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Information propagation speed versus transport capacity in mobile ad hoc wireless networks

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Abstract: We give an estimate of an upper bound of information propagation speed as a function of a targeted transport capacity density. The propagation speed depends on the density of simultaneous emitters and the node mobility model. We assume that nodes move according to an independent random walk. This work makes the junction between the work of Gupta and Kumar (2000) and Grossglauser and Tse (2002). We show that the information speed tends to zero when the transport capacity tends to infinity. We compare our result to the performance of an actual protocol based on angle determination.

Key-words: wireless, networks, mobility, information theory, complex analysis

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Vitesse de propagation de l'information en fonction de la capacité de transport dans les réseaux mobiles ad hoc

Résumé : Nous donnons une borne supérieure de la vitesse de propagation en fonction d'une capacité de transport ciblée. La vitesse de propagation dépend de la densité des émetteurs et de la mobilité des postes. Nous supposons fait la jonction entre les travaux de Gupta et Kumar (2000) et ceux de Grossglauser et Tse (2002). Nous montrons que la vitesse de propagation tend vers zero quand la capacité de transport tend vers l'infini. Nous comparons avec les performances d'un algorithme réaliste basé sur des angles.

Mots-clés : sans fil, réseau, mobilité, théorie de l'information, analyse complexe

1 Introduction

We consider a wireless network with a (potentially) infinite density of nodes on the plan. Each node has a wireless interface of nominal capacity C (expressed in bit per second). We assume that the access protocol generates consecutive sets of simultaneous transmitters such that any of those sets follows a uniform distribution of density equal to λ (expressed in inverse square meters). We consider asymptotic situations where $\lambda \to \infty$. To fix the idea, the access protocol can be ALOHA-like with slotted time as described in [5], but the result could be generalized to more sophisticated access schemes as did Gupta and Kumar in [1]. Indeed they show that in very general settings the efficient transmission radius $r = r(\lambda) = \frac{\alpha}{\sqrt{\lambda}}$ for some constant $\alpha > 0$ depending on protocol, signal propagation, demodulation, *etc.*

If nodes were to transmit data only to their closest neighbors, then the useful capacity density of the network would be equal to $\beta C \lambda$ (in bit per second per square meter) where β, depending on the access scheme, is the average number of retries before any succesful packet delivery. This would lead to an actual capacity density tending to infinity when $\lambda \to \infty$. For the remaining of the paper we will assume $\beta = 1$, the case $\beta < 1$ does not actually impact our result.

But in general data are to be transmitted toward distant destinations; in this case the data must be relayed by other nodes. With shortest path routing, if L is the distance between source and destination, then the data would be relayed on a number of hops equal to $\lceil \frac{L}{r(\lambda)} \rceil \sim \frac{L\sqrt{\lambda}}{\alpha}$ when $\lambda \to \infty$. If all datas were to be moved over an average distance L then each packet would be repeated $L\frac{\sqrt{\lambda}}{\alpha}$ times, and therefore the net capacity density of the network would be divided by this factor, hence $C\alpha\sqrt{\lambda}\frac{1}{L}$, which tends to zero when $L \to \infty$.

Due to the obvious intrication of wireless network with physical space, Gupta and Kumar in [1] introduced the concept of transport capacity. The transport capacity of a connection is equal to the product of the flow throughput with the euclidian distance to destination (expressed in bit meter per second). Similarly the transport capacity density is equal to the useful capacity density multiplied by the average distance to destination of the locally generated traffic. Gupta and Kumar result tells that such density does not depend on the average distances to destinations and is actually equal to $C\alpha\sqrt{\lambda}$, as also shown in [5]. This fundamental result shows that the transport capacity density still tends to infinity when $\lambda \to \infty$.

From now we assume that the nodes are mobile and follow i.i.d motion processes. Grossglauser and Tse [2] have shown that the mobility indeed increases capacity, stressing the fact that wireless networks also show a strong implication with the physical time. If the nodes are confined in a squared network map of size $L \times L$ and if the pairing between sources and destinations follows an uniform distribution, then the data delivery can be actually done with only two transmissions. A fundamental condition is that the motion process is ergodic on the whole network map. The trick is that the packet travels most of the distance in the memory buffer of a mobile relay instead of being moved like a "hot potatoe" as in [1]. We denote v the average node velocity. The consequence is that with Grossglauser Tse scheme, the useful capacity density is now $\frac{C}{2}\lambda$ and the transport density rises to $\frac{C}{2}\lambda\Theta(L)$. But Grossglauser Tse scheme shows two main drawbacks:

1. the transport capacity does not scale well, it would drop when destinations are distributed close to sources instead of being uniformly distributed over the square.

2. The average packet delivery delay which is of order $\frac{L^2}{vr(\lambda)} = \frac{L^2\sqrt{\lambda}}{v\alpha}$, does not significantly drop when the destinations are closer to the sources.

Notice that the average speed of information propagation, defined as the ratio of the average distance to destination over the average delivery time, is of order $\frac{vr(\lambda)}{L}$ with Grossglauser Tse algorithm and tends to zero when L or λ tends to infinity, which suggests that the information propagation speed tends to zero when the transport capacity tends to infinity.

Our aim is to show that, for a given targeted transport capacity density, there exists an upper bound $v_p(\Delta)$ of information propagation. Quantity $v_p(\Delta)$ depends of λ and on the average distance Δ travelled by the packet toward the destination between two transmissions. In this case the transport capacity equals to $C\lambda\Delta$. The speed also depends on the node mobility model. To simplify we assume that store and forward operations take zero time, which means that the propagation speed is *infinite* in Gupta and Kumar model. Assuming non zero time would only introduce minor complications.

When node velocity is bounded by $v_{\max}$, quantity $\frac{\Delta}{\Delta - r(\lambda)} v_{\max}$ is a very trivial estimate of $v_p(\Delta)$. This evaluation is not satisfactory since it does not tend to zero when $\Delta \to \infty$. We will exhibit a non trivial estimate of $v_p(\Delta)$ based on the roots of a multivariate complex function and such that $v_p(\Delta) \to \infty$ when $\Delta \to r(\lambda)$, and $v_p(\Delta) \to 0$ when $\Delta \to \infty$.

Our result makes the link between [1] and [2] and uses the space-time implication of wireless networks like in [3]. The paper is organized as follow: we introduce the models and parameters in the next section, in the section "Methodology" we introduce an upper bound scheme and define the concept of packet journey as in [3]. We develop our mathematical analysis in the section "Journey Analysis". In a last section we compare to an existing scheme based on angles.

2 Models and parameters

We assume that the nodes moves according to an i.i.d. random walk. They move on strait lines at constant speed v and change heading after an exponentially distributed time of parameter τ (expressed in inverse time). The average "free space" distance, *i.e.* the distance traveled in strait lines between two consecutive heading changes, is $\frac{v}{\tau}$. When the node change heading it takes a random direction whose angle is uniformly distributed on $[0, 2\pi]$. If we consider that nodes may have different speed with a maximum speed $v_{\max}$, we consider the optimal case where nodes move at constant speed $v = v_{\max}$.

We assume that the node motions are unpredictable. Therefore the packet delivery delay is a random variable. We denote $T(L)$ the packet delivery delay of a packet to a destination at distance L. To simplify we will assume that the destination does not move, but extension to mobile destination will be given in the generalizations, and in this case L indicates the distance of the destination when it receives the packet.

Definition 1. *Quantity v_p is a packet speed upper bound [3] if for all $c > v_p$:*

$$\lim_{L\to\infty} P\left(T(L) < \frac{L}{c}\right) = 0 \ . \quad (1)$$

We denote $N(L)$ the number of times the packet is transmitted before being delivered to a destination at distance L. Let Δ be a length. We call $v_p(\Delta)$ an upper bound constrained packet speed, defined as follows.

Definition 2. *Quantity $v_p(\Delta)$, in length times inverse time unit, is a constrained packet speed upper bound if for all $c > v_p(\Delta)$:*

$$\lim_{L\to\infty} P\left(T(L) < \frac{L}{c} \mid N(L) < \frac{L}{\Delta}\right) = 0 \ . \quad (2)$$

We notice that when $N(L) < \frac{L}{\Delta}$ then the transport capacity density is greater than $C\lambda\Delta$. We should have $v_p(\Delta) \to \infty$ when $\Delta \to r(\lambda)$, since it corresponds to the classic store and forward scheme without carry phase.

3 Methodology

3.1 Transmission phases versus carry phases

Our aim is to find a packet speed upper bound for all possible schemes. A scheme is characterized by a sequence of transmit and carry phase. A transmit phase is when a relay transmits the packet to a next relay within the radius $r = r(\lambda)$. A transmission phase can be made of several transmissions in series over a sequence of consecutive relays, each of them being within radius r from the previous one. We will suppose that the new relay has a random motion vector whose angle is uniformly distributed in $[0, 2\pi]$. This means that we assume that the transmitter is not aware of the motion vector of the next relay. We will later give a straightforward extension to the case where the transmitter selects with respect to the motion vector of the later.

A carry phase is a period when the relay carries the packet in its memory and proceeds in its random walk before retransmitting the packet to a new relay.

3.2 Upper bound schemes

We consider a source S and a destination D located at a distance L of S. In order to have a lower bound on the delay delivery, we assume that the packet is delivered when it reaches any point on the line containing D and orthogonal to the vector (S, D). We call this line, the "destination wall". we denote $\vec{\mathbf{d}}$ the unitary vector oriented like (S, D). With respect to this model we consider the class of optimal schemes such that :

1. Packet is transmitted to the destination wall when a relay arrive at distance r from the destination wall;

2. otherwise in transmission phase, the packet is transmitted to the closest node to the destination wall, *i.e.* we assume it is always at distance r from the transmitter oriented according to $\vec{\mathbf{d}}$;

3. Transmission phases are made of an arbitrary number of emissions and occur only when the mobile relay changes heading.

It is relatively straightforward to see that the optimal transmission phase would occur on points on the mobile relay trajectory which correspond to local optima with respect to the distance to the destination wall.

3.3 Protocol versus journey analysis

We introduce the concept of journey as in [3]. We consider a given destination wall and the unitary vector $\vec{\mathbf{d}}$. A journey J is a space-time trajectory made of a sequence of carry phases and transmission phases. We assume that the journey ends within transmission distance r of the destination wall. We denote $s(J)$ the start point of the journey, called the source point. We denote $t(J)$ the difference of time between the two end points, which is basically the packet delivery delay. We denote $n(J)$ the number of packet transmissions made on the journey J. In figure 1 and 2 we display the spatial and time trajectory of a journey J with $n(J) = 3$. The fastest journey would take $\frac{L}{r}$ hops.

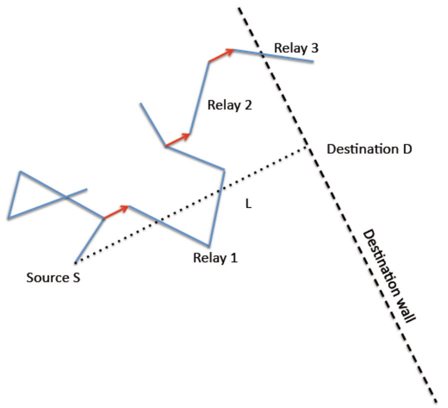

Figure 1: The space trajectory of a journey from source S at distance L to a destination D. Transmission phases are in red.

Let a point S at distance L of the destination wall. Let T be a real number and N an integer. Our aim is to find if there exist a journey J such that $s(J) = S$, $t(J) < T$ and $n(J) \leq N$. We deduce that if there exist is a protocol which can deliver the packet from S to D in less than T time, with less than N transmissions. Therefore whatever the protocol, the probability to deliver the packet in less than T time with less than N transmissions is always smaller than the probability that there exists a journey J such $t(J) < T$, under the condition that $s(J) = S$ and $n(J) \leq N$. We call this probability $P(L, T, N)$. Since by [2], we know that there is always a journey with $n(J) \leq N$ (provided that $N \geq 2$), This is also the probability that there exists a journey J such $t(J) < T$ and $n(J) \leq N$, under the condition $s(J) = S$.

Our aim is to estimate an upper bound of the quantity $P(L, T, N)$. We denote by $E(L, T, N)$ the average number of journeys J that starts from a source S at distance L to the destination wall, such that $t(J) < T$ and $n(J) \leq N$. We have $P(L, T, N) \leq E(L, T, N)$. We will evaluate $E(L, T, N)$.

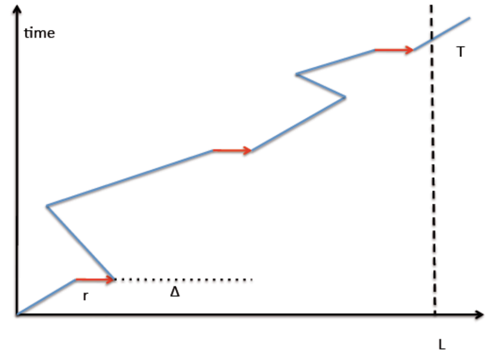

Figure 2: The time trajectory of a journey from source S at distance L to a destination D with average inter transmission Δ.

4 Journey Analysis

4.1 Journey density function

We develop a tool comparable inspired from [3], but adapted to dense networks. We are going to estimate the space-time density of journey starting from a point S at distance L to the destination wall. We call $d(J)$ the end point of of a journey J. The are several journeys that can connect S to the destination wall. We want to know the average number of journeys that satisfies certain conditions on $t(J)$ and $n(J)$. We call $f(x,t,n)$ the journey density function, *i.e.* the density function of the average number of journeys such that $(S,d(J)).\vec{\mathbf{d}} = x$, $t(J) = t$ and $n(J) = n$. This is a density because the variables x and t stand on continuous space, contrary to the variable n which is discrete. We have the inequality

$$E(L,T,N) \leq \sum_{i=1}^{N} \int_0^T dt \iint_{x>L-r} f(x,t,i)dx \ , \tag{3}$$

which translates the fact that at some time between 0 and T, the packet has been within r of the distance wall.

4.2 Journey space-time Laplace transform

We denote the Laplace transform

$$w(\rho,\theta,u) = \sum_n \int_0^\infty dt \iint f(x,t,n)\exp(-\theta t - \rho x)u^n dx \ .$$

Via inverse laplace transform we will get

$$\begin{aligned} E(L,T,N) &= \frac{1}{(2i\pi)^3} \oint \frac{du}{(1-u)u^{N+1}} \\ &\times \iint w(\rho,\theta,u)\frac{e^{-T\theta-(L-r)\rho}d\theta d\rho}{\theta\rho} \end{aligned} \tag{4}$$

We will build function $w(\rho,\theta,u)$ via enumeration methods.

A transmission phase is made of an arbitrary number of emissions, all made within radius r from the last relay and oriented according to $\vec{\mathbf{d}}$. Therefore the space-time Laplace transform of a transmission phase is equal to $\frac{1}{1-ue^{-r\rho}}$. Notice that variable θ is not involved in the formula, since the transmission phase is supposed to take zero time.

A carry phase is made of an arbitrary, but non zero, number of free space moves. If the free space move is made at speed v heading with angle ϕ with $\vec{\mathbf{d}}$ during time t then its space-time Laplace transform is $\exp(-vt\rho\cos(\phi)-\theta t)$. Integrating on an exponential time distribution of parameter τ for t, and for ϕ uniform on $[0,2\pi]$ we get the space-time transform of one free space move equal to $\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}$.

Since a journey J is an alternance of free space moves interleaved by transmission phase, it can be described in term of the formalism of grammar by $J=X^*(FX^*)^*$ where the symbol X means an emission, and F a free space move. The Laplace transform is $\frac{1}{1-ue^{-r\rho}}\left(1-\frac{1}{1-ue^{-r\rho}}\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}\right)^{-1}$. To get $w(\rho,\theta,u)$ one must multiply by the Laplace transform of the last interrupted free space move, namely $\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}$:

$$w(\rho,\theta,u)=\frac{1}{(1-ue^{-\rho r})\sqrt{(\theta+\tau)^2-\rho^2v^2}-\tau}\ . \tag{5}$$

Let $D(\rho,\theta,\omega)=(1-e^{-\omega-\rho r})\sqrt{(\theta+\tau)^2-\rho^2v^2}-\tau=\frac{1}{w(\rho,\theta,,e^{-\omega})}$. We call Kernel $\mathcal{K}$ the set of tuples (ρ,θ,ω) such that $D(\rho,\theta,\omega)=0$.

Theorem 1. *For all $(\rho,\theta,\omega)\in\mathcal{K}$, for all $\omega'>\omega$, there exist A such that*

$$E(L,T,N)\leq A\exp(\theta T+\rho L+\omega' N)\ .$$

Proof. See appendix. □

Theorem 2. *For all $(\rho,\theta,\omega)\in\mathcal{K}$ the ratio $-\frac{\theta}{\rho+\frac{\omega}{\Delta}}$ is an information speed upper bound of $v_p(\Delta)$, as long as it is positive.*

Proof. For a given inter-transmission distance Δ, the number of transmissions on a distance L should be smaller or equal to $N=\frac{L}{\Delta}$. Let $\omega''>\omega'>\omega$, according to Theorem 1 there exist A such that we have $P(L,T,N)\leq A\exp(\theta T+\rho L+\omega'\frac{L}{\Delta})$. Let $c(\rho,\theta,\omega)=-\frac{\theta}{\rho+\frac{\omega}{\Delta}}$, we have

$$\begin{aligned}P(T<\frac{L}{c(\rho,\theta,\omega")}\quad|\quad N<\frac{N}{\Delta})&\leq E(L,\frac{L}{c(\rho,\theta,\omega")},\frac{L}{\Delta})\\&\leq A\exp(\theta\frac{L}{c(\rho,\theta,\omega'')}+\rho L+\omega'\frac{L}{D})\\&= A\exp((\omega'-\omega'')\frac{L}{\Delta})\end{aligned}$$

which tends to zero when $L\to\infty$, and since $c(\rho,\theta,\omega'')$ tends to $c(\rho,\theta,\omega)$ when ω'' tends to ω, the quantity $c(\rho,\theta,\omega)$ is a propagation speed upper bound of $v_p(\Delta)$. □

The Kernel is made of (ρ, θ, ω) such that $\theta \geq 0$, $(\theta+\tau)^2 > \tau^2 + v^2\rho^2$ and

$$\omega = -r\rho - \log\left(1 - \frac{\tau}{\sqrt{(\theta+\tau)^2 - \rho^2 v^2}}\right) .$$

Thus the upper bound ratio has expression

$$R(\rho,\theta) = \frac{-\theta}{(1-\frac{r}{\Delta})\rho - \frac{1}{\Delta}\log\left(1 - \frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}\right)} . \tag{6}$$

We notice that the expression $\log\left(\frac{1}{1-\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}}\right)$ is equal to the logarithm of the Laplace transform of the space time vector between the position where a relay receives the packet and the position where it forwards it.

When $\Delta \to \infty$ the ratio becomes $-\frac{\theta}{\rho}$ and the minimum ratio comes when ρ and τ tend to zero. But in this case $\theta = O(\rho^2)$, therefore the minimum ratio is zero. When $\Delta = r$ the ratio becomes

$$R(\rho,\theta) = \frac{\theta r}{\log\left(1 - \frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}\right)} ,$$

which is always negative, therefore there is no speed upper bound. This is confirmed by the fact that the speed is indeed infinite since the journey exclusively made of emission segments arrive to D in zero time.

4.3 Kernel Analysis in the Basic Case

We want to give an analytic expression for the minimum value of $R(\rho,\theta)$. We denote $x = (\theta+\tau)^2 - v^2\rho^2$ the Minkowski norm of vector $(\theta+\tau,\rho)$. Let $\ell(x) = -\log(1-\frac{\tau}{\sqrt{x}})$. We have

$$R(\rho,\theta) = \frac{\theta\Delta}{(\Delta-r)\rho + \ell(x)} .$$

Therefore the optimal value of (ρ,θ) only depends on the difference $\Delta - r$. At the optimal point we must have $\frac{\partial}{\partial\theta}R(\rho,\theta) = 0$ and $\frac{\partial}{\partial\rho}R(\rho,\theta) = 0$. This leads to the Lagrange system:

$$\left\{\begin{array}{rcl} (\Delta-r)\frac{\rho}{\theta} + \frac{1}{\theta}\ell(x) - 2(\theta+\tau)\ell'(x) & = & 0 \\ \Delta - r - 2v^2\rho\ell'(x) & = & 0 \end{array}\right.$$

with $x = (\theta+\tau)^2 - \rho^2 v^2$. The resultant of this system is:

$$2\left((\theta+\tau)\theta - v^2\rho^2\right)\ell'(x) = \ell(x)$$

which can be rewritten in $x - \frac{\ell(x)}{2\ell'(x)} = (\theta+\tau)\tau$ which gives a parametric description of the optimal tuples for $x > \tau^2$:

$$\left\{\begin{array}{rcl} \theta & = & \frac{1}{\tau}(x - \frac{\ell(x)}{2\ell'(x)}) - \tau \\ \rho & = & -\frac{1}{v}\sqrt{(\theta+\tau)^2 - x^2} \\ \Delta & = & r + 2v^2\rho\ell'(x) \end{array}\right.$$

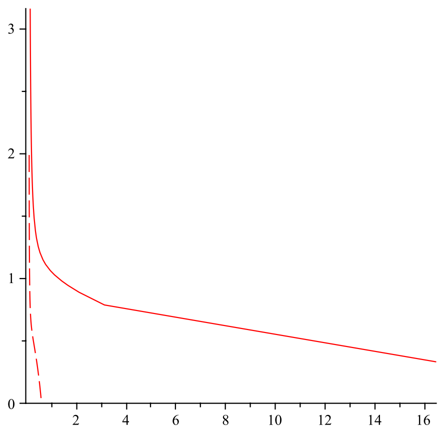

Figure 3: Information upper bound speed versus Δ, compared to actual speed of angle protocol (dashed), for $r = 0.1$, $v = 1$, $\tau = 2$.

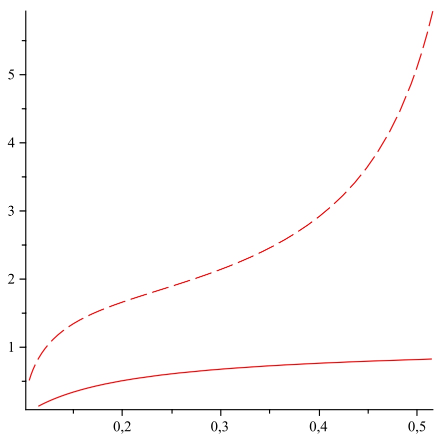

Figure 4: Information slowness versus Δ, compared to actual slowness of angle protocol (dashed), for $r = 0.1$, $v = 1$, $\tau = 2$.

Via straightforward application of analytic expansion when $\Delta \to \infty$ we get

$$v_p = \frac{2v^2}{\tau\Delta}\left(1 - \log 2 + 2\log(\frac{\Delta\tau}{v}) + O(\log\log(\frac{\Delta\tau}{v}))\right) ,$$

and when $\Delta \to r$:

$$v_p = v\frac{r}{\Delta - r}\left(1 + O(\Delta - r)\right)$$

4.4 Speed aware next relaying

In this section we suppose that in a transmission phase the node transmits to the relay which optimizes its motion vector toward the destination. For the upper bound speed we simply assume that the motion vector of the relay exactly heads to the destination. In this case the Laplace transform of the space-time displacement between the reception and the next packet transmission is

$$1+\frac{1}{\theta+\tau+\rho v}\frac{1}{1-\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}}$$

and we have therefore $R(\rho,\theta)=\frac{\theta\Delta}{(\Delta-r)\rho+\ell(\rho,\theta)}$ with

$$\ell(\rho,\theta)=\log\left(1+\frac{1}{\theta+\tau+\rho v}\frac{1}{1-\frac{\tau}{\sqrt{(\theta+\tau)^2-\rho^2v^2}}}\right) .$$

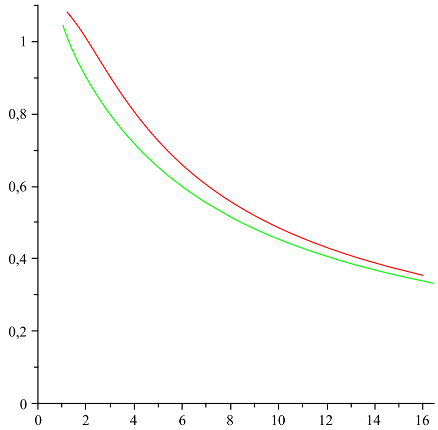

Figure 5: Information speed versus Δ in speed aware next relaying (red) compared to basic relaying (green), for $r=0.1$, $v=1$, $\tau=2$.

Since $\frac{\partial}{\partial\theta}R(\rho,\theta)=0$ and $\frac{\partial}{\partial\rho}R(\rho,\theta)=0$, we have

$$\left\{\begin{array}{rcl} -(\Delta-r)\rho-\ell+\theta\frac{\partial}{\partial\theta}\ell & = & 0\\ (\Delta-r)\rho+\rho\frac{\partial}{\partial\rho}\ell & = & 0\end{array}\right.$$

The Kernel equation is

$$\ell=\theta\frac{\partial}{\partial\theta}\ell+\rho\frac{\partial}{\partial\rho}\ell . \quad (7)$$

But contrary to the previous case, the equation is not separable and we cannot apply Lagrange parametric reduction. Therefore we have to rely on a Euler finite element resolution.

4.5 Moving destination

The previous analysis was done under the hypothesis that the destination D of the packet is fixed. The analysis can be easily extended to the moving destination case. In this case the distance L must be understood as the distance between the original position of the source S at the packet generation and the final position of D, when the later eventually receives the packet. The modification consists into multiplying the journey space-time Laplace transform $w(\rho, \theta, u)$ by the space Laplace transform of the destination trajectory, *i.e.* $\frac{1}{\sqrt{(\theta+\tau)^2-\rho^2v^2}-\tau}$, namely $w(\rho, \theta, 0)$. Therefore the new laplace transform $\tilde{w}(\rho, \theta)$ has expression

$$\tilde{w}(\rho, \theta, u) = w(\rho, \theta, 0)w(\rho, \theta, u) \ .$$

In this case the new laplace transform has the same Kernel $\mathcal{K}$, the roots of $\frac{1}{w(\rho,\theta,0)}$, (*i.e.* $\sqrt{(\theta+\tau)^2 - \rho^2v^2} - \tau$) being screened by the roots of $D(\rho, \theta, \omega)$.

5 A sub-optimal angle protocol

This scheme has been introduced in [4]. The principle (for fixed destination) is the following, assuming θ_c is a non negative protocol parameter strictly smaller than π:

- The relay keeps and carries the packet as long as the angle between its heading and the bearing to the destination is smaller than θ_c.
- Otherwise it transmits the packet to the next hop to the node the closest to the destination.
- If the relay is in range of the destination it transmits the packet to the destination.

The angle protocol is shown to satisfy:

$$\left\{ \begin{array}{rcl} \Delta & = & r + \frac{\theta}{\pi}\frac{\sin(\theta)}{\pi-\theta}\frac{v}{\tau} \\ T & = & \frac{\theta}{\pi-\theta}\frac{1}{\tau} \ , \end{array} \right.$$

where T indicates here the average time between two consecutive transmissions. Notice that Δ cannot exceed $r + \frac{v}{\tau}$. The average propagation speed is therefore $v_p = \frac{\pi-\theta}{\theta} r\tau + \frac{\sin(\theta)}{\pi} v$.

6 Conclusion

We have identified an upper bound propagation speed of information which is function of the transmission inter-distance Δ. This result makes the link between Gupta and Kumar result (where $\Delta \to r(\lambda)$) and Grossglauser and Tse scheme where $\Delta \to \infty$. This confirms the strong implication of space-time in the evaluation of transport capacity of a wireless network.

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Appendix

of theorem 1. we proceed via inverse Laplace 4 on complex tuples (u, θ, ρ):

$$E(L,T,N) = \frac{1}{(2i\pi)^3}\int_{|u|=u_0}\frac{du}{(1-u)u^{N+1}} \quad (8)$$
$$\times \iint_{\Re(\theta)=\theta_0,\Re(\rho)=\rho_0} w(\rho,\theta,u)$$
$$\times \frac{e^{-T\theta-(L-r)\rho}d\theta d\rho}{\theta\rho} \quad (9)$$

where (u_0, θ_0, ρ_0) is a real tuple which belongs to the definition domain of $w(\rho, \theta, u)$. Let $(\rho_0, \theta_0, \omega_0) \in \mathcal{K}$ and let $u_0 = e^{-\omega'}$ with $\omega' > \omega_0$. Thus (ρ, θ, u) with $|u| = e^{-\omega'}$, $\Re(\theta) = \theta_0$ and $\Re(\rho) = \rho_0$ is in the definition domaine of $w(\rho, \theta, u)$. The following inequality holds:

$$E(L,T,N) \leq \frac{\exp(\theta_0 T + \rho_0 L + \omega' N)}{1-u_0}\int_{|u|=u_0} du$$
$$\times \iint_{\Re(\theta)=\theta_0,\Re(\rho)=\rho_0} |w(\rho,\theta,u)|\frac{d\theta d\rho}{\theta\rho}$$

The function $\frac{w(\rho,\theta,u)}{\theta\rho}$ is absolutely integrable. Thus we don't need a sophisticated integration path as in [3]. Indeed since we have $|w(\rho, \theta, u)| \leq w(\Re(\rho), \Re(\theta), |u|)$, a careful analysis gives

$$|w(\rho,\theta,u)| \leq \max\{\frac{1}{(1-u_1)\sqrt{||\theta|^2-|\rho|^2v^2|}}, w(|u|,\Re(\theta),\Re(\rho))\}$$

which confirms the absolute integrability of $\frac{w(\rho,\theta,u)}{\theta\rho}$. Thus

$$E(L,T,N) \leq A\exp(\theta_0 T + \rho_0 L + \omega' N)$$

with

$$A \leq \frac{u_0}{1-u_0}\frac{1}{(2\pi)^2}\iint_{\Re(\theta)=\theta_0,\Re(\rho)=\rho_0}\frac{d\theta d\rho}{\theta\rho}$$
$$\times \max\{\frac{1}{(1-u_0)\sqrt{||\theta|^2-|\rho|^2v^2|}}, w(\rho_0,\theta_0,u_0)\} \ .$$

□

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