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in a financial market with discrete demand shocks**

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Strategic complementarity of information acquisition in a financial market with discrete demand shocks

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Abstract

A simple model of financial market with rational learning and without friction is presented in which the value of private information increases with the mass of informed individuals, contrary to the property presented by Grossman and Stiglitz (1980). The key assumption is the possibility of independent discrete shocks on the fundamental value and on an exogenous demand.

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1 Introduction

Private information about the fundamental value of an asset is valuable because it increases the profit from trading that arises from the gap between the agent's expected value of the asset and its market price. When more agents are informed, the market price should reflect that information, thus reducing the gap between private information and public price, and the value of private information. This mechanism is "common sense" but there cannot be a general proof of it. Grossman and Stiglitz (1980) described that effect as a conjecture (the second among seven), but they merely illustrate it through an example in the CARA-Gauss model. The present paper provides another example of a financial market. In that example, the conjecture is false: an increase of private information may increase its value. Their property will have a simple interpretation.

The issue is particularly relevant if private information is endogenous and can be obtained at some cost. If the value of information for an informed agent increases with information gathering of other agents, the strategic complementarity may generate multiple equilibria and sudden switches between market regimes of low information gathering and trading volume, and regimes of frenzies in information and trading.

Private information is spread in a financial market through the flow of trade, and this flow must contain some noise (because of the no-trade theorem). The value of private information depends on the difference between the fundamental and the market price. In the CARA-Gauss model, both the trade of the informed agents and the noise have a Gaussian distribution. A higher mass of informed agents increases the signal to noise ratio in the observed order flow, the market price is a better signal of the fundamental and the value of information decreases. A similar mechanism operates for "standard" single-peaked distributions. If there are more informed agents, large volumes of observed trades will be attributed more to informed agents when the tails of the noise distribution are decreasing, and the market will be more informative.

When quantum jumps of the fundamental or the noise trade, the single-peaked assumption is not appropriate. The possibility of such jumps is the essential assumption here. When there are not too many informed agents, moderate trades carry their

usual information function and large trades are dismissed as being driven by a noise shock. In this case, a higher number of informed agent increases the signal to noise ratio and lowers the value of private information. However, if the number of informed agents keeps rising, then a large trade by informed agents may become confused with a noise shock. The signal to noise ratio falls and the value of private information rises. A model of this effect is presented here¹.

2 The model

The payoff of the asset (the fundamental) is equal to $\theta \in \{\theta_1, \theta_2\}$, with $\theta_1 < \theta_2$. We normalize $\theta_0 = 0$, $\theta_1 = 1$, but the notation $\{\theta_1, \theta_2\}$ will be useful for the text. The high state $\theta = \theta_2$ has a prior probability μ_θ .

Three types of non-atomistic agents trade in a one-period perfectly competitive market for the asset.

- An exogenous demand is equal to $\zeta + \eta$ where $\zeta \in \{\zeta_1, \zeta_2\}$. Without loss of generality, we assume that $\zeta_1 = 0$ and $\zeta_2 = 1$. The impact of this normalization will be discussed later. The probability of $\zeta = 1$ is μ_ζ .
- Second, there is a continuum of mass I of informed agents who know the value of θ . They are risk-neutral but constrained in their trade. The trade constraints are normalized such that an agent cannot trade more than one unit of the asset, in absolute value². Hence, each agent trades $2\theta - 1$, and the demand by informed agents is $I(2\theta - 1)$.
- Third, there is a large mass of uninformed agents who are risk-neutral.

As is well known, the information conveyed by the market equilibrium is equivalent to the observation of the order flow which is the sum of the demands of the informed and the noise traders. This order flow is equal to

$$y(\hat{\theta}, \hat{\zeta}, \eta) = I(2\hat{\theta} - 1) + \hat{\zeta} + \eta, \quad (1)$$

where $(\hat{\theta}, \hat{\zeta})$ is the true state.

¹ It seems that the present model is the first one to show how strategic complementarity in information can arise solely through the observation of the price in a frictionless financial market with fully rational agents who trade for the long-term. In Froot *et al.* (1992), the value of some private signal increases with the number of agents owning the signal, but there is a friction in the market where the price does not fully reflect the information generated by trades. Trade for the short-term will be discussed later.

²One could also consider a constraint on the monetary value of the transaction.

Because of the risk-neutral uninformed agents, the equilibrium price is such that

$$p = E[\theta|y] = P(\theta = \theta_2|y)(\theta_2 - \theta_1) + \theta_1. \quad (2)$$

Let $\pi(\theta, \zeta)$ be the prior probability of (θ, ζ) , and $\mu(\theta, \zeta|y)$ this probability (belief) after the observation of the equilibrium price. Since η has a Gaussian distribution, the post equilibrium belief can be written

$$\mu(\theta, \zeta|y; I) = A \exp\left(-\frac{1}{2\sigma^2}(y - I(2\theta - 1) - \zeta)^2\right) \pi(\theta, \zeta), \quad (3)$$

where the constant A is determined such that the sum of the probabilities is equal to one and y is determined by (1).

Substituting in (3) y by its expression in (1) of y in (3), the posterior distribution over (θ, ζ) is given by

$$\mu(\theta, \zeta|\hat{\theta}, \hat{\zeta}, \eta; I) = A \exp\left(-\frac{1}{2\sigma^2}\left(2I(\hat{\theta} - \theta) + \hat{\zeta} - \zeta + \eta\right)^2\right) \pi(\theta, \zeta), \quad (4)$$

and from (2), using here $\theta_1 = 0$, $\theta_2 = 1$, the price of the asset is

$$p(\hat{\theta}, \hat{\zeta}, \eta; I) = \mu(\theta_2, \zeta_2|\hat{\theta}, \hat{\zeta}, \eta; I) + \mu(\theta_2, \zeta_1|\hat{\theta}, \hat{\zeta}, \eta; I). \quad (5)$$

In computing the value of information, before the market opens, the agent knows that two possibilities may arise.

- If he will learn that the true state is $\hat{\theta} = \theta_2$ (with the prior probability μ_θ), then he will buy one unit of the asset (up to his constraint), and get a profit of $1 - p(\hat{\theta}_2, \hat{\zeta}, \eta; I)$. The price $p(\hat{\theta}_2, \hat{\zeta}, \eta; I)$ depends on the independent random variables $\hat{\zeta}$ and η . The first is binary and equal to ζ_2 with probability μ_ζ , and η has the density $\phi(\eta)$. Taking the expectation over all possible realizations of (ζ, η) and using (5), the expected profit, contingent on learning that $\theta = \theta_2$, is equal to

$$B(I) = \int \left(1 - \mu_\zeta p(\theta_2, \zeta_2, \eta; I) - (1 - \mu_\zeta) p(\theta_2, \zeta_1, \eta; I)\right) d\phi(\eta) d\eta.$$

- If the agent will learn that the true state is $\hat{\theta} = \theta_1$, then he will sell one unit of the asset and get a profit equal to its price $p(\hat{\theta}_1, \hat{\zeta}, \eta; I)$ which depends on the random variables $\hat{\zeta}$ and η . As in the previous case, the agent computes the expectation of the profits for this strategy.

Summing over the two possible events $\hat{\theta} = \hat{\theta}_2$, $\hat{\theta} = \hat{\theta}_1$, with probabilities μ_θ and $1 - \mu_\theta$ respectively, the value of information is a function of I that is defined by

$$V(I) = \mu_\theta \int \left(1 - \mu_\zeta p(\theta_2, \zeta_2, \eta; I) - (1 - \mu_\zeta) p(\theta_2, \zeta_1, \eta; I) \right) d\phi(\eta) d\eta \\ + (1 - \mu_\theta) \int \left(\mu_\zeta p(\theta_1, \zeta_2, \eta; I) + (1 - \mu_\zeta) p(\theta_1, \zeta_1, \eta; I) \right) \phi(\eta) d\eta.$$

Let us consider first the case where the mass of informed agents takes an “intermediate value”, here $I = 1/4$. The difference between the order flow of informed agents in the low and the high state is equal to $1/2$, which is half of the difference between the mean of the distribution of the demand with and without shock respectively. If a jump of the order flow is driven by information about the fundamental, its value is $1/2$, whereas if it is driven by a demand shock, its value will be around 1. When the variance of the noise term η is small, the two shocks which are independent, can be well (but not perfectly) separated by the order flow in equilibrium. The market price is on average close to the fundamental. That is the meaning of the next result.

Proposition 1

If $I = 1/4$, the price is close to the fundamental in the following sense: for any $\alpha > 0$, $\epsilon > 0$, there exists σ^* such that if $\sigma < \sigma^*$,

$$P\left(|p(\theta, \zeta, \eta; \frac{1}{4}) - \theta| > \alpha\right) < \epsilon,$$

where the probability P is measured with the prior distribution on (θ, ζ, η) .

Proof: We omit the variable I (which is equal to $1/4$) from the arguments of μ and p in their expressions (6) and (5). The posterior belief takes the form

$$\mu(\theta, \zeta | \hat{\theta}, \hat{\zeta}, \eta) = A \exp\left(-\frac{1}{2\sigma^2} \left(z(\theta, \zeta; \hat{\theta}, \hat{\zeta}) + \eta\right)^2\right) \pi(\theta, \zeta), \quad \text{with} \quad (6)$$

$$z(\theta, \zeta; \hat{\theta}, \hat{\zeta}) = 0.5(\hat{\theta} - \theta) + \hat{\zeta} - \zeta.$$

For $(\theta, \zeta) \neq (\hat{\theta}, \hat{\zeta})$, and since $z(\hat{\theta}, \hat{\zeta}; \hat{\theta}, \hat{\zeta}) = 0$,

$$\frac{\mu(\theta, \zeta | \hat{\theta}, \hat{\zeta}, \eta)}{\mu(\hat{\theta}, \hat{\zeta} | \hat{\theta}, \hat{\zeta}, \eta)} = A \exp\left(-\frac{z(\theta, \zeta; \hat{\theta}, \hat{\zeta})}{\sigma^2} \left(\frac{z(\theta, \zeta; \hat{\theta}, \hat{\zeta})}{2} + \eta\right)\right).$$

If for some $\beta > 0$,

$$\frac{\mu(\zeta, \zeta | \hat{\theta}, \hat{\zeta}, \eta)}{\mu(\hat{\zeta}, \hat{\zeta} | \hat{\theta}, \hat{\zeta}, \eta)} > \beta, \quad (7)$$

then for some $\beta' > 0$

$$\left| \frac{z(\theta, \zeta; \hat{\theta}, \hat{\zeta})}{\sigma^2} \left(\frac{z(\theta, \zeta; \hat{\theta}, \hat{\zeta})}{2} + \eta\right) \right| < \beta'. \quad (8)$$

There exists a strictly positive M such that $|z(\theta, \zeta; \hat{\theta}, \hat{\zeta})| > M$ over all $(\theta, \zeta; \hat{\theta}, \hat{\zeta})$ with $(\theta, \zeta) \neq (\hat{\theta}, \hat{\zeta})$ (since there are 12 such values), and (8) implies that

$$\left| \frac{z(\theta, \zeta; \hat{\theta}, \hat{\zeta})}{2\sigma} + \frac{\eta}{\sigma} \right| < \sigma \frac{\beta'}{M},$$

By a similar argument, there is A and σ_1^* such that if $\sigma < \sigma_1^*$,

$$\left| \frac{\eta}{\sigma} \right| > \frac{A}{\sigma}. \quad (9)$$

The random variable η/σ is Gaussian with mean 0 and variance 1. Hence, for any $\epsilon > 0$, there exists $\sigma^* < \sigma_1^*$ such that if $\sigma > \sigma_2^*$, $P(|\eta/\sigma|) < \epsilon$. (Choose $\sigma^* < \sigma_1^*$ such that $2(1 - F(A/\sigma^*)) < \epsilon$).

If (7) applies for any $(\theta, \zeta) \neq (\hat{\theta}, \hat{\zeta})$ then, because the sum of the probabilities of all (θ, ζ) is equal to one,

$$\mu(\theta, \zeta | \hat{\theta}, \hat{\zeta}, \eta) < \frac{\beta}{3\beta + 1} < \beta, \text{ and } \mu(\hat{\theta}, \hat{\zeta} | \hat{\theta}, \hat{\zeta}, \eta) > 1 - 3\beta. \quad (10)$$

Recall that these inequalities hold for any $(\theta, \zeta, \hat{\theta}, \hat{\zeta})$ and over a set of η of probability at least $1 - \epsilon$.

Since the price is given by $p(\hat{\theta}, \hat{\zeta}, \eta) = \mu(\hat{\theta}, \zeta_2 | \hat{\theta}, \hat{\zeta}, \eta) + \mu(\hat{\theta}, \zeta_2 | \hat{\theta}, \hat{\zeta}, \eta)$, under previous inequality,

$$P(|p - \hat{\theta}| < 4\beta) > 1 - \epsilon.$$

The proof is concluded by choosing $\beta = \alpha/4$, where α is the parameter in the proposition.

Proposition 1 shows that when the variance of the noise term η is sufficiently small, the price is arbitrarily close to the fundamental with a probability arbitrarily close to one. Since the payoff of information rests on the difference between the price and the fundamental, the value of information must be arbitrarily small. From Proposition 1, and since the gain from trade is bounded by 1, it follows immediately that $V(1/4) < \alpha + \epsilon$ where α and ϵ are arbitrarily small.

Proposition 2

For any $\gamma > 0$, there exists σ^ such that if $\sigma < \sigma^*$, $V(1/4) < \gamma$.*

We now consider the case where the mass of informed agents is 1/2. Since in equilibrium, the net demand of an informed agent (who is perfectly informed) does not

depend on the mass I of informed agents, the total net demand of these agents is proportional to I . When $I = 1/2$, a shift of θ from θ_1 to θ_2 increases by one the demand of the informed and this impact on the order flow is the same as the impact of an exogenous demand shock from ζ_1 to ζ_2 . The two shocks cannot be separated by the observation of the order flow when only one of them occurs. The market price remains bounded away from 0 or 1 if the variance σ^2 of the noise η becomes arbitrarily small and private information retains a value bounded away from zero. This argument is formalized in the next result.

Proposition 3 $V(1/2) > \frac{2\mu_\theta\mu_z(1-\mu_\theta)(1-\mu_z)}{\mu_\theta(1-\mu_\zeta) + (1-\mu_\theta)\mu_\zeta}$.

Proof: The profit from trading by a non-informed agent is based on the observation of the variable y given in (1), which, when $I = 1/2$, is equivalent to $y' = y + 0.5 = \hat{\theta} + \hat{\zeta} + \eta$. His (maximum) expected profit is zero. The signal y' is less informative than the signal $\hat{\theta} + \hat{\zeta}$ in the sense of Blackwell (1951). If an agent could observe the signal $\hat{\theta} + \hat{\zeta}$ without the garble η , and trade at the equilibrium price (5), his expected profit would be strictly positive. The value of information is the difference between the expected profit from trade by an informed agent and the non-informed agent. Therefore, this value is bounded below by the difference between the expected trade profit of an informed agent and an agent who observes $\hat{\theta} + \hat{\zeta}$.

The agent who observes $\hat{\theta} + \hat{\zeta}$ has perfect information in the states (1, 1) and (0, 0). Hence, we can restrict the comparison with the payoff of the perfectly informed agent to the states with $\hat{\theta} \neq \hat{\zeta}$.

$$V\left(\frac{1}{2}\right) > \mu_\theta(1-\mu_z)\left(1 - P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1)\right) + (1-\mu_\theta)\mu_\zeta P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1).$$

By application of Bayes' rule,

$$P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1) = \frac{(1-\mu_\zeta)\mu_\theta}{(1-\mu_\zeta)\mu_\theta + \mu_\zeta(1-\mu_\theta)}.$$

Substituting in the previous expression, one finds the inequality in Proposition 3.

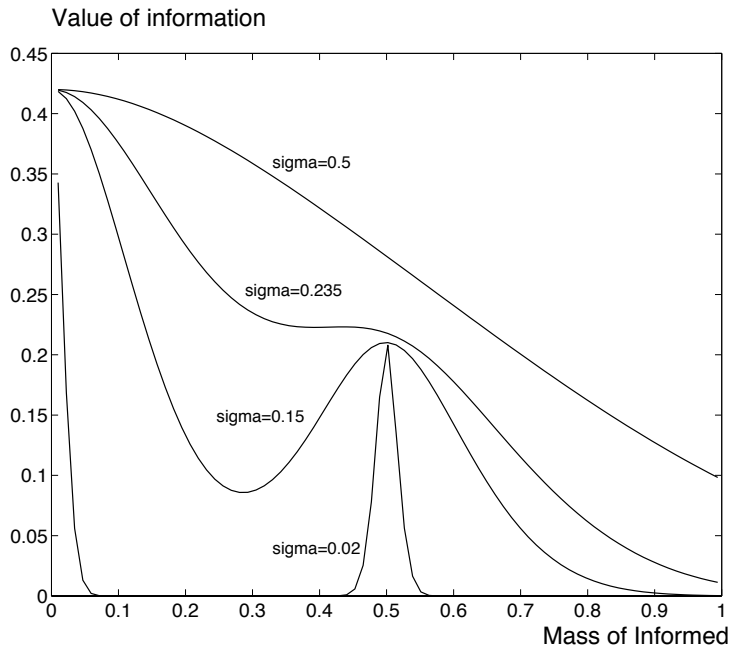
The next result follows immediately from the previous propositions.

Theorem 1

Let $V(I)$ be the value of information (before the market opens). There exists $\bar{\sigma}$ such that if $\sigma < \bar{\sigma}$, $V(1/4) < V(1/2)$, and the value of information is increasing in some interval contained in (0, 1).

An example

The function $V(I)$ is computed numerically and represented for four values of the variance parameter σ in Figure 1. Propositions 1 and 2 are illustrated by the case $\sigma = 0.02$: when I is in some range between 0 and $1/2$, the order flow provides very good information on the fundamental and the value of information is very small, not distinguishable from 0 in the figure. Following Proposition 3 and given the parameters of the example, for all values of σ , $V(1/2) > 0.21$. When $I = 0$, the market provides no information, and information has its maximum value. When I increases from 0, the market provides some information, and the value of private information must be decreasing from its maximum. If σ is sufficiently small, the *ex ante* value of information may decrease to a very low level. But if I increases toward $1/2$, the market provides less information when only one shock occurs and the value of information increases. In the region $I > 1/2$, the case is similar to the one for small I and $V(I)$ is decreasing.



Parameters: $\mu_\theta = \mu_\zeta = 0.7$.

Figure 1: The value of information as a function of the mass of informed agents

Conclusion

The result of the paper was established for normalized values of the discrete changes, but its extension to arbitrary values of the jumps is a trivial exercise. The main assumption for the argument of the paper is the possibility of independent quantum jumps in both the fundamental and the exogenous demand. When quantum jumps are restricted to either the fundamental or the exogenous demand, the standard property of a decreasing value of information seems to hold. The mechanism that generates here the strategic complementarity in information acquisition is simple. It remains to be seen if other mechanisms could generate such a strategic complementarity when agents trade in a frictionless market for the long-term, that is to hold until the revelation of the fundamental. As shown in Chamley (2007) in a model with a Gaussian distribution of noise, when agents trade for the short-term and hold their position for a limited amount of time, strategic complementarity may hold.

REFERENCES

- Blackwell, D. (1951). "The comparison of experiments, in *Proceedings, Second Berkeley Symposium on Mathematical Statistics and Probability*, pp. 93-102. Univ. of California Press, Berkeley.
- Chamley, C. (2007). "Complementarities in Information Acquisition With Short-Term Trades," *Theoretical Economics*, forthcoming.
- Froot, K., D. Scharfstein and J. Stein (1992). "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation," *Journal of Finance*, **47**, 1461-1484.
- Grossman, S. J., and J. E. Stiglitz (1980). "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, **70**, 393-408.