



A Simple Layered RGB BRDF Model

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Abstract

Many natural objects, and in general layered materials, have non-linear reflection behaviour along the wavelengths. An accurate representation of phenomena such as interference and colour separation generally requires a fine spectral representation of light instead of the commonly used RGB components. In this article, we introduce and experiment with a general approach to create similar and convincing effects, with a simple RGB BRDF model. We present a model for both specular and a diffuse reflection.

keywords : BRDF, reflection model, interference, spectral effects

1 Introduction

The bi-directional reflectance distribution function (BRDF) has proven its efficiency to describe complex light interactions with surfaces. A large range of material appearances can be represented by this local approach, as it can be noticed in the high quality of the nowadays computer generated images. BRDF models have played an essential role in creating realistic illumination of the virtual worlds.

In order to simulate effects such as old metallic objects or those with a thin transparent layer, with a wavelength-dependent refraction index, it is necessary to make the BRDFs dependent on the wavelength, thereby adding an additional dimension. Most previous work in this area [17, 6, 18, 10], is based on a fine spectral representation of the light. That work is focused on accuracy for a physical simulation of the light reflection.

Because RGB is the most commonly used light representation in computer graphic, as the final display process uses the same encoding, we want to explore the possibility of creating such effects with a limited number of wavelength samples, only three samples (645 nm -R, 525 nm -G, 445 nm -B as an example). With this common restriction, the resulting models can be merged more easily in the current image production process.

Thus, our main goal is to create a realistic and a convincing model, even if it is not an accurate physical model. We will mainly focus on the simplicity, and on the low number of parameters.

1.1 Overview

In this paper, we introduce a new BRDF model, based on the commonly used RGB representation of light. This model allows computing some prominent wavelength-dependent effects.

We begin by reviewing the existing work on the topic, which is mainly focused on creating accurate models for such effects, in general based on a fine spectral representation. Then, we will present our general approach to create interference and colour decomposition with a simple RGB basis. With this approach, we will describe two possible implementations, one for the specular case, one for the diffuse case. Then we will show, by some results, the possible range of appearance that such approach allows to compute.

2 Previous Work

A lot of work during the past thirty years in order to create BRDF models that have a convincing behaviour, from the well-known Phong model [15] and its adaptation by Lafortune et al. [12] to make it more physically plausible or to extend it in a more general way [13]. Many complex representations have also been developed to simulate the influence of the underlying geometry (micro-facet model [19, 1]), or the subsurface scattering [4], or to obtain an accurate representation by a real physical model [5]. This last one has an accurate wavelength behaviour, but can't represent phenomena like interference and colour separation.

To allow fast computation and an easy integration in global illumination problem (for an easy stochastic sampling), Ward [20] and Schlick [16] have introduced simpler models, between empirical and physical. Together with the

Phong model and its variants, these are today the most commonly used due to their efficiency and their simplicity.

But despite the complexity of, for example, the model by He et al. [5], none of these representations can simulate a number of non-linear wavelength effects. One of these effects is the diffraction due to a non-planar surface when the size of underlying micro-geometry reaches the wavelength size, like those we can see on a CD-ROM surface for example. Stam [17] has introduced a general reflection model for these effects, and Sun et al. [18] a model restricted to the CD-ROM simulation. All these methods require multiple wavelength samples to have convincing results and cannot deal with the phenomena due to a layered surface.

Icart et al. [11, 10] have introduced physically based models for multiple layered surfaces, for both layers with parallel boundaries [11] and those with uncorrelated one [10]. These models are based on solid physical assumptions, by are quite complex and require a fine spectral representation of the light. Hirayama et al [7, 9, 8] provide also an accurate model for materials with parallel layers and an approximation [6] for rough surfaces. Their model is based on recursive traversal of all the layers and require also this fine discretisation of the wavelength.

Since our goal is to provide a simpler model that can compute similar wavelength effects for layered materials, with the RGB base for the colour representation, all these previous approaches are too complex for our purpose and require too many wavelength samples. We will focus on the creation of a heuristic model that allow computing convincing appearance, and on the reduction of the complexity of the previous solutions.

3 General Presentation

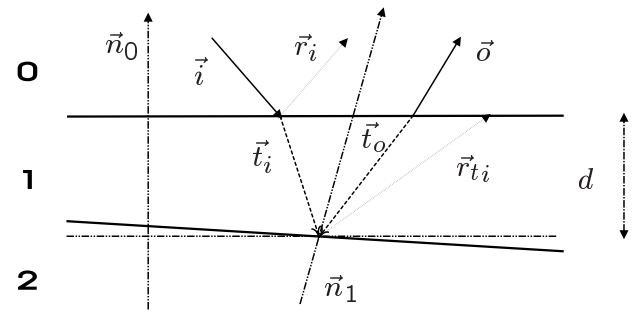
In this section, we present our approximations and the general form of our BRDF model. The detailed implementations will be described in the following sections.

3.1 Wavelength Effects

We can determine two wavelength effects due to the propagation in a layer. The first one is the phase change between the light directly reflected and the part that has traversed the layer. The second is a local prism configuration that can create some colour separation if the refraction index is varying with the wavelength. In our BRDF model, we want to integrate these two effects.

The phase difference influences the interference between the directly reflected energy and the energy due to the travel through the layer. The two energies can be added or subtracted. The size of the layer is the main influence for the phase change [3].

The colour separation is a well-known effect of the prism, for a material with a non-constant refraction index. This effect can also occur in a layered material if the two boundaries of the layer are not parallel. This configuration creates locally the same geometry than a prism.



0 represent the vacuum, 1 the layer and 2 the object surface

Figure 1. Local Layer geometry

3.2 Local Configuration

To integrate these two effects, we will use the layer configuration described in Figure 1. The material is composed of a thin layer with uncorrelated boundaries. We make also the classical assumption, that the layer size is small enough to neglect the distance between the directly reflected ray and the ray resulting from propagation through the layer. Then, we suppose that locally, the layer size d is constant and the normal of the two boundaries (between vacuum and layer - from 0 to 1 - and between the layer and the object - from 1 to 2) are uncorrelated. When the two normals are not aligned, this creates a local prism configuration.

Along the surface, we can make varying the size of the layer to create different interference effects. We can change also the relative position of the two normals. The normal variation can be correlated to the d variation, then we have only one parameter to make local change. These two parameters can be also independent for, as an example, creating colour decomposition with a constant-height layer. This independence will increase the range of possible reflection effects.

Note that, in such configuration, the multiple reflections in the layer are quickly absorbed [6]. We will use then only the first one.

3.3 Phase Computation

The phase change for a wave after a travel trough a layer with two perfectly parallel boundary surfaces is commonly expressed as [3]

$$\Phi = 2\pi \frac{r_1(\lambda)d}{\lambda} \langle \vec{i}, -\vec{n}_0 \rangle, \quad (1)$$

where $\langle \cdot, \cdot \rangle$ is the dot product, λ is the wavelength, and $r_1(\lambda)$ the refraction index of the layer. In this formula $2d \langle \vec{i}_i, -\vec{n}_0 \rangle$ represent the distance covered by the ray in the layer along the main axes (i.e. \vec{n}_0). This expression is only valid for the mirror direction of the incoming light (i.e. \vec{r}_i).

For our case, we need to consider every possible outgoing direction. We can then express more generally the distance covered by the ray by

$$d (\langle \vec{i}_i, -\vec{n}_0 \rangle + \langle \vec{i}_o, -\vec{n}_0 \rangle), \quad (2)$$

where \vec{i}_0 depends on the normal \vec{n}_1 of the lower boundary surface. Eq. 1 is now only valid for the special case where the outgoing direction \vec{o} correspond to the mirror one. The more general phase change can be expressed as

$$\Phi (\vec{i}, \vec{o}, \lambda) = \pi \frac{r_1(\lambda)d}{\lambda} \langle \vec{i} + \vec{i}_o, -\vec{n}_0 \rangle \quad (3)$$

3.4 Resulting Energy

Once we know the phase difference, we can compute the resulting energy due to interference between the directly reflected energy I_r and the transmitted energy I_t . It is commonly [3] expressed as

$$I_r + I_t + 2 \cos(2\Phi) \sqrt{I_r I_t} \quad (4)$$

We choose then as a general representation of our BRDF models:

$$\rho (\vec{i}, \vec{o}, \lambda) = \frac{R (\vec{i}, \vec{o}, \lambda) + T (\vec{i}, \vec{o}, \lambda) + 2 \cos (2\Phi (\vec{i}, \vec{o}, \lambda)) \sqrt{R (\vec{i}, \vec{o}, \lambda) T (\vec{i}, \vec{o}, \lambda)}}{2} \quad (5)$$

In this equation, R represents a BRDF for the direct reflection and T a BRDF for the transmission part.

We have to be careful when we choose the two functions R and T , as we want to be able to compute a convincing solution with only three components. If we choose Dirac-based functions, we will generate, in the best case, the three distinct RGB colours for the three different wavelengths. To allow some smooth transition between the colour, we use a lobe representation, similar to the Phong models [15], to simulate the wavelength dependency of the material.

4 Phong-like Model

As said in the previous section, using Phong lobe is the easiest way to create smooth transition between wavelength. We will use then two independent Phong models, one for the reflection, one for the transmission. This allows us to represent the effects of two uncorrelated and rough boundaries of the layer.

Note that in the following description, we will point out each time that a parameter is wavelength-dependent. By default, a parameter is wavelength-independent.

Note also that we will use the assumption of an increasing refraction index from outside (0) to the inside (2). With this condition, any ray can be refracted. It can be easily removed, by adding a test on the maximum refracted angle.

4.1 Reflected Part

The reflected part $R(\vec{i}, \vec{o}, \lambda)$ is a Phong reflection weighted by the amount of energy that is directly reflected. It can be then written as

$$R (\vec{i}, \vec{o}, \lambda) = \mathcal{F} (-\vec{i}, \lambda) \langle \vec{r}_i, \vec{o} \rangle^{e_0} \quad (6)$$

where $\mathcal{F} (-\vec{i})$ is the Fresnel term. For efficiency purposes, we chose the Schlick approximation [16]:

$$\mathcal{F} (\vec{i}, \lambda) = f(\lambda) + (1 - f(\lambda))(1 - \langle \vec{i}, \vec{n}_0 \rangle)^5 \quad (7)$$

where $f(\lambda)$ is the Fresnel coefficient (i.e. Fresnel term at normal direction):

$$f(\lambda) = \left(\frac{r_0 - r_1(\lambda)}{r_0 + r_1(\lambda)} \right)^2 \quad (8)$$

In this equation r_0 is the refraction index of air (or vacuum) and $r_1(\lambda)$ is the refraction index for the layer for the current wavelength. Considering that $r_0 \simeq 1$ this reflection is fully determined by only 4 parameters, the RGB refraction indexes, and the Phong exponent.

4.2 Transmitted Part

We choose the same representation for the transmitted part $\mathcal{T} (\vec{v}, \lambda)$. As the sum of the reflected and the transmitted energy is always 1, the transmission term can be simply obtained by $\mathcal{T} = 1 - \mathcal{F}$, or :

$$\mathcal{T} (\vec{v}, \lambda) = (1 - f(\lambda)) \left(1 - (1 + \langle \vec{v}, \vec{n} \rangle)^5 \right) \quad (9)$$

The transmitted part can be then written as

$$T (\vec{i}, \vec{o}, \lambda) = \left(1 - \mathcal{F} (\vec{i}, \lambda) \right) \mathcal{T} (\vec{i}_o, \lambda) \langle \vec{r}_{ii}, -\vec{i}_o \rangle^{e_1} \quad (10)$$

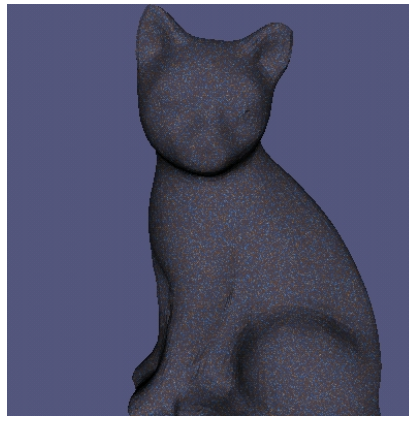
Note that this function has a null value for the three following conditions.

$$\begin{aligned} \langle \vec{i}_i, \vec{n}_1 \rangle &\geq 0 \\ \langle \vec{i}_o, \vec{n}_1 \rangle &\geq 0 \\ \langle \vec{r}_{ii}, \vec{i}_o \rangle &\geq 0 \end{aligned}$$

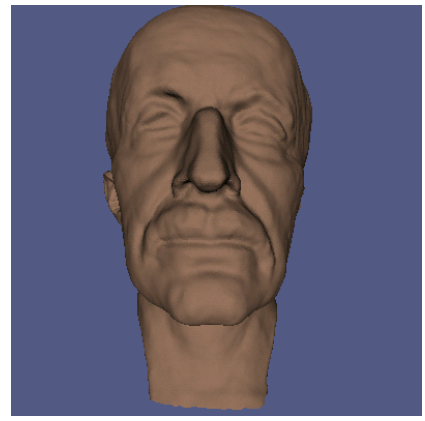
They correspond to the case where there is no possible reflection in the layer, due to the local configuration. When



1-30nm / refraction index 1.5



1-210nm / refraction index 1.3



67-73nm / refraction index 1.4

Figure 2. Diffuse part

the boundaries of the layer are parallels, these three conditions are always true.

Like for the reflected part, the transmitted part is fully determined by only 4 parameters, the RGB refraction indexes, and the Phong exponent.

4.3 Discussion of the Parameters

There is two parameters do determine the local configuration and five parameters to describe the BRDF behaviour.

The two parameters for the local configuration are the distance, and the normal variation between the two boundaries. The distance influences the colour of the reflection, a low one will create in general a red colour. If we increase it, blue and green colour will appear. The normal variation controls the creation of the colour-separation (rainbow-effect). The greater is normal variation, the better the separation.

The colour decomposition can appear only if, at least, two of the three refraction indexes are different. In general, the refraction index is decreasing with increasing wavelength (i.e. the blue index would be higher than the green one, which would in turn be higher than the red one).

Thus, the two Phong exponents will control the roughness of the two boundaries of the layer, and the interpolation between the different colours. Low exponents will result in a blurry colour separation, and too high value will no create the smooth transition that we are expecting.

5 Diffuse Reflection

Diffuse reflection increases in general the realistic feeling of a scene. It allows describing the more generally perceived colour of the objects. As we have for the specular

part, we want to investigate if we can create a diffuse model on which the layer has an influence. For this case, only the thickness of the layer has an effect on the resulting colour, since colour separation is a purely directional effect.

A diffuse approximation of the Phong-like model can be obtained by averaging it along the incident and reflected directions. But the resulting integration has no trivial solution, and this approach can't then provide a simple expression. However, we can note that resulting equation is similar to the original one Eq. 5:

$$\rho_d = \overline{R} + \overline{T} + \overline{2\cos(2\Phi)\sqrt{RT}}$$

where $\overline{\cdot}$ represents the averaging operator. The resulting diffuse coefficient is still the sum of the reflected part with the transmitted part, and with the interference part. We approximate then the diffuse reflection with the same model:

$$\rho_d(\lambda) = R_d(\lambda) + T_d(\lambda) + \frac{\overline{2\cos(2\Phi)\sqrt{R_d(\lambda)T_d(\lambda)}}}{2\cos(2\Phi(\lambda))\sqrt{R_d(\lambda)T_d(\lambda)}} \quad (11)$$

where R_d is the diffuse term for the reflected part, T_d is diffuse term for the refracted part, and Φ is an average phase change.

To define R_d , we simply compute the average value of the reflected part in the Phong-like model (Eq. 6), when the Phong exponent in zero $e_0 = 0$. Thus, we obtain :

$$R_d(\lambda) = \frac{1}{21} \frac{-38r_0r_1(\lambda) + 21(r_1(\lambda)^2 + r_0^2)}{(r_1(\lambda) + r_0)^2} \quad (12)$$

Considering that there is no absorption in the layer, we can define then $T_d = 1 - R_d$, i.e. :

$$T_d(\lambda) = \frac{80}{21} \frac{r_0r_1(\lambda)}{(r_1(\lambda) + r_0)^2} \quad (13)$$

To compute an equivalent phase change between the diffuse reflected and the transmitted part, we can use the previous expression of the phase change (Eq. 3) at the normal direction. In fact, the average refracted cosine (i.e. the average value of $\langle \vec{i}_i, -\vec{n}_0 \rangle$) is:

$$\frac{1}{4} \frac{\ln \left(\frac{r_1(\lambda)+r_0}{r_1(\lambda)-r_0} \right) (r_1(\lambda)^2 - r_0^2) + 2r_1(\lambda)r_0}{r_1(\lambda)r_0} \quad (14)$$

When $r_0 \simeq 1$, this value quickly converges to 1 when $r_1(\lambda)$ is increasing (for $r_1 = 1.5$, like glass material, this expression is equal to .85).

Then, we choose as the equivalent phase change the following value

$$\Phi(\lambda) = 2\pi \frac{r_1(\lambda)d}{\lambda} \quad (15)$$

and the diffuse coefficient becomes

$$\rho_d(\lambda) = 1 + 2 \cos\left(4\pi \frac{r_1(\lambda)d}{\lambda}\right) \sqrt{T_d(\lambda)(1 - T_d(\lambda))}. \quad (16)$$

By combining the diffuse reflection and the specular reflection, we can create a complete model of a layered surface.

6 Results

In this section, we will present some images from our implementations of the general RGB BRDF model. These were implemented in a ray-tracing renderer. These results will show the possible range of effects that such approach can provide. We will perform some tests for both the diffuse reflection and the Phong model.

6.1 Variation of the Layer Thickness

We use Perlin noise [14] to compute the variation of the layer size. Then, we can control the colour variation by adjusting the range of the layer height. The more visible effects appear for diffuse model, as shown in Figure 2 and Figure 5¹.

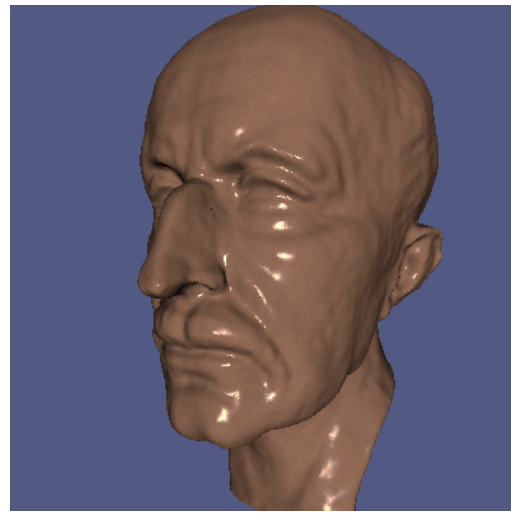
As explained in the section 4.3, a low layer size will result in general in a red colour, and a higher value in blue or green values, as you can see in Figure 5.

With only the height variation, we can create the appearance of old-metal (Figure 2-left), clay (Figure 2-right), or some more coloured effects (Figure 2-centre).

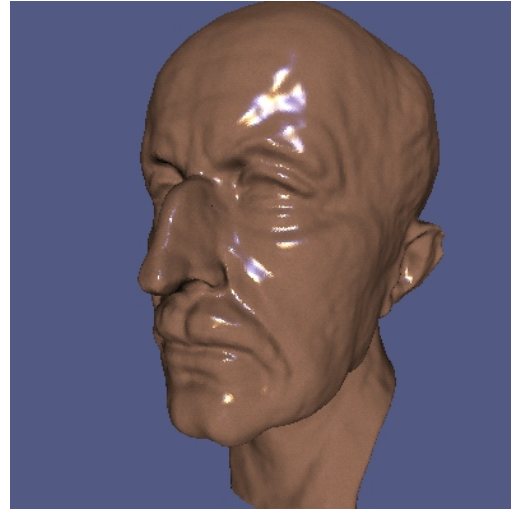
6.2 Normal Variation

By introducing a difference between the normal orientation of the two layer boundaries, we create both deviation of

¹The figures 2 to 5 are also available in the color plate



parallel boundaries



constant deviation 10°

Phong exponents: $e_0 = 90, e_1 = 170$
 RGB refraction indexes: 1.4, 1.45, 1.6
 Height range: 66-76 nm / Phong Vs Diffuse : 1-0.7

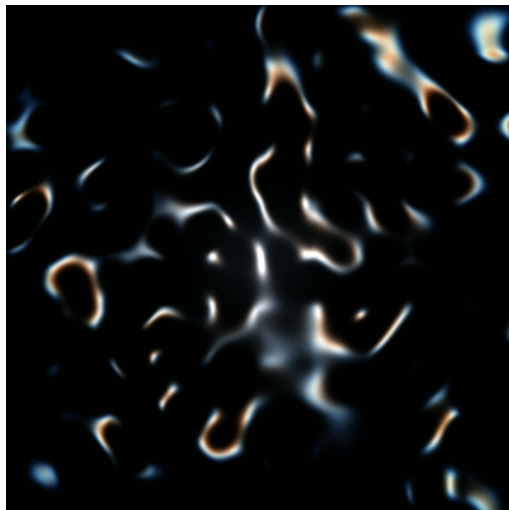
Figure 3. Normal variation

the reflected direction, (Figure 3), but also a colour separation when the RGB refraction indices are different (Figure 3 and Figure 5). In these examples, the difference between the two normals is constant over the surfaces.

6.3 Combining Both Effects

We can combine also the normal deviation and the height variation independently to create more effects. The height will control the base colour, and the normal deviation the

colour decomposition (Figure 3 and Figure 5).



Phong model only

Figure 4. Normal / layer size correlated

But the normal deviation can be also correlated to the variation of the layer thickness (Figure 4), as for bump-mapping [2]. This allows creating more effects, like those visible when there is a thin layer of oil on a surface.

7 Conclusion

In this paper, we have presented a simple and a general approach to integrate wavelength dependent effects for a layered surface, to a traditional RGB BRDF approximation of the local light reflection. This approach allows to compute the interferences between the light that is directly reflected and the part the light that have traversed the layer, but also to create colour separation when the refraction index is non-uniform in the layer, and when the two boundaries of the layer are not parallel.

We have presented two implementations of this approach, a Phong-like specular reflection, and a diffuse model. Even if this models is not completely physically based, these implementations show that realistic effects can be achieved by adjusting a small set of intuitive parameters. This allows for computing a large range of surface appearances that are based on layered materials.

Future work

Since this approach has not been design to be physically accurate, but only to create convincing effects, it has several drawbacks. However, it will be interesting to use this model to fit some measured BRDF data to it. Due to the low number of parameters, it will be easier to fit the measured data

to this representation. A simple non-linear optimisation approach should be sufficient.

Although this model show that wavelength dependent effects are possible by using only the RGB representation of colour, a more accurate representation can be investigated. By using the projection function from a spectral representation of the light to the RGB component, a more physical model can be developed. The main problem is here to find the correct approximation to be able to find simple solution to the resulting integration.

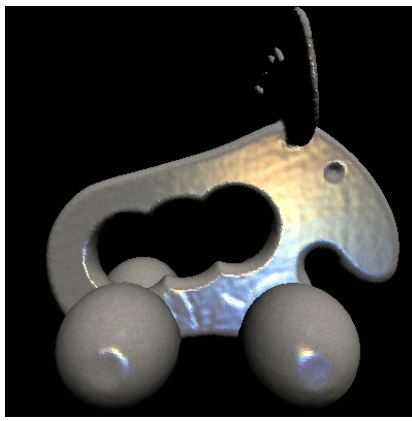
Also, as we provide a simple RGB model, some hardware accelerated methods can be investigated. Our general approach can be integrated more easily in the standard graphic pipe-line.

Acknowledgements

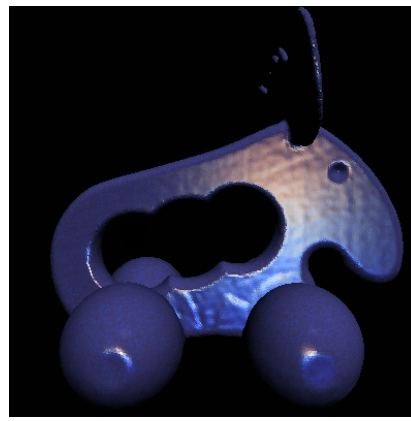
Special thanks to Lionel Bastard, for the productive discussions. He provides a lot of remarks and comments during the development of this model.

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1-30 nm



79-106 nm



146-200 nm

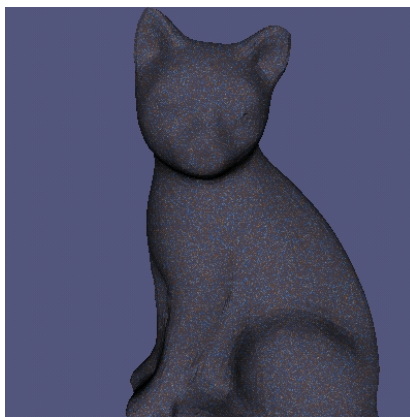
For these examples, the RGB refraction indexes are respectively 1.5, 1.6 and 1.8, the constant normal deviation is 14° , the Phong exponents are $e_0 = e_1 = 130$ and the weight of the Specular part and the diffuse part are respectively 1 and 0.5.

Figure 5. Combining specular and diffuse part

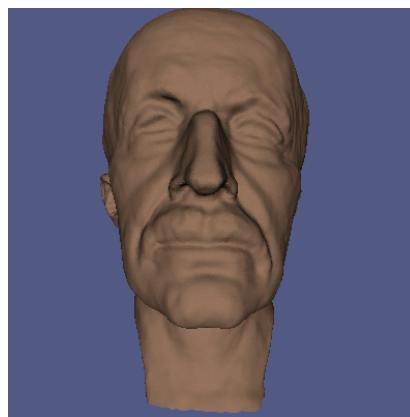
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1-30nm / refraction index 1.5

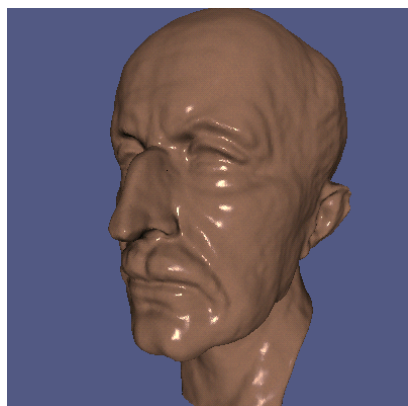


1-210nm / refraction index 1.3

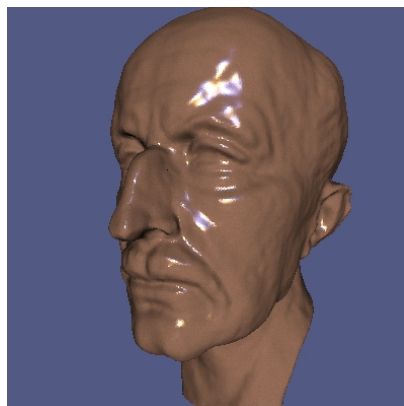


67-73nm / refraction index 1.4

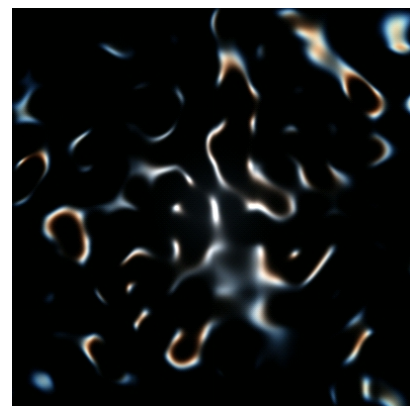
Figure 2. Diffuse part



parallel boundaries



constant deviation 10°

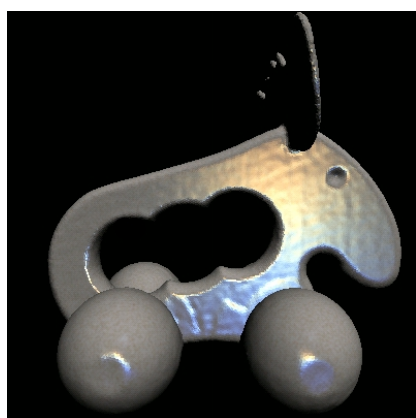


Phong model only

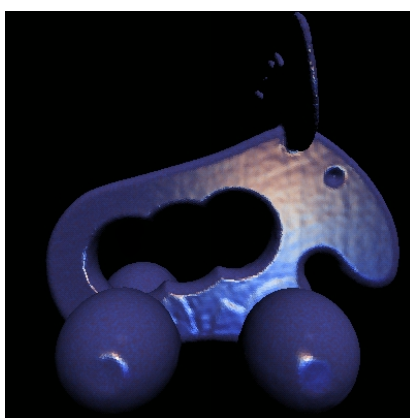
Phong exponents: $e_0 = 90$, $e_1 = 170$ / Height range: 66-76 nm
 RGB refraction indexes: 1.4, 1.45, 1.6 / Phong Vs Diffuse : 1-0.7

Figure 3.
Normal variation

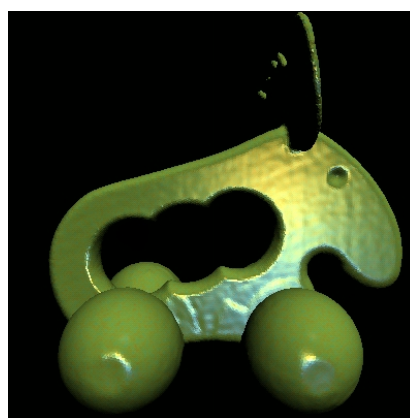
Figure 4.
Normal / layer size correlated



1-30 nm



79-106 nm



146-200 nm

For these examples, the RGB refraction indexes are respectively 1.5, 1.6 and 1.8, the constant normal deviation is 14° , the Phong exponents are $e_0 = e_1 = 130$ and the weight of the Specular part and the diffuse part are respectively 1 and 0.5.

Figure 5. Combining specular and diffuse part