes as shown in  
285 FIG. 5(c). The evolution with  $f$  is similar to that of the electrodes, but indicates a stronger  
286 leveling-off as  $f$  increases.

287 FIG. 6 shows the magnetic field induced inside the fluid during the typical experiment.  
288 The measurements are taken in the sleeve placed at  $40^\circ$  latitude. The induced azimuthal field  
289 in FIG. 6 (a) is measured at 6 different radii (given in section II B 2). Its intensity reaches  
290 60 mT near the inner sphere and gets larger than the imposed dipole in some locations.  
291 Note the simple evolution with  $f$ , which contrasts with that of the electric potentials and  
292 velocities in that it increases with an exponent close to 1. The induced meridional field  
293 (FIG. 6) is some 20 times weaker. It is dominated by fluctuations, and does not change  
294 sign when  $f$  does. Note that the evolution with  $f$  is not monotonic. Similar behaviors are  
295 observed at latitudes  $10^\circ$  and  $-20^\circ$ .

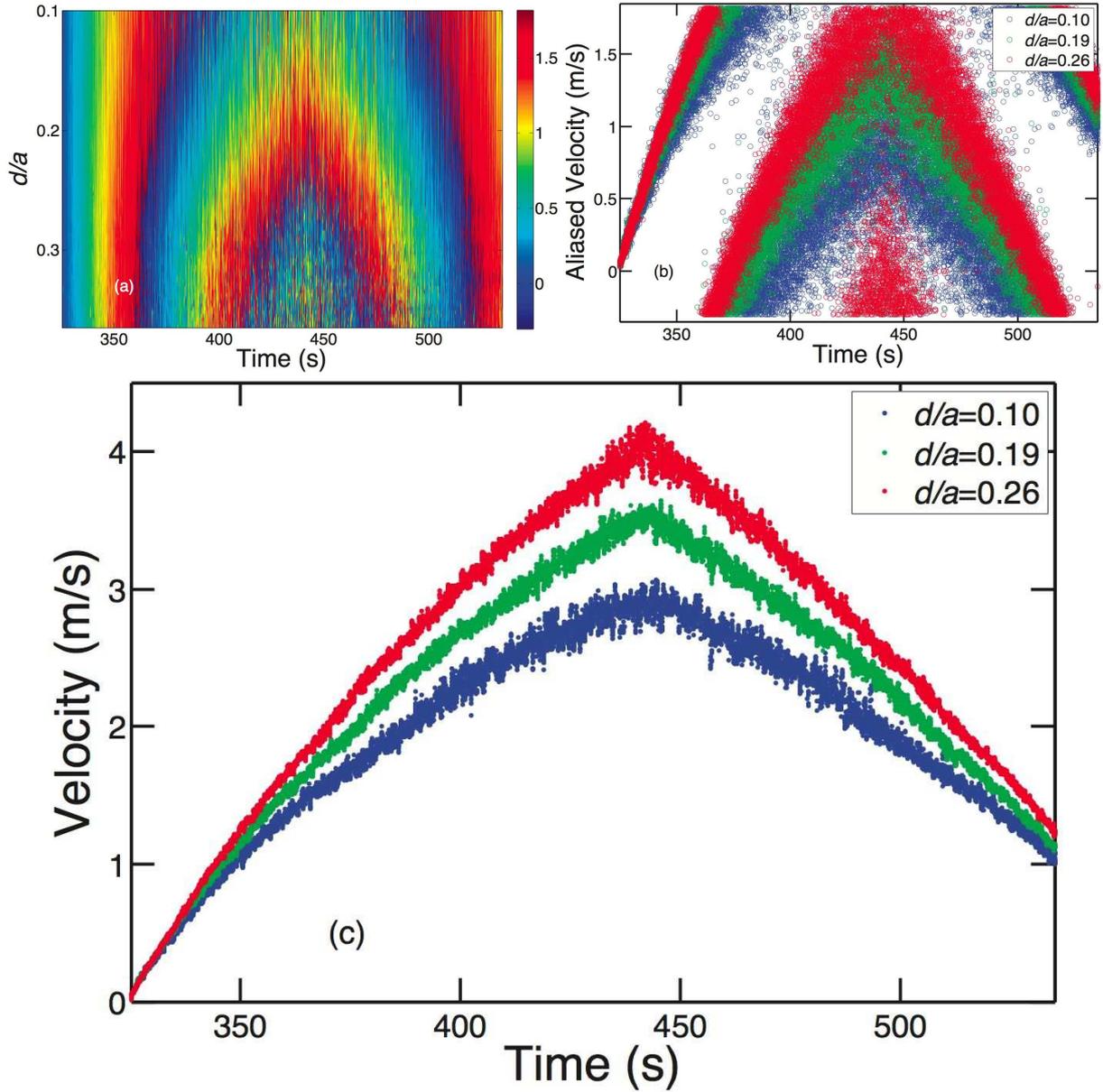


FIG. 5. (Color online) UDV measurements performed along the ray number 6 (see FIG. 3) during the second half of the typical experiment when the inner sphere was rotated from rest to -30 Hz and then back to rest. (a) Spatio-temporal representation of the measured velocity, given by the color scale (in m/s). (b) Velocity at three distances from the probe as a function of time, extracted from the spatio-temporal shown in (a). The velocity profiles are clearly aliased since the profiles are discontinuous. (c) After applying a median time-filtering window of 0.2 s and unfolding the profiles, the correct velocities are retrieved as a continuous function of time.

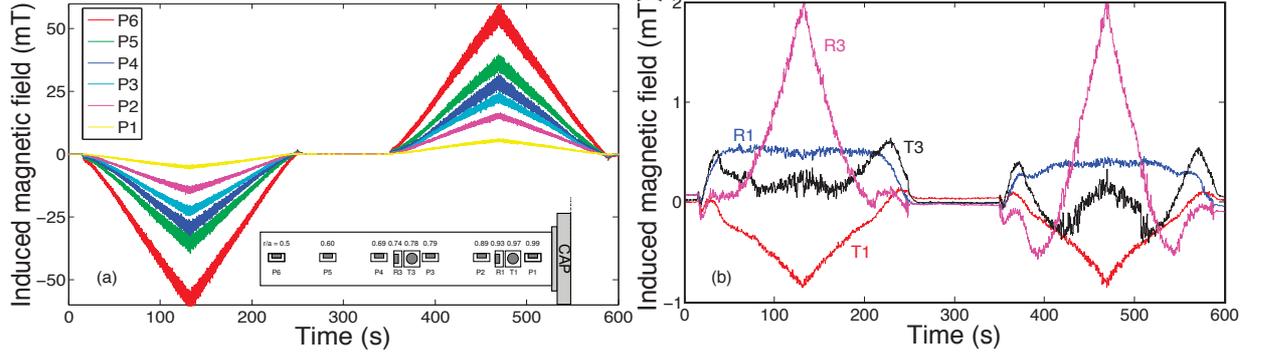


FIG. 6. (Color online) (a) Azimuthal  $b_\phi$ , (b) radial  $b_r$  and orthoradial  $b_\theta$  induced magnetic field at a latitude of  $40^\circ$  in the sleeve at different radial positions recorded during the two triangles sequence of FIG. 4. A top view of the sleeve at the bottom of (a) gives the radial position and the orientation of the various Hall magnetometers. The intensity of the induced azimuthal field reaches 60 mT near the inner sphere and has the sign of  $-f$ . The fluctuations reach about 10% of the mean. The meridional components of the induced magnetic field are much weaker and dominated by fluctuations, which have been filtered out here (0.2 Hz low-pass filter).

## 296 D. Power scaling

297 The power dissipated by the flow is shown in FIG. 7 as a function of the rotation frequency  
298  $f$ . It is computed from the product  $\Gamma \times 2\pi f$ , where  $\Gamma$  is the torque retrieved from the motor  
299 drive. We subtracted the power measured with an empty shell (dash-dot curve) to eliminate  
300 power dissipation in the mechanical set-up. The dissipation in the fluid reaches almost 8 kW  
301 for the highest rotation frequency of the inner sphere ( $f = \pm 30$  Hz). The small spread of  
302 the data dots indicates that power fluctuations are small. The continuous line is the record  
303 of power versus  $f$  when the inner sphere is ramped from 0 to  $-30$  Hz as in FIG. 4. The  
304 corresponding increase in kinetic energy only slightly augments power dissipation.

305 Power dissipation is found to scale as  $f^{2.5}$ , which does not differ from the scaling obtained  
306 in the laminar numerical study of section VI. There, it is explained as the result of the  
307 balance between the magnetic torque on the inner sphere and the viscous torque on the  
308 outer sphere, assuming that the fluid angular velocity below the outer viscous boundary  
309 layer is of the order of the inner sphere angular velocity. Although the outer boundary layer  
310 displays strong fluctuations, the situation is completely different from Taylor-Couette water  
311 experiments [48].

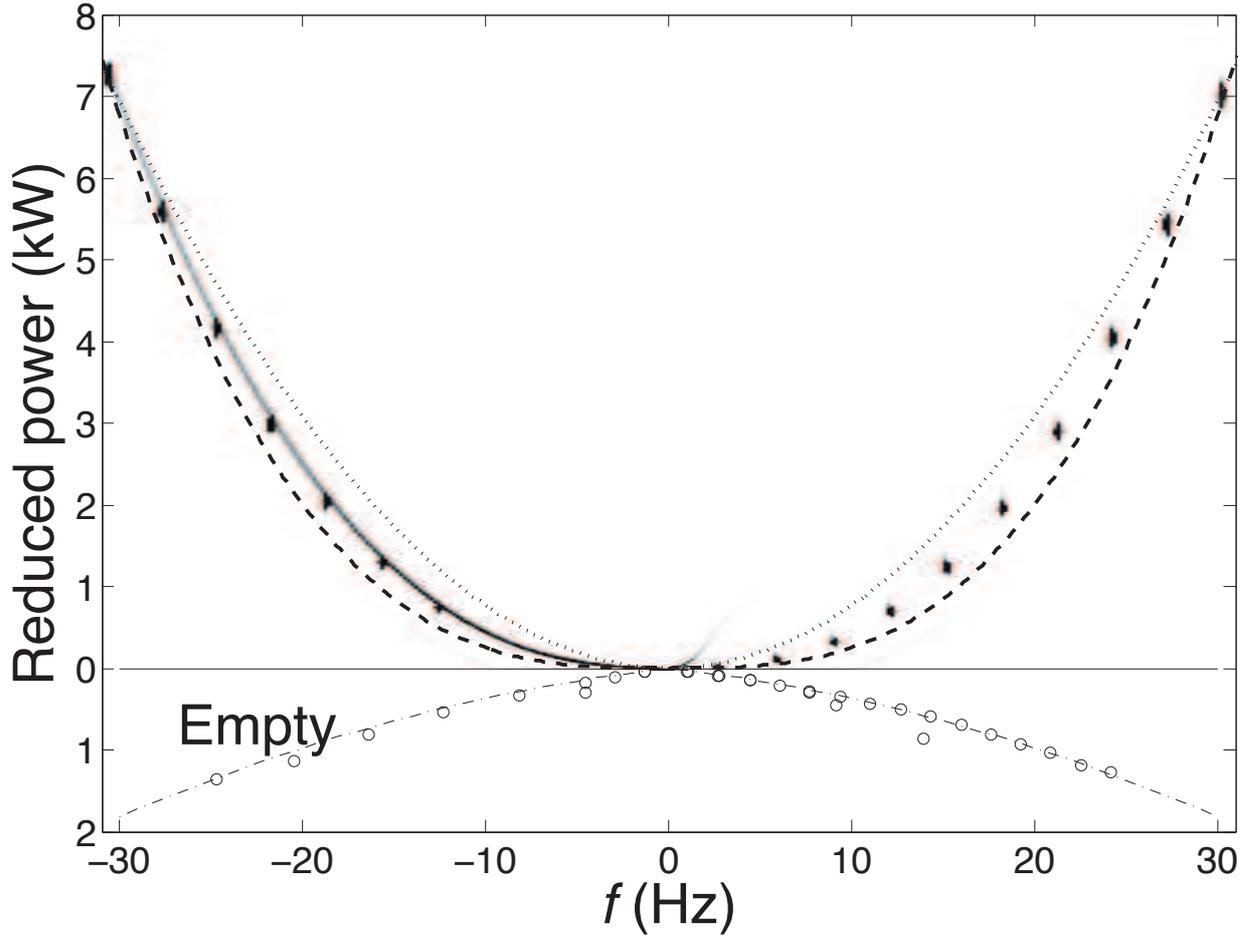


FIG. 7. Power dissipated by the flow in *DTS*. The data dots are from measurements of the motor torque for plateaus at given  $f$ . The dissipation in the mechanical set-up has been removed. It is obtained by rotating the inner sphere before filling the shell with sodium. It is drawn here upside-down in the lower panel (empty symbols) and can be fit by  $\mathcal{P}_{\text{empty}}(\text{W}) = 4 \times |2\pi f| + 0.03 \times (2\pi f)^2$  (dash-dot curve). Dissipation in the flow scales as  $f^{2.5}$ , and is here compared with  $f^2$  (dotted line) and  $f^3$  (dashed line).

### 312 III. GOVERNING EQUATIONS

A spherical shell of inner radius  $b$  and outer radius  $a$  is immersed in an axisymmetric dipolar magnetic field  $\mathbf{B}_d$ :

$$\mathbf{B}_d(r, \theta, \varphi) = B_0 \left(\frac{b}{r}\right)^3 [2 \cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta],$$

313 where  $(r, \theta, \varphi)$  are spherical coordinates. The outer boundary is kept at rest and the inner  
 314 sphere rotates with the constant angular velocity  $\Omega = 2\pi f$  along the same axis as the dipole  
 315 field that it carries. We assume that the electrically conducting fluid filling the cavity is  
 316 homogeneous, incompressible and isothermal. We further assume that the flow inside the  
 317 cavity is steady.

318 The inner body consists of a magnetized innermost core enclosed in an electrically con-  
 319 ducting spherical solid envelope of finite thickness  $d_b$ . We choose  $b$  as unit length,  $b\Omega$  as unit  
 320 velocity,  $\rho b^2 \Omega^2$  as unit pressure, and  $b^2 \Omega B_0 / \eta = \text{Rm} B_0$  as unit of induced magnetic field  $\mathbf{b}$   
 321 ( $\mathbf{B} = \mathbf{B}_d + \text{Rm} \mathbf{b}$ ). Then, the equations governing the flow  $\mathbf{u}$  and the induced magnetic field  
 322 are:

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\nabla \cdot \mathbf{b} = 0 \tag{3}$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \Lambda ((\mathbf{B}_d \cdot \nabla) \mathbf{b} + (\mathbf{b} \cdot \nabla) \mathbf{B}_d) + \text{Re}^{-1} \nabla^2 \mathbf{u} \tag{4}$$

$$\nabla^2 \mathbf{b} = -\nabla \times (\mathbf{u} \times \mathbf{B}), \tag{5}$$

323 where  $p$  is a modified pressure. The notation  $\Lambda$  refers to the Elsasser number, classically  
 324 used for rotating flows in the presence of a magnetic field. That number  $\Lambda$  compares the  
 325 magnetic and inertial forces in the vicinity of the magnetized inner sphere. In the shell  
 326 interior, the two forces are better compared by a "local" Elsasser number:  $\Lambda_l = (b/r)^6 \Lambda$   
 327 (with  $(b/a)^6 \simeq 1.83 \cdot 10^{-3}$ ). Finally, it is of interest to introduce the Hartmann number  
 328  $\text{Ha}$  that compares the magnetic and viscous forces. We have  $\text{Ha} = (\Lambda \text{Re})^{1/2}$ . In the shell  
 329 interior, the number  $(b/r)^3 \text{Ha}$  is more appropriate to compare the two forces. Typical values  
 330 of these dimensionless numbers can be found in TABLE II.

331 The set of equations (2-5), where the non linear terms are neglected, was the subject  
 332 of the analytical study of Dormy et al. [31] that described how the differential rotation

TABLE II. Typical values of the dimensionless numbers in the *DTS* experiment, computed for  $f = \Omega/2\pi = 25$  Hz.

Re	$b^2\Omega/\nu$	$1.3 \cdot 10^6$
Rm	$b^2\Omega/\eta$	10
$\Lambda$	$\sigma B_0^2/\rho\Omega$	1.9
Ha	$(\text{Re}\Lambda)^{1/2}$	$1.6 \cdot 10^3$

333 between the fluid interior and the outer sphere drives an influx of electrical currents from  
 334 the mainstream into the outer viscous Hartmann boundary layer. Electrical currents flow  
 335 along the viscous boundary layer and return to the conducting inner body along a free shear  
 336 layer located on the magnetic field line tangent to the outer boundary at the equator. As  
 337 these electrical currents cannot flow exactly parallel to the magnetic field line, they produce  
 338 a Lorentz force, which sustains "super-rotation" of the fluid. Recent studies have extended  
 339 the analysis to the case of a finitely conducting outer sphere [49, 50]. On increasing the  
 340 conductance of the container, Dormy et al. (2010) found that more and more electrical  
 341 currents leak into the solid boundary and the super-rotation rate gets as large as  $O(\text{Ha}^{1/2})$ .  
 342 Though the analytical results have set the stage for the interpretation of the experimental  
 343 results, the neglected non linear effects are crucial in the *DTS* experiment, even for the  
 344 smallest rate of rotation of the solid inner body.

345 Upon reversal of  $\Omega$ ,  $u_\varphi$  and  $b_\varphi$  change into  $-u_\varphi$  and  $-b_\varphi$  whilst the other components of  
 346  $\mathbf{u}$  and  $\mathbf{b}$  are kept unchanged.

## 347 IV. DIFFERENTIAL ROTATION

### 348 A. Transition between the Ferraro and geostrophic regimes

349 In this section, we use the UDV records to delve into the geometry of isorotation surfaces.

350 The L number associated to each dipolar magnetic field line enters the equation of the  
351 surfaces spanned by dipolar lines of force:

$$r = L \sin^2 \theta . \quad (6)$$

352 Accordingly,  $L$  gives the radius of the intersection of the magnetic field line with the  
353 equatorial plane. The notation L refers to the L-value (or L-shell parameter) widely used to  
354 describe motions of low energy particles in the Earth's magnetosphere. FIG. 8 shows that,  
355 for  $L \leq 2.7$ , the angular velocity measured along rays 2 and 3, which are the most appropriate  
356 to map the azimuthal velocity field, is, to a large extent, a function of L only. Thus, the  
357 angular velocity does not vary along magnetic field lines near the inner sphere, where the  
358 magnetic field is the strongest. We interpret this result as a consequence of Ferraro's theorem  
359 of isorotation. The latter is written:

$$\mathbf{B}_d \cdot \nabla \left( \frac{u_\varphi}{s} \right) = 0. \quad (7)$$

360 It is obtained from the  $\varphi$  component of the induction equation for steady fields, ignoring  
361 magnetic diffusion. Although often invoked in the framework of ideal MHD (where magnetic  
362 diffusion is negligible), Ferraro's law does not require a large Rm [51]. It implies that there  
363 is no induced magnetic field and that, as a consequence, the magnetic force is exactly zero.  
364 More precisely, deviations from this law lead to the induction of a magnetic field, which  
365 produces a magnetic force that tends to oppose this induction process. Writing  $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1$ ,  
366 where  $\mathbf{u}_0$  obeys the equation (7), we obtain  $b \approx u_1$  from (5). Then, the momentum equation  
367 (4) yields  $u_1 \approx (\text{Re}\Lambda)^{-1} u_0 = \text{Ha}^{-1} u_0$  (as numerically verified in [35]) when the inertial term,  
368 on the left hand side, can be neglected. Ferraro's law of isorotation, though, is not the only  
369 way to cancel the magnetic force. In the presence of electric currents parallel to the magnetic  
370 field, the magnetic force remains zero and the equation (7) can be violated [50, 51]. For the  
371 geometry of the *DTS* experiment, it cannot happen along the innermost dipolar field lines  
372 that join the two hemispheres, without touching the outer sphere. Indeed, symmetry with  
373 respect to the equatorial plane E implies that the currents do not cross E.

374 Thus, the observation of a velocity field obeying Ferraro's law is a symptom that magnetic  
 375 forces predominate in that region. Note that the fact that the two legs of the profile along  
 376 ray 2 show similar velocities even for large  $L$  only probes the symmetry of the flow with  
 377 respect to the equatorial plane.

379 Now, FIG. 9 shows that for  $s \geq 0.6$  the azimuthal velocity is largely a function of  $s$   
 380 only. There, the Proudman-Taylor theorem holds and azimuthal flows are geostrophic as  
 381 the inertial forces predominate. In contrast with the case of a rotating outer sphere (see  
 382 Figure 7 in [39]), there is no region of uniform rotation: zonal velocities are  $z$ -independent  
 383 but vary with the distance to the  $z$  axis.

384 The transition between the Ferraro and geostrophic regimes (FIG. 10) occurs at smaller  
 385 distances from the axis as the rotation frequency of the inner core increases (unfortunately,  
 386 we cannot get reliable UDV data for larger  $f$ ). It takes place where the local Elsasser number  
 387  $\Lambda_l$ , which compares the magnetic and inertial forces, is of order 1. It is noteworthy that  
 388 the Elsasser number  $\Lambda$  defines the location (cylindrical radius) where  $\Lambda_l = 1$ . The surface  
 389  $\Lambda_l = 1$  separates two regions of the fluid cavity. Inside this surface, the magnetic forces  
 390 predominate whether outside it the rotation forces are the most important ones. Finally,  
 391 the value of  $\Lambda$  largely defines the geometry of isorotation surfaces.

392 In the geostrophic region, magnetic stress integrated on the geostrophic cylinders remains  
 393 strong enough to overcome the viscous friction at the outer boundary and to impart a rapid  
 394 rotation to the fluid but becomes weaker than the Reynolds stress (which can be represented  
 395 as a Coriolis force). As a result, the fluid angular velocity is still of the order of the angular  
 396 velocity of the inner sphere and the velocities are predominantly geostrophic.

## 397 B. Inversion of velocity profiles

398 Flow velocity is constrained by its projection on the several ultrasonic rays that we  
 399 shoot. We invert the Doppler velocity profiles for the large scale mean flow, assuming that  
 400 the steady part of the flow is symmetric about the axis of rotation and with respect to the  
 401 equatorial plane. A poloidal/toroidal decomposition,

$$\mathbf{u} = u_\varphi \mathbf{e}_\varphi + \nabla \times (u_p \mathbf{e}_\varphi), \quad (8)$$

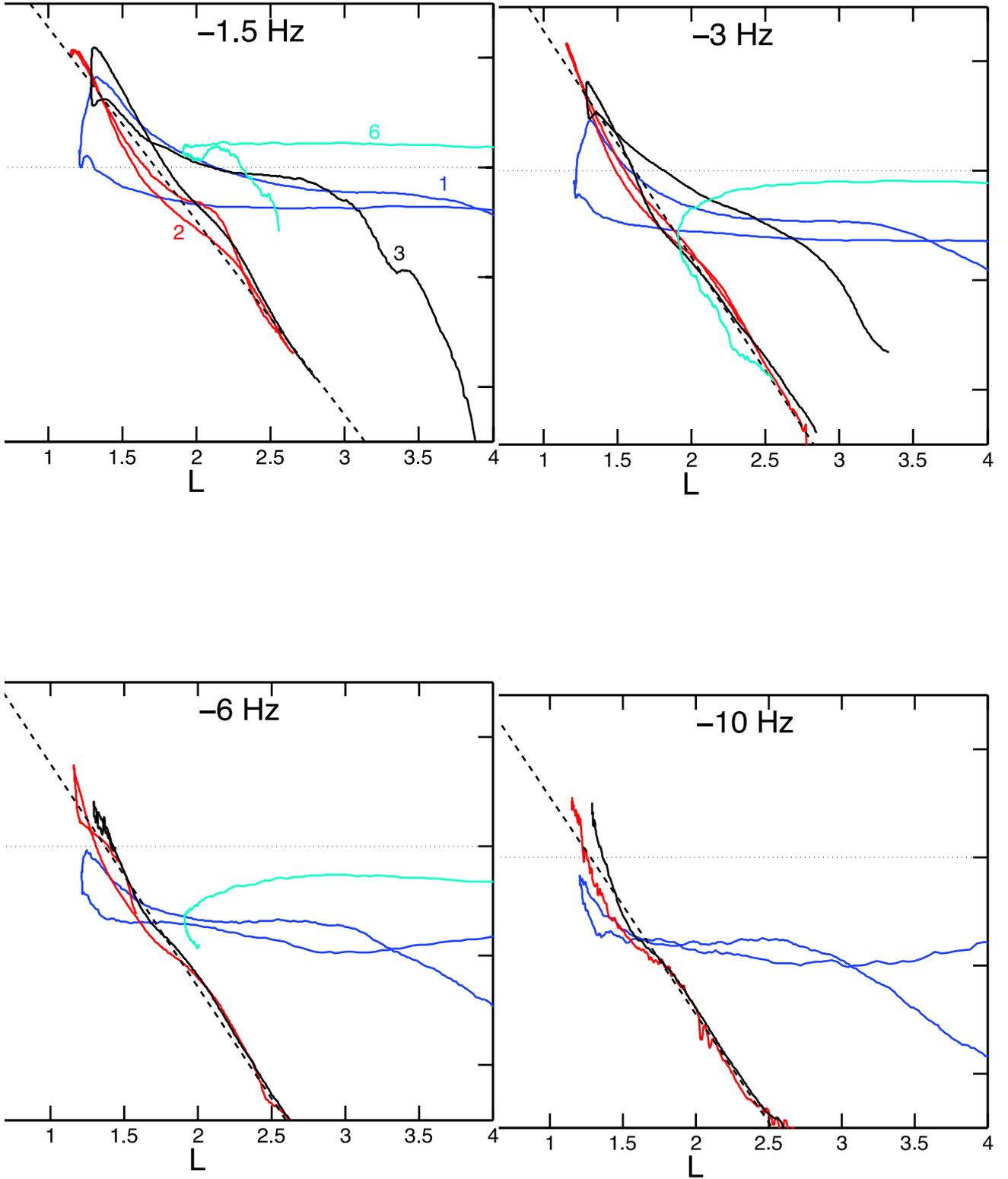


FIG. 8. (Color online) Rotation frequency of the fluid sodium over the inner sphere rotation frequency as a function of the magnetic field lines  $L$  for four ultrasonic velocity profiles (trajectories 1, 2, 3 and 6, with the same color code as in FIG 3) and four inner sphere rotation frequencies ( $f = -1.5, -3, -6$  and  $-10$  Hz). The dashed line is a straight line to help the eye.

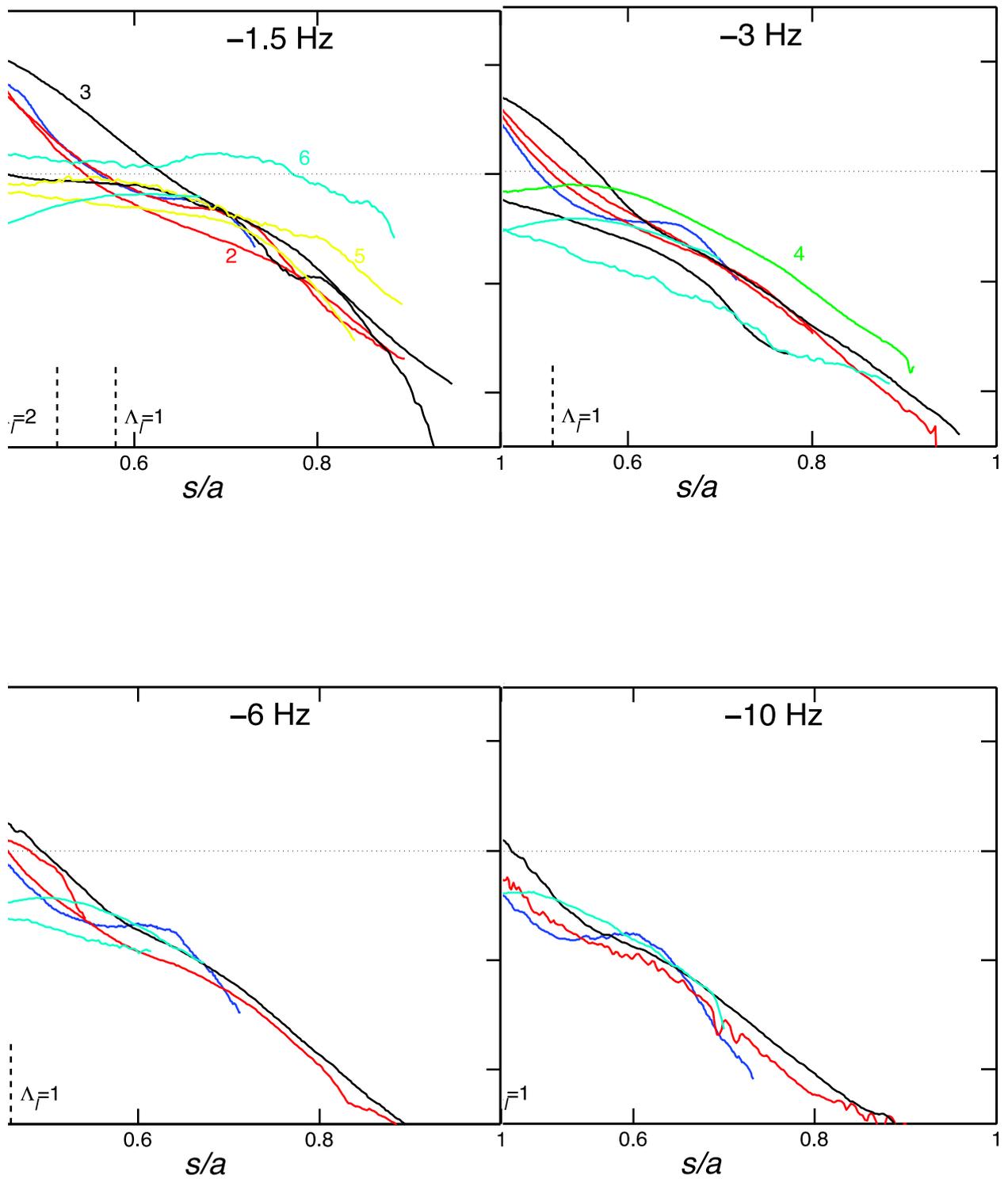


FIG. 9. (Color online) Rotation frequency of the fluid sodium normalized by the inner sphere rotation frequency as a function of  $s$ , for various ultrasonic velocity profiles and four inner sphere rotation frequencies ( $f = -1.5, -3, -6$  and  $-10$  Hz). The colors of the profiles (numbers) follow the conventions laid out in FIG. 3.

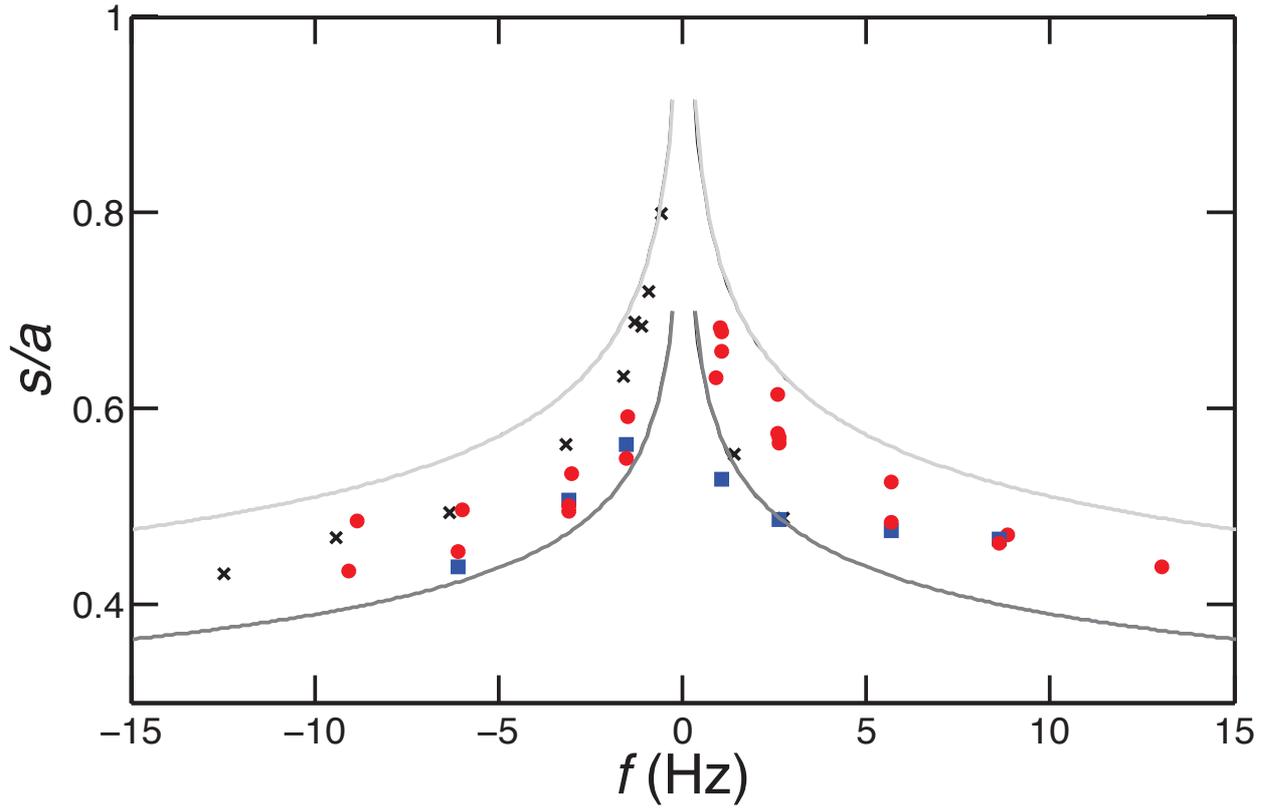


FIG. 10. (Color online) Normalized cylindrical radius  $s/a$  along the UDV trajectories number 1 (blue square), 2 (red circle) and 3 (black cross) where  $f_{fluid} = f$  (*i.e.*  $f^* = 1$ ) as a function of the inner sphere rotation frequency. Pale line :  $\Lambda_l = 0.5$ , Dark line :  $\Lambda_l = 2.5$ .

402 is employed. We first consider the azimuthal velocity  $u_\varphi$ , which is expanded in associated  
 403 Legendre functions with odd degree and order 1, *i.e.*

$$u_\varphi(r, \theta) = \sum_{l=0}^{l_{max}} u_\varphi^l(r) P_{2l+1}^1(\cos \theta) . \quad (9)$$

404 The functions  $u_\varphi^l(r)$  are decomposed into a sum from  $k = 0$  to  $k_{max}$  of Chebyshev poly-  
 405 nomials of the second kind on the interval  $[0, 1]$  mapped onto the interval  $[b/a, 1]$ , *i.e.* the  
 406 fluid domain. The azimuthal velocity is not constrained to vanish at the inner and outer  
 407 boundaries, in order to account for the presence of thin unresolved boundary layers.

408 Azimuthal velocities are more than 10 times larger than the poloidal (*i.e.* meridional)  
 409 velocities. Nevertheless, the latter projects onto the ultrasound rays. We take the difference  
 410 of the profiles acquired for  $f$  and  $-f$  in order to eliminate this small contribution (the  
 411 meridional circulation does not change sign while the azimuthal velocity does).

412 FIG. 11 shows the isovalues of angular frequency  $f^*$  inverted for  $f = \pm 3$  Hz, with  
 413  $l_{max} = 3$  and  $k_{max} = 7$ . A crescent of super-rotation is present near the inner sphere.  
 414 There, isorotation contours roughly follow magnetic field lines, in agreement with Ferraro's  
 415 theorem, as anticipated above. At larger cylindrical distance from the inner sphere, the  
 416 flow becomes geostrophic: the contour lines are vertical. We note that angular velocities  
 417 just above the north pole of the inner sphere do not comply with Ferraro's law. Instead,  
 418 velocities decrease to quite low values inside the cylinder tangent to the inner sphere. Such  
 419 violations have been shown to occur when the electric conductivity of boundaries is high  
 420 [50, 51]. We speculate that we might be in this situation inside the tangent cylinder because  
 421 the opening of the sphere at the top and bottom (see FIG. 3) replaces the poorly conducting  
 422 stainless steel wall by sodium.

423 FIG. 11 compares the synthetic angular velocity profiles to the observed Doppler velocity  
 424 profiles along the various rays. Note that super-rotation is clearly visible in the raw profiles.  
 425 The drop in velocity just above the inner sphere is constrained by profiles 4 (green) and 6  
 426 (cyan), but its vertical extent is not.

### 427 C. $f_{fluid}$ deduced from differences in electric potential and from UDV

428 As in the previous study of *DTS* with rotating outer sphere [39], we observe that the  
 429 amplitudes of the differences in electric potential  $\Delta V$ 's vary linearly with  $\Delta V_{40}$ , the pro-

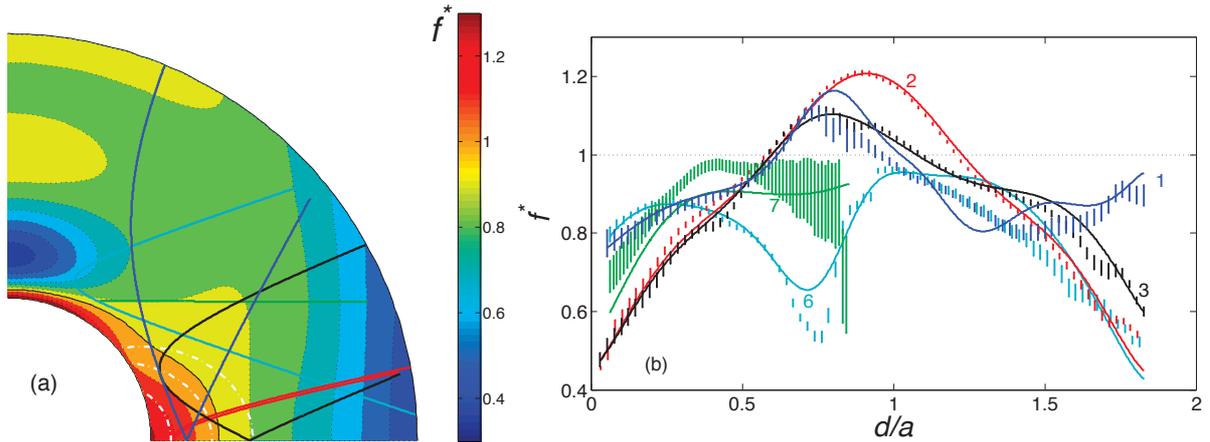


FIG. 11. (Color online) (a) Reconstructed isovalue map of fluid angular frequency  $f^*$  (the fluid angular frequency normalized by  $f$ ) at  $f = \pm 3$  Hz in a meridional plane, assuming axisymmetry and symmetry with respect to the equator. Three dipolar field lines (dash-dot white) are superimposed in the angular velocity maps. Super-rotation ( $f^* \gtrsim 1$ ) is clearly visible near the inner sphere, where the Ferraro law of isorotation applies. Contours become vertical further away, where geostrophy dominates. The fluid frequency is higher than 0.4 everywhere except in thin unresolved boundary layers. The color lines are the projection in the upper half ( $s, z$ ) plane of the ultrasonic rays used in the inversion (see FIG. 3). (b) Comparison between the measured ultrasonic Doppler  $f^*$  (shown by their error bars) and the synthetic profiles (solid lines) computed from the angular frequency map of (a) for  $f = \pm 3$  Hz. The  $x$ -axis gives the distance along the ray (in  $a$  units). The corresponding rays are plotted in (a) with the same color code (and indicated with trajectory numbers referring to FIG. 3).

430 portionality factor increasing from the equator toward the poles due in particular to the  
 431 increase of  $B_r$  in formula (1). We show however in the present study that measuring the  
 432 electric potential does not yield a reliable indicator of the angular velocity  $f^*$  using formula  
 433 (1). In FIG. 12, we compare the normalized fluid angular velocity  $f^*$  retrieved from the  
 434  $\Delta V$ 's, for four different latitudes, to  $f^*$  obtained directly by UDV at the nearest measured  
 435 point, around  $d/a = 0.1$ . The frequencies  $f^*$  obtained from  $\Delta V$  and from UDV in FIG. 12,  
 436 would be similar if both measurement techniques were only sensitive to  $u_\varphi$  in the interior  
 437 below the outer viscous boundary layer. The strong discrepancy between these two sets of  
 438 frequencies reveals instead that the outer boundary layer in *DTS* cannot simply be reduced  
 439 to a Hartmann layer, outside of which the meridional currents  $j_\theta$  can be neglected. We

440 further discuss this point in the numerical part VI.

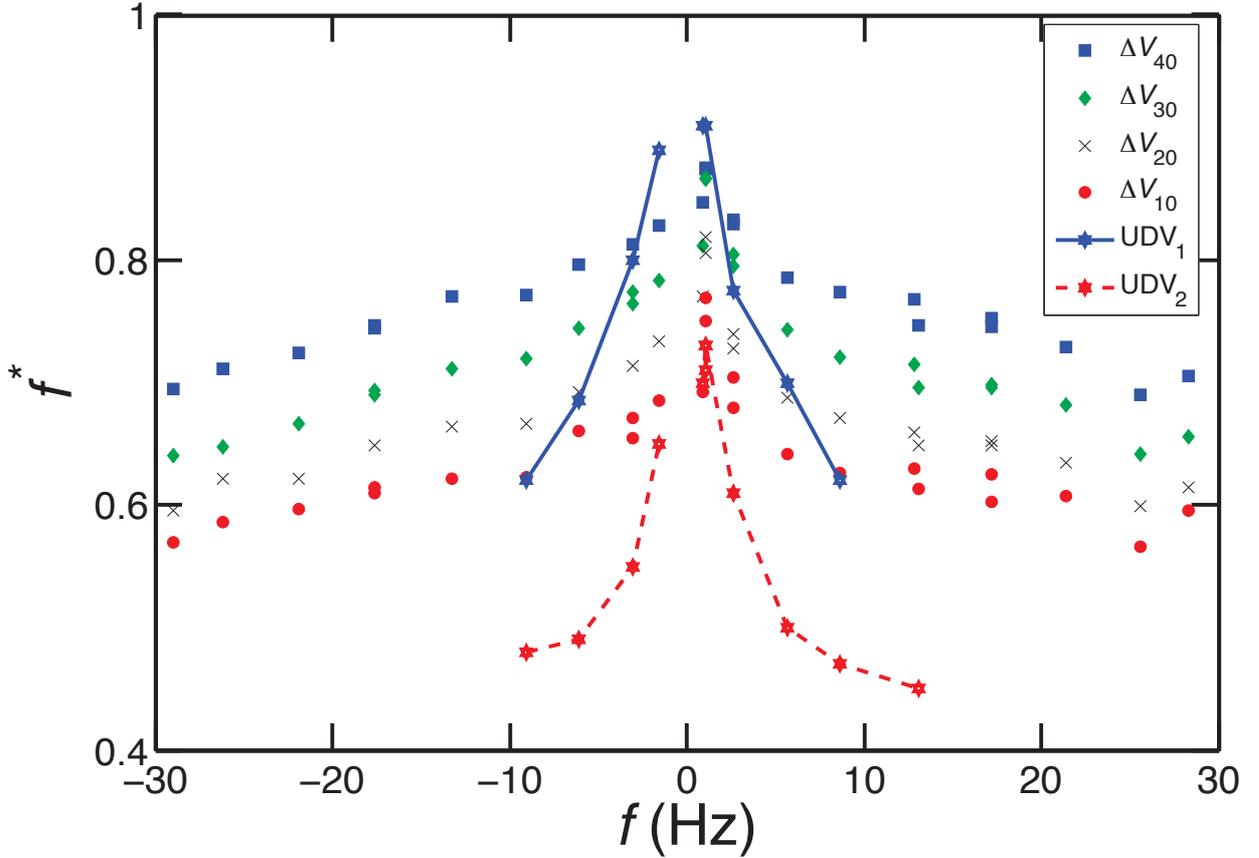


FIG. 12. (Color online)  $f^*$  deduced from the measurements of  $\Delta V$  using formula (1). Blue line :  $f^*$  value obtained with UDV measurements on the trajectory number 1 at the distance  $d/a = 0.1$ . Dashed red line :  $f^*$  value obtained with UDV measurements on the trajectory number 2 for  $d/a = 0.1$ .

## 441 V. MERIDIONAL CIRCULATION

442 The meridional circulation is constrained from Doppler velocity profiles of the radial ve-  
 443 locity (shot along the radial direction), from profiles shot in a meridional plane, and from the  
 444 projection of the meridional velocity on "azimuthal" shots. The latter is obtained by taking  
 445 the sum of the profiles acquired for  $f$  and  $-f$ , in order to eliminate the azimuthal contribu-  
 446 tion. The same is done for the radial and meridional profiles to remove any contamination  
 447 from azimuthal velocities.

448 The poloidal velocity scalar  $u_P$  of equation (8) is expanded in associated Legendre func-  
 449 tions with even degree and order 1, *i.e.*

$$u_P(r, \theta) = \sum_{l=0}^{l_{max}} u_P^l(r) P_{2l}^1(\cos \theta). \quad (10)$$

450 The radial  $u_r$  and orthoradial  $u_\theta$  components of velocity are then obtained as:

$$u_r(r, \theta) = \sum_{l=0}^{l_{max}} \frac{u_P^l(r)}{r} \frac{1}{\sin \theta} \frac{d}{d\theta} (\sin \theta P_{2l}^1(\cos \theta)). \quad (11)$$

451

$$u_\theta(r, \theta) = - \sum_{l=0}^{l_{max}} \left( \frac{u_P^l(r)}{r} + \frac{du_P^l(r)}{dr} \right) P_{2l}^1(\cos \theta). \quad (12)$$

452 The functions  $u_P^l(r)$  are decomposed into a sum of  $\sin(k\pi(r - b/a)/(1 - b/a))$  from  $k = 0$   
 453 to  $k_{max}$ . The radial velocity is thus constrained to vanish at the inner and outer (rigid)  
 454 boundaries, but the orthoradial velocity is not, in order to account for the presence of thin  
 455 unresolved boundary layers. FIG. 13 shows the streamlines of the meridional circulation  
 456 inverted for  $f = \pm 3$  Hz, with  $l_{max} = 4$  and  $k_{max} = 8$ . The fluid is centrifuged from the  
 457 inner sphere in the equatorial plane and moves north in a narrow sheet beneath the outer  
 458 boundary. It loops back to the inner sphere in a more diffuse manner. Meridional velocities  
 459 are more than ten times weaker than azimuthal velocities.

460 FIG. 14 compares the synthetic radial and meridional profiles to the observed Doppler  
 461 velocity profiles along the various rays. Velocities are normalized by  $2\pi fa$ .

462 Over a decade (from  $f = 1.5$  Hz to  $= 15$  Hz), radial velocities are consistently centrifugal  
 463 at  $10^\circ$  latitude and centripetal at  $40^\circ$ , and are roughly proportional to  $f$ . The radial profiles  
 464 at  $20^\circ$  are more complex and evolve with  $f$ , indicating a non-monotonic evolution of the  
 465 meridional circulation, also evidenced by the records of the  $r$  and  $\theta$  components of the  
 466 induced magnetic field inside the fluid (see FIG. 6). FIG. 15 compiles the *rms* value of  
 467 radial velocity at  $20^\circ$  for various  $f$ . Note that the fluctuations are larger than this value,  
 468 which is almost 50 times smaller than azimuthal velocities.

## 469 VI. COMPARISON WITH NUMERICAL SIMULATIONS

470 Two previous numerical studies are particularly relevant to our work. Hollerbach et al.  
 471 studied exactly the *DTS* configuration but for values of  $\Lambda$  much larger than its value in the  
 472 experiment [29]. They focus their study on the modification of the linear solution by inertial

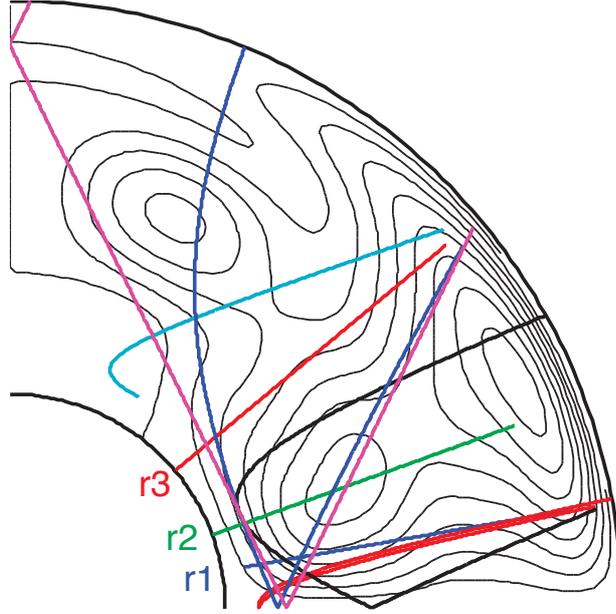


FIG. 13. (Color online) Reconstructed stream lines of the meridional circulation at  $f = \pm 3$  Hz in a meridional plane, assuming axisymmetry and symmetry with respect to the equator. The interval between lines is  $1.6 \times 10^{-3}$ . The fluid is centrifuged away from the inner sphere in the equatorial region and moves up to the pole along the outer boundary. The color lines are the projection in the upper half  $(s, z)$  plane of the ultrasonic rays used in the inversion.

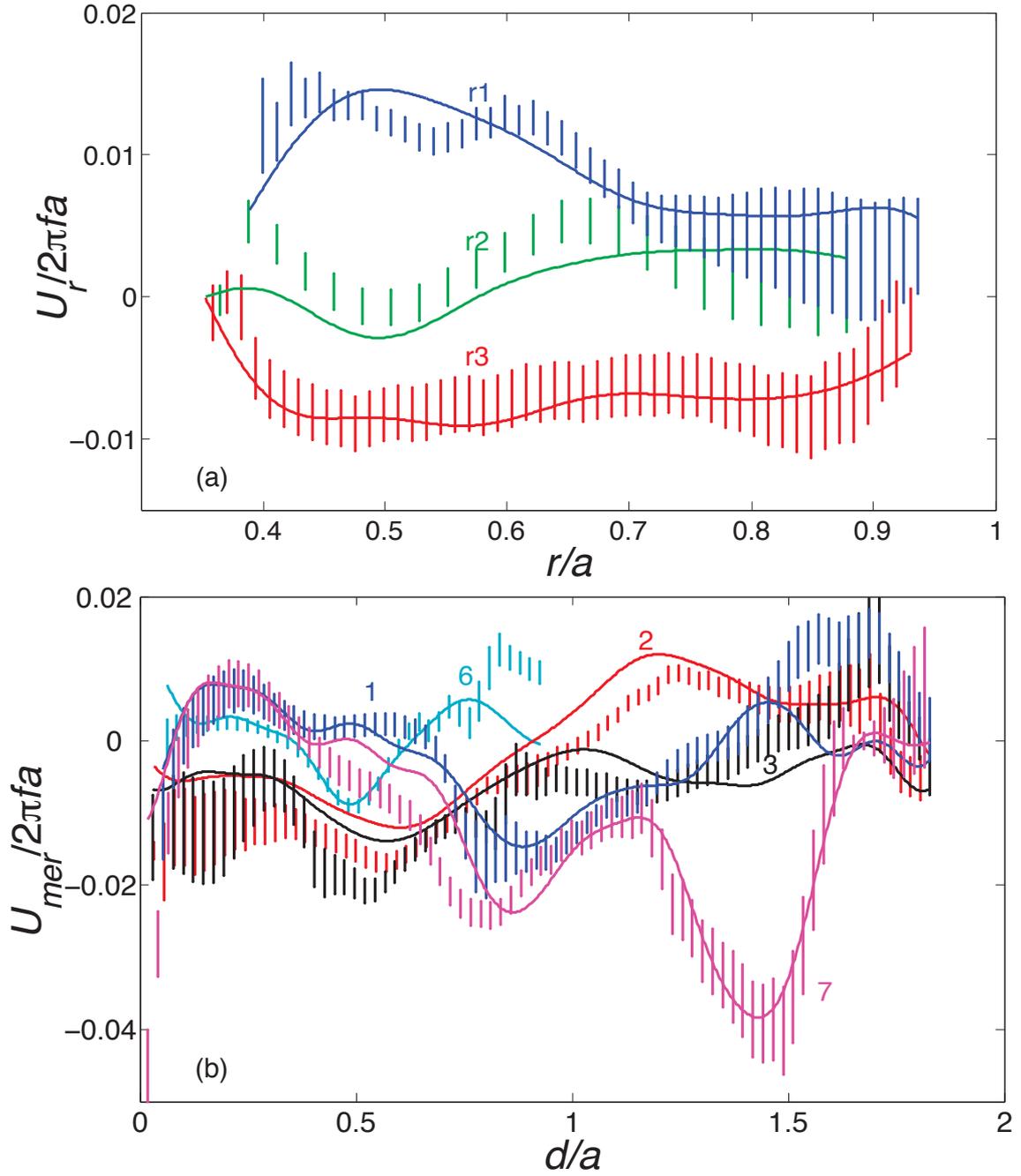


FIG. 14. (Color online) Comparison between the measured ultrasonic Doppler velocity profiles (shown by their error bars) and the synthetic profiles (solid lines) computed from the meridional circulation map of FIG. 13 for  $f = \pm 3$  Hz. (a) Radial profiles along the radial direction  $r_1$ ,  $r_2$  and  $r_3$  shown in FIG. 13. (b) "Azimuthal" profiles. The contribution from the azimuthal flow has been removed by taking the sum of profiles acquired for  $f$  and  $-f$ . The  $x$ -axis gives the distance along the ray (in  $a$  units) and the  $y$ -axis is the velocity measured along the ray, adimensionalized by  $2\pi f a$ . The corresponding rays are plotted in FIG. 13 with the same color code (for the grayscale version, the trajectory numbers in (b) refers to those in FIG. 3).

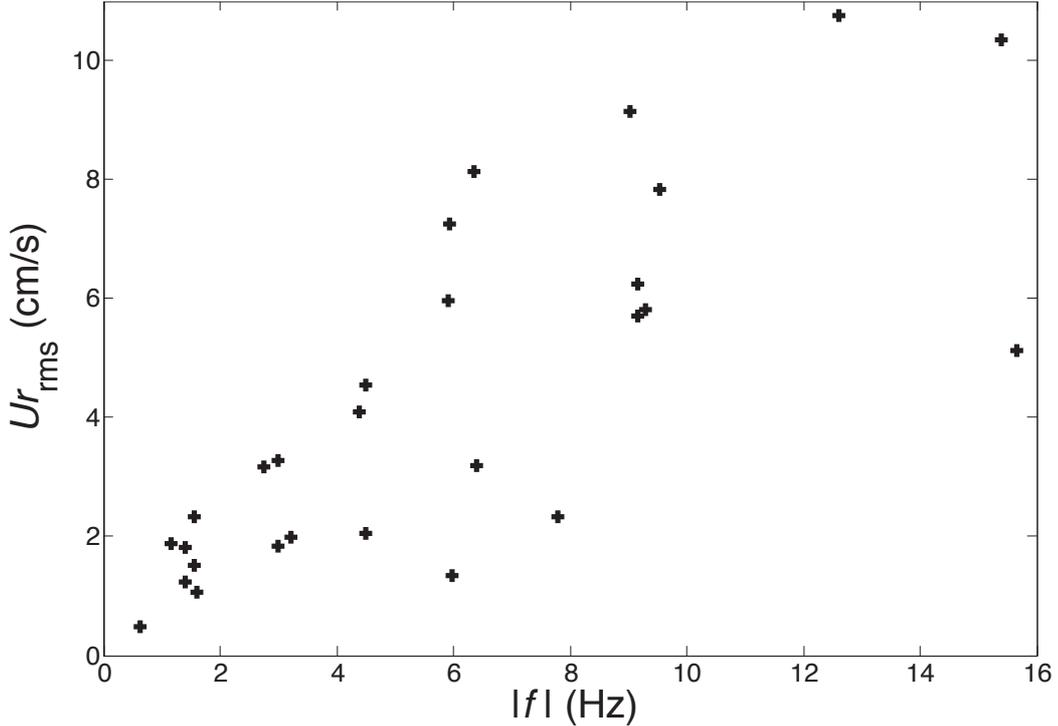


FIG. 15. Compilation of the *rms* radial velocity amplitude as a function of the absolute value of  $f$ . The *rms* velocity is computed from Doppler velocimetry profiles shot at a latitude of  $20^\circ$ , from 3 cm beneath the outer shell down to the inner sphere (to avoid spurious values close to the outer boundary). Radial velocity is roughly proportional to  $f$  but there is a large dispersion, as the shape of the profiles changes with  $f$ . Note that for  $f = 10$  Hz, the tangential velocity on the inner sphere reaches 465 cm/s.

473 effects, stressing that the magnetic field line tangent to the outer sphere at the equator loses  
474 its significance in the non linear regime. As a result of the relatively large value of  $\Lambda$ , the  
475 inertial effects remain too weak -when the outer sphere is at rest- to make a geostrophic  
476 region arise at large distances from the axis. The solutions of Garaud [52] (see the figures  
477 7 and 11) for a slightly different problem do show the transition between a Ferraro and a  
478 geostrophic regions. In her model, which pertains to the formation of the solar tachocline,  
479 a dipolar magnetic field permeates a thick spherical shell as in *DTS*, the rotation of the  
480 outer boundary is imposed and the rotation of the inner boundary is a free parameter: a  
481 condition of zero torque is imposed on that boundary. Numerical models [29, 39] of the  
482 *DTS* experiment when the outer sphere is rotating also clearly show a Ferraro region near

483 the inner sphere where the magnetic field is strong and a geostrophic region in the vicinity  
 484 of the equator of the outer sphere. We argue below that all these results obtained for a  
 485 rotating outer sphere provide us with a useful guide to interpret the numerical solutions  
 486 when the outer sphere is at rest.

#### 487 **A. The numerical model**

488 The model consists of four nested spherical layers (see FIG. 16). The fluid layer is enclosed  
 489 between a weakly conducting outer container and a central solid sphere comprised of an inner  
 490 insulating core and of a strongly conducting outer envelope.

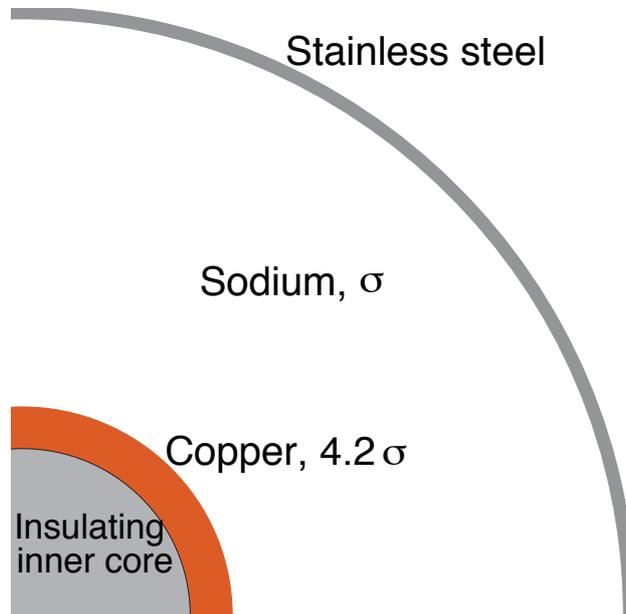


FIG. 16. (Color online) Geometry of the numerical model. The relative conductance of the solid outer shell is  $\sigma_b \delta / \sigma a = 1/336$ , with  $\sigma_b$  and  $\delta$  respectively the conductivity and the thickness of the outer sphere. It reproduces the experimental value with  $\sigma_b$  chosen as the conductivity of stainless steel at  $140^\circ\text{C}$ . The conductivity ratio between the layers 2 and 3 reproduces the ratio (4.2) between the conductivity of copper and sodium.

491 The velocity field is decomposed as stated in the definitions (8) and (9). The variables  
 492  $u_\varphi^l(r)$  and  $u_p^l(r)$  are then discretized in radius. Analogous decompositions of variables de-  
 493 noted  $b_\varphi^l(r)$  and  $b_p^l(r)$  are employed to represent the induced magnetic field. The truncation  
 494 level  $l_{max}$  (see (9)) is 120 and at least 450 unevenly spaced points are used in the radial

495 direction. Specifically, the density of points strongly increases close to the boundaries in  
496 order to resolve the viscous boundary layers.

497 The equations (4) and (5), modified to include all the non linearities and the time deriva-  
498 tives of  $\mathbf{u}$  and  $\mathbf{b}$ , are transformed into equations for  $u_\varphi^l$ ,  $u_p^l$ ,  $b_\varphi^l$  and  $b_p^l$ . We treat the non linear  
499 terms explicitly. To advance from one time step to the next, we use an Adams-Bashforth  
500 method. Diffusive terms, however, are treated implicitly. Finally, Laplace's equation in  
501 spherical coordinates separates which makes it easy to write the magnetic boundary condi-  
502 tions.

503 The dimensionless numbers  $Re$  and  $\Lambda$  are chosen so that steady solutions exist and are  
504 stable, with  $Pm \ll 1$  ( $Pm$  enters the definition of the unit induced field). We strive to repro-  
505 duce the experimental values of  $\Lambda$  and  $R_m$ . Solutions are obtained after time-stepping the  
506 equations until a stationary or periodic state is reached. They have been successfully com-  
507 pared to solutions obtained with another numerical code PARODY, which is not restricted  
508 to axisymmetric variables [26, 53].

509 It is not possible to simulate the Reynolds number of the experiment, which is about  $10^6$ .  
510 For the experimental range of  $\Lambda$ , steady solutions are obtained with  $Re \sim 10^3$ .

## 511 B. Steady axisymmetric solutions

512 FIG. 17 displays a typical solution for the angular and meridional velocities that illustrates  
513 well the experimental results. The fluid rotates faster than the magnetized inner body in its  
514 vicinity. There, the angular velocity is constant along magnetic field lines of force. Further  
515 away of the inner core, the zonal shear becomes almost geostrophic. In addition to these  
516 features that we have retrieved from the experimental results, the numerical solution displays  
517 recirculation in the outer boundary layer at high latitude. There, the interior flow largely  
518 consists in rigid rotation and the boundary layer has the characteristics of a Bödewadt layer  
519 with a region of enhanced angular rotation.

520 For large enough  $Re$  (*e.g.*  $(a/b)^2 Re = 10^4$  with  $(b/a)^2 Ha = 20$ ), circular waves are  
521 present in the Bödewadt layer, above  $60^\circ$  of latitude. They propagate towards the axis.  
522 Similar waves had been reported before in simulations of the flow between a rotating and  
523 a stationary disk in the absence of a magnetic field [54]. There, they eventually die out.  
524 Thus, the persistence of propagation of circular waves in the boundary layer attached to the

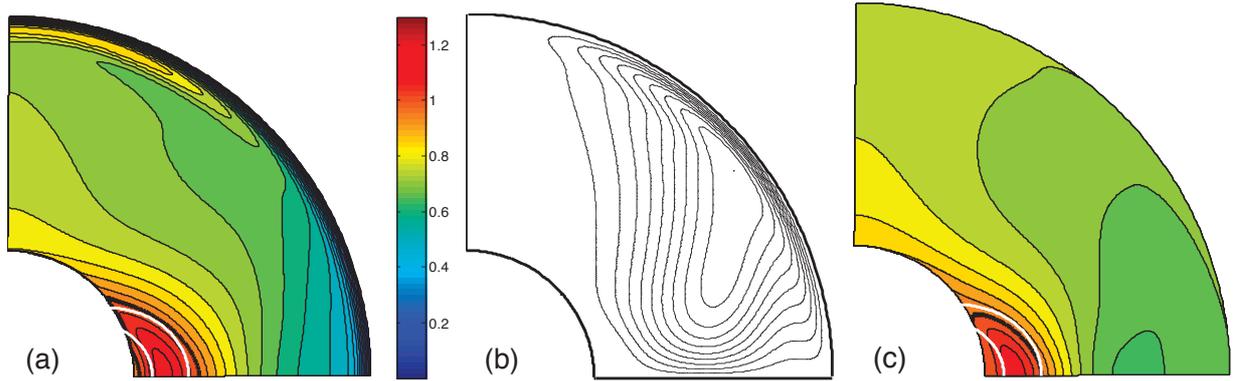


FIG. 17. (Color online) (a) Angular and (b) meridional velocity in a meridional plane for  $\text{Re} = 9.5 \cdot 10^2$ ,  $\text{Ha} = 163$ , and  $\text{Pm} = 10^{-3}$ . (c) angular velocity estimated from  $V$ , using (1). Two dipolar field lines (white) are superimposed in the angular velocity maps, and the thick black contour line is where the angular velocity is unity.

525 sphere at rest may be attributed to the presence of a magnetic field. On the other hand,  
 526 these waves arise for larger  $\text{Re}$  as  $\text{Ha}$  is augmented. Their emergence delimits the domain  
 527 of steady solutions.

528 We have checked that the thickness of the outer boundary layer in the numerical solution  
 529 scales as  $\Omega^{-1/2}$ . Note that it corresponds to 3 mm for  $\Omega = 1.5 \text{ s}^{-1}$  and the viscosity of  
 530 liquid sodium. The fluid rotation is driven by the electromagnetic torque acting at the inner  
 531 boundary against the viscous torque at the outer boundary. We have found that both the  
 532 viscous torque on the inner surface and the electromagnetic torque on the outer surface are  
 533 negligible. Comparing different simulations, we have also checked that the main viscous  
 534 torque scales as  $\sim \Omega^{3/2}$ , as expected from the thickness of the Bödewadt layer. Thus, the  
 535 power required to drive the fluid rotation scales as  $\Omega^{5/2}$ , as does the experimentally measured  
 536 power, and torque measurements do not give indications on turbulence (see section IID).

537 The angular rotation just below the outer viscous layer scaled by the inner core angular  
 538 rotation decreases with  $\text{Re}$  in agreement with the experimental results. On the other hand,  
 539 the angular rotation that would be inferred from the electric potential differences calculated  
 540 at the outer surface using expression (1) increases with  $\text{Re}$ . FIG. 17(c) displays the angu-  
 541 lar velocity as estimated from the electric potential, according to equation (1). It can be  
 542 compared to FIG. 17(a). The actual shear is well retrieved where the magnetic force pre-  
 543 dominates, in the region where Ferraro's law of isorotation holds. There, the electric current

544 density  $\mathbf{j}$  is limited by the strength of the magnetic force, which needs to be balanced by an-  
545 other force. That restriction makes it possible to neglect  $\mathbf{j}$  in Ohm's law. Then, predictions  
546 made from (1) are correct. On the other hand, the actual shear is not well recovered in the  
547 geostrophic region where the electric current density is not limited by the strength of the  
548 magnetic field. There, the frozen-flux relation (1) can be violated. We thus explain why the  
549 electric potential measurements at the surface of the *DTS* experiment do not yield a good  
550 prediction of the angular velocity immediately below the outer viscous boundary layer.

551 Our first discussion [30] of the electric potential measurements was based on a numerical  
552 model calculated for the experimental values of  $Ha$  and thus for too large values of  $\Lambda$ . As a  
553 result, the magnetic force, in the numerical model, was dominant in the entire fluid layer and  
554 the frozen-flux relationship (1) was verified, at least away from the equator where  $B_r = 0$ .  
555 However, equation (1), becomes less and less valid as  $Re$  is increased and  $\Lambda$  decreased, in  
556 agreement with the divergence that has been experimentally observed (see the FIG. 12)  
557 between the angular velocity calculated from (1) and the actual velocity.

558 Incidentally, cranking up the rotation of the magnetized inner sphere stabilizes the fluid  
559 circulation, at least within a certain parameter range. We have calculated the time-averaged  
560 solution (not shown) for the same parameters as the steady solution illustrated by FIG. 17,  
561 but for a lower  $Re$ . Both the flow and the induced magnetic field are periodic for this set of  
562 parameters. A second meridional roll, which is centripetal in the equatorial plane, turns up  
563 in the outer region. There, it creates a disk-shaped region where the rotation is slow and  
564 the solution is strikingly different from the almost geostrophic solution (FIG. 17) obtained  
565 for a slightly larger value of  $Re$ .

566 **C. Comparison between numerical simulations and experimental results**

567 We find that reproducing the Elsasser number  $\Lambda$ , rather than a combination of  $\Lambda$  and  $\text{Re}$   
568 such as the Hartmann number  $\text{Ha} = (\text{Re}\Lambda)^{1/2}$ , is the key factor to recover the experimental  
569 results. The parameters for the solution displayed in FIG. 17 correspond to  $\Lambda = 28$ , which  
570 is the appropriate value for experiments with  $\Omega = 1.5 \text{ s}^{-1}$ . With  $\text{Pm} = 10^{-3}$ , the value of  
571 the magnetic Reynolds number is about right. It remains too small for the poloidal field to  
572 be much different from the imposed dipole field (again for the parameters of FIG. 17).

573 FIG. 18 shows that numerical solutions are able to satisfactorily reproduce the ultrasonic  
574 measurements of angular velocity, obtained for the same values of  $\Lambda$ , as expected from the  
575 similitude of the angular velocity maps 11 and 17. The simulated velocities have weaker  
576 amplitude than the measured ones in much of the fluid though. We have checked that  
577 increasing  $\text{Re}$ , whilst keeping  $\Lambda$  constant, favours enhanced corotation between the fluid  
578 and the inner core. As our calculations are for much smaller  $\text{Re}$  than the values realized in  
579 the experiment, that result may explain the remaining discrepancy between measured and  
580 simulated velocities.

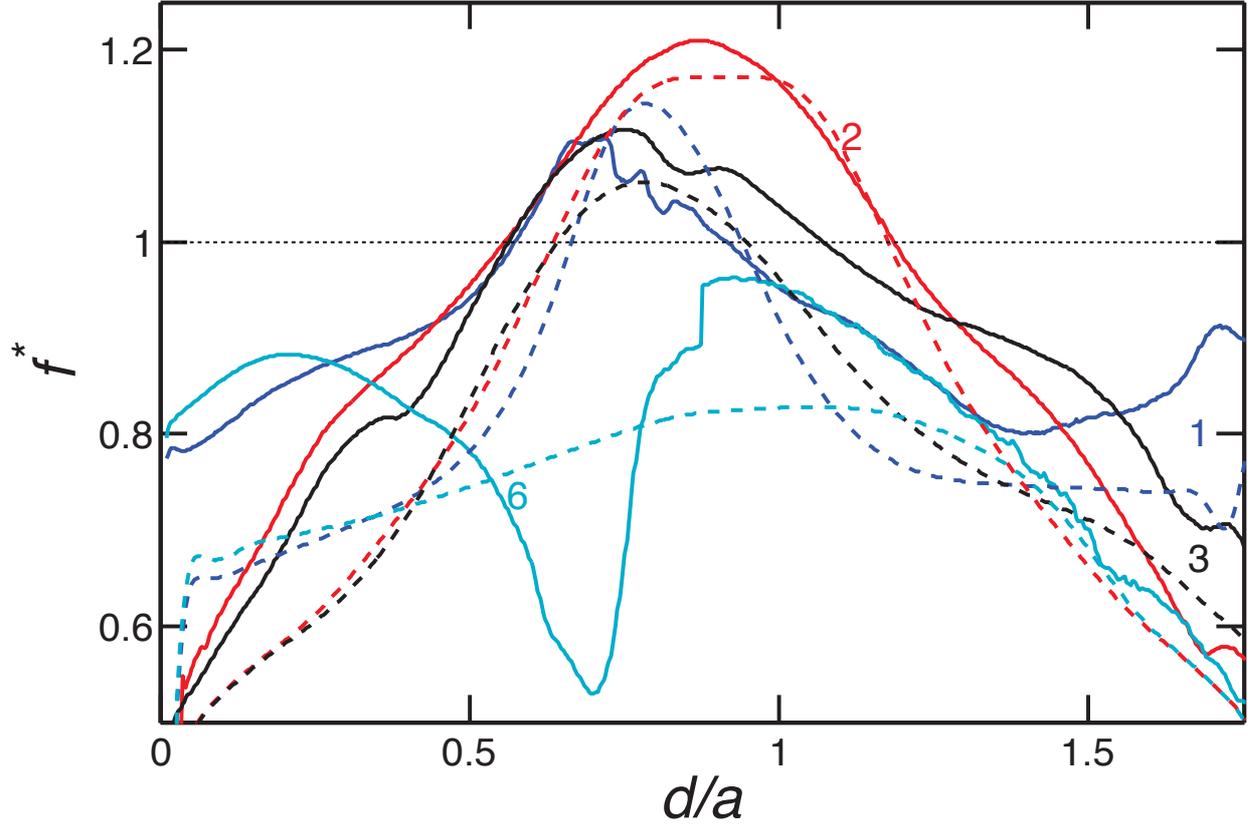


FIG. 18. (Color online) Angular velocity along the ultrasonic rays as a function of the distance from the probe: measured (solid lines, 3 Hz,  $\Lambda = 16$ ) and retrieved from a time-averaged numerical solution (dashed lines,  $\text{Re} = 1.5 \cdot 10^3$ ,  $\text{Ha} = 163$ ,  $\text{Pm} = 10^{-3}$ ,  $\Lambda = 18$ ). The color lines refers to those used to define the ultrasonic beams in the FIG. 3 (the numbers also refers to the ultrasonic beams numbers defined in the FIG. 3). The error bars of the experimental data are shown in FIG. 11.

## 581 VII. DISCUSSION AND CONCLUSION

582 In the presence of an imposed magnetic field, which favors solid body rotation, the inertial  
583 forces largely reduce to a Coriolis force, even for large Reynolds numbers. Experimental  
584 results can thus be interpreted using a single dimensionless number, the Elsasser number.  
585 In that respect, experimental results obtained with global rotation [39] provide a better  
586 guide to interpreting the present results than the linear situation studied by Dormy et al.  
587 [28, 31]. We estimate that, in *DTS*, the rotation frequency  $f$  should be less than 0.1 Hz for  
588 the latter to be approached.

589 Experiments have been conducted with the inner sphere rotating in the range -30 Hz  
590  $\leq f \leq 30$  Hz. We have been able to map extensively the shear in the fluid cavity from  
591 ultrasonic Doppler velocimetry for  $|f| \leq 10$  Hz. Our observations provide a very clear  
592 experimental illustration of Ferraro's law of isorotation, demonstrating the predominance  
593 of magnetic forces near the inner sphere. They also exhibit a strong super-rotation: in the  
594 region where magnetic forces dominate, the fluid angular velocity gets 30% larger than that  
595 of the inner sphere. This contrasts with the results obtained by Dormy et al. [28] when  
596 global rotation is present, which indicate that the phenomenon of super-rotation is hindered  
597 by the Coriolis force. The experimental results obtained in our previous study with global  
598 rotation [39] could not address this issue and we plan to run additional experiments for that  
599 purpose.

600 The experiments also display a clear violation to Ferraro's law: quite low angular velocities  
601 are observed just above the inner sphere, where the magnetic field is strongest (see FIG. 11).  
602 We suspect that this is due to the presence of sodium at rest at the top and bottom of the  
603 cylinder tangent to the inner sphere. Indeed, such violations have been shown to occur when  
604 the electric conductivity of boundaries is high [50, 51].

605 We could follow the evolution of induced magnetic field, electric potentials and power  
606 across the full range of forcing. In a first approximation, all observables associated with the  
607 azimuthal flow (which dominates) can be described by a universal solution, both velocities  
608 and induced magnetic field scaling with  $f$ . In a second approximation, the increase of the  
609 dimensional fluid velocity with  $f$  thins the viscous boundary layer at the outer sphere and  
610 increases friction accordingly, thus reducing the adimensional velocity of the fluid inside  
611 the sphere. At the same time, the effective Coriolis force that results from the non-linear

612  $(\mathbf{u} \cdot \nabla)\mathbf{u}$  term increases with respect to the (linear) Lorentz force: the geostrophic region  
 613 extends further towards the inner sphere. This explains that the fluid velocity increases with  
 614  $f$  less rapidly than  $f$  (FIG. 5) at large  $f$  whilst the torque instead increases more rapidly  
 615 than  $f$  (FIG. 4) (the electric potentials follow an intermediate trend). The outer friction  
 616 torque is balanced by the magnetic torque at the inner boundary. This is consistent with  
 617 an increase of the induced magnetic field, near the solid inner body, that is steeper than  $f$   
 618 (see FIG. 6). On the other hand, the description of Nataf and Gagnière [55] pertains to the  
 619 region where the shear is geostrophic. There, the increased torque at the outer boundary is  
 620 balanced by the magnetic torque on the geostrophic cylinders in the interior, which results  
 621 from the shearing of the imposed dipolar field. The direct measurement of the velocity (up to  
 622 10 Hz, see FIG. 9) shows that the adimensionalized shear does not change significantly with  
 623  $f$  even though the velocity itself decreases. In addition, the induced azimuthal magnetic  
 624 field that we measure inside the sphere (FIG. 6), for the whole range of  $f$ , increases more  
 625 rapidly than  $f$ . At large  $f$ , we observe that  $b_\varphi$  gets larger than the imposed dipolar field in  
 626 much of the fluid layer. Eventually, this induced field is large enough to modify the overall  
 627 magnetic field, and the resulting flow.

628 This last regime, only achieved because the magnetic Reynolds number is large enough,  
 629 is probably the most interesting one. Unfortunately, we cannot directly measure the flow  
 630 velocities with the ultrasound technique at these very large  $f$ . Less direct techniques are  
 631 now required to investigate the zonal shear for  $f > 10$  Hz. Inertial waves modified in the  
 632 presence of the dipolar and the induced magnetic fields have been inferred from records of  
 633 the electric potential along parallels at the surface [41] and of the magnetic field along a  
 634 meridian. Both their period and their wavenumber vary with the geometry of the differential  
 635 rotation in the cavity. Hopefully, it will be possible to invert the zonal shear from the records  
 636 of magneto-inertial waves.

637 Guided by the numerical model, we find that electric field measurements are difficult to  
 638 interpret, particularly in the equatorial region where the radial magnetic field  $B_r$  vanishes.  
 639 The frozen-flux approximation (1) holds when there is a mechanism that keeps under control  
 640 the strength of the electrical currents [56]. This is the reason why the magnetic Reynolds  
 641 number  $Rm$  is not relevant to discuss the validity of the frozen-flux approximation in our  
 642 quasi-steady experiment. That approximation has predictive power, instead, in regions  
 643 where the magnetic force is dominating. In the *DTS* experiment, it corresponds to the

644 inner region close to the magnet where  $\Lambda \geq 1$ .

645 In a geophysical context, a similar approach is routinely used [57] to invert the velocity  
646 field at the Earth's core surface from models of the time changes of the geomagnetic field,  
647 the so-called secular variation. Taking the example of a quasi steady state, this geophysical  
648 application has been criticized from a strictly kinematic standpoint [58]. We reckon in-  
649 stead that it is necessary to consider the balance of forces to decide whether the frozen-flux  
650 hypothesis holds, at least for a quasi steady state as illustrated by the *DTS* experiment.

651 Features of the experiment that only depend upon dimensionless numbers that do not  
652 involve diffusivities have been simulated numerically. An analogous explanation has been  
653 put forward to explain the intriguing successes of geodynamo simulations [5].

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660 by Christian Chillet. We thank two anonymous referees for useful comments.

661 **Appendix: Angular and meridional velocity along the ultrasonic oblique rays**

662 The seven oblique ultrasonic rays shot in *DTS* are sketched in FIG. 3. We define the  
 663 declination  $D$  as the angle between the beam and the meridional plane ( $D$  counted positively  
 664 eastwards), the inclination  $I$  as the angle between the projected beam in the meridional plane  
 665 and the radial direction ( $I$  counted positively upwards) and  $\lambda$  as the latitude of the ultrasonic  
 666 probe. Using those definitions, TABLE III give the characteristics of the beams.

TABLE III. Latitude  $\lambda$ , inclination  $I$ , and declination  $D$  (in degrees) at the origin of the shots (on the outer sphere) of the oblique ultrasonic beams in *DTS*.

Trajectory number and color	$\lambda$	$I$	$D$
1, blue	40	21.1	11.7
2, red	10	2.2	23.9
3, black	10	12.5	-20.6
4, green	-20	20	-13.5
5, yellow	-20	21.1	-11.7
6, cyan	-40	21.1	11.7
7, magenta	-40	-24	0

667 **1. Angular velocity**

668 Along these oblique beams, the projection  $u(d)$  ( $d$  is the distance from the probe) of the  
 669 velocity is a combination of the components  $u_r$ ,  $u_\theta$  and  $u_\varphi$  of the total velocity field. Velocity  
 670  $u(d)$  is counted positive in the shooting direction. We assume that the mean fluid flow is  
 671 axisymmetric, and also  $(u_r, u_\theta) \ll u_\varphi$ , the meridional velocities amplitude in *DTS* being less  
 672 than 10% the amplitude of the azimuthal velocities. Using projections along the beam, we  
 673 retrieve the angular velocity  $\omega(d)$  along trajectories 1 to 6 using the following relationship

$$\omega(d) = -\frac{u(d)}{a \cos \lambda \sin D}. \quad (\text{A.1})$$

674 **2. Meridional velocity**

675 We have also exploited the observation that the meridional velocity does not change sign  
 676 when the rotation of the inner sphere is reversed - it remains centrifugal in the equatorial  
 677 plane - whereas the angular velocity does change sign. Thus, combining measurements  
 678 obtained with two opposite rotation rates of the inner core, we can separate azimuthal and  
 679 meridional velocities.

680 Assuming now that the mean meridional velocity is axisymmetric and using projections,  
 681 we can retrieve the radial velocity

$$u_r(d) = \frac{u(d)r(d)}{d - a \cos D \cos I}, \quad (\text{A.2})$$

682 and the orthoradial velocity

$$u_\theta(d) = \frac{u(d)r(d)s(d)}{a[a \cos D \cos \lambda \sin I - d \cos^2 D \cos(\lambda + I) \sin I + d \sin^2 D \sin \lambda]}, \quad (\text{A.3})$$

683 where  $r(d) = \sqrt{x^2 + y^2 + z^2}$  is the spherical radius and  $s(d) = \sqrt{x^2 + y^2}$  is the cylindrical  
 684 radius at the measurement point. They  $(x, y, z)$  coordinates of the measurement point are  
 685 given by:

$$x(d) = a \cos \lambda - d \cos D \cos(\lambda + I) \quad (\text{A.4})$$

$$y(d) = -d \sin D \quad (\text{A.5})$$

$$z(d) = a \sin \lambda - d \cos D \sin(\lambda + I) \quad (\text{A.6})$$

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