Ecosystem Viable Yields

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Abstract

The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the ecosystem approach by 2010. However, at the same Summit, the signatory States undertook to restore and exploit their stocks at maximum sustainable yield (MSY), a concept and practice without ecosystemic dimension, since MSY is computed species by species, on the basis of a monospecific model. Acknowledging this gap, we propose a definition of "ecosystem viable yields" (EVY) as yields compatible i) with biological viability levels for all time and ii) with an ecosystem dynamics. To the difference of MSY, this notion is not based on equilibrium, but on viability theory, which offers advantages for robustness. For a generic class of multispecies models with harvesting, we provide explicit expressions for the EVY. We apply our approach to the anchovy–hake couple in the Peruvian upwelling ecosystem.

Key words: control theory; state constraints; viability; ecosystem management; Peruvian upwelling ecosystem; yields.

1 Introduction

Following the World Summit on Sustainable Development (Johannesburg, 2002), the signatory States undertook to restore and exploit their stocks at maximum sustainable yield (MSY, see

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(Clark, 1990)). Though being criticized for decades, MSY remains a reference. Criticisms of MSY, like (Larkin, 1977), point out that MSY relies upon a single variable stock description (the species biomass), without age structure nor interactions with other species; what is more, computations are made at equilibrium. In fisheries, one of the more elaborate method of fixing quotas, the ICES (International Council for the Exploration of the Sea) precautionary approach (ICES, 2004), does not assume equilibrium (it projects abundances one year ahead) and assumes age structure; it remains however based on a monospecific dynamical model. Thus, in fisheries, yields are usually defined species by species.

On the other hand, more and more emphasis is put on multispecies models (Hollowed, Bax, Beamish, Collie, Fogarty, Livingston, Pope, and Rice, 2000) and on ecosystem management. For instance, the World Summit on Sustainable Development encouraged the application of the ecosystem approach by 2010. Also, sustainability is a major goal of international agreements and guidelines to fisheries management (FAO, 1999; ICES, 2004).

Our interest is in providing conceptual insight as what could be *sustainable yields for ecosystems*. In this, we follow the vein of (Katz, Zabel, Harvey, Good, and Levin, 2003) which introduces the concept of *Ecologically Sustainable Yield* (ESY), or of (Chapel, Deffuant, Martin, and Mullon, 2008) which defines yield policies in a viability approach. A general discussion on the ecosystem approach to fisheries may be found in (Garcia, Zerbi, Aliaume, Chi, and Lasserre, 2003).

Our emphasis is on providing formal definition and practical methods to design and compute such yields. For this purpose, our approach is not based on equilibrium calculus, nor on intertemporal discounted utility maximization but on the so-called viability theory, as follows.

On the one hand, the ecosystem is described by a dynamical model controlled by harvesting. On the other hand, building upon (Béné, Doyen, and Gabay, 2001), constraints are imposed: catches are expected to be above given production thresholds, and biomasses above safety biological thresholds. Sustainability is the property that such constraints can be maintained for all time by appropriate harvesting strategies.

Such problems of dynamic control under constraints refer to viability (Aubin, 1991) or invariance (Clarke, Ledayev, Stern, and Wolenski, 1995) frameworks, as well as to reachability of target sets or tubes for nonlinear discrete time dynamics in (Bertsekas and Rhodes, 1971).

We consider sustainable management issues formulated within such framework as in (Béné, Doyen, and Gabay, 2001; Béné and Doyen, 2003; Eisenack, Sheffran, and Kropp, 2006; Mullon, Cury, and Shannon, 2004; Rapaport, Terreaux, and Doyen, 2006; De Lara, Doyen, Guilbaud, and Rochet, 2007; De Lara and Doyen, 2008; Chapel, Deffuant, Martin, and Mullon, 2008).

A viable state is an initial condition for the ecosystem dynamical system such that appropriate harvesting rules may drive the system on a sustainable path by maintaining catches and biomasses above their respective production and biological thresholds. We provide a way to characterize production thresholds (yields) such that the present initial conditions are a viable state. These yields are sustainable in the sense that they can be indefinitely maintained, while making possible that the ecosystem remains in an ecologically viable zone; we coin them *ecosystem viable yields*.

The paper is organized as follows. In Section 2, we introduce generic harvested nonlinear ecosystem models, and we present how preservation and production constraints are modelled. Thanks to an explicit description of viable states, we are able to characterize sustainable yields. These latter are not defined species by species, but depend on the whole ecosystem dynamics and on all biological thresholds. In Section 3, an illustration in ecosystem management and numerical applications are given for the hake–anchovy couple in the Peruvian upwelling ecosystem between the years 1971 and 1981. We conclude in Section 4 with possible extensions of the notion of ecosystem viable yields to more general ecosystem models. In the appendix, Section A is devoted to recalls on discrete–time viability and its possible use for sustainable management, while Section B contains the mathematical proofs.

2 Ecosystem viable yields

After a brief recall on the notion of maximum sustainable yield (for monospecific models), we introduce a class of generic harvested nonlinear ecosystem models, then present how to define maximum sustainable yields for this class. Next, we provide an explicit description of viable states, for which production and biological constraints can be guaranteed for all times under appropriate management. This makes possible to define *ecosystem viable yields*, compatible with biological and conservation constraints. We end up discussing relations between ecosystem viable yields and maximum sustainable yields.

2.1 A brief recall on maximum sustainable yield

We briefly sketch the principles leading to the notion of *maximum sustainable yield* (MSY) (see (Clark, 1990) in continuous time and (De Lara and Doyen, 2008) in discrete time).

Consider a single population described by its total biomass $B(t)$ at time t. Suppose that the time evolution of the biomass is given by a dynamical equation, a differential equation $\dot{B}(t) = \mathtt{Biol}_c\big(B(t)\big)$ in continuous time or a difference equation $B(t+1) = \mathtt{Biol}_d\big(B(t)\big)$ in discrete time. From this, build a Schaefer model (Schaefer, 1954) by substracting a catch term $h(t)$, giving $\dot{B}(t) = \mathtt{Biol}_c\big(B(t)\big) - h(t)$ or $B(t+1) = \mathtt{Biol}_d\big(B(t) - h(t)\big)$. In general, to each biomass level B_{E} (below the carrying capacity) corresponds a catch level $h_{\mathrm{E}} = \mathtt{Sust}(B_{\mathrm{E}})$ for which the biomass B_{E} is at equilibrium, solution of $\mathtt{Biol}_c\big(B_{\mathrm{E}}\big) - h_{\mathrm{E}} = 0$ or $\mathtt{Biol}_d\big(B_{\mathrm{E}} - h_{\mathrm{E}}\big) = B_{\mathrm{E}}$. The *maximum sustainable yield* is the largest of such equilibrium catches: $\textsc{msy} = \max_{B_{\mathrm{E}}} \mathtt{Sust}(B_{\mathrm{E}})$.

2.2 Ecosystem biomass dynamical model

For simplicity, we consider two species. However, the following Proposition 2 may be easiliy extended to N–dimensional systems as long as each species is harvested by a specific device: one species, one harvesting effort.

Each species is described by its biomass: the two–dimensional state vector (y, z) represents the biomasses of both species. The two–dimensional control (v, w) comprises the harvesting effort for each species, respectively. The catches are thus vy and wz (measured in biomass).[1]

The discrete–time control system we consider is

$$\left\{ \begin{array}{rcl} y(t+1) & = & y(t)R_y\big(y(t), z(t), v(t)\big) \\ z(t+1) & = & z(t)R_z\big(y(t), z(t), w(t)\big) \ , \end{array} \right. \tag{1}$$

where t stand for time (typically, periods are years), and where $R_y : \mathbb{R}^3 \to \mathbb{R}$ and $R_z : \mathbb{R}^3 \to \mathbb{R}$ are two functions representing growth factors (the growth rates being $R_y - 1$ and $R_z - 1$).

This model is generic in that no explicit or analytic assumptions are made on how the growth factors R_y and R_z indeed depend upon both biomasses (y, z).

[1] In fact, any expression of the form $c(y, v)$, instead of vy, would fit for the catches in the following Proposition 2 as soon as $v \mapsto c(y, v)$ is strictly increasing and goes from 0 to $+\infty$ when v goes from 0 to $+\infty$. The same holds for $d(z, w)$ instead of wz.

2.3 Maximum sustainable yields

An *equilibrium* of the ecosystem model (1) is a couple $(y_{\mathrm{E}}, z_{\mathrm{E}})$ of biomasses (state) and a couple $(v_{\mathrm{E}}, w_{\mathrm{E}})$ of harvesting efforts (control) satisfying

$$\begin{cases} y_{\mathrm{E}} & = & y_{\mathrm{E}} R_y\big(y_{\mathrm{E}}, z_{\mathrm{E}}, v_{\mathrm{E}}\big) \\ z_{\mathrm{E}} & = & z_{\mathrm{E}} R_z\big(y_{\mathrm{E}}, z_{\mathrm{E}}, w_{\mathrm{E}}\big) \ , \end{cases} \tag{2}$$

The *maximum sustainable yields*, MSY_y for species y and MSY_z for species z, are given by

$$\mathrm{MSY}_y := \max_{v_{\mathrm{E}}, w_{\mathrm{E}}} v_{\mathrm{E}} y_{\mathrm{E}} \ \text{ and } \ \mathrm{MSY}_z := \max_{v_{\mathrm{E}}, w_{\mathrm{E}}} w_{\mathrm{E}} z_{\mathrm{E}} \ . \tag{3}$$

They must be jointly defined because the ecosystem equilibrium equations (2) couple all variables.

After having presented maximum sustainable yields, we propose another way to define sustainability as the ability to respect preservation and production thresholds for all times, building upon the original approach of (Béné, Doyen, and Gabay, 2001).

2.4 Preservation and production sustainability

Let us be given

- on the one hand, *minimal biomass thresholds* $B_y^\flat \geq 0$, $B_z^\flat \geq 0$, one for each species,
- on the other hand, *minimal catch thresholds* $C_y^\flat \geq 0$, $C_z^\flat \geq 0$, one for each species.

A couple (y_0, z_0) of initial biomasses is said to be a *viable state* if there exist appropriate harvesting efforts (controls) $\big(v(t), w(t)\big)$, $t = t_0, t_0 + 1, \ldots$ such that the state path $\big(y(t), z(t)\big)$, $t = t_0, t_0 + 1, \ldots$ starting from $\big(y(t_0), z(t_0)\big) = (y_0, z_0)$ satisfies the following goals:

- preservation (minimal biomass thresholds)

$$\text{biomasses: } \ y(t) \geq B_y^\flat \ , \quad z(t) \geq B_z^\flat \ , \quad \forall t = t_0, t_0 + 1, \ldots \tag{4}$$

- and production requirements (minimal catch thresholds)

$$\text{catches: } \ v(t)y(t) \geq C_y^\flat \ , \quad w(t)z(t) \geq C_z^\flat \ , \quad \forall t = t_0, t_0 + 1, \ldots \tag{5}$$

The set of all viable states is called the *viability kernel* (Aubin, 1991). Characterizing viable states makes it possible to test whether or not minimal biomasses and catches can be guaranteed for all time.

The following definition summarizes useful and natural properties required for the growth factors in the ecosystem model.

Definition 1 *We say that growth factors R_y and R_z in the ecosystem model* (1) *are* nice *if the function $R_y : \mathbb{R}^3 \to \mathbb{R}$ is continuously decreasing*[2] *in the harvesting effort v and satisfies $\lim_{v\to+\infty} R_y(y,z,v) \leq 0$, and if $R_z : \mathbb{R}^3 \to \mathbb{R}$ is continuously decreasing in the harvesting effort w, and satisfies $\lim_{w\to+\infty} R_z(y,z,w) \leq 0$.*

The following Proposition 2 gives an explicit description of the viable states, under some conditions on the minimal thresholds. Its proof is given in § B.1 in the Appendix.

Proposition 2 *Assume that the growth factors in the ecosystem model* (1) *are nice. If the biomass thresholds $B_y^\flat$, $B_z^\flat$, and the catch thresholds $C_y^\flat$, $C_z^\flat$ are such that the following growth factors values are greater than one*

$$R_y(B_y^\flat, B_z^\flat, \frac{C_y^\flat}{B_y^\flat}) \geq 1 \ \text{ and } \ R_z(B_y^\flat, B_z^\flat, \frac{C_z^\flat}{B_z^\flat}) \geq 1 \ , \tag{6}$$

then viable states are biomasses couples (y,z) such that

$$y \geq B_y^\flat, \quad z \geq B_z^\flat, \quad yR_y(y,z,\frac{C_y^\flat}{y}) \geq B_y^\flat, \quad zR_z(y,z,\frac{C_z^\flat}{z}) \geq B_z^\flat \ . \tag{7}$$

Let us comment the assumptions of Proposition 2. That the growth factors are decreasing with respect to the harvesting effort is a natural assumption. Conditions (6) mean that, at the point $(B_y^\flat, B_z^\flat)$ and applying efforts $u^\flat = \frac{C_y^\flat}{B_y^\flat}$, $v^\flat = \frac{C_z^\flat}{B_z^\flat}$, the growth factors are greater than one, hence both populations grow; hence, it could be thought that computing viable states is useless since everything looks fine. However, if all is fine at the point $(B_y^\flat, B_z^\flat)$, it is not obvious that this also goes for a larger domain. Indeed, the ecosystem dynamics given by (1) has no monotonocity properties that would allow to extend a result valid for a point to a whole domain. What is more,

[2]In all that follows, a mapping $\varphi : \mathbb{R} \to \mathbb{R}$ is said to be increasing if $x \geq x' \Rightarrow \varphi(x) \geq \varphi(x')$. The reverse holds for decreasing. Thus, with this definition, a constant mapping is both increasing and decreasing.

if continuous-time viability results mostly relies upon assumptions at the frontier of the constraints set, this is no longer true for discrete-time viability.

We shall explicitely draw a viability kernel in the next section, for a discrete-time Lotka–Volterra model for the hake–anchovy couple in the Peruvian upwelling ecosystem.

2.5 Ecosystem viable yields

Considering that minimal biomass conservation thresholds are given first (for prominent biological issues), we shall now examine conditions for the existence of minimal catch thresholds

First, we define (when they exist) the *ecosystem viable yields.*

Definition 3 *Let biomass conservation thresholds $B_y^\flat \geq 0$, $B_z^\flat \geq 0$ be given. Suppose that the growth factors in the ecosystem model* (1) *are nice, and that they take values greater than one in the absence of harvesting, namely:*

$$R_y(B_y^\flat, B_z^\flat, 0) \geq 1 \ \text{ and } \ R_z(B_y^\flat, B_z^\flat, 0) \geq 1 \ . \tag{8}$$

Define equilibrium catches as the largest nonnegative[3] *catches $C_y^{\flat,\star}, C_z^{\flat,\star}$ such that*

$$R_y(B_y^\flat, B_z^\flat, \frac{C_y^{\flat,\star}}{B_y^\flat}) = 1 \ \text{ and } \ R_z(B_y^\flat, B_z^\flat, \frac{C_z^{\flat,\star}}{B_z^\flat}) = 1 \ . \tag{9}$$

For a couple (y_0, z_0) of biomasses, define (when they exist) the ecosystem viable yields $C_y^{\flat,\star}(y_0, z_0)$ *and $C_z^{\flat,\star}(y_0, z_0)$ by*

$$\begin{cases} C_y^{\flat,\star}(y_0, z_0) & := & \max\{C_y \in [0, C_y^{\flat,\star}] \mid y_0 R_y(y_0, z_0, \frac{C_y}{y_0}) \geq B_y^\flat\} \\ C_z^{\flat,\star}(y_0, z_0) & := & \max\{C_z \in [0, C_z^{\flat,\star}] \mid z_0 R_z(y_0, z_0, \frac{C_z}{z_0}) \geq B_z^\flat\} \ , \end{cases} \tag{10}$$

The term *ecosystem viable yields* is justified by the following Proposition 4.

Proposition 4 *Assume that the growth factors in the ecosystem model* (1) *are nice. For a couple (y_0, z_0) of biomasses above preservation thresholds – that is, $y_0 \geq B_y^\flat$ and $z_0 \geq B_z^\flat$ – and satisfying*

$$y_0 R_y(y_0, z_0, 0) \geq B_y^\flat \ \text{ and } \ z_0 R_z(y_0, z_0, 0) \geq B_z^\flat \ , \tag{11}$$

[3]Such catches are nonnegative because the growth factors in the ecosystem model (1) are nice, hence continuously decreasing in the harvesting effort, and by (8).

the ecosystem viable yields $C_y^{\flat,\star}(y_0,z_0)$ *and* $C_z^{\flat,\star}(y_0,z_0)$ *in* (10) *are well defined.*

What is more, consider catches $C_y^\flat$ *and* $C_z^\flat$ *lower than these ecosystem viable yields, that is,* $0 \leq C_y^\flat \leq C_y^{\flat,\star}(y_0,z_0)$ *and* $0 \leq C_z^\flat \leq C_z^{\flat,\star}(y_0,z_0)$*. Then, starting from the initial biomasses* $(y(t_0),z(t_0)) = (y_0,z_0)$*, there exists appropriate harvesting paths which provide, for all time, at least the* sustainable yields $C_y^\flat$ *and* $C_z^\flat$ *and which guarantee that biomass conservation thresholds* $B_y^\flat \geq 0$*,* $B_z^\flat \geq 0$ *are respected for all time.*

From the practical point of view, the upper quantities $C_y^{\flat,\star}(y_0,z_0)$ and $C_z^{\flat,\star}(y_0,z_0)$ in (10) cannot be seen as catches targets, but rather as *crisis limits*. Indeed, the closer to them, the more vulnerable, since the initial point is close to the viability kernel frontier.

Notice that the yield $C_y^{\flat,\star}(y_0,z_0)$ depends, first, on both species biomasses (y_0,z_0), second, on both conservation thresholds $B_y^\flat$ and $B_z^\flat$, third, on the ecosystem model by the growth factor R_y; the same holds for $C_z^{\flat,\star}(y_0,z_0)$. Thus, these yields are designed jointly on the basis of the whole ecosystem model and of all the conservation thresholds; this is why we coined them *ecosystem* viable yields.

This observation may have practical consequences. Indeed, the catches guaranteed for one species depend not only on the biological threshold of the same species, but on the other species. For instance, in the Peruvian upwelling ecosystem, it is customary to increase the biological threshold of the anchovy before an El Niño event, but without explicitely considering to lower catches of other species. Our analysis stresses the point that thresholds have to be designed globally to guarantee sustainability for the whole ecosystem.

2.6 Ecosystem viable yields and maximum sustainable yields

Now, we show how *ecosystem viable yields* are related to *maximum sustainable yields* defined in (3).

Say that the maximum sustainable yields MSY_y and MSY_z are *viable maximum sustainable yields* if the corresponding biomasses equilibrium values y_{E} and z_{E} are such that $y_{\mathrm{E}} \geq B_y^\flat$ and $z_{\mathrm{E}} \geq B_z^\flat$. In this case, MSY_y and MSY_z are ecosystem viable yields for the couple $(y_{\mathrm{E}},z_{\mathrm{E}})$ of initial biomasses: indeed, the stationary harvest strategy $v(t) = v_{\mathrm{E}}$ and $w(t) = w_{\mathrm{E}}$ drives the ecosystem model (1) at equilibrium $(y_{\mathrm{E}},z_{\mathrm{E}})$ which satisfies the conservation thresholds $y_{\mathrm{E}} \geq B_y^\flat$ and $z_{\mathrm{E}} \geq B_z^\flat$.

3 Numerical application to the hake–anchovy couple in the Peruvian upwelling ecosystem (1971–1981)

We provide a viability analysis of the hake–anchovy Peruvian fisheries between the years 1971 and 1981. For this, we shall consider a discrete-time Lotka–Volterra model for the couple anchovy (prey y) and hake (predator z), then provide an explicit description of viable states.

We warn the reader that our emphasis is not on developing a "knowledge" biological model to "faithfully" describe the complexity of the Peruvian upwelling ecosystem. This formidable task is out of our competencies, and is not necessary for our analysis. Indeed, our approach makes use, not of "knowledge" models, but of "action" models; these are small, compact, models which capture essentials features of the system in what concern decision-making. In our case, we needed a compact model able to put in consistency biomass and catches yearly data between the years 1971 and 1981. We chose a discrete–time Lotka–Volterra model, despite well-known criticisms as candidate for a "knowledge" biological model (Hall, 1988; Murray, 2002), but for its compactness qualities and for the reasonable fit (see Figure 1).

3.1 Viable states and ecosystem sustainable yields for a Lotka–Volterra system

Consider the following discrete–time Lotka–Volterra system of equations with density–dependence in the prey

$$\begin{cases} y(t+1) &= Ry(t) - \dfrac{R}{\kappa}y^2(t) - \alpha y(t)z(t) - v(t)y(t) \ , \\ z(t+1) &= Lz(t) + \beta y(t)z(t) - w(t)z(t) \ , \end{cases} \tag{12}$$

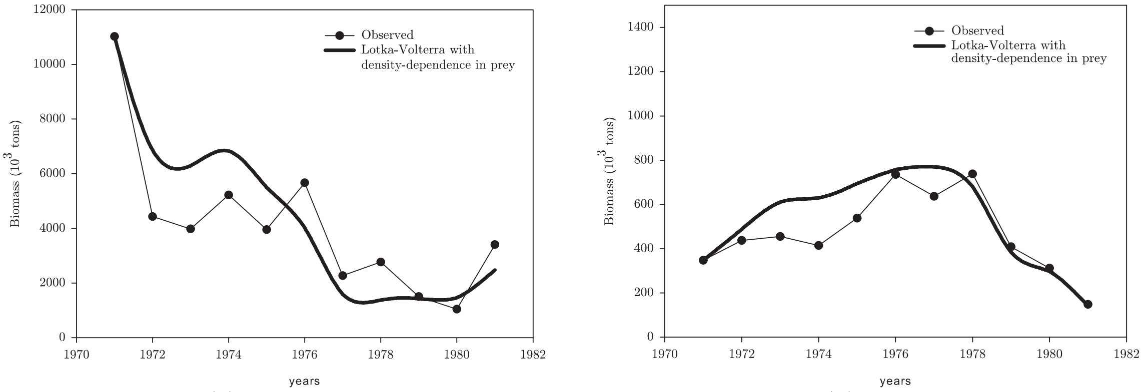
where $R > 1$, $0 < L < 1$, $\alpha > 0$, $\beta > 0$ and $\kappa = \frac{R}{R-1}K$, with $K > 0$ the carrying capacity for prey. In the dynamics (1), we identify $R_y(y, z, v) = R - \frac{R}{\kappa}y - \alpha z - v$ and $R_z(y, w) = L + \beta y - w$.

By Proposition 4, we obtain that, for any initial point (y_0, z_0) such that

$$y_0 \geq B_y^\flat \ , \quad z_0 \geq B_z^\flat \ , \quad y_0(R - \frac{R}{\kappa}y_0 - \alpha z_0) \geq B_y^\flat \ , \tag{13}$$

the ecosystem sustainable yields are given by

$$\begin{cases} C_y^{\flat,\star}(y_0, z_0) &= \min\left\{B_y^\flat(R - \frac{R}{\kappa}B_y^\flat - \alpha B_z^\flat) - B_y^\flat, y_0(R - \frac{R}{\kappa}y_0 - \alpha z_0) - B_y^\flat\right\} \\ C_z^{\flat,\star}(y_0, z_0) &= B_z^\flat(L + \beta B_y^\flat - 1) \ . \end{cases} \tag{14}$$



(a) Anchovy (b) Hake

Figure 1: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey (1971–1981). Model parameters are $R = 2.25$ year^{-1}, $L = 0.945$ year^{-1}, $\kappa = 67\ 113\ \times 10^3$ $\texttt{t}$ ($K = 37\ 285\ \times 10^3$ $\texttt{t}$), $\alpha = 1.220 \times 10^{-6}$ $\texttt{t}^{-1}$, $\beta = 4.845 \times 10^{-8}$ $\texttt{t}^{-1}$.

In other words, if viably managed, the ecosystem could produce at least $C_y^{\flat,\star}(y_0, z_0)$ and $C_z^{\flat,\star}(y_0, z_0)$, while respecting biological thresholds $B_y^\flat$ and $B_z^\flat$.

3.2 A viability analysis of the hake–anchovy Peruvian fisheries between the years 1971 and 1981

The period between the years 1971 and 1981 has been chosen because the competition between the fishery and hake was reduced due to low anchovy catches, and because of the absence of strong El Niño events. We have 11 couples of biomasses, and the same for catches. The 5 parameters of the model are estimated minimizing a weighted residual squares sum function using a conjugate gradient method, with central derivatives. Estimated parameters and comparisons of observed and simulated biomasses are shown in Figure 1.

We consider values of $B_y^\flat = 7\ 000\ 000$ $\texttt{t}$ and $B_z^\flat = 200\ 000$ $\texttt{t}$ for minimal biomass thresholds (IMARPE, 2000, 2004). Conditions (13) are satisfied and the expressions (14) give the ecosystem viable yields (EVY)

$$C_y^{\flat,\star}(y_0, z_0) = 5\ 399\ 000\ \texttt{t} \quad \text{and} \quad C_z^{\flat,\star}(y_0, z_0) = 56\ 800\ \texttt{t} \ . \tag{15}$$

In other words, such yields were theoreticaly achievable in a sustainable way starting from year

1971. In reality, the catches of year 1971 were very high and the biomasses trajectories were outside of the biological thresholds for fourteen years.

The 4 250 000 t anchovy quota and the 55 000 t hake quota, respectively, established for the year 2006 (PRODUCE, 2005), or to the 5 000 000 t anchovy quota and the 35 000 t hake quota, respectively, established for the year 2007 (PRODUCE, 2006) are rather close to the EVY $C_y^{\flat,\star}(y_0, z_0) = 5\ 399\ 000$ t and $C_z^{\flat,\star}(y_0, z_0) = 56\ 800$ t. Thus, our approach provides reasonable figures.[4]

4 Conclusion

We have defined the notion of sustainable yields for ecosystem, and provided ways to compute them by means of a viability analysis of generic ecosystem models with harvesting. Our analysis stresses the point that yields should certainly be designed globally, and not species by species as in the current practice, to guarantee sustainability for the whole ecosystem.

Our results have then been applied to a Lotka–Volterra model using the anchovy–hake couple in the Peruvian upwelling ecosystem. Despite simplicity[5] of the models considered, our approach has provided reasonable figures and new insights: it may be a mean of designing sustainable yields from an ecosystem point of view.

The framework we propose is not restricted to two populations, each described by its global biomass, but it may be adapted to several species, each described by a vector of abundances at age, or by vectors of abundances at age for each patch in a spatial model, etc. Suppose that the time evolution is given by a dynamical equation reflecting ecosystemic interactions and driven by catches. Fix minimal safety levels (reference points) for biological indicators like spawning stock biomass,

[4] At this stage, we do not claim that the figures may be proposed as yields for the present management of hake–anchovy Peruvian fisheries. Indeed, our computations of EVY rely upon a dynamical model adjusted for some thirty years ago. To propose EVY, we should first dispose of a dynamical model adapted to the current situation, because it ought to reflect the new ecosystem functioning and the depleted state of stocks (Ballón, Wosnitza-Mendo, Guevara-Carrasco, and Bertrand, 2008). This is beyond the scope of this paper.

[5] In addition to hake, there are other important predators of anchovy in the Peruvian upwelling ecosystem, such as mackerel and horse mackerel, seabirds and pinnipeds, which were not considered. Also, anchovy has been an important prey of hake, but other prey species have been found in the opportunistic diet of hake (Tam, Purca, Duarte, Blaskovic, and Espinoza, 2006)

abundances at specific ages, etc. (such reference points for biological indicators like spawning stock biomass are generally given by international bodies like the ICES, or nationally). Ecosystem viable yields are minimal harvests for each species which can be guaranteed for all times while respecting the above minimal safety levels for biological indicators for all times too.

Thus, control and viability theory methods have allowed us to introduce ecosystem considerations, such as multispecies and multiobjectives, and have contributed to integrate the long term dynamics, which is generally not considered in conventional fishery management.

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A Discrete–time viability

Let us consider a nonlinear control system described in discrete–time by the dynamic equation

$$\begin{cases} x(t+1) = f\big(x(t), u(t)\big) \ \text{ for all } \ t \in \mathbb{N}, \\ x(0) = x_0 \ \text{ given}, \end{cases} \tag{16}$$

where the *state variable* $x(t)$ belongs to the finite dimensional state space $\mathbb{X} = \mathbb{R}^{n_{\mathbb{X}}}$, the *control variable* $u(t)$ is an element of the *control set* $\mathbb{U} = \mathbb{R}^{n_{\mathbb{U}}}$ while the *dynamics* f maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

A controller or a decision maker describes "acceptable configurations of the system" through a set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ termed the *acceptable set*

$$\big(x(t), u(t)\big) \in \mathbb{D} \ \text{ for all } \ t \in \mathbb{N} \ , \tag{17}$$

where $\mathbb{D}$ includes both system states and controls constraints.

The *state constraints set* $\mathbb{V}^0$ associated with $\mathbb{D}$ is obtained by projecting the acceptable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \mathrm{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,,\ (x,u) \in \mathbb{D}\}\,. \tag{18}$$

Viability is defined as the ability to choose, at each time step $t \in \mathbb{N}$, a control $u(t) \in \mathbb{U}$ such that the system configuration remains acceptable. More precisely, the system is viable if the following feasible set is not empty:

$$\mathbb{V}(f,\mathbb{D}) := \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\,(u(0),u(1),\ldots) \ \text{and}\ (x(0),x(1),\ldots) \\ \text{satisfying}\ (16)\ \text{and}\ (17) \end{array} \right\}\,. \tag{19}$$

The set $\mathbb{V}(f,\mathbb{D})$ is called the *viability kernel* (Aubin, 1991) associated with the dynamics f and the acceptable set $\mathbb{D}$. By definition, we have $\mathbb{V}(f,\mathbb{D}) \subset \mathbb{V}^0 = \mathrm{Proj}_{\mathbb{X}}(\mathbb{D})$ but, in general, the inclusion is strict. For a decision maker or control designer, knowing the viability kernel has practical interest since it describes the states from which controls can be found that maintain the system in an acceptable configuration forever. However, computing this kernel is not an easy task in general.

We now focus on some tools to achieve viability. A subset $\mathbb{V}$ is said to be *weakly invariant* for the dynamics f in the acceptable set $\mathbb{D}$, or a *viability domain* of f in $\mathbb{D}$, if

$$\forall x \in \mathbb{V}\,, \quad \exists u \in \mathbb{U}\,, \quad (x,u) \in \mathbb{D}\ \text{and}\ f(x,u) \in \mathbb{V}\,. \tag{20}$$

That is, if one starts from $\mathbb{V}$, an acceptable control may transfer the state in $\mathbb{V}$. Moreover, according to viability theory (Aubin, 1991), the viability kernel $\mathbb{V}(f,\mathbb{D})$ turns out to be the union of all viability domains, or also the largest viability domain:

$$\mathbb{V}(f,\mathbb{D}) = \bigcup \Big\{ \mathbb{V},\ \mathbb{V} \subset \mathbb{V}^0,\ \mathbb{V}\ \text{viability domain for}\ f\ \text{in}\ \mathbb{D} \Big\}\,. \tag{21}$$

A major interest of such a property lies in the fact that any viability domain for the dynamics f in the acceptable set $\mathbb{D}$ provides a *lower approximation* of the viability kernel. An *upper approximation* $\mathbb{V}_k$ of the viability kernel is given by the so called *viability kernel until time k associated with f in $\mathbb{D}$*:

$$\mathbb{V}_k := \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\,(u(0),u(1),\ldots,u(k))\ \text{and}\ (x(0),x(1),\ldots,x(k)) \\ \text{satisfying}\ (16)\ \text{for}\ t=0,\ldots,k-1 \\ \text{and}\ (17)\ \text{for}\ t=0,\ldots,k \end{array} \right\}\,. \tag{22}$$

We have

$$\mathbb{V}(f,\mathbb{D}) \subset \mathbb{V}_{k+1} \subset \mathbb{V}_k \subset \mathbb{V}_0 = \mathbb{V}^0 \quad \text{for all} \quad k \in \mathbb{N} \ . \tag{23}$$

It may be seen by induction that the decreasing sequence of viability kernels until time k satisfies

$$\mathbb{V}_0 = \mathbb{V}^0 \quad \text{and} \quad \mathbb{V}_{k+1} = \{x \in \mathbb{V}_k \mid \exists u \in \mathbb{U}\,,\ (x,u) \in \mathbb{D} \ \text{ and } \ f(x,u) \in \mathbb{V}_k\} \ . \tag{24}$$

By (23), such an algorithm provides approximation from above of the viability kernel as follows:

$$\mathbb{V}(f,\mathbb{D}) \subset \bigcap_{k \in \mathbb{N}} \mathbb{V}_k = \lim_{k \to +\infty} \downarrow \mathbb{V}_k \ . \tag{25}$$

Conditions ensuring that equality holds may be found in (Saint-Pierre, 1994). Notice that, when the decreasing sequence $(\mathbb{V}_k)_{k \in \mathbb{N}}$ of viability kernels up to time k is stationary, its limit is the viability kernel. Indeed, if $\mathbb{V}_k = \mathbb{V}_{k+1}$ for some k, then $\mathbb{V}_k$ is a viability domain by (24). Now, by (19), $\mathbb{V}(f,\mathbb{D})$ is the largest of viability domains. As a consequence, $\mathbb{V}_k = \mathbb{V}(f,\mathbb{D})$ since $\mathbb{V}(f,\mathbb{D}) \subset \mathbb{V}_k$ by (23). We shall use this property in the following Sect. B.

B Viable control of generic nonlinear ecosystem models with harvesting

For a generic ecosystem model (1), we provide an explicit description of the viability kernel. Then, we shall specify the results for predator–prey systems, in particular for discrete-time Lotka–Volterra models.

The acceptable set $\mathbb{D}$ is defined by *minimal biomass thresholds* $B_y^\flat \geq 0$, $B_z^\flat \geq 0$ and *minimal catch thresholds* $C_y^\flat \geq 0$, $C_z^\flat \geq 0$:

$$\mathbb{D} = \{\, (y,z,v,w) \in \mathbb{R}^4 \mid y \geq B_y^\flat,\ z \geq B_z^\flat,\ vy \geq C_y^\flat,\ wz \geq C_z^\flat \,\} \ . \tag{26}$$

B.1 Expression of the viability kernel

The following Proposition 5 gives an explicit description of the viability kernel, under some conditions on the minimal thresholds.

Proposition 5 *Assume that the function $R_y : \mathbb{R}^3 \to \mathbb{R}$ is continuously decreasing in the control v and satisfies $\lim_{v \to +\infty} R_y(y,z,v) \leq 0$, and that $R_z : \mathbb{R}^3 \to \mathbb{R}$ is continuously decreasing in the* *control variable w, and satisfies $\lim_{w\to+\infty} R_z(y,z,w) \leq 0$. If the thresholds in* (26) *are such that the following growth factors are greater than one*

$$R_y(B_y^\flat, B_z^\flat, \frac{C_y^\flat}{B_y^\flat}) \geq 1 \ \text{ and } \ R_z(B_y^\flat, B_z^\flat, \frac{C_z^\flat}{B_z^\flat}) \geq 1 \ , \tag{27}$$

the viability kernel associated with the dynamics f in (1) *and the acceptable set $\mathbb{D}$ in* (26) *is given by*

$$\mathbb{V}(f,\mathbb{D}) = \left\{ (y,z) \mid y \geq B_y^\flat, \ z \geq B_z^\flat, \ yR_y(y,z,\frac{C_y^\flat}{y}) \geq B_y^\flat, \ zR_z(y,z,\frac{C_z^\flat}{z}) \geq B_z^\flat \right\}. \tag{28}$$

Proof. According to induction (24), we have:

$$\begin{aligned}
\mathbb{V}_0 &= \{ (y,z) \Big| y \geq B_y^\flat, z \geq B_z^\flat \}, \\
\mathbb{V}_1 &= \left\{ (y,z) \middle| \begin{array}{c} y \geq B_y^\flat, z \geq B_z^\flat \text{ and, for some } (v,w) \geq 0, \\ vy \geq C_y^\flat, wz \geq C_z^\flat, yR_y(y,z,v) \geq B_y^\flat, zR_z(y,z,w) \geq B_z^\flat \end{array} \right\} \\
&= \left\{ (y,z) \middle| y \geq B_y^\flat, z \geq B_z^\flat, yR_y(y,z,\frac{C_y^\flat}{y}) \geq B_y^\flat, zR_z(y,z,\frac{C_z^\flat}{z}) \geq B_z^\flat \right\}
\end{aligned}$$

because $v \mapsto R_y(y,z,v)$ and $w \mapsto R_z(y,z,w)$ are decreasing,

and thus we may select $v = \dfrac{C_y^\flat}{y}$, $w = \dfrac{C_z^\flat}{z}$.

Denoting $y' = yR_y(y,z,v)$, $z' = zR_z(y,z,w)$, we obtain,

$$\mathbb{V}_2 = \left\{ (y,z) \middle| \begin{array}{l} y \geq B_y^\flat, z \geq B_z^\flat \text{ and, for some } (v,w) \geq 0, \\ vy \geq C_y^\flat, \ wz \geq C_z^\flat \\ y' \geq B_y^\flat, \ y'R_y(y',z',\frac{C_y^\flat}{y'}) \geq B_y^\flat, \ z' \geq B_z^\flat, \ z'R_z(y',z',\frac{C_z^\flat}{z'}) \geq B_z^\flat \end{array} \right\}.$$

We shall now make use of the property, recalled in Sect. A, that when the decreasing sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ of viability kernels up to time k is stationary, its limit is the viability kernel $\mathbb{V}(f,\mathbb{D})$. Hence, it suffices to show that $\mathbb{V}_1 \subset \mathbb{V}_2$ to obtain that $\mathbb{V}(f,\mathbb{D}) = \mathbb{V}_1$. Let $(y,z) \in \mathbb{V}_1$, so that

$$y \geq B_y^\flat, \quad z \geq B_z^\flat \ \text{ and } \ yR_y(y,z,\frac{C_y^\flat}{y}) \geq B_y^\flat, \quad zR_z(y,z,\frac{C_z^\flat}{z}) \geq B_z^\flat \ .$$

Since $R_y : \mathbb{R}^3 \to \mathbb{R}$ is continuously decreasing in the control variable, with $\lim_{v\to+\infty} R_y(y,z,v) \leq 0$, and since $yR_y(y,z,\frac{C_y^\flat}{y}) \geq B_y^\flat$, there exists a $\hat{v} \geq \frac{C_y^\flat}{y}$ (depending on y and z) such that $y' = yR_y(y,z,\hat{v}) = B_y^\flat$. The same holds for $R_z : \mathbb{R}^3 \to \mathbb{R}$ and $z' = zR_z(y,z,\hat{w}) = B_z^\flat$. By (27), we deduce that

$$y'R_y(y',z',\frac{C_y^\flat}{y'}) = B_y^\flat R_y(B_y^\flat, B_z^\flat, \frac{C_y^\flat}{B_y^\flat}) \geq B_y^\flat \ \text{ and } \ z'R_z(y',z',\frac{C_z^\flat}{z'}) = B_z^\flat R_z(B_y^\flat, B_z^\flat, \frac{C_z^\flat}{B_z^\flat}) \geq B_z^\flat \ .$$

The inclusion $\mathbb{V}_1 \subset \mathbb{V}_2$ follows. □

B.2 Proof of Proposition 4

Proof. By (11), and the property that both R_y and R_z are decreasing in the control variable, the quantities (10) exist.

Also since both R_y and R_z are decreasing in the control variable, we obtain that

$$R_y(B_y^\flat, B_z^\flat, \frac{C_y^{\flat,\star}(y_0, z_0)}{B_y^\flat}) \geq R_y(B_y^\flat, B_z^\flat, \frac{C_y^{\flat,\star}}{B_y^\flat}) = 1 \text{ and } R_z(B_y^\flat, B_z^\flat, \frac{C_z^{\flat,\star}(y_0, z_0)}{B_z^\flat}) \geq R_z(B_y^\flat, B_z^\flat, \frac{C_z^{\flat,\star}}{B_z^\flat}) = 1 \ .$$

To end up, the above inequalities and the assumption that $y_0 \geq B_y^\flat$ and $z_0 \geq B_z^\flat$ allow us to conclude, thanks to Proposition 5, that (y_0, z_0) belongs to the viability kernel $\mathbb{V}(f, \mathbb{D})$ given in (28).

In other words, starting from the initial point $(y(t_0), z(t_0)) = (y_0, z_0)$, there exists an appropriate harvesting path which can provide, for all time, at least the catches (10). $\square$

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