

## Power and limits of distributed computing shared memory models

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**Abstract:** Due to the advent of multicore machines, shared memory distributed computing models taking into account asynchrony and process crashes are becoming more and more important. This paper visits some of the models for these systems, and analyses their properties from a computability point of view. Among them, the snapshot model and the iterated model are particularly investigated. The paper visits also several approaches that have been proposed to model crash failures. Among them, the *wait-free* case where any number of processes can crash is fundamental. The paper also considers models where up to  $t$  processes can crash, and where the crashes are not independent. The aim of this survey is to help the reader to better understand recent advances on what is known about the power and limits of distributed computing shared memory models and their underlying mathematics.

**Key-words:** Adversary, Agreement, Asynchronous system, Borowsky-Gafni's simulation, Concurrency, Core, Crash failure, Distributed computability, Distributed computing model, Fault-Tolerance, Iterated model, Liveness, Model equivalence, Obstruction-freedom, Progress condition, Recursion, Resilience, Shared memory system, Snapshot, Survivor set, Task, Topology, Wait-freedom.

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### *Une introduction au modèles de calcul asynchrone pour les mémoires partagées*

**Résumé :** *Ce rapport est une introduction au modèles de calcul asynchrone pour les systèmes à mémoire partagée.*

**Mots clés :** *Calculabilité du réparti, Mémoires partagée, Modèles de calcul asynchrone.*

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# 1 Introduction

**Sequential computing vs distributed computing** Modern computer science was born with the discovery of the Turing machine model, that captures the nature and the power of sequential computing, and with the proofs of equivalence of this model with all other known models of a computer (e.g., Post systems, Church's lambda calculus, etc.). This means that the *functions* that can be computed in one model are exactly the same that the ones that can be computed in another model: these sequential computing models are defined by the same set of computable functions.

An asynchronous distributed computing model consists of a set of processes (individual state machines) that communicate through some communication medium and satisfy some failure assumptions. *Asynchronous* means that the speed of processes is entirely arbitrary: each one proceeds at its own speed which can vary and is always independent from the speed of other processes. Other timing assumptions are also of interest. In a *synchronous* model, processes progress in a lock-step manner, while in a *partially synchronous* system the speed of processes is not as tightly related.

If the components (processes and communication media) cannot fail, and each process is a Turing machine, then the distributed system is equivalent to a sequential Turing machine, from the computability point of view. Namely, processes can communicate to each other everything they know, and they compute locally any (Turing-computable) function. In this sense the power of failure-free distributed computing is the same as the one of sequential computing. Unfortunately the situation is different when processes are prone to failures.

**Asynchronous distributed computing in presence of failures** We consider here the case of the most benign process failure model, namely, the crash failure model. This means that, in addition to proceeding asynchronously, a process may crash in an unpredictable way (premature stop). Moreover, crashes are stable: a crashed process does not recover.

The net effect of asynchrony and process crashes gives rise to a fundamental feature of distributed computing: a process may always have uncertainty about the state of other processes. Processes cannot compute the same global state of the system, to simulate a sequential computation. Actually, in distributed computing we are interested in focusing on *distributed* aspects of computation, and thus we eliminate any restrictions on local, sequential computation. That is, when studying distributed computability (and disregard complexity issues), we model each process by an infinite state machine. We get models whose power is orthogonal to the power of a Turing machine. Namely, each process can compute functions that are not Turing-computable, but the system as a whole cannot solve problems that are easily solvable by a Turing machine (in a centralized manner).

The problems for a distributed system, called *tasks*, are indeed distributed: each process has only part of the input to the problem. After communicating with each other, each process computes a part of the solution to the problem. A task specifies the possible inputs, and which part of the input gets each process. The input/output relation of the task, specifies the legal outputs for each input, and which part of the output can be produced by each process. From a computability point of view, a distributed system where even a single process may crash cannot solve tasks that can be computed by a Turing machine.

**The multiplicity of distributed computing models** In this paper we consider the simplest case, where processes can fail only by crashing. Even in the case of crash failures, several models have been considered in the past, by specifying how many processes can fail, if these failures are independent or not, and if the shared memory can also fail or not. The underlying communication model can also take many forms. The most basic is when processes communicate by reading and writing to a shared memory. However, stronger communication objects are needed to be able to compute certain tasks. Also, some systems are better modeled by message passing channels. Plenty of distributed computing models are encountered in the literature, with combination of these and other assumptions. A "holy grail" quest is the discovery of a basic distributed model that could be used to study essential computability properties, and then generalize or extrapolate results to other models, by systematic reductions and simulations. This would be great because, we would be able to completely depart from the situation of early distributed computing research, and instead of working on specific results suited to particular models only, a basic model allow to have more general positive (algorithms) or negative (lower bounds and impossibility) results.

There is evidence that the basic model where processes communicate by atomically reading and writing a shared memory, and any number of them can crash, is fundamental. This paper considers this base, *wait-free* model, motivated by the following reasons. First, the asynchronous read/write communication model is the least powerful non-trivial

shared memory model. Second, it is possible to simulate an atomic read/write register on top of a message-passing system as soon as less than half of the processes may crash [5, 47] (but if more than half of the processes may crash, a message passing system is less powerful). Third, it has been observed that techniques used to analyze the read/write model can be extended to analyze models with more powerful shared objects (e.g., [28]), and that results about task computability when bounds on the number of failures are known can be reduced to the wait-free case via simulations e.g. [10].

**Safety and liveness properties** As far as safety and liveness properties are concerned, the paper considers mainly linearizability and wait-freedom. *Linearizability* means that the shared memory operations appear as if they have been executed sequentially, each operation appearing as being executed between its start event and its end event [31] (linearizability generalizes the atomicity notion introduced for shared read/write registers in [38] to any object type). *Wait-freedom* means that any operation on a shared object invoked by a non-faulty process (a process that does not crash) does terminate whatever the behavior of the other processes, i.e., whatever their asynchrony and failure pattern [25] (wait-freedom can be seen as starvation-freedom despite any number of process crashes).

**Content of the paper** This survey is on the power and limits of shared memory distributed computing models to solve tasks, in environments where processes can fail by crashing. It takes the approach that the write-snapshot wait-free iterated model is at the center of distributed computing theory. Results and techniques about this model can be extrapolated to the usual read/write models (where registers can be accessed many times) and message passing modes. Also, they can be extrapolated to models where bounds on the number of failures are known, or where failures are correlated.

It first defines what is a task (the distributed counterpart of a function in sequential computing) in Section 2. Then, Section 3 presents and investigates the base asynchronous read/write distributed computing model and its associated snapshot abstraction that makes programs easier to write, analyze and prove. Next, Section 4 considers the iterated write-snapshot model that is more structured than the base write/snapshot model. Interestingly, this model has a nice mathematical structure that makes it very attractive to study properties of shared memory-based distributed computing.

Section 5 considers the case where the previous models are enriched with failure detectors. It shows that there is a tradeoff between the computational structure of the model and the power added by a failure detector. Section 6 considers the case of a very general failure model, namely the adversary failure model. Section 7 discusses the BG simulation (and its variants), that reduces questions about task solvability under other adversaries, to the simplest adversary, namely to the wait-free case. Finally, Section 8 concludes the paper.

## 2 What is a task?

As already indicated, a *task* is the distributed counterpart of the notion of a *function* encountered in sequential computing. In a task  $T$ , each of the  $n$  processes starts with an input value and each process that does not crash has to decide on an output value such the set of output values has to be permitted by the task specification. More formally we have the following where all vectors are  $n$ -dimensional ( $n$  being the number of processes) [30].

**Definition** A task  $T$  is a triple  $(\mathcal{I}, \mathcal{O}, \Delta)$  where  $\mathcal{I}$  is a set of input vectors,  $\mathcal{O}$  is a set of output vectors, and  $\Delta$  is a relation that associates with each  $I \in \mathcal{I}$  at least one  $O \in \mathcal{O}$ .

The vector  $I \in \mathcal{I}$  is the input vector where, for each entry  $i$ ,  $I[i]$  is the private input of process  $p_i$ . Similarly  $O$  describes the output vector where  $O[i]$  is the output that should be produced by process  $p_i$ .  $\Delta(I)$  defines which are the output vectors legal for the input vector  $I$ .

**Solving a task** Roughly speaking, an algorithm  $\mathcal{A}$  wait-free solves a task  $T$  if the following holds. In any run of  $\mathcal{A}$ , each process  $p_i$  starts with an input value  $in_i$  such that  $\exists I \in \mathcal{I}$  with  $I[i] = in_i$  (we say “ $p_i$  proposes  $in_i$ ”) and each non-faulty process  $p_j$  eventually computes an output value  $out_j$  (we say “ $p_j$  decides  $out_j$ ”) such that  $\exists O \in \Delta(I)$  with  $O[j] = out_j$  for all processes  $p_j$  that have computed an output value.

**Examples of tasks** The most famous task is *consensus* [17]. Each input vector  $I$  defines the values proposed by the processes. An output vector  $O$  is a vector whose entries contain the same value and  $\Delta$  is such that  $\Delta(I)$  contains all vectors whose single value is a value of  $I$ . The  $k$ -set agreement task relaxes consensus allowing up to  $k$  different values to be decided [15]. Other examples of tasks are renaming [6] and weak symmetry breaking (see [12] for an introductory survey), and  $k$ -simultaneous consensus [3].

## 3 Base shared memory models

### 3.1 The wait-free read/write model

**Base wait-free model** This computational model is defined by  $n$  sequential asynchronous processes  $p_1, \dots, p_n$  that communicate by reading and writing one-writer/multi-reader (1WMR) reliable atomic registers. Moreover up to  $n - 1$  processes may crash. Given a run of an algorithm, a process that crashes is *faulty* in that run, otherwise it is *non-faulty* (or *correct*).

This is the well-know *wait-free* shared memory distributed model. As processes are asynchronous and the only means they have to communicate is reading and writing atomic registers, it follows that the main feature of this model is the impossibility for a process  $p_i$  to know if another process  $p_j$  is slow or has crashed. This “indistinguishability” feature lies at the source of several impossibility results (e.g., the consensus impossibility [39]).

**The case of an unreliable shared memory** The previous model assumes that the atomic registers are *reliable*: a read or a write always returns and the atomicity behavior is always provided.

In a real distributed system potentially any of its components could fail. As we are interested in benign crash failures (the simplest kind of failures), we might also consider register crash failures, i.e., the case where registers stop working. Two types of such failures can be distinguished: responsive and non-responsive. In the responsive type, a register fails if it behaves correctly until some time, after which every read or write operation returns the default value  $\perp$ . hence, the register behaves correctly until it crashes (if it ever crashes) and then the failure can be detected. Responsive crash is sometimes called *fail-stop*. In the non-responsive type, after a register has crashed, its read and write operations never terminate, they remain pending forever. Non-responsive crash is sometimes called *fail-silent*.

It is possible to build reliable atomic registers on top of crash-prone base atomic registers. More precisely, let us assume that we want to cope with the crash of up to  $t$  unreliable base registers ( $t$ -register-resilience). There are  $t$ -register-resilient wait-free algorithms that build an atomic register [24]. If failure are responsive (resp., non-responsive)  $t + 1$  (resp.,  $2t + 1$ ) base registers are necessary and sufficient. As register crash failures can be overcome, we consider only reliable registers in the rest of the paper.

### 3.2 The snapshot abstraction

Designing correct distributed algorithms is very hard. Thus, it is interesting to construct out of read/write registers communication abstractions of higher level. A very useful abstraction, that can be efficiently constructed out of read/write registers, is a *snapshot* object [1] (more developments on snapshot objects can be found in [4, 7, 32]).

A snapshot abstracts an array of 1WMR atomic registers with one entry per process and provides them with two operations denoted  $X.write(v)$  and  $X.snapshot()$  where  $X$  is the corresponding snapshot object [1]. The former assigns  $v$  to  $X[i]$  (and is consequently also denoted  $X[i] \leftarrow v$ ). Only  $p_i$  can write  $X[i]$ . The latter operation,  $X.snapshot()$ , returns to the invoking process  $p_i$  the current value of the whole array  $X$ . The fundamental property of a snapshot object is that all write and snapshot operations appear as if they have been executed atomically, which means that a snapshot object is linearizable [31]. These operations can be wait-free built on top of atomic read/write registers (the best implementation known so far has  $O(n \log n)$  time complexity [2]). Hence, a snapshot object provides the programmer with a high level shared memory abstraction but does not provide her/him with additional computational power.

### 3.3 A progress condition weaker than wait-freedom

As already indicated, the progress condition associated with an algorithm solving a task in the base shared memory distributed computing model is wait-freedom (any correct process has to decide a value whatever the behavior of the other processes). This is the strongest progress condition one can think of but is not the only one. We present here a weaker progress condition.

**Obstruction-freedom** Wait-freedom is independent of the concurrency pattern. Differently, obstruction-freedom involves the concurrency pattern. It states that, if a correct process executes alone during a long enough period, it has to decide a value [26]. The words “long enough period” are due to asynchrony, they capture the fact that a process needs time to execute the algorithm.

As we can see, there are concurrency patterns in which no process is required to decide when we consider the obstruction-freedom progress condition. The important point to notice is that any algorithm that solves a task with the obstruction-freedom progress condition has to always ensure the task safety property: if processes decide, the decided values have to be correct.

**Obstruction-freedom vs wait-freedom** Obstruction-freedom is trivially weaker than wait-freedom. This has a consequence on task computability. As an example, while it is impossible to wait-free solve the consensus problem in the base read/write (or snapshot) shared memory model, it is possible to solve it in the same model when the wait-freedom requirement is replaced by obstruction-freedom.

More generally, when conflicts are rare, obstruction-freedom can be used instead of wait-freedom.

## 4 The iterated write-snapshot model

### 4.1 The iterated write-snapshot model

Attempts at unifying different read/write distributed computing models have restricted their attention to a subset of *round-based* executions e.g. [11, 29, 40]. The approach introduced in [9] proposes an *iterated* model in which processes execute an infinite sequence of rounds, and in each round communicate through a specific object called *one-shot write-snapshot* object. This section presents this shared memory distributed computing model [43].

**One-shot write-snapshot object** A *one-shot write-snapshot* object abstracts an array  $WS[1..n]$  that can be accessed by a single operation denoted `write_snapshot()` that each process invokes at most once. That operation pieces together the `write()` and `snapshot()` operations presented previously [8]. Intuitively, when a process  $p_i$  invokes `write_snapshot(v)` it is as if it instantaneously executes a  $WS[i] \leftarrow v$  operation followed by an `WS.snapshot()` operation. If several `WS.write_snapshot()` operations are executed simultaneously, then their corresponding writes are executed concurrently, and then their corresponding snapshots are also executed concurrently (each of the concurrent operations sees the values written by the other concurrent operations): they are set-linearizable [42].  $WS[1..n]$  is initialized to  $[\perp, \dots, \perp]$ .

When invoked by a process  $p_i$ , the semantics of the `write_snapshot()` operation is characterized by the following properties, where  $v_i$  is the value written by  $p_i$  and  $sm_i$ , the value (or *view*) it gets back from the operation. A view  $sm_i$  is a set of pairs  $(k, v_k)$ , where  $v_k$  corresponds to the value in  $p_k$ 's entry of the array. If  $WS[k] = \perp$ , the pair  $(k, \perp)$  is not placed in  $sm_i$ . Moreover, we assume that  $sm_i = \emptyset$ , if the process  $p_i$  never invokes `WS.write_snapshot()`. These properties are:

- Self-inclusion.  $\forall i : (i, v_i) \in sm_i$ .
- Containment.  $\forall i, j : sm_i \subseteq sm_j \vee sm_j \subseteq sm_i$ .
- Immediacy.  $\forall i, j : [(i, v_i) \in sm_j \wedge (j, v_j) \in sm_i] \Rightarrow (sm_i = sm_j)$ .
- Termination. Any invocation of `WS.write_snapshot()` by a correct process terminates.

The self-inclusion property states that a process sees its writes, while the containment properties states that the views obtained by processes are totally ordered. Finally, the immediacy property states that if two processes “see each other”, they obtain the same view (the size of which corresponds to their *concurrency* degree of the corresponding `write_snapshot()` invocations).

**The iterated model** In the *iterated write-snapshot model* (IWS) the shared memory is made up of an infinite number of one-shot write-snapshot objects  $WS[1], WS[2], \dots$ . These objects are accessed sequentially and asynchronously by each process, according to the round-based pattern described in Figure 1 where  $r_i$  is  $p_i$ 's current round number.

```

 $r_i \leftarrow 0;$ 
loop forever  $r_i \leftarrow r_i + 1;$ 
    local computations; compute  $v_i$ ;
     $sm_i \leftarrow WS[r_i].write\_snapshot(v_i);$ 
    local computations
end loop.

```

Figure 1: Generic algorithm for the iterated write-snapshot model

**A fundamental result** Let us observe that the IWS model requires each correct process to execute an infinite number of rounds. However, it is possible that a correct process  $p_1$  is unable to receive information from another correct process  $p_2$ . Consider a run where both execute an infinite number of rounds, but  $p_1$  is scheduled before  $p_2$  in every round. Thus,  $p_1$  never reads a value written to a write-snapshot object by  $p_2$ . Of course, in the usual (non-iterated read/write shared memory) asynchronous model, two correct processes can always eventually communicate with each other. Thus, at first glance, one could intuitively think that the base read/write model and the IWS model have different computability power. The fundamental result associated with the IWS model is captured by the following theorem that shows that the previous intuition is incorrect.

**Definition 1** A task is bounded if its set of input vectors  $\mathcal{I}$  is finite.

**Theorem 1** [9] A bounded task can be wait-free solved in the 1WMR shared memory model if and only if it can be wait-free solved in the IWS model.

**Why the IWS model?** The interest of the IWS model comes from its elegant and simple round-by-round iterative structure. It restricts the set of interleavings of the shared memory model without restricting the power of the model. Its runs have an elegant recursive structure: the structure of the global state after  $r + 1$  rounds is easily obtained from the structure of the global state after  $r$  rounds. This implies a strong correlation with topology (see the next section) which allows for an easier analysis of wait-free asynchronous computations to prove impossibility results, e.g. [27, 28]. The recursive structure of runs also facilitates the design and analysis of algorithms (see Section 7 for an example), e.g. [22].

## 4.2 A mathematical view

The properties that characterize the `write_snapshot()` operation are represented in the first picture of Figure 2, for the case of three processes. In the topology parlance, this picture represents a *simplicial complex*, i.e., a family of sets closed under containment. Each set, which is called a *simplex*, represents the views of the processes after accessing the *IS* object. The *vertices* are 0-simplexes of size one; edges are 1-simplexes, of size two; triangles are of size three (and so on). Each vertex is associated with a process  $p_i$ , and is labeled with  $sm_i$  (the *view*  $p_i$  obtains from the object).

The highlighted 2-simplex in the figure represents a run where  $p_1$  and  $p_3$  access the object concurrently, both get the same views seeing each other, but not seeing  $p_2$ , which accesses the object later, and gets back a view with the 3 values written to the object. But  $p_2$  can't tell the order in which  $p_1$  and  $p_3$  access the object; the other two runs are indistinguishable to  $p_2$ , where  $p_1$  accesses the object before  $p_3$  and hence gets back only its own value or the opposite.

These two runs are represented by the 2-simplexes at the bottom corners of the first picture. Thus, the vertices at the corners of the complex represents the runs where only one process  $p_i$  accesses the object, and the vertices in the edges connecting the corners represent runs where only two processes access the object. The triangle in the center of the complex, represents the run where all three processes access the object concurrently, and get back the same view.

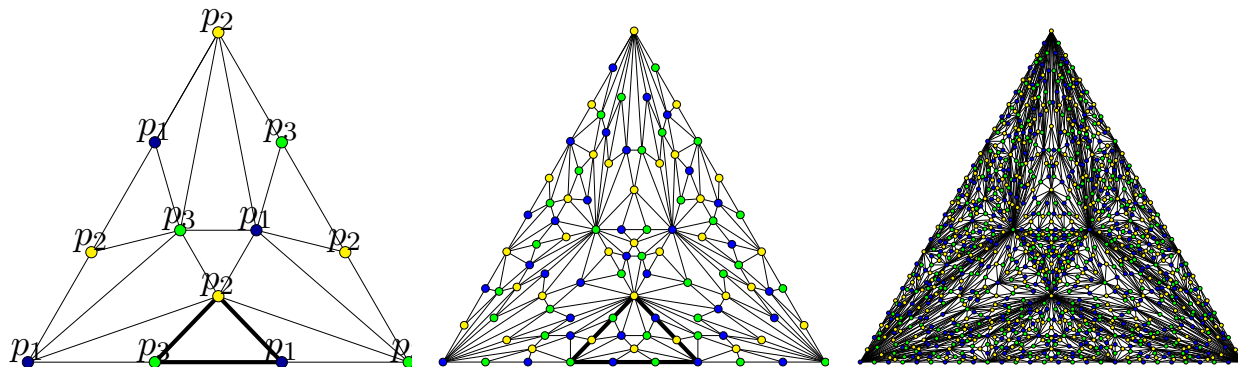


Figure 2: One, two and three rounds in the iterated write-snapshot (IWS) model

Hence, the state of an execution after the first round (with which is associated the write-snapshot object  $WS[1]$ ) is represented by one of the internal triangles (e.g., the one discussed previously that is represented by the bold triangle in the pictures). Then, the state of that execution after the second round (with which is associated the write-snapshot object  $WS[2]$ ) is represented by one of the small triangles inside the bold triangle. Etc. More generally, as shown in Figure 2, one can see that, in the write-snapshot iterated model, at every round, a new complex is constructed recursively by replacing each simplex by a one-round complex.

### 4.3 A recursive write-snapshot algorithm

Figure 3 presents a read/write algorithm that implements the `write_snapshot()` operation. Interestingly, this algorithm is recursive [12, 22]. A proof can be found in [12]. To allow for a recursive formulation, an additional recursion parameter is used. More precisely, in a round  $r$ , a process invokes  $MS.write\_snapshot(n, v)$  where the initial value of the recursion parameter is  $n$  and  $SM$  stands for  $WS[r]$ .

$SM$  is a shared array of size  $n$  (initialized to  $[\perp, \dots, \perp]$  and such that each  $SM[x]$  is an array of  $n$  1WnR atomic registers. The atomic register  $SM[x][i]$  can be read by all processes but written only by  $p_i$ .

Let us consider the invocation  $SM.write\_snapshot(x, v)$  issued by  $p_i$ . Process  $p_i$  first writes  $SM[x][i]$  and reads (not atomically) the array  $SM[x][1..n]$  that is associated with the recursion parameter  $x$  (lines 01-02). Then,  $p_i$  computes the set of processes that have already attained the recursion level  $x$  (line 03; let us note that recursion levels are decreasing from  $n$  to  $n - 1$ , etc.). If the set of processes that have attained the recursion level  $x$  (from  $p_i$ 's point of view) contains exactly  $x$  processes,  $p_i$  returns this set as a result (lines 04-05). Otherwise less than  $x$  processes have attained the recursion level  $x$ . In that case,  $p_i$  recursively invokes  $SM.write\_snapshot(x - 1)$  (line 06) in order to attain and stop at the recursion level  $y$  attained by  $y$  processes.

The cost of a shared memory distributed algorithm is usually measured by the number of shared memory accesses, called *step complexity*. The step complexity of  $p_i$ 's invocation is  $O(n(n - |sm_i| + 1))$ .

### 4.4 Iterative model vs recursive algorithm

It is interesting to observe that the iterative structure that defines the IWS model and the recursion-based formulation of the previous algorithm are closely related notions. In one case iterations are at the core of the model while in the other case recursion is only an algorithmic tool. However, the runs of a recursion-based algorithm are of an iterated nature: in each iteration only one array of registers is accessed, and the array is accessed only in this iteration.

```

operation  $SM.write\_snapshot(x, v)$ :
    %  $x$  ( $n \geq x \geq 1$ ) is the recursion parameter %
(01)  $SM[x][i] \leftarrow v$ ;
(02) for  $1 \leq j \leq n$  do  $aux_i[j] \leftarrow SM[x][j]$  end for ;
(03)  $pairs_i \leftarrow \{(j, v') \mid \exists j \text{ such that } aux_i[j] = v' \neq \perp\}$ ;
(04) if ( $|pairs_i| = x$ )
(05)     then  $sm_i \leftarrow pairs_i$ 
(06)     else  $sm_i \leftarrow SM.write\_snapshot(x - 1, v)$ 
(07) end if;
(08) return( $sm_i$ ).

```

Figure 3: Recursive write-snapshot algorithm (code for  $p_i$ )

## 5 Enriching a system with a failure detector

**The concept of a failure detector** This concept has been introduced by Chandra and Toueg [14] (see [46] for an introductory survey). Informally, a failure detector is a device that provides each process  $p_i$  with information about process failures, through a local variable  $fd_i$  that  $p_i$  can only read. Several classes of failure detectors can be defined according to the kind and the quality of the information on failures that has to be delivered to the processes.

Of course, a non-trivial failure detector requires that the system satisfies additional behavioral assumptions in order to be implemented. The interested reader will find such additional behavioral assumptions and corresponding algorithms implementing failure detectors of several classes in chapter 7 of [47].

**An example** One of the most known failure detectors is the eventual leader failure detector denoted  $\Omega$  [13]. This failure detector is fundamental because it encapsulates the weakest information on failures that allows consensus to be solved in a base read/write asynchronous system.

The output provided by  $\Omega$  to each (non crashed) process  $p_i$  is such that  $fd_i$  always contains a process identity (validity). Moreover, there is a finite time  $\tau$  after which all local failure detector outputs  $fd_i$  contains forever the same process identity and it is the identity of a correct process (eventual leadership). The time  $\tau$  is never explicitly known by the processes. Before  $\tau$ , there is an anarchy period during which the local failure detector outputs can be arbitrary.

**A result** As indicated, the consensus problem cannot be solved in the base read/write system [39] but can be solved as soon as this system is enriched with  $\Omega$ .

On another side (see Theorem 1), the base shared memory model and the IWS model have the same wait-free computability power bounded tasks. Hence a natural question: Is this computability power equivalence preserved when both models are enriched with the same failure detector? Somehow surprisingly, the answer to this question is negative. More precisely, we have the following.

**Theorem 2** [44] *For any failure detector  $FD$  and bounded task  $T$ , if  $T$  is wait-free solvable in the model IWS enriched with  $FD$ , then  $T$  is wait-free solvable in the base shared memory model without failure detector.*

Intuitively, this negative result is due to the fact that the IWS model is too much structured to benefit from the help of a failure detector.

**How to circumvent the previous negative result** A way to circumvent this negative result consists in “embedding” (in some way) the failure detector inside the `write_snapshot` operation. More precisely, the infinite sequence of invocations  $WS[1].write\_snapshot$ ,  $WS[2].write\_snapshot$ , etc., issued by any process  $p_i$  has to satisfy an additional property that depends on the corresponding failure detector. This approach has given rise to the IRIS model described in [45].

## 6 From the wait-free model to the adversary model

*Adversaries* are a very useful abstraction to represent subsets of executions of a distributed system. The idea is that, if one restricts the set of possible executions, the system should be able to compute more tasks. For example, the *condition based approach* [41] restricts the set of inputs of a task (and hence the corresponding executions), and allows to solve more tasks, or to solve tasks faster. Various adversaries have been considered in the past to model failure restrictions, as we shall now describe.

### 6.1 The notion of an adversary

In the wait-free model, any number of process can crash. In practice one sometimes estimates a bound  $t$  on how many processes can be expected to crash. However, often the crashes are not independent, due to processes running on the same core, or on the same subnetwork, for example.

**Wait-freedom** It is easy to see that wait-freedom is the least restrictive adversary, i.e., the adversary that contains all the (non-empty) subsets of processes, namely, the sets of processes that may be alive in some execution. Hence, a wait-free algorithm has to work whatever the number of process crashes.

**$t$ -Faulty process resilience** The  *$t$ -faulty process resilient* failure model (also called  *$t$ -threshold* model) considers that, in any run, at most  $t$  processes may crash. Hence, the corresponding adversary is the set of all the sets of  $(n - t)$  processes plus all their supersets: for each such set, there are executions where the processes that do not crash consist of this set.

**Cores and survivor sets** The notion of  $t$ -process resilience is not suited to capture the case where processes fail in a dependent way. This has motivated the introduction of the notions of *core* and *survivor set* [37].

A core  $C$  is a minimal set of processes such that, in any run, some process in  $C$  does not fail. A survivor set  $S$  is a minimal set of processes such that there is a run in which the set of non-faulty processes is exactly  $S$ . Let us observe that cores and survivor sets are dual notions (any of them can be obtained from the other one).

Computability results of set agreement have been generalized from  $t$ -resilience to cores in [27]. A connection relating adversaries that are superset-closed (i.e.,  $(s \in A) \Rightarrow (\forall s' : s \subseteq s' : s' \in A)$ ) and wait-freedom is presented in [21].

It is easy to see that the notion of survivor sets is more general than the notion  $t$ -threshold resilience. It is also possible to see that more general notions failures are possible:

**Adversaries** The most general notion of adversary with respect to failure dependence has been introduced in [16]. An *adversary*  $A$  is a set of sets of processes. It states that an algorithm solving a task must terminate in all the runs whose the corresponding set of correct processes is (exactly) a set of  $A$ .

As an example, Let us considers a system with four processes denoted  $p_1, \dots, p_4$ . The set  $A = \{\{p_1, p_2\}, \{p_1, p_4\}, \{p_1, p_3, p_4\}\}$  defines an adversary. An algorithm  $A$ -resiliently solves a task if it terminates in all the runs where the set of correct processes is exactly either  $\{p_1, p_2\}$  or  $\{p_1, p_4\}$  or  $\{p_1, p_3, p_4\}$ . This means that an  $A$ -resilient algorithm is not required to terminate in an execution in which the set of correct processes is exactly the set  $\{p_3, p_4\}$  or the set  $\{p_1, p_2, p_3\}$ .

Adversaries are more general than the notion of survivor sets (this is because when we build the adversary corresponding to a set of survivor sets, due the “minimality” feature of each survivor set, we have to include all its supersets in the corresponding adversary).

**On progress conditions** It is easy to see that an adversary can be viewed as a liveness property that specifies the crash patterns in which the correct processes must progress. The interested reader will find more developments on progress conditions in [20, 21, 36, 48].

## 6.2 Simulating the snapshot model in the iterated model

The iterated write-snapshot model (IWS) has been presented in Section 4 where we have seen that the base read/write (or snapshot) shared memory model and the IWS model are equivalent from a wait-free (bounded) task solvability point of view [9].

This section shows that this equivalence remains true when wait-freedom is replaced by an adversary-based progress condition. To that end, this section presents the simulation of the snapshot memory model in a simple variant of the IWS model where wait-freedom is replaced by the adversary defined by survivor sets. This simulation is from [23].

**The snapshot-based algorithm** Let  $\mathcal{A}$  be the snapshot-based algorithm we want to simulate. We assume without loss of generality that it is a *full information* algorithm, i.e., each time a process writes, it writes its full local state. Initially, each process  $p_i$  has an input value denoted  $input_i$ . Then, each process alternates between writing its state in the shared memory, taking a snapshot of the shared memory and taking this snapshot as its new local state.

The algorithm  $\mathcal{A}$  solves a task with respect to an adversary  $A$  if each correct process decides a value based on its current local state in every run that is *fair* with respect to  $A$  (which means that each time a process takes a snapshot it reads new values written by processes of a survivor set of the adversary  $A$ ). This means that as in [23], given a snapshot object  $WS$ , a process writes it only once but can repeatedly invokes  $WS.snapshot()$  until it sees new values from a survivor set of  $A$ .

Given a local state of a process, we assume that the algorithm  $\mathcal{A}$  has a predicate  $undecided(state)$  and a decision function  $decide(state)$ . Once a process has decided, its predicate  $undecided(state)$  remains forever false. The  $k$ th snapshot issued by  $p_i$  is denoted  $snapshot(k, i)$  and its  $k$ th write is denoted  $write(k, i)$ .

As it does not lead to confusion, we use “ $p_i$ ” to denote both the simulated process and the process that simulates it.

**Simulation: the operation**  $WS[r].write\_ \& \_ snapshot(v)$  This is the simulation operation that allows processes to coordinate and communicate. When a process  $p_i$  invokes it, it does the following. The value  $v$  is written in  $WS[r][i]$  and  $p_i$  waits until the set of processes that have written into  $WS[r]$  contains a survivor set of  $A$ . When this happens, a snapshot of  $WS[r]$  is taken and returned. To simplify the presentation we consider that the snapshot value that is returned is an array  $sm[1..n]$ .

```

(01) init:  $r_i \leftarrow 0$ ;  $state_i.clock \leftarrow [0, \dots, 0]$ ;  $state_i[i] \leftarrow (1, input_i)$ .

(02) loop forever
(03)    $r_i \leftarrow r_i + 1$ ;
(04)    $sm_i[1..n] \leftarrow WS[r_i].write\_ \& \_ snapshot(state_i)$ ;
(05)   foreach  $j$  do
(06)      $state_i[j] \leftarrow sm[x][j]$  such that  $\forall y : sm[x][j].clock \geq sm[y][j].clock$ ;
(07)   end for;
(08)   if  $(\sum_{1 \leq j \leq n} state_i[j].clock) = r_i$  then
      % simulation of  $snapshot(k, i)$  (with  $k = state_i[i].clock$ ) which returns  $state_i.val$ 
(09)     if  $undecided(state_i.val)$ 
(10)       then  $state_i[i] \leftarrow (state_i[i].val, state_i[i].clock + 1)$ 
          % simul. of  $write(k, i)$  (with  $k = state_i[i].clock$ ) which writes  $state_i[i].val$ 
(11)       else  $decide(state_i.val)$  if not yet done
(12)     end if
(13)   end if
(14) end loop.

```

Figure 4: The iterated simulation for adversary  $A$  (code for  $p_i$ )

**Simulation: local variables** A process  $p_i$  manages the following local variables.

- $r_i$  is the local current round number of the iterated model (initially 0).

- $state_i[1..n]$  is an array of pairs such that  $state_i[x].clock$  is a clock value (integer) that measures the progress of  $p_x$  as known by  $p_i$  and  $state_i[x].val$  is the corresponding local state of  $p_x$ ;  $state_i[i].val$  is initialized to  $input_i$  (the input value of  $p_i$ ), the initial values of the other  $state_i[j].val$  are irrelevant. The notation  $state_i.clock$  is used to denote the array  $[state_i[1].clock, \dots, state_i[n].clock]$  and similarly for  $state_i.val$ .

**Simulation: behavior of a process** During a round  $r$ , the behavior of a process  $p_i$  is made up of two parts.

- First  $p_i$  writes its current local state  $state_i$  in  $WS[r]$  and saves in  $sm_i$  the view of the global state it obtains for that round (line 04). Then (lines 05-07)  $p_i$  computes its new local state as follows: it saves in  $state_i[j]$  the most recent local state of  $p_j$  it knows (“most recent” refers to the clock values that measure  $p_j$  progress).
- Then the simulation  $p_i$  strives for making the simulation of the simulated process to progress.

The quantity  $\sum_{1 \leq j \leq n} state_i[j].clock$  represents the current date of the global simulation from  $p_i$ 's point of view. If this date is different from  $p_i$ 's current round number  $r$ , then  $p_i$  is late (as far as its round number is concerned) and it consequently proceeds to the next simulation round in order to catch up. (The proof shows that the predicate  $\sum_{1 \leq j \leq n} state_i[j].clock \leq r$  remains invariant.)

If  $\sum_{1 \leq j \leq n} state_i[j].clock = r$ , the round number is OK for  $p_i$  to simulate the invocation  $snapshot(k, i)$  of the simulated process where  $k = state_i[i].clock$ . The corresponding value that is returned is  $state_i.val$ .

Then, if its state is undecided (line 09),  $p_i$  makes the simulation progress by simulating  $write(k, i)$  where  $k = state_i[i].clock + 1$  and the value written is  $state_i[i].val$  (line 10).

If the state of  $p_i$  can allow for a decision,  $p_i$  decides if not yet done (line 11). Let us notice that, as soon as a process  $p_i$  has decided, it continues looping (this is necessary to prevent permanent blocking if  $p_i$  belongs to survivor sets) but its local clock is no longer increased in order to allow the correct processes to decide.

A proof of this simulation can be found in [23]. It is based on the observation that if for two processes the round number is OK, then their  $state$  variables agree.

## 7 Simulating adversary models in the wait-free model

The simulation of Section 6.2 showed that the same tasks can be solved in the iterated write-snapshot (IWS) model and in the base read/write (or snapshot) shared memory model, in the presence of failures. Namely, considering adversaries defined by survivor sets, a task can be solved in the IWS model under some adversary if and only if it can be solved in the base model, under the same adversary. As we have seen, the simplest adversary is the wait-free adversary. So it would be nice to reduce questions about task solvability under other adversaries, to the wait-free case. This is exactly what the BG simulation [10] and its variants (e.g. [19, 33, 34]) allow.

**BG simulation and its variants** Let us consider an algorithm  $\mathcal{A}$  that is assumed to solve a task  $T$  in an asynchronous read/write shared memory system made up of  $n$  processes, and where any subset of at most  $t$  processes may crash. Given algorithm  $\mathcal{A}$  as input, the BG simulation is an algorithm that solves  $T$  in an asynchronous read/write system made up of  $t + 1$  processes, where up to  $t$  processes may crash. Hence, the BG simulation is a wait-free algorithm.

The BG simulation has been used to prove solvability and unsolvability results in crash-prone read/write shared memory systems. It works only for a particular class of tasks called *colorless* tasks. These are the tasks where, if a process decides a value, any other process is allowed to decide the very same value, and if a process has an input value  $v$ , then any other processes can exchange its own input by  $v$ . Thus, for colorless tasks, the BG simulation characterizes  $t$ -resilience in terms of wait-freedom, and it is not hard to see that the same holds for any other adversary (defined by survivor sets).

As an example, let us assume that  $\mathcal{A}$  solves consensus, despite up to  $t = 1$  crash, among  $n$  processes in a read/write shared memory system. Taking  $\mathcal{A}$  as input, the BG simulation builds a  $(t + 1)$ -process (i.e., 2-process) algorithm  $\mathcal{A}'$  that solves consensus despite  $t = 1$  crash, i.e., wait-free. But, we know that consensus cannot be wait-free solved in a crash-prone asynchronous system where processes communicate by accessing shared read/write registers only

[17, 25, 39] in particular if it is made up of only two processes. It then follows that, whatever the number  $n$  of processes the system is made up of, there is no 1-resilient consensus algorithm.

The BG simulation algorithm has been extended to work with general tasks (called *colored* tasks) [19, 33] and for algorithms that have access to more powerful communication objects (e.g., [34] that extends the BG simulation to the 0/1-exclusion objects defined in [18]).

**The idea of the BG simulation** Let  $\mathcal{A}$  be an algorithm that solves a colorless decision task in the  $t$ -resilient model for  $n$  processes. The basic aim is to design a wait-free algorithm  $\mathcal{A}'$  that simulates  $\mathcal{A}$  in a model with  $t + 1$  processes. A simulated process is denoted  $p_j$ , while a simulator process is denoted  $q_j$ , with  $1 \leq j \leq n$ .

Each simulator  $q_j$  is given the code of every simulated process  $p_1, \dots, p_n$ . It manages  $n$  threads, each one associated with a simulated process, and locally executes these threads in a fair way (e.g., using a round-robin mechanism). It also manages a local copy  $mem_i$  of the snapshot memory  $mem$  shared by the simulated processes. The code of a simulated process  $p_j$  contains invocations of  $mem[j].write()$  and of  $mem.snapshot()$ . These are the only operations used by the processes  $p_1, \dots, p_n$  to cooperate. So, the core of the simulation is the design of algorithms that describe how a simulator  $q_i$  simulates these operations. These simulation algorithms are denoted  $sim\_write_{i,j}()$ , and  $sim\_snapshot_{i,j}()$ .

The *safe agreement* object type is at the core of the BG simulation. It provides each simulator  $q_i$  with two operations, denoted  $sa\_propose(v)$  and  $sa\_decide()$ , that  $q_i$  can invoke at most once, and in that order. The operation  $sa\_propose(v)$  allows  $q_i$  to propose a value  $v$  while  $sa\_decide()$  allows it to decide a value. The properties satisfied by an object of the type safe agreement are the following.

- Termination. If no simulator crashes while executing  $sa\_propose(v)$ , then any correct simulator that invokes  $sa\_decide()$  returns from that invocation.
- Agreement. At most one value is decided.
- Validity. A decided value is a proposed value.

**BG simulation and iterated models** Simple implementations of the safe agreement type are described in [10, 35]. We introduce here an alternative implementation, that works in the iterated write-snapshot model (IWS). Actually, an implementation of  $sa\_propose()$  and  $sa\_decide()$  needs two iterations of the base IWS model. We hope this example illustrates the benefits of programming in the iterated model. As the safe agreement object is at the core of the BG simulation, doing so connects the BG simulation with the “iterated model” research line.

As seen in Section 4.1 the semantics of the  $write\_snapshot()$  operation is defined by the properties Self-inclusion, Containment, Immediacy, and Termination which have been described in that section.

**operation**  $sa\_propose(v)$ :

(01)  $sm_i^1 \leftarrow WS[1].write\_snapshot(v)$ ;

(02)  $sm_i^2 \leftarrow WS[2].write\_snapshot(sm_i^1)$ .

Figure 5: Operation  $sa\_propose(v)$  in the iterated model (code for  $q_i$ )

The two underlying write-snapshot objects used to implement a safe agreement object are denoted  $WS[1]$  and  $WS[2]$ . An algorithm implementing the safe agreement operation  $sa\_propose(v)$  is described in Figure 5. When a simulator  $q_i$  invokes  $sa\_propose(v)$  it writes  $v$  into the first write-snapshot object  $WS[1]$  and stores the result in a local variable  $sm_i^1$  (line 01). It then writes this result in the second write-snapshot object, and stores the result in  $sm_i^2$  (line 02).

An algorithm implementing the sister operation  $sa\_decide()$  is described in Figure 6. When a simulator  $q_i$  invokes  $sa\_decide()$ , it inspects  $sm_i^1$  to find out how many processes it saw in the first iteration of its  $sa\_propose()$  invocation. If the set  $sm_i^1$  is such that  $|sm_i^1| = 1$ , it contains only the value proposed by  $q_i$ , and  $q_i$  decides then this value (lines 01, 02 and 09).

Otherwise, in addition to the value proposed by  $q_i$ ,  $sm_i^1$  contains values proposed by other processes. This means that these processes have also invoked  $sa\_propose()$ . Then  $q_i$  enters a wait statement (**repeat** statement, lines 03-05).

```

operation sa_decide():
(01) if ( $|sm_i^1| = 1$ )
(02)   then let  $v =$  value proposed by  $q_i$ 
(03)   else repeat  $sm_i \leftarrow WS[2].read()$ 
(04)     until  $(k, \cdot) \in sm_i$  for each  $k : (k, \cdot) \in sm_i^1$ 
(05)   end repeat;
(06)    $S_i \leftarrow \min_{|view_k|} \{v_k : (k, view_k) \in sm_i\}$ ;
(07)   let  $v = \min\{v_x : (x, v_x) \in S_i\}$ 
(08)   end if
(09)   return( $v$ ).

```

Figure 6: Operation sa\_decide() in the iterated model(code for  $q_i$ )

In this statement  $q_i$  repeatedly reads the second write-snapshot object  $WS[2]$  (line 03<sup>1</sup>) and stores the result in  $sm_i$ , until it sees that all processes in  $sm_i^1$  have written a value to it. If no simulator crashes while executing sa\_propose(), this wait statement terminates and (if it does not crash)  $q_i$  returns from its invocation of sa\_decide(). This is precisely the termination property of the safe consensus object type.

Once  $q_i$  exits the wait statement, its local variable  $sm_i$  includes a pair  $(k, view_k)$ , for each  $p_k$  that appears in  $sm_i^1$ , and  $view_k$  corresponds to the value written by  $p_k$  in the second iteration. Namely,  $view_k$  is the view of  $p_k$  in the first iteration (a set of pairs). Let  $S_i$  be equal to the  $view_k$  with the smallest size such that the pair  $(k, view_k) \in sm_i$  (line 06):

$$S_i \leftarrow \min_{|view_k|} \{view_k : (k, view_k) \in sm_i\}.$$

The main point is that any two simulators  $q_i, q_j$  compute the same set:  $S_i = S_j$ . Thus, they can decide on any deterministically chosen value from  $S_i$ , say the smallest proposed value present in  $S_i$  (line 07):

$$v \leftarrow \min\{v_x : (x, v_x) \in S_i\}.$$

## 8 Conclusion

This paper has presented an introductory survey of recent advances in asynchronous shared memory models where processes can commit unexpected crash failures. To that end the base snapshot model and iterated models have been presented. As far as resilience is concerned, the wait-free model and the adversary model have been discussed. Moreover, the essence of the Borowsky-Gafni's simulation has been described. It is hoped that this survey will help a larger audience of the distributed computing community to understand the power, subtleties and limits of crash-prone asynchronous shared memory models.

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<sup>1</sup>Recall that, as in the simulation of Section 6.2, we are assuming that, although writing a write-snapshot object of the IWS model is done only once, it can be read many times. Hence, we assume here an operation  $WS[2].read()$  that returns asynchronously all the pairs presents in the write-snapshot  $WS[2]$ .

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