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Alireza Esna Ashari — Laurent Mevel

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Optimal Input Design for Subspace-Based Fault Detection and Identification

Alireza Esna Ashari*, Laurent Mevel†

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Abstract: This study focuses on input design for subspace based fault detection and identification methods and investigates its possible advantages over using noise inputs. In several real applications the noise available in environment is the only input to the system and in some cases produce low quality output data for subspace identification and fault detection purposes. Therefore, model order may be underestimated. Due to the nature of subspace based methods, some modes of the system may not appear in the response as the input is not strong enough to excite these modes. In order to improve the result, a method is suggested in literature, that is to use "rotated" input. The rotated input design is proposed in several papers to apply to "ill conditioned systems" in which the vector of different outputs are typically close to collinearity if a white noise is used. In this report, we use this technique to verify possible improvement of subspace-based identification method including output-only, and input-output approaches. Then, for the first time we investigate the possible impacts of the rotated input on subspace base fault detection method. Simulations on a high-purity distillation column shows that this auxiliary input can improve subspace-based fault detection and identification.

Key-words: Fault Detection, Stochastic System Identification, Subspace Identification, Rotated Input, Ill Conditioned Systems.

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* INRIA, Centre Rennes - Bretagne Atlantique
† INRIA, Centre Rennes - Bretagne Atlantique

Centre de recherche INRIA Rennes – Bretagne Atlantique
IRISA, Campus universitaire de Beaulieu, 35042 Rennes Cedex
Téléphone : +33 2 99 84 71 00 — Télécopie : +33 2 99 84 71 71

Conception Optimale d'Entrée pour la Détection et l'Identification de défauts par la méthode des sous-espaces

Résumé : Cette étude se concentre sur la conception d'entrées pour la détection de défauts par méthodes sous-espace. Nous étudions les avantages possibles d'utiliser une entrée autre que le bruit. Dans plusieurs applications réelles, le bruit produit par l'environnement est la seule entrée du système et dans certains cas, produit des données de qualité faible pour l'identification et la détection de défauts par (mais pas seulement) méthodes sous-espace. Par conséquent, l'ordre du modèle peut être sous-estimé. En raison de la nature et de la construction des méthodes à base de sous-espace, certains modes du système peuvent ne pas apparaître dans la réponse, parce que l'entrée n'est pas assez forte pour exciter ces modes. Afin d'améliorer le résultat, une méthode est suggéré dans la littérature. Elle utilise une entrée "tournée". Cette méthode-la est proposé dans plusieurs articles pour s'appliquer aux systèmes "mal conditionnés" dans laquelle le vecteur des sorties différentes sont généralement proches de co-linéarité si un bruit blanc est utilisé. Dans ce rapport, nous utilisons cette technique pour vérifier l"amélioration possible apportée aux méthodes d'identification sous-espace, y compris en sortie-seule, mais aussi en entrées-sorties. De plus, pour la première fois, nous étudions l'impact possible de l'entrée tournée sur la méthode de détection de faute par sous-espace . Des simulations sur une colonne de distillation de haute pureté montrent que cette entrée tournée auxiliaire peut améliorer la détection de défauts sous-espace ainsi que l'identification sous-espace.

Mots-clés : Détection de pannes, système d'identification stochastique, Identification sous-espaces, entrée tournée, systèmes mal conditionnés.

1 Introduction

Over the last decades subspace identification methods have been an active domain of research. This method is based on geometric concepts including the calculation of certain matrices, geometric manipulation of the row spaces and computation of projections of data on certain subspaces. A comprehensive survey of subspace-based identification approaches can be found in [10]. The identification problem consists of obtaining the state-space representation of the system from input-output data using linear algebra tools, up to a similarity transformation. Subspace identification methods can be categorized into two main groups, output-only and input output methods. In first category, only output information is used to calculate the system eigenstructure, while input data is also used in the second type of subspace identification methods [7].

The problem of fault detection is another relevant subject of research that has been investigated using several methods. Based on the subspace identification methods, an approach is developed in [2] to detect changes in the eigenstructure of the system. As this fault detection method is derived from the subspace eigenstructure identification, it inherits its merits and also difficulties.

One important issue in all these problems is to know or calculate the system order, in order to obtain correct results. All the subspace-base approaches of identification and fault detection include a common step of performing singular value decomposition (SVD) on a data matrix. This is usually determined by the number of “large” singular values. Small singular values are considered as the effect of noise on data. In several practical applications, the natural unknown and unmeasured excitation is usually considered as the only input to the system that excite the modes and produces output data. In some applications, this input cannot stimulate some modes, and consequently the corresponding singular value would be small enough to be considered as the effects of noise. This problem is more severe for a type of systems called “ill-conditioned systems”. These systems are type of multi-input multi-output systems for which the direction of the input is important and for some directions the outputs are very larger than the others.

For these systems some singular values are very small when the common inputs are applied to the system even if there is no noise affected the system. Therefore, it is very hard to select the real nonzero singular values in real applications in presence of noise. This may affect our estimation of the system order if the real order is not given. In order to overcome this problem, “rotated inputs” are proposed in literature for identification tests for which the best angles between the inputs are calculated and applied to the system [8, 9]. Application of this predesigned test input helps to increase the ratio between real singular values and the rest of singular values due to the noise.

In this work we examine the rotated input together with different subspace identification methods including output-only and input-output approaches. The objective is to study the effect of the rotated input on these two categories of identification methods. The other important object is to consider possible advantages of using the rotated input on subspace-based fault detection methods. In some cases, the change of the residual due to the fault is hardly distinguishable. It is desired to increase this change using an input.

2 System Identification

2.1 Subspace System Identification

Consider the discrete time model in state space form:

$$x_{k+1} = Ax_k + w_k, \quad (1)$$
$$y_k = Cx_k, \quad (2)$$

where $x_k \in \Re^n$, $y_k \in \Re^r$, $A \in \Re^{n\times n}$ and $C \in \Re^{r\times n}$ are the state vector, the output vector, the state transient matrix, and the output matrix, respectively. The state noise w_k is unmeasured and and Gaussian, zero mean, white and with covariance Σ_w. A subset of the r sensors may be used for reducing the size of the matrices in the identification process. These sensors are called projection channels or reference sensors. Let r_0 be the number of reference sensors ($r_0 \leq r$) and p and q are chosen parameters with $n \leq qr_0 \leq (p+1)r$. From the output data $y_k, \quad k = 1, \cdots N+p+q$ a matrix $\mathcal{H}_{p+1,q} \in \Re^{(p+1)r\times qr_0}$ is built according to a chosen SSI algorithm, see e.g. [4] for an overview. The matrix $\mathcal{H}_{p+1,q}$ is called "subspace matrix" in the following and has asymptotically the following factorization property

$$\mathcal{H}_{p+1,q} = \mathcal{O}_{p+1}\mathcal{Z}_q, \quad (3)$$

where $\mathcal{O}_{p+1}$ is the observability matrix,

$$\mathcal{O}_{p+1} = \begin{pmatrix} C^T & (CA)^T & \cdots & (CA^p)^T \end{pmatrix}^T, \quad (4)$$

and $\mathcal{Z}_p$ depends on the selected SSI algorithm. The observability matrix $\mathcal{O}_{p+1}$ is obtained from a thin Singular Value Decomposition (SVD) of the matrix $\mathcal{H}_{p+1,q}$ and its truncation at the desired model order n. Considering the SVD of $\mathcal{H}_{p+1,q}$

$$\begin{aligned} \mathcal{H}_{p+1,q} &= U\Delta V^T, \\ &= \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_0^T \end{pmatrix}, \end{aligned} \quad (5)$$

one may obtain

$$\mathcal{O}_{p+1} = U_1 \Delta_1^{\frac{1}{2}}. \quad (6)$$

The output matrix C is then found in the first block row of the observability matrix $\mathcal{O}_{p+1}$. The state transition matrix A is obtained from the shift invariance property of $\mathcal{O}_{p+1}$, namely as the least squares solution of

$$\mathcal{O}^{\uparrow}_{p+1} A = \mathcal{O}^{\downarrow}_{p+1}, \quad (7)$$

where

$$\mathcal{O}^{\uparrow}_{p+1} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{pmatrix}, \quad \mathcal{O}^{\downarrow}_{p+1} = \begin{pmatrix} CA \\ CA^2 \\ \vdots \\ CA^p \end{pmatrix}. \quad (8)$$

The eigenstructure of system (1)-(2) is represented by $(\lambda, \varphi_\lambda)$, where λ denotes the eigenvalues and $\varphi_\lambda = C\phi_\lambda$ where ϕ_λ is the eigenvector corresponds to λ. For simplicity, let p and q be given and skip the subscripts related to p and q of $\mathcal{H}_{p+1,q}$, $\mathcal{O}_{p+1}$ and $\mathcal{Z}_p$ in the following. Also, the subscripts of the zero matrix 0_{st} of size $s \times t$ and identity matrix I_s of size $s \times s$ may be skipped, when their

2.2 Ill-conditioned systems

In this section, we briefly introduce the systems which are considered "ill-conditioned" for identification purposes. A comprehensive discussion on this topic is given in [9], however for the systems that have only two inputs and two outputs. In this study, we focus on this type of systems for the sake of simplicity. Here we summarize the discussion given in [9]. Ill-conditioned processes are defined as multivariable processes whose transfer function matrices have high condition number at zero frequency (steady-state gain) or even at higher frequencies. This type of systems is a challenge for identification methods, including subspace-based methods. Here, we particularly focus on the difficulties we face using these methods. The main problem is the identification of the system order and consequently the signal subspace dimension. In all subspace identification methods we need to identify the order of the system by the number of big singular values of $\mathcal{H}$ which are usually much bigger than the other singular values corresponding to noise [10]. In identification of the ill-conditioned systems some of the singular values of the matrix $\mathcal{H}$ corresponding to the system become very small and thus they may be considered to be the effect of noise. Therefore, we do not count these singular values when we calculate the order of the system. To illustrate the properties of an ill-conditioned system, consider the transfer function of the system

$$y(s) = G(s)u(s), \tag{9}$$

and the SVD of $G(s)$

$$G(s) = \Upsilon(s)\Sigma(s)\Omega^T(s), \tag{10}$$

where the 2×2 orthogonal matrices $\Upsilon(s)$ and $\Omega(s)$ can be written as

$$\Upsilon(s) = \begin{pmatrix} cos\varphi(s) & -sin\varphi(s) \\ sin\varphi(s) & cos\varphi(s) \end{pmatrix}, \tag{11}$$

$$\Omega^T(s) = \begin{pmatrix} cos\theta(s) & -sin\theta(s) \\ sin\theta(s) & cos\theta(s) \end{pmatrix}. \tag{12}$$

The system $G(s)$ is ill-conditioned if the singular value $\sigma_1(s)$ is much larger than the singular value $\sigma_2(s)$. This helps us to calculate the outputs and approximate it as follows

$$\begin{aligned} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} \sigma_1 cos\varphi(u_1 cos\theta - u_2 sin\theta) - \sigma_2 sin\varphi(u_1 sin\theta + u_2 cos\theta) \\ \sigma_1 sin\varphi(u_1 cos\theta - u_2 sin\theta) + \sigma_2 cos\varphi(u_1 sin\theta + u_2 cos\theta) \end{pmatrix}, \\ &\approx \begin{pmatrix} \sigma_1 cos\varphi(u_1 cos\theta - u_2 sin\theta) \\ \sigma_1 sin\varphi(u_1 cos\theta - u_2 sin\theta) \end{pmatrix}. \end{aligned} \tag{13}$$

In (13) the dependence on s is temporarily dropped. From (13) we can conclude that

$$y_2(s) \approx y_1(s) tan\varphi(s). \tag{14}$$

The important consequence of (14) is that the two outputs of $G(s)$ become almost collinear at all frequencies. In [9] it is shown that if the input to such a system is white noise, it is difficult to distinguish the smallest singular value of $\mathcal{H}_{p+1,q}$ corresponding to the system from the other small nonzero singular values corresponding to noise. This justifies the failure of white inputs to be the input that disclose the correct system order of the ill-conditioned systems.

2.3 Subspace System Identification using Rotated Input

In literature, the method of rotated input design is proposed to solve the difficulties of system order identification discussed in Section 2.2. In this section, we present the method in [9] for the systems with two inputs and two outputs. Considering the singular value decomposition of the system $G(s)$ introduced in (10), one may obtain

$$y(s) = G(s)u(s) = \sum_{i=1}^{2} \sigma_i(s)\psi_i(s)\omega_i^T(s)u(s) = \sum_{i=1}^{2} \sigma_i(s)\psi_i(s)\xi_i(s), \qquad (15)$$

where $\xi(s) \triangleq \omega_i^T(s)u(s)$. Using (12), it can be concluded that

$$\xi_1(s) = u_1(s)cos\theta(s) - u_2(s)sin\theta(s), \qquad (16)$$

$$\xi_2(s) = u_1(s)sin\theta(s) - u_2(s)cos\theta(s). \qquad (17)$$

Assuming that $\Omega(s) \approx \Omega$ (see [9]) we obtain

$$\xi_1(t) = u_1(t)cos\theta - u_2(t)sin\theta, \qquad (18)$$

$$\xi_2(s) = u_1(t)sin\theta - u_2(t)cos\theta. \qquad (19)$$

It is shown that the outputs need to be as uncorrelated as possible in order to avoid collinearity problems and identify the order of the state subspace more precisely. Designing the inputs in a certain way can satisfy this need. The key point is that no terms in the summation in (15) should be negligible. Hence, we try to make each term of the summation contribute equally to the magnitude of y. As $\psi_i(s)$ are orthonormal, the criterion becomes

$$\int_0^{\infty} |\sigma_i(j\omega)|^2 \, |\xi_i(j\omega)|^2 \, d\omega = constant, \qquad (20)$$

for all $i = 1, \cdots, 2$. This occurs if

$$|\sigma_i(j\omega)| \, |\xi_i(j\omega)| = constant, \qquad (21)$$

or equivalently

$$\frac{|\xi_2(j\omega)|}{|\xi_1(j\omega)|} = \frac{|\sigma_1(j\omega)|}{|\sigma_2(j\omega)|} = \kappa(\omega) \approx \kappa. \qquad (22)$$

Using Parseval's Theorem, we can translate the relation in time domain

$$\frac{\|\xi_2(t)\|}{\|\xi_1(t)\|} \approx \kappa, \qquad (23)$$

where $\|.\|$ represent 2-norm. As the process model $G(s)$ is not known, neither Ω nor κ is known. However, if it is known that the process being identified is ill-conditioned, it is not necessary to know the value of κ. Instead, if we have

$$\frac{\xi_2(t)}{\xi_1(t)} = \kappa, \qquad (24)$$

then (23) is satisfied. Using this assumption and (18)–(19) it follows that

$$u_2(t) \approx \frac{\kappa cos\theta - sin\theta}{cos\theta + \kappa sin\theta} u_1(t). \tag{25}$$

However, we have $\kappa >> 1$ if the system is ill-conditioned and (25) can be rewritten as follows

$$u_2(t) \approx cot\theta \; u_1(t), \tag{26}$$

where θ is the rotation angles. This angle can be computed experimentally by trial and error, by dividing the interval $[0°, 180°]$ into equal parts, selecting different test values for θ, selecting u_1 as a random signal and then calculating u_2. We use the algorithm proposed in [9] to calculate the optimal value of θ with a minor difference of using random signals instead of a $PRBS$ signal.

1. Pick a value for θ from the set of test values.
2. Select the input u_1 as a zero mean white noise.
3. Compute $u_2(t)$ as follows
$$u_2(t) \approx cot\theta \; u_1(t) + e(t),$$
where $e(t)$ is zero mean white noise with small amplitude.
4. Perform the subspace identification and compute the singular values of $\mathcal{H}_{p+1,q}$.
5. Save the ratio between the second and third singular values of $\mathcal{H}_{p+1,q}$ as a measure of separation. Go to step 1 if the exist other values for θ to be tested,
6. Find the maximum separation of the pair of singular values. The corresponding θ is the solution.

Note that the addition of $e(t)$ is necessary as the inputs must not be exactly collinear, although they are highly correlated (see [9] for more information).

2.4 Output-Only vs. Input-Output Methods

The subspace identification methods can be categorized into two main groups: output-only and input-output approaches. In some practical implementations, the only excitation to the system is natural noise which is not measurable. Therefore the only information that can be used to identify the system is the output data. In some other application, handling both known and unknown inputs can be taken into account. Input-output method takes advantage of available knowledge on the inputs. These two different approaches lead to various Hankel matrices $\mathcal{H}$. In this study, we compare an output-only method with an input-output approach when the rotated input is implemented. The corresponding $\mathcal{H}$ for the output-only method is

$$\mathcal{H}_{p+1,q} = \begin{pmatrix} R_{l+1} & R_{l+2} & \cdots & R_{l+q} \\ R_{l+2} & R_{l+3} & \cdots & R_{l+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{l+p+1} & R_{l+p+2} & \cdots & R_{l+p+q} \end{pmatrix}, \tag{27}$$

for covariance driven methods where

$$R_j = E(y_{k+j} y_k^T), \tag{28}$$

and $E(.)$ is the expected value of $(.)$. The integer l reflects the assumed correlation in measurement noise sequence. Considering no measurement noise as in (2) we have $l = -1$ while if we change (2) to $y_k = Cx_k + v_k$, it can have other values. Similarly, we can use a data driven method in which

$$\mathcal{H}_{p+1,q} = E(\mathcal{Y}^+_{k,p+1} \mathcal{Y}^{-\,T}_{k,q}), \tag{29}$$

where

$$\mathcal{Y}^+_{k,p+1} = \begin{pmatrix} y_k \\ \vdots \\ y_{k+p} \end{pmatrix}, \quad \mathcal{Y}^-_{k,q} = \begin{pmatrix} y_{k-l-1} \\ \vdots \\ y_{k-l-q} \end{pmatrix}. \tag{30}$$

For the input-output method, we use the method introduced in [10]

$$\mathcal{H} = Y_f /_{U_f} W_p \Pi_{U_f^\perp}, \tag{31}$$

where the operations Π_B and $\Pi_{B^\perp}$ on matrix B are

$$\Pi_B = B^T (BB^T)^{-1} B, \tag{32}$$

$$\Pi_{B^\perp} = I - \Pi_B. \tag{33}$$

Here, $W_p = \begin{pmatrix} U_p \\ Y_p \end{pmatrix}$, where U_p, U_f and Y_p are the Hankel matrices of past input, future input and past output, respectively (see [10]). Also, $B^\dagger$ for an arbitrary matrix B is a basis for the orthogonal complement of the rows space of B. The Oblique projection is defined as follows

$$Y_f /_{U_f} W_p = Y_f / U_f^\perp (W_p / U_f^\perp)^\dagger W_p. \tag{34}$$

In (34) orthogonal projection is used which is defined as

$$A/B = A\Pi_B, \tag{35}$$

for any arbitrary matrices A and B.

3 Fault Detection

The problem of fault detection and isolation has recently received much attention and has been investigated with several approaches [3], [6]. In many practical applications, the fault detection problem

3.1 Subspace Based Fault Detection

In [2] a statistical fault detection method was described, which can be used with subspace algorithms satisfying factorization property (3). This fault detection method consists in comparing characteristics of a reference state with a subspace matrix $\hat{\mathcal{H}}$ computed on a new data sample $(y_k)_{k=1,\ldots,N+p+q}$, corresponding to an unknown, possibly damaged state, assuming that $\hat{\mathcal{H}}$ is a consistent estimate of $\mathcal{H}$.

To compare the states, the left null space matrix S of the observability matrix of the reference state is computed, which is also the left null space of the subspace matrix at the reference state because of factorization property (3). The characteristic property of a system in the nominal state is $S^T\hat{\mathcal{H}} = 0$ and the residual vector

$$\zeta_1 \stackrel{\text{def}}{=} \sqrt{N} vec(S^T\hat{\mathcal{H}}), \tag{36}$$

describes the difference between the state of matrix $\hat{\mathcal{H}}$ and the reference state.

Let θ be a vector containing a canonical parameterization of the actual state of the system (see [3] for details) and θ_0 the parameterization of the nominal state. The damage detection problem is to decide whether the subspace matrix $\hat{\mathcal{H}}$ from the (possibly damaged) system (corresponding to θ) is still well described by the characteristics of the reference state (corresponding to θ_0) or not. This is done by testing between the hypotheses

$$\begin{array}{lcll} \mathrm{H}_0: \ \theta & = & \theta_0 & \text{(reference system)}, \\ \mathrm{H}_1: \ \theta & = & \theta_0 + \delta/\sqrt{N} & \text{(faulty system)}, \end{array} \tag{37}$$

where δ is unknown but fixed. This is called the local approach, and the following proposition is used to test between both hypotheses.

Proposition 3.1 ([3]) *The residual ζ_1 is asymptotically Gaussian for large N, and the test between the hypotheses H_0 and H_1 is achieved through the χ^2-test*

$$\chi_1^2 = \zeta_1^T \Sigma_1^{-1} \mathcal{J}_1 (\mathcal{J}_1^T \Sigma_1^{-1} \mathcal{J}_1)^{-1} \mathcal{J}_1^T \Sigma_1^{-1} \zeta_1, \tag{38}$$

and comparing it to a threshold, where $\mathcal{J}_1$ and Σ_1 are consistent estimates of the sensitivity and covariance of ζ_1. Both can be estimated in the reference state under the assumption that the covariance of the input noise w_k of the system does not change between the reference state and the possibly damaged state.

The computation of the Jacobian $\mathcal{J}_1$ needs a parameterization of the system, where the eigenvalues and mode shapes of the reference system must be known, and is explained in detail in [3]. In [1] an empirical non-parametric version of the test is proposed, where $\mathcal{J}_1$ is set as the identity matrix.

The computation of the covariance matrix Σ_1 depends on

$$\Sigma_{\mathcal{H}} \stackrel{\text{def}}{=} \text{cov}(\sqrt{N} vec\mathcal{H}),$$

which is dependent on the chosen subspace algorithm. For simplicity, $\Sigma_{\mathcal{H}}$ will still be called *covariance of the subspace matrix*. A method to calculate $\Sigma_{\mathcal{H}}$ is proposed in [5]. Finally, the covariance matrix Σ_1 can be obtained from

$$\Sigma_1 = (I \otimes S^T)\Sigma_{\mathcal{H}}(I \otimes S), \tag{39}$$

due to (36), where $\otimes$ denotes the Kronecker product.

3.2 Subspace Based Fault Detection using Rotated Input

The interesting question is whether or not the method introduced in Section 2.3 can be implemented to improve the subspace-based fault detection method in Section 3.1. This fault detection method is a stochastic approach and there always exist the probability of receiving false alarms. This issue is more serious when the residual does not change for some systems and some certain faults. It is desired to design an input such that the residual is more sensitive to the fault and changes considerably due to a fault. One important case is ill-conditioned systems in which the effect of some modes on the residual is very low. It means that the change in some part of the eigenstructure is not taken into account when we perform the detection test. It is clear that if the remaining part changes then we may detect the fault, but then the residual may show a smaller change. The detection method in Section 3.1 is based on calculating S, the left kernel of the observability matrix. The selection of S is also affected if we don't know the order of the system in advance, as it is obtained from the singular value decomposition of the observability matrix and we select n first left singular vectors. In the next step we compute the SVD of $\hat{\mathcal{H}}$ and only consider the n first columns to make the residual. Therefore the first guess is that the rotated input can improve the fault detection method in two directions:

1. to identify the order of the system and consequently to obtain the matrix S.
2. to strengthen the effect of weak modes on the residual.

We will try to justify this preliminary guess in Section 4. The Hankel matrix we use for fault detection part is

$$\mathcal{H}_{p+1,q} = \mathcal{Y}^+ \mathcal{Y}^{-\,T} (\mathcal{Y}^- \mathcal{Y}^{-\,T})^{-1} \mathcal{Y}^-, \tag{40}$$

where $\mathcal{Y}^+$ and $\mathcal{Y}^+$ are introduced in (30). To calculate the $\mathcal{H}_{p+1,q}$ we perform the following QR decomposition

$$\begin{pmatrix} \mathcal{Y}^- \\ \mathcal{Y}^+ \end{pmatrix} = RQ = \begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}, \tag{41}$$

and it can be shown that

$$\mathcal{H}_{p+1,q} \approx R_{21}. \tag{42}$$

To compute $\mathcal{Y}^+$ and $\mathcal{Y}^+$ we implement the rotated input design introduced in Section 2.3 and find the correct order of the system.

4 Numerical Examples

In order to verify the efficiency of the rotated input together with subspace identification and fault detection method, we select an ill-conditioned system which has been studied in several papers e.g. [9] and [8]. This system is a high-purity distillation column in LV control configuration, that is the reflux and vapor flow rates, L and V, are the manipulated variables, and the distillate and bottom concentrations, x_D and x_B are the controlled variables. The transfer matrix of the system is

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \frac{87.8}{1+194s} & -\frac{87.8}{1+194s} + \frac{1.4}{1+15s} \\ \frac{108.2}{1+194s} & -\frac{108.2}{1+194s} - \frac{1.4}{1+15s} \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix},$$

We perform three simulations. In the fist one the rotated input is used to perform the identification experiment, however with output-only method. Thus, the input u_2 is the same as u_1 but rotated by different angles θ. From Figure 1 it is concluded that the ratio between the second and the third singular values of the Hankel matrix $\mathcal{H}$ has the maximum value at $\theta = 54°$. This shows that the ratio at this point is larger than $\theta = 0$ which is actually the ordinary white noise input. The identification test using white noise input and measurements corrupted by Gaussian white noise produces the singular value plot of Figure 2 for the matrix , shows that the system order is 1. Note that small singular values whose value is less than 10^{-5} are not shown in this plot. In figure 3 the singular value plot of the Hankel matrix is demonstrated for the rotated input with $\theta = 54°$. Although we managed to push up the singular value using the rotated input, the difference between the first and the second singular values is still too much. Therefore, it is not clear from this plot that the order of the system should be 2.

In the second experience, we repeat the test for input-output method. From Figure 4, the optimal rotation angle is $\theta = 45°$. While the second singular value is much smaller than the first one using white noise (Figure 5) we can see that the rotated input can push up the second singular value as shown in Figure 6. From this figure we can clearly deduce that the order of the system is 2.

In the last experiment, we apply the rotated input method to the fault detection approach. For the fault detection approach, we assume that the system has been changed to

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \frac{87.8}{1-10s} & -\frac{87.8}{1-10s} + \frac{1.4}{1+15s} \\ \frac{108.2}{1-10s} & -\frac{108.2}{1-10s} - \frac{1.4}{1+15s} \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix},$$

and we create the output data using the nominal and this faulty model. At 500-th sample the fault is occurred. Figure 7 depicts the residual corresponding to this experiment. The green line is the residual when it is created using white noise input and therefore the order of the system is assumed to be 1, and the blue line shows this residual when the rotated input is implemented and system order is taken 2. We can clearly see that the rotated input can raise the level of the residual in the faulty case. It will help to better decide whether or not the fault has been occurred.

It will be more helpful for the faults to which the residual is not sensitive enough and does not change significantly. A clear example is the fault which leads to the following system

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \frac{87.8}{1+100s} & -\frac{87.8}{1+100s} + \frac{1.4}{1+15s} \\ \frac{108.2}{1+100s} & -\frac{108.2}{1+100s} - \frac{1.4}{1+15s} \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix}.$$

The residuals are represented in Figure 8 which shows that white noise cannot reveal this fault, but using the rotated input the residual changes significantly when fault happens. It clearly shows an improvement. The last question remains is to verify if this improvement is the effect of selecting the correct order of the system or the impact of strengthening the weak modes. In the last experiment, we assume that we have already the correct order of the system to be used with the white noise. In Figure 9 the residual is plotted for the three case considering the second fault we introduced above. The new red line shows this the case we we give the correct system order to the traditional fault detection method with white noise. It clearly shows that the result is almost the same as the previous experiment in which the method finds out the order itself (green line).

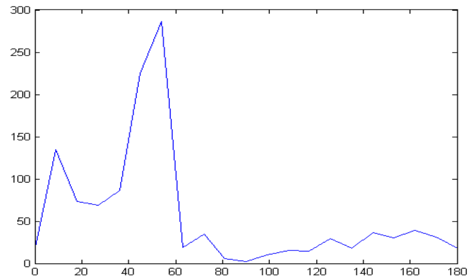

Figure 1: The ratio between the second and third singular values, changing θ, output-only method.

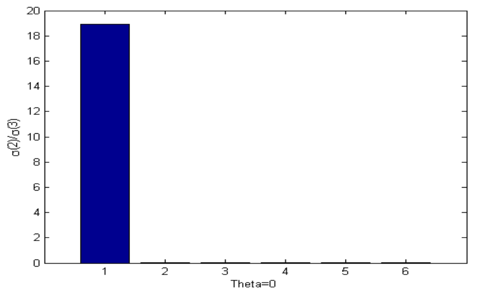

Figure 2: Major singular values using white noise, output-only method. Order of the system is obtained as 1.

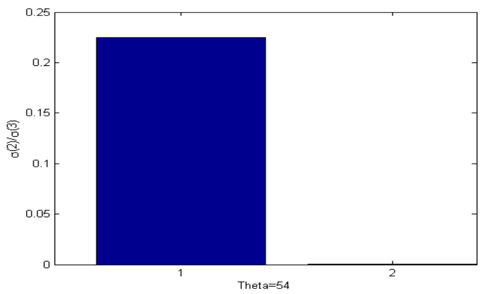

Figure 3: Major singular values using rotated input, output-only method. Order of the system can hardly be accepted as 2.

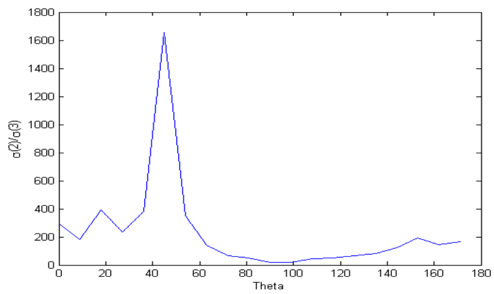

Figure 4: The ratio between the second and third singular values, changing θ, input-output method.

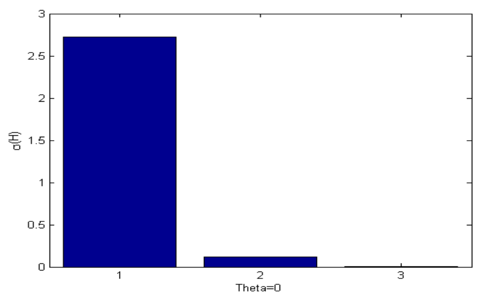

Figure 5: Major singular values using white noise, input-output method. Order of the system is obtained as 1.

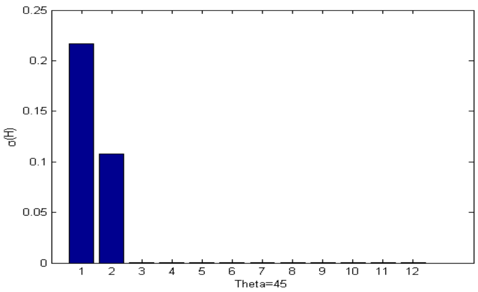

Figure 6: Major singular values using rotated input, input-output method. Order of the system is obtained as 2.

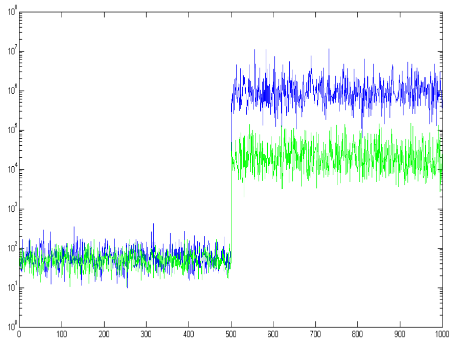

Figure 7: Residual signal using rotated input (blue) and white noise (green) in the case of a major fault. The order of S is 2 using the rotated input , and 1 using white noise.

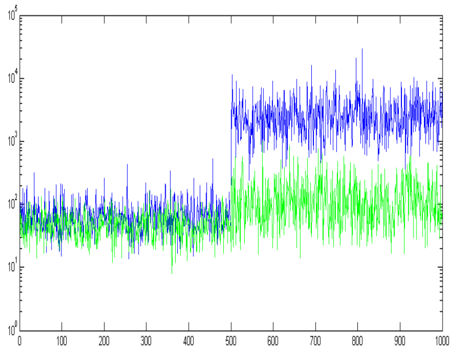

Figure 8: Residual signal using rotated input (blue) and white noise (green) in the case of a minor fault. The order of S is 2 using the rotated input , and 1 using white noise.

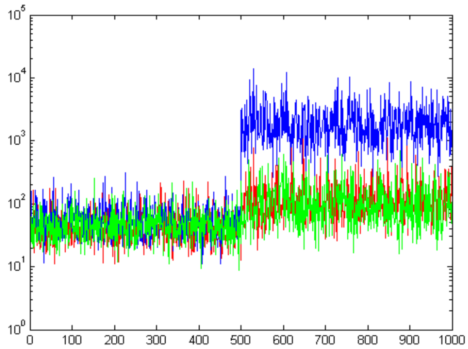

Figure 9: Residual signal using rotated input (blue), white noise given the correct system order, and white noise which finds out the system order (green) in the case of a minor fault.

5 Conclusions

The importance of estimating the correct order of the system and strengthening the weak modes in subspace-based fault detection and identification is discussed. It has been shown that for a group of systems, called ill-conditioned systems, model order may be underestimated using random inputs. Instead, we may use the rotated inputs to better excite some local modes and get better results. This inputs are used together with output-only and input-output subspace identification methods. Simulation results show that input-output methods give better results using the rotated inputs approach. We can push up the singular values corresponding to the local modes, which can be considered as the effect of noise and neglected if we do not use a rotated input. Also, we investigated the possible impacts of the rotated input on subspace base fault detection method. Simulations results show that we can increase the sensitivity of the residual to the fault if we are performing the detection test on ill-conditioned systems after excitation by the rotated input. It is shown that this improvement is more the effect of strengthening the weak modes than the correct estimation of system order.

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Centre de recherche INRIA Rennes – Bretagne Atlantique
IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)

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Centre de recherche INRIA Sophia Antipolis – Méditerranée : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
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