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Abstract: Almost all models of the (New) Economic Geography have focused on interregional transportation costs to understand industrial location, considering regions as dots without intraregional transportation costs. We introduce a distinction between interregional and intraregional transportation costs. This allows assessing more precisely the effects of different types of transport policies. Focusing on two regions (a core and a periphery), we show that improving the quality of the interregional infrastructure, or of the intraregional infrastructure in the core region, leads to an increased concentration of activity in the core region. However, if we reduce intraregional transportation costs in the periphery, some firms transfer from the core to the periphery. From an efficiency point of view, we observe that, in absence of regulation, the concentration of firms is too high in the center. We show what set of policies improves the equilibrium.

JEL Classification : R11; R12; R42; R48; R58

Key-words : Economic geography ; Industrial location ; Transportation costs; Intraregional ; Interregional ; Concentration ; Transport Policies

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1. Introduction

At the end of the nineteenth century, Marshall (1890) explained that "a lowering of tariffs, or of freights for the transport of goods, tends to make each locality buy more largely from a distance what it requires; and thus tends to concentrate particular industries in special localities." Indeed, during the industrial revolution, Marshall witnessed a key moment in the history of geography. As Bairoch (1989, 1997) explains, throughout the nineteenth century, transportation costs have been divided by ten, and at the very same time, inequalities between countries emerged: the standard deviation of GDP per capita in Europe has been multiplied by 7.5. A very detailed analysis of this phenomenon is given by Lafourcade and Thisse (2011). The reduction of trade costs is one of the causes of these inequalities.

In this paper, we distinguish between interregional and intraregional transportation costs. This allows assessing more precisely the effects of different transport policies on the regional distribution of activities.

Even if Marshall had a good intuition of the relation between reduction of transportation costs and concentration of activities, the main contributions began to emerge in the 80's. Three main models have been developed to study the effect of interregional trade on industrial location. The first one has been developed by Krugman (1980) (but it is usually called the Dixit-Stiglitz-Krugman model). He sets the basis of interregional trade in an imperfect competition framework with increasing returns to scale. He considers two sectors (agriculture and industry) that are both present in two regions. A manufactured good can be sold in the two regions, but to sell it "abroad", a transportation cost must be paid by the consumer. The author concludes that a decrease of the transport costs will affect positively the two regions (by diminishing the price level in both regions). Nevertheless, the impact will be higher for the big region than for the small one, and so there will be an increase in inequalities between regions.
This model has been further developed by Helpman and Krugman (1985). Consider 2 regions A and B, and 2 production factors: labor is homogenous and can be used either in the agricultural or the industrial sectors. Labor is immobile, whereas capital is mobile between the two regions. Each worker has a unit of capital he can invest either in region A or B. But the returns he gets from the capital are spent in his own region. Defining equilibrium as the equality of returns in the two regions, they managed to highlight what they referred to as the

“Home-Market Effect”. This means that the share of industry in the employment of the central region is bigger than its share in the population. The reason is that, in presence of increasing returns and transportation costs, firms will prefer to locate near the biggest market. This effect will be even more intense when trade costs are reduced.

The last main contribution is by Krugman (1991). He analyzes which workers move between regions, and how wages are set endogenously. Workers and firms move between regions comparing their expected utilities and profits. Krugman observes that, below a certain threshold, the reduction of transportation costs will automatically lead to the concentration of all the industrial activity in one of the two regions.

These three models have two points in common: first, they all find that the reduction of interregional transportation costs will increase inequalities between regions. Second, they all use the same assumption: they consider regions as dots, without dimension, in which there are no intraregional transportation costs (see Behrens and Thisse, 2007).

This last point is disturbing since a whole field of economics has focused on cities: urban economics. Many contributions have been made, but most of them were neglected by interregional trade economists. The first attempt to unify this field was the paper of Tabuchi (1998), in which the author proposes a synthesis of Alonso (1964) and Krugman (1991). Other papers have contributed to the linkage of these two growing fields. We can think of the paper by Puga (1999),where he observes that, with congestion costs, the “tomahawk curve” of Krugman (1991) becomes a bell-shaped curve The idea is that the congestion costs will reduce the incentive for firms to remain in the center when interregional trade costs are reduced. Cavailhès and al. (2006) go one step further and investigate the way the structure of cities is affected by interregional trade, shifting from a monocentric to a polycentric configuration to reduce congestion costs.

However, all these contributions have neglected a very important scale: “the region”. As observe Behrens and Thisse (2007), these contributions have gone from the interregional scale to the urban one, skipping the region. We help filling this gap, making the difference between interregional and intraregional infrastructures. Such a distinction has already been made by Martin and Rogers (1995). They use this distinction to analyze FDIs (Foreign Direct Investments) in developing countries. They observe that an improvement of the international infrastructure will motivate firms to move to developed countries, whereas an improvement of the regional infrastructure in the periphery will lead to a transfer of firms from the developed country to the developing one. We want to use such a distinction, between interregional and intraregional transportation costs, to assess more precisely the effects of different transport policies.

There are several examples of the effects of infrastructure on regional inequalities. Vickerman (1991, 1999) has studied the effects of the European transport policy on regional inequalities. An historical example is provided by Cohen (2004). He explains that during the French colonization of Algeria, many roads were built to connect distant villages to central cities. These roads allowed firms from the center to sell their products to the villages. Far from improving the situation, these roads emptied the remote villages and increased the spatial polarization of activities.

It is crucial to understand the various mechanisms at stake, in order to implement a transport policy taking into account the cohesion objective. Another interesting objective is the normative study of such a situation. Indeed, is the geographical equilibrium an optimum from a Pareto point of view? Few contributions have focused on the normative point of view of the economic geography, and none of them made a distinction between interregional and intraregional transportation costs.

This paper has two sections. In the first section, we start from an existing model of intraregional trade (the Helpman and Krugman’s one), and introduce intraregional transportation costs. With these intraregional costs, we can assess the effects of different transport policies on industrial location. In a second section, we take a normative point of view. We compare the spatial equilibrium to the Pareto optimal one. We show that the geographical equilibrium is not optimal. We next examine policies like road pricing that improve the efficiency of the equilibrium. A numerical example with transport policies for African countries illustrates the model. In a final part of the paper, we draw some conclusions from our model. Detailed mathematical proofs of the propositions are relegated in the appendixes.

2. A theoretical model to study the impact of interregional and intraregional transportation costs on industrial location

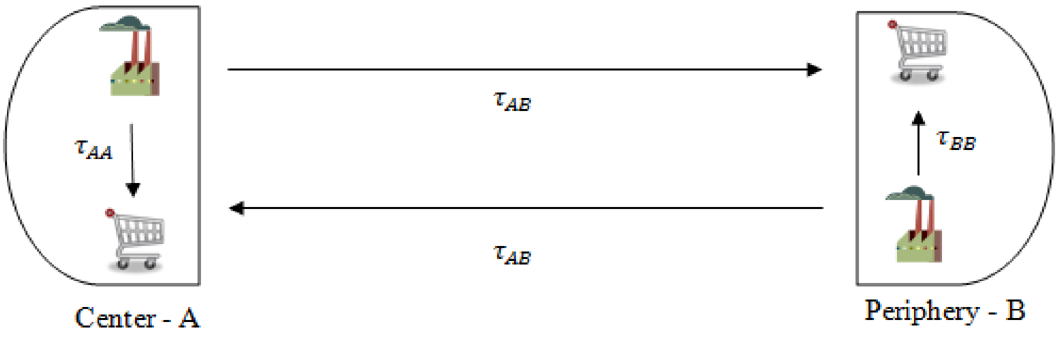
2.1. Description of the model

Most previous models in Economic Geography neglected the intraregional transportation costs. One of the exceptions is the model developed by Martin and Rogers (1995), in which they make a distinction between international and domestic trade costs. If their paper is a first step in the understanding of the impact of the intraregional trade costs, they do not use a normative framework: they do not look for efficiency improving policies.

In this paper, we add to the Helpman and Krugman's model (1985), intraregional transportation costs. The advantage of this model is that it has an analytical solution, what it is not the case of the Krugman (1991) model. Our reasoning is very close to the one proposed by Martin and Rogers (1995). We change the mathematical notations so as to obtain results more directly.

We consider 2 regions: A (the center) and B (the periphery), where A has a bigger share of the population. In each region, there are two sub-regions: factories on one side and houses (with shops) on the other side.

Figure 1. Structure of the regions and transportation costs



The structure of these regions allows us to have intraregional as well as interregional transportation costs.

2.2. Main Assumptions

Before solving the equilibrium, we need some assumptions. We consider a model of two regions: the center and the periphery. In the economy, there are L workers. A share θ of the workers are in the central region, with $\theta \geq 1/2$. In this model there are two kinds of factors: labor which is immobile and capital which is mobile, all members of the population own one unit of capital. As in Helpman and Krugman (1985), wages in the two regions are set to 1: $w_A = w_B = 1$.

More precisely, we define the different unit transportation costs within regions (AA, BB) and between regions (AB, BA). The intraregional transportation costs are given by τ_{AA} and τ_{BB}, whereas interregional ones are given by $\tau_{AB} = \tau_{BA}$.

We suppose that the intraregional transportation costs are more important in the poor region (given the low quality of transport infrastructure) than in the rich region. Moreover, we assume that interregional transportation costs are much higher than intraregional ones. We then have the following inequality:

$$\tau_{BA} = \tau_{AB} > \tau_{BB} > \tau_{AA}.$$

2.3. Equilibrium

The objective of this model is to determine the value of λ, which is the share of the industry located in the central region. As labor is immobile, intraregional commuting costs do not affect the location of production. We focus then on transportation costs. To obtain the value λ we must first specify some elements concerning production and demand.

2.3.1. Production

The cost function of a firm is defined as $C(q) = f\ r(\lambda) + cq$, so that there are increasing returns. Each firm needs f units of capital and each of the L workers have a unit of capital. All firms have the same cost function and produce each a different variety. Denote by λ the share of the capital invested in A, Since capital is perfectly mobile between the two regions, the number of firms in regions A and B are:

(1) $$n_A = \frac{\lambda L}{f} \text{ and } n_B = \frac{(1-\lambda)L}{f}.$$

Given we have assumed that the capital is mobile, the spatial equilibrium is defined by the equalization of returns :

$$r_A(\lambda^*) = r_B(\lambda^*) = r(\lambda^*).$$

The returns are spent in the region of the owner. We obtain the value of the income of each region:

(2) $$Y_A = [1 + r(\lambda)]\theta L \quad and \quad Y_B = [1 + r(\lambda)](1-\theta)L.$$

Without iceberg transportation cost, the profit equation for a representative firm i is given by $\pi_i = p_i q(p_i) - fr(\lambda) - cq(p_i)$. Profit maximization leads to the equilibrium price $p_i\left(1 - \frac{1}{\varepsilon_i}\right) = c$, where $\varepsilon_i = -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$. With the utility function defined in the Section 2.3.2, we have: $\varepsilon_i = \sigma$, where σ is the elasticity of substitution between varieties.

Now, we introduce iceberg transportation costs as the percentage of the good that is lost because of transportation costs. If we want to receive a quantity q of a product, it will be necessary to ship τq of the product, with $\tau > 1$. We then infer the price p_{lm}, paid by a consumer living in the region *m*, and purchasing a product made in region *l* :

(3) $$p_{lm} = \tau_{lm} p_i = \frac{\tau_{lm} c\sigma}{\sigma - 1} \text{ where } l = A, B \text{ and } m = A, B,$$

where τ_{lm} represents the iceberg cost.

2.3.2. Demand

As in the Dixit-Stiglitz-Krugman model, we have the following utility function: $U = M^{\mu} A^{1-\mu}$ where μ, the part of the income spent on the composite good, is such that $\mu < 1$. M is the composite good produced by the industry and A is the numeraire good. This numeraire good can be transported without cost between regions and every unit of labor that is not used to produce the industrial goods can generate one unit of the numeraire good. This allows us to set the price of the numeraire good and the wage equal to 1 in both regions, as in Krugman's model. The share of revenue dedicated to the composite good is μY_m, where Y_m is the total income in region m. Maximizing $\mathcal{L} = M + \lambda(\mu Y_m - q_{lm}(i)\tau_{lm} p_i)$, we obtain the demand for a variety i, made in l and consumed in m:

(4) $$q_{lm}(i) = \frac{(\tau_{lm} p_i)^{-\sigma}}{\sum_j (\tau_{lm} p_j)^{-(\sigma-1)}} \mu Y_m \text{ ,where } l = A, B \text{ and } m = A, B.$$

In the rest of this model, we will use the notation: $\boldsymbol{\phi_{lm} = \tau_{lm}}^{-(\sigma-1)}$**.** We observe that ϕ_{lm} takes a value between 0 and 1, if $\sigma > 1$. When ϕ_{lm} is near 1, there are low barriers for trade.

We add some assumptions on the values of ϕ :

(5) $$\phi_{AA} > \phi_{BB} > \phi_{AB} = \phi_{BA} \text{ and}$$

(6) $$1 - \theta > \frac{\phi_{AB}}{\phi_{BB}}.$$

The first hypothesis is just the transportation cost inequality written in terms of ϕ. The second assumption will be justified soon, we need it to make sure the share of industry in the region belongs to [0,1]. In other words, this inequality means that interregional transportation costs must be much higher than the intraregional ones. This assumption seems reasonable.

The total demand for the variety i produced in A is given by the sum of the demand for this variety by the region A and by the region B. The revenues in the equation (4) refer to the equation (2). Since the prices are given by the equation (3) and the number of firms by (1), we find:

(7) $$q_A^i(\lambda) = \frac{\mu(\sigma-1)}{c\sigma}\left(\frac{\phi_{AA}(1+r_A(\lambda))\theta L}{\phi_{AA}\lambda L + \phi_{AB}(1-\lambda)L} + \frac{\phi_{AB}(1+r_B(\lambda))(1-\theta)L}{\phi_{AB}\lambda L + \phi_{BB}(1-\lambda)L}\right).$$

The first part is the demand for this variety from the consumers of the region A, whereas the second part is the demand from the consumers of region B

2.3.3. Determining the equilibrium

In the long term, the profits are just high enough to cover the cost of the capital. So the profit of the firm, producing the variety i, in the region A is:

(8) $$\Pi_A(i) = p^*_{AA}(i)q_{AA}(i) + p^*_{AB}(i)q_{AB}(i) - c\left[\tau_{AA}q_{AA}(i) + \tau_{AB}q_{AB}(i)\right] - fr(\lambda) = 0$$

Let define the aggregate production as $q_A(i) = \tau_{AA}q_{AA}(i) + \tau_{AB}q_{AB}(i)$. Then we have:

(9) $$r_A(\lambda) = \frac{cq_A(i)}{f(\sigma-1)},$$

which can, using the demand functions defined by the equation (7), also be written as:

(10) $$r_A(\lambda) = \frac{\mu}{\sigma f}\left(\frac{\phi_{AA}(1+r_A(\lambda))\theta}{\phi_{AA}\lambda + \phi_{AB}(1-\lambda)} + \frac{\phi_{AB}(1+r_B(\lambda))(1-\theta)}{\phi_{AB}\lambda + \phi_{BB}(1-\lambda)}\right).$$

Symmetrically, we find $r_B(\lambda)$:

(11) $$r_B(\lambda) = \frac{\mu}{\sigma f}\left(\frac{\phi_{AB}(1+r_A(\lambda))\theta}{\phi_{AA}\lambda + \phi_{AB}(1-\lambda)} + \frac{\phi_{BB}(1+r_B(\lambda))(1-\theta)}{\phi_{AB}\lambda + \phi_{BB}(1-\lambda)}\right).$$

The spatial equilibrium is obtained when the returns in the two zones are the same. Therefore we are looking for the λ value such that r_A and r_B are equal:

$$\frac{\phi_{AA}\theta}{\phi_{AA}\lambda + \phi_{AB}(1-\lambda)} + \frac{\phi_{AB}(1-\theta)}{\phi_{AB}\lambda + \phi_{BB}(1-\lambda)} = \frac{\phi_{AB}\theta}{\phi_{AA}\lambda + \phi_{AB}(1-\lambda)} + \frac{\phi_{BB}(1-\theta)}{\phi_{AB}\lambda + \phi_{BB}(1-\lambda)}.$$

After simplifications, we find λ (the share of industry in the central region):

(12) $$\lambda = \frac{\Psi}{(\boldsymbol{\phi_{AA}} - \boldsymbol{\phi_{AB}})(\boldsymbol{\phi_{AB}} - \boldsymbol{\phi_{BB}})},$$

where $\Psi = (1-\theta)(\phi_{BB} - \phi_{AB})\phi_{AB} + \theta(\phi_{AB} - \phi_{AA})\phi_{BB}$.

Proposition 1: If the interregional transportation cost is sufficiently large, compared to the intraregional cost ($\boldsymbol{\phi_{BB}}(1-\theta) \geq \boldsymbol{\phi_{AB}}$), there exists a unique interior equilibrium where the industrial activity is shared between the two regions ($\lambda \in [0,1]$). If the interregional cost is too low ($\boldsymbol{\phi_{BB}}(1-\theta) < \boldsymbol{\phi_{AB}}$), there is a corner solution and all the industrial activity is in the center ($\lambda = 1$).

The proof of this proposition is relegated to Appendix 1.

2.4. Comparative statics

One of the advantages of building our model on Helpman and Krugman (1985) is that we have an analytical solution for the equilibrium. This allows comparative statics. Indeed, we want to know the effects of improving the different types of infrastructure on the industrial location patterns.

In this part, we modify the quality of a specific type of road and we evaluate its impact on the distribution of industrial activity. We assume that the funds (and resources) for the realization of this infrastructure are external. For instance, the funds could come from an international development agency (World Bank) or be part of a federal effort to help peripheral regions (Regional investment Fund in the EU). We show in Appendix 2 the following proposition.

Proposition 2: Both the improvement of the quality of interregional infrastructure and the reduction of intraregional transportation costs in the center lead to a higher concentration of industries in the center. However, lowering the peripheral intraregional transportation costs increases the attractiveness of the periphery for firms.

Proposition 2 can be explained using the notion of market potential. As it was defined by Harris (1954), and then extended by Head and Mayer (2004), the market potential is like a weighted sum of the different potential sales on the national market and the surrounding markets where a firm would like to export its product. The weights are inversely proportional to the trade costs.

Since the center is bigger than the periphery, it has naturally a higher market potential, which attracts firms. Reducing the interregional transportation costs will increase the market potential of the center, because central firms will have a better access to the periphery. Because of increasing returns, firms wish to move to the center. The reasoning is the same with intraregional transportation cost in the center. However, a decrease of the intraregional transportation cost in the periphery will increase the weight of the periphery in its market potential. The market potential of the periphery will then increase, and will consequently attract firms that will relocate from the center to the periphery.

3. Is this equilibrium efficient and can we improve it?

In this section, we first look for the efficiency benchmark: what geographical distribution of industrial activity maximizes the sum of utilities in the two regions. The equilibrium we described in the previous section is then compared to the benchmark. Next we look for policies that could bring the equilibrium closer to the efficiency benchmark.

3.1. Computation of the indirect utility functions in the two regions

In order to maximize overall efficiency, we need an analytical expression for the utility in both regions. Recall that $U = M^{\mu} A^{1-\mu}$ where $\mu < 1$. Moreover, since A is the numeraire, then the budget constraint reduces to: $P.M + A = 1$. Substituting into the utility function, we obtain: $U = M^{\mu}(1 - PM)^{1-\mu}$.

Maximizing the utility, with respect to M, and injecting the optimal value of M in the utility function, we obtain the indirect utility function:

(13) $$V = \frac{\mu^{\mu}\left(1-\mu\right)^{(1-\mu)}}{P^{\mu}}.$$

Moreover, we know that maximizing V is equivalent to maximizing any increasing transformation of V. We will then rely on the following indirect utilities in this section:

(14) $$V_A = P_A^{-\mu} \text{ and } V_B = P_B^{-\mu}.$$

In the new economic geography literature, the price index in the region A is given by:

(15) $$P_A = \frac{c\sigma}{\sigma - 1}(\phi_{AA} n_A + \phi_{AB} n_B)^{\frac{-1}{\sigma-1}}.$$

Using equation (1) for the number of firms, we obtain equation (16), which can be rewritten as equation (17).

(16) $$P_A = \frac{c\sigma}{\sigma - 1}\left(\phi_{AA}\frac{\lambda L}{f} + \phi_{AB}\frac{(1-\lambda)L}{f}\right)^{\frac{-1}{\sigma-1}},$$

or

(17) $$P_A = K\left(\phi_{AA}\lambda + \phi_{AB}\left(1-\lambda\right)\right)^{\alpha},$$

where $K = \frac{c\sigma}{\sigma-1}\left(\frac{L}{f}\right)^{\frac{-1}{\sigma-1}}$ and $\alpha = \frac{-1}{\sigma-1}$.

We then have the indirect utility functions for an agent living in A (V_A) and in B (V_B) :

(18) $$V_A = K^{-\mu}(\phi_{AA}\lambda + \phi_{AB}(1-\lambda))^{-\alpha\mu}.$$

Similarly, we obtain:

(19) $$V_B = K^{-\mu}(\phi_{AB}\lambda + \phi_{BB}(1-\lambda))^{-\alpha\mu}.$$

3.2. Maximization of the total welfare in the economy

We start by looking for the maximum of the unweighted sum of utilities in the two regions. There are two justifications for this. First one can allow for lump sum redistributive transfers by a federal government. Second one can see the maximization of utilities as the basis for an efficient bargaining between the two regions where the two regions share the gains of a better equilibrium via transfers among themselves.

Since a share θ of the population is in the center (and $1-\theta$ in the periphery), the welfare function in the economy is then given by:

$$W = \theta V_A + (1-\theta)V_B,$$

$$W = \theta\left[K^{-\mu}\left(\phi_{AA}\lambda + \phi_{AB}(1-\lambda)\right)^{-\alpha\mu}\right] + (1-\theta)\left[K^{-\mu}\left(\phi_{AB}\lambda + \phi_{BB}(1-\lambda)\right)^{-\alpha\mu}\right].$$

We look for the value λ^o that maximizes the welfare function:

$$\max_{\lambda} W \Leftrightarrow \max_{\lambda}\left\{W = \theta\left[\left(\phi_{AA}\lambda + \phi_{AB}(1-\lambda)\right)^{-\alpha\mu}\right] + (1-\theta)\left[\left(\phi_{AB}\lambda + \phi_{BB}(1-\lambda)\right)^{-\alpha\mu}\right]\right\}.$$

The first order condition gives us the following value:

(20) $$\lambda^o = \frac{\zeta\phi_{BB} - \phi_{AB}}{\zeta\phi_{BB} - \phi_{AB} + \phi_{AA} - \zeta\phi_{AB}},$$

where $\zeta = \left[\frac{(1-\theta)(\phi_{BB} - \phi_{AB})}{\theta(\phi_{AA} - \phi_{AB})}\right]^{\frac{-1}{\alpha\mu+1}}$.

We can note that $1-\theta<\zeta<1$, and as a consequence: $\lambda^o\in[0,1]$. The proof of these properties is relegated to Appendix 3.

3.3. Comparison of the optimal and equilibrium values of the shares in industrial activity

The question here is whether the concentration in the center is too high when the regulator doesn't intervene. To answer this question, we must compare the equilibrium value λ^{Eq} and the optimal value λ^o. Recall that:

$$\lambda^{Eq}=\frac{\theta\phi_{BB}}{\phi_{BB}-\phi_{AB}}+\frac{\theta\phi_{AB}-\phi_{AB}}{\phi_{AA}-\phi_{AB}},$$

and that :

$$\lambda^o=\frac{\zeta\phi_{BB}}{\phi_{AA}-\phi_{AB}+\zeta\left(\phi_{BB}-\phi_{AB}\right)}+\frac{-\phi_{AB}}{\phi_{AA}-\phi_{AB}+\zeta\left(\phi_{BB}-\phi_{AB}\right)}.$$

After some calculations (see Appendix 5), it can be shown that $\lambda^{Eq}>\lambda^o$. The results are summarized in:

Proposition 3: At equilibrium, the spatial concentration of industrial activity in the center is too high compared to the first best optimum.

That the spatial equilibrium leads to a higher concentration than the first best optimum can easily be understood. In fact, without any intervention, there is a kind of "snowball effect" (the expression was first used by Myrdal in 1957). For a given level of infrastructures, the center will attract firms. With these new firms, the market potential of the central region will increase, attracting more firms and so on… If nothing is done by the regulator, there will be a concentration of firms in the center that will be higher than the first-best optimum.

What would become of Prop. 3 when one gives a higher weight to peripheral citizens and there would be no lump sum redistribution possible or no efficient bargaining between the two regions? In Appendix 4 it is shown that a higher weight to peripheral citizens decreases the value of the optimal share of industry in the center.. This is obvious since, giving a higher importance to peripheral citizens, the concentration of firms in the center reduces the price index in the periphery.

In order to improve total welfare, the concentration of firms in the center should be reduced. This means that the government must intervene so as to favor the transfer of firms from the center to the periphery.

3.4. Policies to reach the optimal location : the role of road pricing

In order to decentralize the social optimum λ^o, we will use a set of incentives. In principle one could use different instruments; the regulator could tax the firms in the center and/or subsidize the firms in the periphery. Here, we focus on instruments that tax or subsidize the use of transport infrastructures. This can be understood as a form of road pricing or as a shadow cost used to compute the optimal size of different transport infrastructures. These incentives will allow us to match λ^o and λ^{Eq}.

The intuition is clear from the outset, we will have to tax the use of the interregional road, tax the use of the intraregional road in the center and/or subsidy the use of the intraregional road in the periphery. The question is then to know the precise value of these taxes or subsidies.

3.4.1. Taxation of the use of the interregional road

We want to tax the interregional infrastructure. This is equivalent to reducing the value of ϕ_{AB}, which is the "freeness of trade". To do this, we will look for the value t such that $\phi_{AB} = \phi_{AB} - t_{AB}$. We want to reach the value λ^o, so we look for t, that solves :

$$\frac{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)\phi_{AB}+\theta\left(\phi_{AB}-\phi_{AA}\right)\phi_{BB}}{\left(\phi_{AA}-\phi_{AB}\right)(\phi_{AB}-\phi_{BB})}=\lambda^o.$$

After some calculations, and solving the polynomial function in ϕ_{AB}, we find :

$$\text{(21)} \quad \phi_{AB}=\frac{1}{2(\theta+\lambda^o-1)}\left[\lambda^o\phi_{AA}+(\lambda^o-1)\phi_{BB}-\sqrt{\left((1-\lambda^o)^2\phi_{BB}-\lambda^o\phi_{AA}\right)^2-4\phi_{AA}\phi_{AB}(\lambda^o-\theta)(\lambda^o+\theta-1)}\right]$$

Two remarks are in order:

Remark 1: From ϕ_{AB}, we can find the value of t_{AB}.

Remark 2: Reducing the value of ϕ_{AB} to $\phi_{AB}-t$ is equivalent to increasing the iceberg cost from τ_{AB} to $\tau_{AB}+k$. The value of k is then given by the following expression:

$$k=\left(\phi_{AB}-t_{AB}\right)^{\frac{1}{1-\sigma}}-\tau_{AB}.$$

3.4.2. Taxation of the use of intraregional infrastructure in the center

We want to tax the use of intraregional infrastructure in the center. This is equivalent to reducing the value of ϕ_{AA}, which is the “freeness of trade”. To do this, we will look for the value t such that $\phi_{AA}=\phi_{AA}-t_{AA}$. We want to reach the value λ^{o}, so we look for t, that solves:

$$\frac{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)\phi_{AB}+\theta\left(\phi_{AB}-\phi_{AA}\right)\phi_{BB}}{\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)}=\lambda^{o}.$$

After some calculations, and solving the polynomial function in ϕ_{AA}, we find

(22) $$\phi_{AA}=\phi_{AB}+\frac{(1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB}}{\lambda^{o}(\phi_{AB}-\phi_{BB})+\theta\phi_{BB}}.$$

And the above remarks become:

Remark 1: From ϕ_{AA}, we can deduct the value of t_{AA}.

Remark 2: Reducing the value of ϕ_{AA} to $\phi_{AA}-t$ is equivalent to increasing the iceberg cost from τ_{AA} to $\tau_{AA}+k$. The value of k is then given by the following expression:

$$k=\left(\phi_{AA}-t\right)^{\frac{1}{1-\sigma}}-\tau_{AA}.$$

3.4.3. Subsiding of the use of the intraregional road in the periphery

We want to subsidize the use of the intraregional infrastructure in the periphery. This is equivalent to increasing the value of ϕ_{BB}, which is the "freeness of trade". To do this, we will look for the value of the subsidy s such that $\phi_{BB} = \phi_{BB} + s_{BB}$.

We want to reach the value λ^o, so we look for ϕ'_{BB}, that solves:

$$\frac{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)\phi_{AB}+\theta\left(\phi_{AB}-\phi_{AA}\right)\phi_{BB}}{\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)}=\lambda^o.$$

After some calculations, we obtain:

(23) $$\phi_{BB}=\frac{\lambda^o\phi_{AA}\phi_{AB}+\phi_{AB}^2(1-\theta-\lambda^o)}{(1-\lambda^o)\phi_{AB}+(\lambda^o-\theta)\phi_{AA}}.$$

Remark 1 : From ϕ_{BB}, we can deduct the value of s_{BB}.

Remark 2 : Increasing the value of ϕ_{BB} to $\phi_{BB}+s$ is equivalent to reducing the iceberg cost from τ_{BB} to $\tau_{BB}-k$. The value of k is then given by the following expression:

$$k=\tau_{BB}-\left(\phi_{BB}+s_{BB}\right)^{\frac{1}{1-\sigma}}.$$

We have developed a set of incentives such that the equilibrium will be the optimal one. To do this, we focused on road pricing. One can reach the optimum in three different ways: taxing the use of the interregional infrastructure; taxing the use of the intraregional infrastructure in the center or subsidizing the use of the intraregional infrastructure in the periphery. Note that subsidizing the intraregional infrastructure in the periphery does not mean building a road. Indeed, the creation of the infrastructure is costly, while a subsidy is in principle a mere transfer of resources that corrects incentives and is not consuming real resources (except for the transaction costs). We illustrate our model below with a numerical example.

3.4.4. A numerical example: Mozambique and Malawi

In order to illustrate the mechanisms of the model, we take a numerical example. Since comprehensive data on interregional and intraregional transportation costs for road transport are difficult to gather, we use from UNCTAD (2004). In their report, they calculate very accurate values for transportation costs in Africa. We take the example of two countries, instead of two regions. This choice is mainly due to the availability of data, but especially because it does not change anything concerning the assumptions and results. The two selected countries are Mozambique (the center) and Malawi (the periphery).

First, we must calibrate our model. To do so, we match real data with the various parameters we use in the model. Table 1a displays values for the central country, whereas table 1b does it for the periphery.

Table 1a. Numerical values for the parameters of Mozambique

Region A – The core	**Mozambique**
Population	**19M**
Share of the population θ	**0.6**
Infrastructure quality index	**23.1**
ϕ_{AA}	**0.9**

Table 1b. Numerical value for the parameters of Malawi

Region B – The Periphery	**Malawi**
Population	**13M**
Share of the population $1-\theta$	**0.4**
Infrastructure quality index	**20.4**
ϕ_{BB}	**0.79**

Once these parameters have been calibrated, we must set the values of other parameters, that are not always known (for instance σ). Others, like the interregional transportation cost, can be found in UNCTAD (2004) .

Table 2. Other parameters to run the simulation

Other parameters	
w wage	**1**
σ (elasticity of substitution)	**6**
Share of transport cost in the price of goods sold in the other region	**22%**
Iceberg cost τ_{AB}	**1.28**
ϕ_{AB}	**0.29**
μ	**0.6**

Note that several values have been tested for the elasticity of substitution, and they do not change dramatically the results. We can now compute the spatial equilibrium and the spatial optimum. It is interesting to note that the simulated spatial equilibrium ($\lambda = 0.75$) is very close to the real value ($\lambda = 0.74$).

Table 3. Spatial equilibrium and spatial optimum

Spatial equilibrium and optimum	
Spatial equilibrium λ^{Eq}	**0.75**
Intermediate parameter ζ	**1.59**
Optimal concentration λ^{o}	**0.69**

As predicted by Proposition 3, there is a significant difference between the spatial equilibrium and the optimal concentration. The next step is to calculate the values of the different taxes or subsidies to reach the optimal concentration.

Table 4. Optimal taxes and subsidies on use of infrastructure to reach the optimum and their impact on the transportation costs

Road pricing to reach the equilibrium	
Optimal ϕ_{AA}	**0.74**
Optimal tax t_{AA}	**0.16**
Optimal ϕ_{AB}	**0.20**
Optimal tax t_{AB}	**0.09**
Optimal ϕ_{BB}	**0.91**
Optimal subsidy s_{BB}	**0.12**

From these new values of transportation costs, we can deduct the impact on the product prices:

Table 5. Impact of taxes on transportation costs

Share of the transport cost in the price of a shipped product	Before tax	After tax
Interregional	**22%**	**27%**
Intraregional in the center	**2%**	**6%**
Intraregional in the periphery	**4%**	**2%**

This numerical example illustrates the different values predicted by our model. The values of the various taxes or subsidies are not unrealistic. These numerical simulations confirm that the predictions of our model are in line with the theoretical results. At equilibrium, the concentration of activities in the center is too high and the use of interregional road taxes can restore the optimum.

CONCLUSION

If a whole literature has studied the impact of transportation costs on the geographical distribution of activities, this paper is among the first to analyze the specific roles of interregional and intraregional transport costs .

Several conclusions can be drawn from this paper. First, the improvements of the different categories (interregional/intraregional) of infrastructures have different effects on industrial location. We confirm Martin and Rogers (1995analysis: if the decision maker whishes to use the transport policy to reduce regional inequalities, then s/he must improve the quality of the peripheral intraregional infrastructure. The second conclusion follows from the normative analysis. Indeed, we show that the spatial equilibrium is far from being Pareto optimal. Without any intervention, the spatial equilibrium will lead to a concentration of firms in the center that is too high. The third result is that it is possible to use tax and subsidy instruments such as road pricing to reach the optimal share of firms in the center.

Yet, this model could be improved in several ways. First, it was designed in such a way that interregional and intraregional transportation costs were independent. In reality, mattters are more complex and intraregional infrastructures may affect directly the interregional transportation costs. These network effects must be taken into account if we want a more realistic view of the effects of infrastructures on industrial location. Second, the introduction of congestion costs within regions would probably give interesting results. As it was explained by Lafourcade and Thisse (2011), congestion costs moderate the spatial polarization of activities. In our model, they would affect strongly the optimal taxes and subsidies. With these improvements, future research allows to understand more properly the dynamics of spatial activities.

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Appendix 1. Proof of Proposition 1

We want to prove that $\phi_{AB} < (1-\theta)\phi_{BB}$, then $\lambda \in [0,1]$

Proof of $\lambda > 0$

We know that: $(\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB}) < 0$. If $\lambda > 0$, then we must have $\Psi \leq 0$. Let's prove it by contradiction. If $\Psi > 0$, then we have

$$\Psi = (1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB} + \theta(\phi_{AB}-\phi_{AA})\phi_{BB} > 0$$
$$\Leftrightarrow (1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB} > \theta(\phi_{AA}-\phi_{AB})\phi_{BB}.$$

Since $1-\theta < \theta$ and $\phi_{BB}-\phi_{AB} < \phi_{AA}-\phi_{AB}$ and since $\phi_{AB} < \phi_{BB}$, the previous line ca not be true. This means that $\Psi \leq 0$, and then $\lambda \geq 0$.

Proof of $\lambda < 1$

We know that $(\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB}) < 0$. If $\lambda \leq 1$, then we must have :

$$(1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB} + \theta(\phi_{AB}-\phi_{AA})\phi_{BB} > (\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB})$$
$$\Leftrightarrow (1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB} + (\phi_{AB}-\phi_{AA})[\theta\phi_{BB}+\phi_{AB}-\phi_{BB}] > 0.$$

Since $(1-\theta)(\phi_{BB}-\phi_{AB}) > 0$ and $(\phi_{AB}-\phi_{AA}) < 0$, we must have $\theta\phi_{BB}+\phi_{AB}-\phi_{BB} < 0$ which is true if $\phi_{AB} < (1-\theta).\phi_{BB}$.

Proof that if $\phi_{AB} > (1-\theta)\phi_{BB}$, then $\lambda > 1$

We know that: $(\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB}) < 0$. If $\lambda > 1$, then we must have:

$$(1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB} + \theta(\phi_{AB}-\phi_{AA})\phi_{BB} < (\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB})$$
$$\Leftrightarrow (\phi_{BB}-\phi_{AB})(\phi_{AA}-\theta\phi_{AB}) < \theta(\phi_{AA}-\phi_{AB})\phi_{BB}.$$

Since $\phi_{AB} > (1-\theta)\phi_{BB}$, then $-\phi_{AB} < (\theta-1)\phi_{BB}$, so we have

$$(\phi_{BB}-\phi_{AB})(\phi_{AA}-\theta\phi_{AB}) < \theta\phi_{BB}(\phi_{AA}-\theta\phi_{AB}).$$

Moreover, we can note that: $\phi_{AA}-\theta\phi_{AB}<\theta\left(\phi_{AA}-\phi_{AB}\right)$. So we can conclude that we have: $\left(\phi_{BB}-\phi_{AB}\right)\left(\phi_{AA}-\theta\phi_{AB}\right)<\theta\left(\phi_{AA}-\phi_{AB}\right)\phi_{BB}$, which means that if $\phi_{AB}>\left(1-\theta\right)\phi_{BB}$, then $\lambda>1$.

Appendix 2. Proof of Proposition 2

Impact of ϕ_{AB} : Improving the quality of the interregional infrastructure is like an increase in ϕ_{AB} .

$$\frac{\partial\lambda}{\partial\phi_{AB}}=\frac{\left[\left(1-\theta\right)\left(\phi_{BB}-2\phi_{AB}\right)+\theta\phi_{BB}\right]\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)-\Psi[\phi_{AA}+\phi_{BB}-2\phi_{AB}]}{\left[\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)\right]^2}$$

We have: $\frac{\partial\boldsymbol{\lambda}}{\partial\phi_{\boldsymbol{AB}}}>0$ if the condition $\frac{\phi_{AA}}{\phi_{AB}}>2\theta$ is respected (which always holds with our assumptions). Using this hypothesis, we observe that the improvement of the infrastructure between regions will strengthen the concentration in the center.

Impact of ϕ_{AA} : We measure the effects of an improvement of infrastructures in the center. We expect that it will lead to a higher concentration in the center.

$$\frac{\partial\lambda}{\partial\phi_{AA}}=\frac{-\theta\phi_{BB}\left[\phi_{AA}-\phi_{AB}\right]\left[\phi_{AB}-\phi_{BB}\right]-[\phi_{AB}-\phi_{BB}]\Psi}{\left[\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)\right]^2}$$

We obtain: $\frac{\partial\boldsymbol{\lambda}}{\partial\phi_{\boldsymbol{AA}}}>0$. As predicted, we conclude that the higher quality of infrastructure in the center will increase the concentration.

Impact of ϕ_{BB} : Since the other actions on infrastructures have led to a higher concentration, we expect that the reduction of transport costs in the periphery will lead to a reduction of the concentration in the center.

$$\frac{\partial\lambda}{\partial\phi_{BB}}=\frac{\left[(1-\theta)\phi_{AB}+\theta\left(\phi_{AB}-\phi_{AA}\right)\right]\left[\phi_{AA}-\phi_{AB}\right]\left[\phi_{AB}-\phi_{BB}\right]+[\phi_{AA}-\phi_{AB}]\Psi}{\left[\left(\phi_{AA}-\phi_{AB}\right)\left(\phi_{AB}-\phi_{BB}\right)\right]^{2}}$$

We find that $\frac{\partial\lambda}{\partial\phi_{BB}}<0$ if we have $\phi_{AB}>\theta\phi_{AA}$ (which holds with our hypotheses) .The better quality of infrastructure in the periphery will lead to a relocation of firms from the center to the periphery.

Appendix 3. Remarks concerning the value of ζ

First, we want to show that $0<\zeta<1$

We can rewrite ζ as:

$$\zeta=\left[\frac{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)}{\theta\left(\phi_{AA}-\phi_{AB}\right)}\right]^{\frac{-1}{\alpha\mu+1}}=\left[\Omega\frac{\phi_{BB}}{\phi_{AA}}\right]^{\frac{-1}{\alpha\mu+1}}.$$

Since $0<\Omega<1$ and $\phi_{BB}<\phi_{AA}$, we can conclude than $0<\zeta<1$.

Second, we want to show that $\zeta>1-\theta$

Let's prove it by contradiction. Then we make the hypothesis that $1-\theta>\zeta$.
So we must have:

$$1-\theta>\left[\frac{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)}{\theta\left(\phi_{AA}-\phi_{AB}\right)}\right]^{\frac{-1}{\alpha\mu+1}}\Leftrightarrow 1-\theta>(1-\theta)^{\frac{-1}{\alpha\mu+1}}\left[\frac{\left(\phi_{BB}-\phi_{AB}\right)}{\theta\left(\phi_{AA}-\phi_{AB}\right)}\right]^{\frac{-1}{\alpha\mu+1}}$$

$$\Leftrightarrow (1-\theta)^{\frac{1}{\alpha\mu+1}+1}>\left[\frac{\theta\left(\phi_{AA}-\phi_{AB}\right)}{\left(\phi_{BB}-\phi_{AB}\right)}\right]^{\frac{1}{\alpha\mu+1}}.$$

Since $\frac{1}{\alpha\mu+1}=\frac{\sigma-1}{\sigma-1-\mu}$ and since $0<1-\theta<1$ then we have : $(1-\theta)^{\frac{1}{\alpha\mu+1}}>(1-\theta)^{\frac{1}{\alpha\mu+1}+1}$.

We can then rewrite the inequality:

$$(1-\theta)^{\frac{1}{\alpha\mu+1}}>\left[\frac{\theta\left(\phi_{AA}-\phi_{AB}\right)}{\left(\phi_{BB}-\phi_{AB}\right)}\right]^{\frac{1}{\alpha\mu+1}}\Leftrightarrow 1-\theta>\frac{\theta\left(\phi_{AA}-\phi_{AB}\right)}{\left(\phi_{BB}-\phi_{AB}\right)}\Leftrightarrow 1>\frac{\theta\left(\phi_{AA}-\phi_{AB}\right)}{(1-\theta)\left(\phi_{BB}-\phi_{AB}\right)}.$$

And this is false. So by contradiction, we can say that $1-\boldsymbol{\theta}<\zeta$

Third, with $1-\boldsymbol{\theta}<\boldsymbol{\zeta}<1$, then we have $\boldsymbol{\lambda}^o \in [0,1]$

We know that $1-\theta<\zeta$. Since we have shown that $(1-\theta)\phi_{BB}>\phi_{AB}$, we now have: $\zeta\phi_{BB}>\phi_{AB}$.

This allows us to say that:

$$\boldsymbol{\lambda^o = \frac{\zeta\phi_{BB}-\phi_{AB}}{\zeta\phi_{BB}-\phi_{AB}+\phi_{AA}-\zeta\phi_{AB}}} \in [0,1].$$

Appendix 4. Impact of the weights on the optimal value of industry share

We want to assess the impact of the weights in the total utility function on the optimal value of the industry share. To do this, we normalize the weight for region A to 1 and give a weight η to region B. This way, the indirect utility function to be maximized is then:

$$W=\theta V_A+(1-\theta)\eta V_B$$

$$\max_{\lambda} W \Leftrightarrow \max_{\lambda}\left\{W=\theta\left[\left(\phi_{AA}\lambda+\phi_{AB}(1-\lambda)\right)^{-\alpha\mu}\right]+(1-\theta)\eta\left[\left(\phi_{AB}\lambda+\phi_{BB}(1-\lambda)\right)^{-\alpha\mu}\right]\right\}$$

The first order condition gives us the following value:

$$\boldsymbol{\lambda^o = \frac{\zeta\phi_{BB}-\phi_{AB}}{\zeta\phi_{BB}-\phi_{AB}+\phi_{AA}-\zeta\phi_{AB}}},$$

where $\zeta=\left[\frac{\eta(1-\theta)(\phi_{BB}-\phi_{AB})}{\theta(\phi_{AA}-\phi_{AB})}\right]^{\frac{-1}{\alpha\mu+1}}$.

It is easy to show that $\frac{\partial\zeta}{\partial\eta}<0$. Moreover, one can prove that $\frac{\partial\lambda^o}{\partial\zeta}>0$. Knowing the signs of these two derivatives, we can then conclude that an increase in η will lead to a reduction of λ^o.

Appendix 5. Proof of Proposition 3

The spatial equilibrium is given by the value λ^{Eq}:

$$\lambda^{Eq} = \frac{(1-\theta)(\phi_{BB}-\phi_{AB})\phi_{AB}+\theta(\phi_{AB}-\phi_{AA})\phi_{BB}}{(\phi_{AA}-\phi_{AB})(\phi_{AB}-\phi_{BB})}$$

$$\lambda^{Eq} = \frac{\theta\phi_{BB}}{\phi_{BB}-\phi_{AB}}+\frac{\theta\phi_{AB}-\phi_{AB}}{\phi_{AA}-\phi_{AB}}.$$

We want to compare λ^{Eq} with the optimal share of firms λ^{o} that can be rewritten as:

$$\lambda^{o} = \frac{\zeta\phi_{BB}}{\phi_{AA}-\phi_{AB}+\zeta(\phi_{BB}-\phi_{AB})}+\frac{-\phi_{AB}}{\phi_{AA}-\phi_{AB}+\zeta(\phi_{BB}-\phi_{AB})}$$

First, we can compare the second parts of the two equations. Since $\zeta(\phi_{BB}-\phi_{AB})>0$ and $\theta\phi_{AB}>0$, then we can observe that:

$$\frac{\theta\phi_{AB}-\phi_{AB}}{\phi_{AA}-\phi_{AB}} > \frac{-\phi_{AB}}{\phi_{AA}-\phi_{AB}+\zeta(\phi_{BB}-\phi_{AB})}.$$

Second, we want to compare the first parts of the equations. Let's prove by contradiction that

$$\frac{\theta\phi_{BB}}{\phi_{BB}-\phi_{AB}} > \frac{\zeta\phi_{BB}}{\phi_{AA}-\phi_{AB}+\zeta(\phi_{BB}-\phi_{AB})}$$

To do so, we make the hypothesis that:

$$\frac{\theta\phi_{BB}}{\phi_{BB}-\phi_{AB}} < \frac{\zeta\phi_{BB}}{\phi_{AA}-\phi_{AB}+\zeta(\phi_{BB}-\phi_{AB})}$$

This would mean that :

$$\theta\left[\phi_{AA}-\phi_{AB}+\zeta\left(\phi_{BB}-\phi_{AB}\right)\right] < \zeta\left(\phi_{BB}-\phi_{AB}\right)$$

$$\Leftrightarrow \theta\left(\phi_{AA}-\phi_{AB}\right)+\zeta\left(\theta-1\right)\left(\phi_{BB}-\phi_{AB}\right) < 0$$

$$\Leftrightarrow \frac{\theta\left(\phi_{AA}-\phi_{AB}\right)}{(1-\theta)(\phi_{BB}-\phi_{AB})} < \zeta$$

$$\Leftrightarrow 1 < \zeta.$$

This is a contradiction since we know that $0 < \zeta < 1$. So this means that we have:

$$\frac{\theta\phi_{BB}}{\phi_{BB} - \phi_{AB}} > \frac{\zeta\phi_{BB}}{\phi_{AA} - \phi_{AB} + \zeta(\phi_{BB} - \phi_{AB})}.$$

These two inequalities lead us to the conclusion that: $\boldsymbol{\lambda}^{\boldsymbol{Eq}} > \boldsymbol{\lambda}^{\boldsymbol{o}}$.