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**JEL Codes: E58, E52**

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# Imprecision of Central Bank Announcements and Credibility

Daniel Laskar\*

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## Abstract

We consider a model where the central bank faces a credibility problem in its announcements, but also cares about its credibility and, therefore, wants to make truthful announcements. We show that, although the central bank would be able to perfectly transmit its information to the private sector through precise announcements, the central bank may nonetheless prefer to make imprecise announcements. This choice of the central bank would be suboptimal from the point of view of society. However, if the central bank gives enough weight to making truthful announcements, this suboptimality disappears, because the central bank would then prefer precise announcements to imprecise announcements.

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Keywords: central bank transparency, central bank announcements, imprecise announcements, credibility.

## 1 Introduction

The basic economic reason in favor of central bank (CB) transparency is that more transparency allows the private sector (PS) to have more information. A more informed PS should make better decisions, which should be beneficial. Of course, things may be more complicated. The theoretical literature on CB transparency has exhibited different situations where such an argument may not hold and where, as a consequence, transparency may be harmful

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rather than beneficial. Thus, according to this literature, the results could depend on the model considered and on the specific assumptions made<sup>1</sup>.

Nonetheless, consider a situation where the basic argument in favor of CB transparency holds: it is beneficial to have a more informed PS because the PS can make more efficient decisions. There still remains the problem of how the CB can transmit its information to the PS in a credible way. Credibility of monetary policy is an issue which has received a lot of attention in the literature. This literature has underlined that the CB may have an incentive to manipulate the PS's expectations and that, consequently, its announcements about future monetary policy may not be credible. This leads to some inefficiency when the CB cannot commit to its future monetary policy<sup>2</sup>. But the same credibility problem may also prevent the CB to transmit its information to the PS.

Thus, suppose the CB has some private information on some variable, and that it would be useful to transmit this information to the PS. This variable could concern the objectives of the CB or an underlying shock. Suppose the CB tries to transmit its private information to the PS by telling the PS what its information on this underlying variable is, or, more indirectly, by making an announcement on a variable which depends on this underlying variable, like telling to the PS what its future monetary policy will be. Then, the credibility problem which affects the CB could make these announcements not credible. As a consequence, it is possible that no information to the PS could be transmitted in such a way.

Two remedies to this issue of transmitting the information to the PS could be considered. The first consists in having some signalling cost of making announcements. If the cost is large enough, it may become possible for the CB to transmit its private information through such announcements<sup>3</sup>. The second remedy is to have imprecise announcements. For, even if there are no signalling costs and, therefore, even if announcements are "cheap talk", some information to the PS could still be transmitted through imprecise announcements<sup>4</sup>.

It is likely that both aspects should play a role in the real world. First,

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<sup>1</sup>For some recent survey of the theoretical and empirical literature, see, for example, Blinder et al. (2008) and van der Cruijssen and Eijffinger (2007).

<sup>2</sup>Standard references on these issues are Barro and Gordon (1983) and Kydland and Prescott (1977).

<sup>3</sup>Models of signalling have been applied to monetary policy. For models where, through the signalling of the CB, the PS learns the CB's type, see, for example, Palmqvist (1999) and Vickers (1986).

<sup>4</sup>See the cheap talk model of Crawford and Sobel (1982). Stein (1989) has applied this kind of analysis to monetary policy when the CB has some private information on the exchange rate target.

the CB usually cares about its credibility and therefore is trying to make announcements which will appear to be true ex post<sup>5</sup>. Second, the CB does not always make precise quantitative announcements. A part of the CB's announcements consists in some rather qualitative imprecise statements<sup>6</sup>.

The aim of the present paper is to develop an analysis which simultaneously and explicitly considers these two aspects, and examine how they interact. If there are signalling costs due to the credibility concern of the CB, it may be possible to make credible precise announcements. Then, does this mean that the CB will choose to make these precise announcements? The CB may actually prefer to make imprecise announcements. But to examine this point, we must consider how imprecise announcements can be equilibria when there are some signalling costs. This means that the analysis, which exhibits these imprecise announcements as the only possible equilibria in the cheap talk case, has to be reconsidered in the presence of such costs. These are the issues that we will consider.

In order to simplify the analysis, to be able to solve the model analytically, and to easily relate this analysis to the previous studies found in the literature, we consider a simple static model of credibility of monetary policy. The CB has some useful private information on some underlying variable affecting monetary policy, and makes some announcement about this variable. As the CB is concerned by its credibility, it is assumed that the CB bears some cost when it does not transmit its information in a truthful way. The model is presented in Section 2.

As indicated above, we consider equilibria with precise announcements, and also equilibria with imprecise announcements. These are considered in Section 3. In Section 4, we examine the issue of the choice of the CB between these equilibria. In particular, we want to know whether, and under what conditions, the CB prefers imprecise rather than precise announcements. This is important because, as we will see, from the point of view of society, imprecise announcements are less efficient than precise announcements. Finally, Section 5 concludes.

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<sup>5</sup>Thus, Blinder (2000) has sent a questionnaire on issues related to CB credibility to the heads of all CBs which are members of the BIS. The answers (84 central bankers responded) indicate that "living up to its word" is ranked first in importance.

<sup>6</sup>For example, the Fed has made rather imprecise qualitative statements on its future policy (see Rudebusch and Williams (2006)).

## 2 Model

We consider a very standard and simple static model of monetary policy, where a credibility issue arise because the CB has a desired level of employment which is greater than the level of employment desired by the private sector<sup>7</sup>.

The private sector (PS) is made of a large number of identical sectors and, in each sector, the wage setters want to stabilize employment according to the utility function

$$U^{PS} = -n^2 \quad (1)$$

As all sectors are identical, the variable  $n$  can be taken to represent aggregate employment. In (1), the desired level of employment of the PS has been normalized to zero. Once the nominal wage has been fixed, employment is determined by labor demand according to

$$n = -\alpha(w - \pi) + \eta \quad (2)$$

where  $w$  represents the (log of the) nominal wage (which, as all sectors are alike, is the same in all sectors),  $\pi$  is the inflation rate, and  $w - \pi$  is the (log of the) real wage (because the last period (log of the) price level  $p_{-1}$  has been normalized to zero). Equation (2) indicates that employment is a decreasing function of the real wage. The variable  $\eta$  is a zero-mean random variable which represents a shock which affects employment (a productivity shock). We assume that the CB has some private information about  $\eta$  and, for simplicity, the CB will be assumed to know  $\eta$ <sup>8</sup>.

The timing of the decisions of the CB and of the PS is as follows. At the beginning of the period, the CB makes some announcement about the shock  $\eta$ . After having heard the CB's announcement, the PS determines the nominal wage variable  $w$ , and, subsequently, the CB determines the inflation rate  $\pi$ . The CB has the utility function

$$U^{CB} = -(n - \mu)^2 - \psi(\pi - \bar{\pi})^2 - \xi(\eta^{an} - \eta)^2 \quad (3)$$

---

<sup>7</sup>The model is taken from Rogoff (1985) and is in the line of the models of credibility of monetary policy of Barro and Gordon (1983) and Kydland and Prescott (1977).

<sup>8</sup>We have chosen to present the analysis by considering that the variable on which the CB has private information is the shock  $\eta$  (Therefore, in terms of signalling theory, the "type" of the CB is given by the value of  $\eta$ ). However, we could as well have taken some other variable which affect monetary policy. For example, we could have assumed that the variable on which the CB has private information is the CB's inflation target  $\bar{\pi}$  (see (3) just below), as in the signalling model of Palmqvist (1999). Then, the CB's "type" would be given by the value of  $\bar{\pi}$ . It could easily be seen that, in this case, we would end up with exactly the same formal analysis and results as those we obtain here with the variable  $\eta$ .

The CB wants to stabilize the level of employment  $n$  around a desired level  $\mu > 0$ , which is therefore greater than the level of employment desired by the PS<sup>9</sup>. The CB also wants to stabilize inflation around some desired level  $\bar{\pi}$ .

The variable  $\eta^{an}$  is the "announced value" of  $\eta$ . It is assumed that the PS can observe  $\eta$  *ex post*, and the term  $\xi(\eta^{an} - \eta)^2$  in (3) represents the cost incurred by the CB, due to a loss of credibility, when the CB makes its announcement. This cost is equal to zero when the CB transmits its information in a truthful way, by simply announcing the true value  $\eta$ . But, in general, when the CB makes some other announcements, the CB will bear some credibility cost, because the lack of truthfulness in the announcement will impair the credibility of the CB<sup>10</sup>.

In the case of precise announcements, where the CB announces a value of  $\eta$ , then  $\eta^{an}$  is simply this announced value. In the case of imprecise announcements, where the CB announces that  $\eta$  belongs to some interval, it will be assumed, for simplicity, that  $\eta^{an}$  is the mean of this interval. This announced value  $\eta^{an}$  would correspond to the value of  $\eta$  expected by the PS if the PS did not take into account any other information than what the CB says. Note that even if  $\eta$  did in fact belong to the announced interval, the CB, in general, would still be penalized, because, by being imprecise, the CB would not be telling the whole truth. Thus, if we have  $\eta^{an} \neq \eta$ , the PS would be misled by such an announcement, because the imprecise announcement would tend to give wrong expectations to the PS, which is penalized.

Let  $z^e$  the expectation that the PS has on any variable  $z$  after the CB's announcement. Being small, each wage setter takes expected inflation  $\pi^e$  as given. Maximizing the expected utility  $EU^{PS}$  under the constraint (2) gives the nominal wage

$$w = \pi^e + \frac{\eta^e}{\alpha} \quad (4)$$

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<sup>9</sup>A usual arguments involve the presence of distortions due to taxes.

<sup>10</sup>Our model takes as exogenous this cost due to a loss of credibility, and does not try to explain why the CB would care to loose credibility by making untruthful announcements. This cost takes into account the fact that, in the real world, CBs do care about their credibility and about making truthful announcements.

Some quadratic cost of deviating from an announced policy because the CB cares about its credibility, is introduced in Gerbasch and Hahn (2008), who use this cost function to study the trade-off between gains from commitment and losses of flexibility implied by forward guidance.

From (2) and (4), we get<sup>11</sup>

$$n = \alpha (\pi - \pi^e) + \eta - \eta^e \quad (5)$$

Then, the CB chooses  $\pi$  which maximize  $U^{CB}$ . From (3) and (5), the first order condition is

$$\alpha (n - \mu) + \psi (\pi - \bar{\pi}) = 0 \quad (6)$$

which, using (5), can be written

$$\pi = \frac{1}{\alpha^2 + \psi} [\alpha^2 \pi^e + \psi \bar{\pi} + \alpha \mu - \alpha (\eta - \eta^e)] \quad (7)$$

Taking the expected value of each member of (7) and subtracting to (7) gives

$$\pi - \pi^e = -\frac{\alpha}{\alpha^2 + \psi} (\eta - \eta^e) \quad (8)$$

From (5) and (8) we get

$$n = \frac{\psi}{\alpha^2 + \psi} (\eta - \eta^e) \quad (9)$$

From (1), (3), (6) and (9) we get

$$U^{CB} = -\frac{\alpha^2 + \psi}{\psi} \left[ \frac{\psi}{\alpha^2 + \psi} (\eta - \eta^e) - \mu \right]^2 - \xi (\eta^{an} - \eta)^2 \quad (10)$$

$$U^{PS} = -\left( \frac{\psi}{\alpha^2 + \psi} \right)^2 (\eta - \eta^e)^2 \quad (11)$$

The rest of the paper will be devoted to the issue of the announcement strategy of the CB.

### 3 Equilibria

We will first examine whether there exists an equilibrium where the CB announces a precise value of  $\eta$ . Then, we will also look for equilibria where the CB makes some imprecise announcements.

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<sup>11</sup>In the literature on CB transparency, the analysis has often relied on an expectations-augmented Phillips curve model, given by the equation  $n = \alpha (\pi - \pi^e) + \varepsilon$ , where the shock  $\varepsilon$  is exogenously given. However, as we see here, the shock  $\varepsilon$  should depend on transparency, because, according to (5),  $\varepsilon$  is equal to  $\eta - \eta^e$ , and therefore depends on the expectations of the private sector (more generally, on this point, see Laskar (2010)).

### 3.1 Precise announcements

We consider the case where the CB announces some precise value  $\eta^{an}$ . This announced value is a function of the true value  $\eta$ . From (10), we see that the CB has an incentive to manipulate the expectation  $\eta^e$  of the PS. The CB would like  $\eta^e$  to be equal to  $\eta - \frac{\alpha^2 + \psi}{\psi} \mu$ , and therefore to be smaller than  $\eta$  by an amount proportional to the gap  $\mu$  between the desired levels of employment of the CB and of the PS. The reason is that, according to (4), a lower value of  $\eta^e$  decreases the nominal wage  $w$ . This tends to increase employment and therefore to make employment closer to the CB's employment target. This means that the CB would like the PS to believe that the employment shock  $\eta$  is smaller than what it is in reality.

Thus, it seems natural to look for an equilibrium where the CB's strategy is  $\eta^{an} = \eta - \theta$ , where  $\theta > 0$  is some given "bias" in the CB's announcement<sup>12</sup>. At the equilibrium, the PS takes the strategy of the CB as given and forms its expectations accordingly. This implies

$$\eta^e = \eta^{an} + \theta \quad (12)$$

At the equilibrium, the CB knows that the expectation  $\eta^e$  of the PS is given by (12). From (10) and (12), we get

$$U^{CB} = -\frac{\alpha^2 + \psi}{\psi} \left[ \frac{\psi}{\alpha^2 + \psi} (\eta - \eta^{an} - \theta) - \mu \right]^2 - \xi (\eta^{an} - \eta)^2 \quad (13)$$

The equilibrium condition is that the strategy  $\eta^{an} = \eta - \theta$  is an optimal strategy when the CB maximizes  $U^{CB}$  given by (13). From (13), the first order condition  $\frac{dU^{CB}}{d\eta^{an}} = 0$  for maximizing  $U^{CB}$  yields  $\eta^{an} = \eta - \frac{\psi\theta + (\alpha^2 + \psi)\mu}{\psi + (\alpha^2 + \psi)\xi}$ . Consequently, the strategy  $\eta^{an} = \eta - \theta$  is an equilibrium if and only if we have  $\frac{\psi\theta + (\alpha^2 + \psi)\mu}{\psi + (\alpha^2 + \psi)\xi} = \theta$ , which is equivalent to  $\theta\xi = \mu$ . Therefore, we get:

**Proposition 1** *In the cheap talk case  $\xi = 0$ , there is no equilibrium with precise announcements.*

*In the case  $\xi > 0$ , there is an equilibrium with precise announcements. It is given by  $\eta^{an} = \eta - \tilde{\theta}$ , where the equilibrium announcement bias  $\tilde{\theta}$  is given by*

$$\tilde{\theta} = \frac{\mu}{\xi} \quad (14)$$

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<sup>12</sup>In fact, this particular form does not restrict the generality of the analysis. For, because of the linear quadratic structure of the model, we would only have to consider the linear CB's announcement strategy  $\eta^{an} = \theta_0 + \theta_1\eta$ , where  $\theta_0$  and  $\theta_1$  are coefficients. But, then, it can easily be shown that such a strategy can be an equilibrium only if we have  $\theta_1 = 1$ . Therefore, no other equilibrium strategies would be found by using this extended set of strategies.

The result can be interpreted in the following way. If the CB announced a value of  $\eta^{an}$  marginally smaller than  $\eta - \theta$ , the CB would have the marginal benefit of more favorable expectations of the PS, but it would also have to bear the marginal cost due to the corresponding greater loss of credibility. The marginal benefit is obtained by differentiating the first term of the right hand side of (13). Using the equality  $\eta^{an} = \eta - \theta$ , this gives a marginal benefit equal to  $2\mu$ . The marginal cost is obtained by differentiating the second term of (13), which (using  $\eta^{an} = \eta - \theta$ ) gives a marginal cost equal to  $2\xi\theta$ . An equilibrium is obtained when the marginal cost is equal to the marginal benefit.

### 3.2 Imprecise announcements

Precise announcements exist in the case  $\xi > 0$  but may not be the unique equilibrium. As the analysis of Crawford and Sobel (1982) suggests, there may also exist equilibria with imprecise announcements. Crawford and Sobel (1982) only considered the cheap talk case  $\xi = 0$ , but some similar analysis and results may also hold in the case  $\xi > 0$ . Therefore, we will also look for equilibria with imprecise announcements in the context of our model.

As in the illustrative example of Crawford and Sobel (1982), we will make a uniform distribution assumption in order to simplify the analysis. We will assume that  $\eta$  is uniformly distributed on  $[\eta_m, \eta_M]$ <sup>13</sup>. Then, consider a partition  $(a_0, a_1, \dots, a_{N-1}, a_N)$  of  $[\eta_m, \eta_M]$ , where we have  $a_0 = \eta_m < a_1 < \dots < a_{N-1} < a_N = \eta_M$ . In the same way as for the equilibrium with perfect precision, the CB's announcement strategy may have some bias. Consider a given bias  $\theta > 0$ . Then, the strategy of the CB consists in announcing that  $\eta$  belongs to the interval  $[a_{i-1} - \theta, a_i - \theta]$  whenever  $\eta$  belongs to the interval  $]a_{i-1}, a_i]$ , for  $i = 1, \dots, N$ <sup>14</sup>.

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<sup>13</sup>As a uniform distribution has a finite support, such an assumption, however, leads to the undesirable property that the CB may announce values outside the support. For, as the equilibrium with perfect precision has underlined, the CB's announcements may exhibit some bias  $\theta$  which leads the CB to announce a value which is lower than the true value. This implies that the announced value would be outside the support when  $\eta$  is close enough (i.e. by an amount smaller than  $\theta$ ) to the lower limit  $\eta_m$  of this support. Therefore, we may rather view the uniform distribution assumption as a limit case. We could assume that the support of  $\eta$  is infinite but that the distribution is very close to a uniform distribution. Formally, we could assume that, inside the interval  $[\eta_m, \eta_M]$ , the density is constant, and that the probability that  $\eta$  does not belong to this interval is equal to  $\varepsilon$ . Then, we could consider the uniform distribution as a limit case case when  $\varepsilon$  goes to zero.

<sup>14</sup>It would remain to specify the strategy when  $\eta$  is equal to  $\eta_m = a_0$ . We could assume, for example, that, in this case, the CB announces that  $\eta$  belongs to  $[a_0, a_1]$ . (However,

As the PS knows the CB's strategy, it takes into account this bias  $\theta$  when it determines its expectations, and therefore the PS knows the true interval to which  $\eta$  belongs. Therefore, because of the uniform distribution assumption on  $[\eta_m, \eta_M]$ , when  $\eta$  belongs to  $]a_{i-1}, a_i]$  we have

$$\eta^e = \frac{a_{i-1} + a_i}{2} \quad (15)$$

On the other hand, we have previously assumed that the "announced value"  $\eta^{an}$ , which enters the loss function (10), is equal to the mean of the announced interval. As this announced interval is  $[a_{i-1} - \theta, a_i - \theta]$ , this gives

$$\eta^{an} = \frac{a_{i-1} + a_i}{2} - \theta \quad (16)$$

From (15) and (16), we see that (12) and therefore also (13) are still valid in the case of imprecise announcements.

For each given value of the bias parameter  $\theta > 0$ , we will look for partition equilibria  $(a_0, a_1, \dots, a_{N-1}, a_N)$ . Note that we can limit the analysis to the case  $\theta < \tilde{\theta}$  because any partition equilibrium associated to a bias  $\theta \geq \tilde{\theta}$ , if it existed, would be more imprecise than the equilibrium with precise announcements without having a smaller bias. It would therefore never be chosen by the CB<sup>15</sup>.

We obtain the following result (see Appendix 1 for the proof):

**Proposition 2** *To each value of the announcement bias  $\theta$  such that  $0 \leq \theta < \tilde{\theta}$ , we can associate an equilibrium partition<sup>16</sup>  $(a_0^*, a_1^*, \dots, a_{N^*-1}^*, a_{N^*}^*)$ . The number  $N^*$  of intervals of the equilibrium partition is*

$$N^* = \left\langle -\frac{1}{2} + \frac{1}{2} \left( 1 + \frac{2}{B} \right)^{\frac{1}{2}} \right\rangle \quad (17)$$

where  $\langle x \rangle$  denotes the smallest integer greater than or equal to  $x$ , and where  $B$  is given by

$$B = \frac{1}{\eta_M - \eta_m} \frac{\alpha^2 + \psi}{(\alpha^2 + \psi)\xi + \psi} (\mu - \xi\theta) > 0 \quad (18)$$

other assumptions, like announcing the value  $\eta_m$ , would lead to the same results, because the probability that  $\eta$  is equal to  $\eta_m$  is equal to zero.)

<sup>15</sup>From (20) below, it will be clear that both the imprecision and the bias of the announcement decrease the expected utility of the CB.

<sup>16</sup>As in Crawford and Sobel (1982), there is actually a finite number of equilibria obtained for all values of  $N$  such that  $N \leq N^*$ . However, as in Crawford and Sobel (1982), all equilibria corresponding to  $N < N^*$  are dominated by the equilibrium obtained for  $N = N^*$  (i.e. they are worse for both players). They will therefore be disregarded for the static comparative analysis that we will do.

Let  $\sigma_\eta^2$  be the residual variance of  $\eta$  that the PS expects to have after having heard the CB's announcement. We have

$$\sigma_\eta^2 = \frac{1}{3} \left[ \frac{1}{4N^{*2}} + B^2(N^{*2} - 1) \right] (\eta_M - \eta_m)^2 \quad (19)$$

When we have  $\xi > 0$ , the residual variance  $\sigma_\eta^2$  is a decreasing function of the bias  $\theta$ : the greater the bias, the smaller the imprecision of the announcement. Thus, there is a trade-off between the imprecision and the bias of the announcement. The imprecision goes to zero when  $\theta$  goes to  $\tilde{\theta}$ .

The residual variance  $\sigma_\eta^2$  is a decreasing function of the weight  $\xi$ <sup>17</sup>.

Proposition 2 exhibit similarities with the results obtained in the cheap talk analysis of Crawford and Sobel (1982). In fact, it can be shown that, in the cheap talk case  $\xi = 0$ , through some adequate change of variables, the present model can be put under the form of the illustrative model of Crawford and Sobel (1982) (see Appendix 1). Furthermore, in the case  $\xi > 0$ , the analysis leads to the same kind of solutions as in the case  $\xi = 0$ , provided that some parameter of the model is modified. The change concerns parameter  $B$ , which, according to (18), depends on  $\theta$  and  $\xi$ .

Thus, this result generalizes the result of Crawford and Sobel (1982) to the case  $\xi > 0$ . As (18) indicates, in the cheap talk case  $\xi = 0$ , the equilibrium is the same whatever the bias  $\theta$  is<sup>18</sup>, but, in the case  $\xi > 0$ , parameter  $B$ , and therefore the equilibrium, depend on the bias  $\theta$ .

As indicated in Proposition 2, the imprecision, given by  $\sigma_\eta^2$ , is smaller when the bias  $\theta$  increases. In order to interpret this result, first note that, in the cheap talk case  $\xi = 0$ , imprecise announcements can be an equilibrium because it may not be advantageous for the CB to announce a lower interval and have the private sector have the implied lower expected value for  $\eta$ . For, from (10), it is beneficial for the CB to have  $\eta^e$  lower than  $\eta$  by some amount, but this can be overdone. If  $\eta^e$  is too low, this could be worse. Such a case will occur when, broadly speaking, intervals are large enough. Second, when we have  $\xi > 0$ , there is an additional cost to announce the lower interval. Consequently, a partition with intervals of smaller lengths could become an equilibrium because of this additional cost when we have  $\xi > 0$ . The greater  $\theta$  is, the greater this cost is (because the corresponding marginal cost is equal to  $2\xi\theta$ , as we have previously seen), and, therefore,

<sup>17</sup>Note that, as the inequality  $\theta < \tilde{\theta} = \frac{\mu}{\xi}$  has to be satisfied, then, for a given  $\theta$ , the weight  $\xi$  has to satisfy the inequality  $\xi < \frac{\mu}{\theta}$ .

<sup>18</sup>Therefore, in the cheap talk case, it is not useful to introduce such a bias in the analysis.

the smaller the lengths of intervals can be. This implies a smaller imprecision of the announcements when  $\theta$  increases. This means that there is a trade-off between the imprecision and the bias of the announcement.

As the imprecision goes to zero when  $\theta$  goes to  $\tilde{\theta}$ , the equilibrium with precise announcements is the limit case of the equilibria with imprecise announcements when  $\theta$  goes to  $\tilde{\theta}$ .

Finally, for a given bias  $\theta$ , as Proposition 2 indicates, we have the rather intuitive result that, when the weight  $\xi$  increases, the imprecision of the announcement is reduced.

## 4 Optimal choice of the central bank

### 4.1 Results

In this section we assume  $\xi > 0$  (which means that CB announcements are not cheap talk)<sup>19</sup>. We consider the choice of the CB between the equilibria we have considered. We consider the *ex ante* choice of the CB, when the CB does not yet know the value of the shock  $\eta$ .

These equilibria are obtained by taking a value of  $\theta$  belonging to the closed interval  $[0, \tilde{\theta}]$ , the value  $\theta = \tilde{\theta}$  corresponding to the equilibrium with precise announcements. For all the equilibria considered, we have  $\eta^{an} = \eta^e - \theta$ . Replacing  $\eta^{an}$  by  $\eta^e - \theta$  in (10), and noting that we have  $E(\eta^e - \eta) = 0$  and  $E(\eta^e - \eta)^2 = \sigma_\eta^2$ , we get

$$EU^{CB} = - \left( \frac{\psi}{\alpha^2 + \psi} + \xi \right) \sigma_\eta^2 - \xi \theta^2 - \frac{\alpha^2 + \psi}{\psi} \mu^2 \quad (20)$$

The expected utility of the CB is a decreasing function of  $\sigma_\eta^2$  and of  $\theta$ . Therefore, the CB dislikes both imprecision and bias. The CB dislikes imprecision for two reasons. First, imprecise announcements lead the PS to make less efficient decisions. Second, when there is some imprecision in the announcement, the CB does not tell the whole truth and is therefore penalized.

The CB dislikes a bias in the announcement because of its credibility concern (the CB does not tell the truth). And there is no gain for the CB in having an announcement bias because, at the equilibrium, the PS corrects

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<sup>19</sup>In the cheap talk case  $\xi = 0$ , the CB's choice is trivial because only one equilibrium remains. For the equilibrium with perfect precision does not exist, and there is only one partition equilibrium because, from (18), the bias  $\theta$  has no effect. This is the equilibrium found in Crawford and Sobel (1982).

for the bias: the CB cannot manipulate the expectations of the private sector by introducing an announcement bias.

In the last section, we have seen that there is a trade-off between imprecision and bias. This implies that the CB has to make a choice between the imprecision and the bias of its announcements. In this section we will consider this choice and determine the optimal amount of bias and imprecision that the CB would like. The CB chooses  $\theta \in [0, \tilde{\theta}]$  which maximizes  $EU^{CB}$ , where  $\sigma_\eta^2$  is the function of  $\theta$  given by (17), (18) and (19).

In order to be able to solve this problem analytically, we will limit our analysis to the case where the credibility problem of the CB is, in some sense, small. It will be assumed to be small relatively to the amount of uncertainty that the PS has on the variable on which the CB has some private information. More precisely, from (10), we can see that the CB would like to have the PS believe that  $\eta^e$  is equal to  $\eta - \frac{\alpha^2 + \psi}{\psi} \mu$  instead of  $\eta$ . This gap  $\frac{\alpha^2 + \psi}{\psi} \mu$  is at the origin of the credibility problem of the CB. We will assume that the magnitude of this gap is small when compared to the amount of uncertainty that the PS has on  $\eta$ . Under our assumption of a uniform distribution for  $\eta$ , this amount of uncertainty can be represented by the length  $\eta_M - \eta_m$  of the support of this distribution. Therefore, we will assume that  $\frac{1}{\eta_M - \eta_m} \frac{\alpha^2 + \psi}{\psi} \mu$  is small. As, from (18), we have  $0 < B \leq \frac{1}{\eta_M - \eta_m} \frac{\alpha^2 + \psi}{\psi} \mu$ , this implies that  $B$  is small. Then, we find (see Appendix 2 for a proof):

**Proposition 3** *Let  $\hat{\theta}$  and  $\hat{\sigma}_\eta^2$  be the values of  $\theta$  and  $\sigma_\eta^2$  chosen by the CB. Define  $\bar{\xi} \equiv \frac{6\mu}{\eta_M - \eta_m} > 0$ . Then, we have:*

- *In the case  $0 < \xi < \bar{\xi}$ , the CB prefers to have imprecise announcements (we have  $\hat{\sigma}_\eta^2 > 0$ ). In this case, the bias  $\hat{\theta}$  is equal to  $\bar{\theta} = \frac{1}{6}(\eta_M - \eta_m) = \frac{\mu}{\xi}$  (and therefore we have  $0 < \bar{\theta} < \tilde{\theta}$ ), and the imprecision  $\hat{\sigma}_\eta^2$  is a decreasing function of  $\xi$ , satisfying  $\lim_{\xi \rightarrow \bar{\xi}} \hat{\sigma}_\eta^2 = 0$ .*

- *In the case  $\xi \geq \bar{\xi}$ , the CB prefers to have precise announcements (we have  $\hat{\sigma}_\eta^2 = 0$ ). In this case, we have  $\hat{\theta} = \tilde{\theta} = \frac{\mu}{\xi}$ . Therefore the bias is a decreasing function of  $\xi$ , and goes to zero when  $\xi$  goes to infinity.*

In the case  $0 < \xi < \bar{\xi}$ , the bias  $\tilde{\theta} = \frac{\mu}{\xi}$ , which would be required to have the equilibrium with precise announcements, is too high. The CB prefers the partition equilibrium with the bias  $\bar{\theta} = \frac{\mu}{\xi}$ , which is lower than  $\tilde{\theta}$ . In this case, an increase in  $\xi$  reduces the imprecision without changing the bias. When  $\xi$  reaches the threshold value  $\bar{\xi}$ , the bias  $\tilde{\theta}$  becomes equal to  $\bar{\theta}$ , and the CB prefers the equilibrium with precise announcements. Further increases in  $\xi$  has only the effect of reducing the bias, which is equal to  $\tilde{\theta} = \frac{\mu}{\xi}$ , because the

announcement with perfect precision is always preferred by the CB in this case.

## 4.2 Implications

The cheap talk case appears as an extreme case where precise announcements do not exist. But, as soon as the CB cares about its credibility, precise announcements can be an equilibrium. However, we have shown that the CB may nonetheless choose to make imprecise announcements. Thus, our model is consistent with the fact that, in the real world, we may observe both imprecise announcements and a credibility concern by the CB.

However, as Proposition 3 also indicates, precise announcements can also be an equilibrium. It is not necessary to have an infinite weight  $\xi$ , and therefore to have truthful announcements, in order to have precise announcements. It is enough that this weight becomes larger than or equal to some threshold value  $\bar{\xi}$ .

This last result has welfare implications. For, as it is usually the case in the literature, let us assume that society cares about employment and inflation, and, for simplicity, consider the case where it weights these two objectives in the same way as the CB. This means that society's utility function  $U^s$  can be obtained by replacing  $\xi$  by zero in the expressions giving the CB's utility function. From (20), we therefore have

$$EU^s = -\frac{\psi}{\alpha^2 + \psi}\sigma_\eta^2 - \frac{\alpha^2 + \psi}{\psi}\mu^2 \quad (21)$$

Equality (21) underlines that society only cares about the imprecision of the announcement. Society does not care about the bias of the announcement because it does not have the credibility concern that the CB has and, as we have seen, the presence of an announcement bias does not change the behavior of the PS because the PS corrects for the bias (we have  $\eta^e = \eta^{an} + \theta$ ). Consequently, society always prefers precise announcements to any equilibrium with imprecise announcements. This implies that, as long as the CB chooses to make imprecise announcements, CB announcements are suboptimal. This suboptimality gets smaller when the weight  $\xi$  gets larger. Such a suboptimality completely disappears when the weight  $\xi$  becomes larger than or equal to  $\bar{\xi}$ .

As a consequence, if we are in a situation where we have  $\xi < \bar{\xi}$ , any factor which increases the weight given by the CB to transmitting its information in a truthful way, may be beneficial. In fact, it has been argued that the observed shift of CBs toward more transparency could be explained by the

previous move of CBs toward more independence. The reason is that this made it more necessary for the CBs to explain their policies and therefore to be more transparent. This kind of argument is usually given as the other main reason for having CB transparency, beside the economic reason. From this point of view, the present analysis allows us to relate in some way these two kinds of arguments for transparency: the economic reason and the shift toward more CB independence. The increased pressure put on an independent CB to give its information in a transparent way may be interpreted as an increase in the weight  $\xi$  in our model. Our analysis can therefore be used to examine the implications of this stronger requirement for transparency, due to greater CB independence, in a model which takes into account some economic consequences of transparency. From our analysis, as we could expect, this greater concern about being transparent makes the CB more transparent, in the sense that, from proposition 3, either the bias or the imprecision of CB announcements decreases when the weight  $\xi$  increases. When we start from a situation where we have  $\xi < \bar{\xi}$ , this increase in the CB's pressure for transparency due to the CB's increased independence is beneficial, because it takes the form of reduced imprecision. If its effect on  $\xi$  is large enough, it could even completely eliminate the suboptimality of announcements by leading the CB to make precise announcements<sup>20</sup>.

## 5 Conclusion

We have considered a model where the CB has some useful private information but where a credibility problem may prevent the CB to transmit this information to the PS. We have assumed that the CB, which is concerned by its credibility, has to bear some cost when it does not truthfully reveal its private information. We have shown that this makes precise announcements of the CB credible, and that this would allow the CB to transmit its private information to the PS. The required marginal cost of deviating from the announcement is produced by some announcement bias (a greater bias giving a greater marginal cost). This implies that precise announcements exhibit some announcement bias.

However, beside this equilibrium with precise announcements, there are also equilibria with imprecise announcements. To each level of some an-

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<sup>20</sup>Independent CBs may also exhibit a less acute credibility problem. This effect will reinforce the effect, going through a larger weight  $\xi$ , that we have just considered. For example, if more independence also implies a smaller value of  $\mu$  ( $\mu$  represents the gap between the CB's and the PS's employment targets), then the value of  $\bar{\xi}$ , given in Proposition 3, decreases, which also makes the condition  $\xi \geq \bar{\xi}$  more easily fulfilled.

nouncement bias, there corresponds some imprecision of announcements. The greater the bias is, the more precise the announcements can be. The reason is that a greater bias implies a greater marginal cost of deviating and, consequently, more precise announcements become credible. At the limit, when the bias becomes large enough, precise announcements become possible.

As a consequence, there is a trade-off between the imprecision and the bias of the CB's announcements. As both the bias and the imprecision are costly to the CB, this implies that the CB has to make some optimal choice between the imprecision and the bias of its announcements.

We have studied this optimal choice of the CB. In order to be able to solve this problem analytically, we have limited the analysis to the case where the credibility problem of the CB is small, in proportion to the amount of uncertainty concerning the variable on which the CB has some private information. We have found that when the weight given to the credibility concern (which makes untruthful announcements costly) is smaller than some threshold value, then the CB prefers to make imprecise announcements. And, the lower this weight is, the more imprecise the announcements chosen by the CB is. Although precise announcements can be an equilibrium, the CB prefers to make imprecise announcements as long as this weight is not too large. Thus, in this case, we have both imprecise announcements and some credibility concern of the CB, which are two features which can be observed in the real world.

On the other hand, when the weight given to the credibility concern is greater than or equal to this threshold value, precise announcements are preferred by the CB. This means that it is not necessary to have an infinite weight, and consequently to constrain the CB to truthfully reveal its information, if we want the CB to make precise announcements. What is required is only that the CB prefers to eliminate all imprecision in its announcements.

From the point of view of society, only the imprecision of the CB's announcements matter, because the PS can correct for the bias but cannot correct for the imprecision of announcements. Therefore, when the CB prefers to make imprecise announcements, this is suboptimal from the point of view of society. However this suboptimality disappears when the CB prefers precise announcements, which occurs when the CB gives a large enough weight to truthfully revealing its information.

Our analysis also permits us to relate two factors which, in the literature, were given as separate explanations to the observed shift of CBs toward more transparency. The first is that the previous shift toward CB independence required CBs to explain their policies to the public, and, for that, required them to be more transparent. In our model, this can be interpreted as an

increase in the weight given to truthfully transmitting the information that CBs have. The second factor concerns the beneficial economic effects of transparency. This beneficial effect is taken into account in our model because, in this model, more information allows the PS to make better decisions. Our analysis underlines that the first factor leads the CB to transmit its useful private information to the PS with more precision, which is beneficial from the point of view of society. Furthermore, if CB independence increases the weight given to more truthful announcements by a sufficient amount, this could lead to full transmission of the CB's information through precise announcements.

The model we used was a simple static model of monetary policy, in which a credibility issue arises. This has allowed us to simplify the analysis and to compare it to other existing studies. However, it would be useful to try to deal with the same kind of issues in a dynamic model. As we know<sup>21</sup>, a dynamic model of monetary policy may lead to credibility issues even in the absence of a gap between the employment targets of the CB and of the wage setters (while introducing such a gap was necessary to get a credibility issue in our static model).

## APPENDICES

### 1. Proof of Proposition 2

For a given  $\theta$ , the equilibrium condition is that the CB does not (strictly) prefer to announce that  $\eta$  belongs to an other interval than  $[a_{i-1} - \theta, a_i - \theta]$  when  $\eta$  belongs to  $]a_{i-1}, a_i]$ . And, by continuity, this has to hold also when  $\eta$  belongs to the closed interval  $[a_{i-1}, a_i]$ . For a partition  $(a_0, \dots, a_N)$ , there are only  $N$  discrete possible values for  $\eta^{an}$  which are given by  $\eta^{an} = \frac{a_{j-1} + a_j}{2}$ ,  $j = 1, \dots, N$ . Therefore, a partition is an equilibrium if and only if, when  $\eta$  belongs to  $[a_{i-1}, a_i]$ , the value  $\eta^{an} = \frac{a_{i-1} + a_i}{2}$  is at least as good as any other value  $\eta^{an} = \frac{a_{j-1} + a_j}{2}$ ,  $j = 1, \dots, N, j \neq i$ . But, as the function  $U^{CB}$  given by (13) is a concave quadratic function of  $\eta^{an}$ , this is actually equivalent to having  $\eta^{an} = \frac{a_{i-1} + a_i}{2}$  be at least as good as  $\eta^{an} = \frac{a_i + a_{i+1}}{2}$  and  $\eta^{an} = \frac{a_{i-2} + a_{i-1}}{2}$ , which are the two possible adjacent values for  $\eta^{an}$ . These two conditions can be written  $U^{CB} \left( \frac{a_{i-1} + a_i}{2} \right) - U^{CB} \left( \frac{a_i + a_{i+1}}{2} \right) \geq 0$ ,  $i = 1, \dots, N - 1$ , and  $U^{CB} \left( \frac{a_{i-1} + a_i}{2} \right) - U^{CB} \left( \frac{a_{i-2} + a_{i-1}}{2} \right) \geq 0$ ,  $i = 2, \dots, N$ , for all  $\eta$  belonging to  $[a_{i-1}, a_i]$ . Using (13),

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<sup>21</sup>See Clarida et al. (1999) in the context of the new Keynesian model which has been widely used in the literature.

the first of these two conditions is equivalent to  $f(i, \eta) \geq 0$  for  $i = 1, \dots, N-1$ , and for all  $\eta$  belonging to  $[a_{i-1}, a_i]$ , where we define

$$f(i, \eta) \equiv \left( \xi + \frac{\psi}{\alpha^2 + \psi} \right) (a_{i+1} + 2a_i + a_{i-1} - 4\eta) + 4(\mu - \xi\theta) \quad (22)$$

As  $f$  is a decreasing function of  $\eta$ , the inequality  $f(i, \eta) \geq 0$  holds for all  $\eta \in [a_{i-1}, a_i]$  if and only if we have  $f(i, a_i) \geq 0$ . Therefore the first condition can be written  $f(i, a_i) \geq 0$  for  $i = 1, \dots, N-1$ .

In the same way, it can easily be seen that the second condition is equivalent to  $f(i, a_i) \leq 0$  for  $i = 1, \dots, N-1$ .

Consequently, the equilibrium condition can be written  $f(i, a_i) = 0$  for  $i = 1, \dots, N-1$ . Using (22), this leads to the difference equation

$$a_{i+1} - 2a_i + a_i = -4(\eta_M - \eta_m) B \quad (23)$$

where  $B$  is given by (18) of proposition 2. If we introduce the "normalized" variables  $\eta' \equiv \frac{\eta_M - \eta}{\eta_M - \eta_m}$  and  $a'_i \equiv \frac{\eta_M - a_{N-i}}{\eta_M - \eta_m}$ , then  $\eta'$  belongs to  $[0, 1]$  and  $(a'_0, a'_1, \dots, a'_N)$  is a partition of  $[0, 1]$  where we have  $a'_0 = 0$ ,  $a'_N = 1$  and  $a'_0 < a'_1 < \dots < a'_N$ , as in Crawford and Sobel (1982). From (23), the  $a'_i$  satisfy the difference equation  $a'_{i+1} - 2a'_i + a'_i = 4B$ ,  $i = 1, \dots, N-1$ . This is a difference equation formally identical to the one of the illustrative example of Crawford and Sobel (1982) (equation 21 p. 1441). However, note that, from (18), in our model, the constant term  $4B$  on the right hand side of this difference equation depends on the weight  $\xi$  and on the bias  $\theta$ <sup>22</sup>.

According to the solutions found in the illustrative example of Crawford and Sobel (1982), this leads to a finite number of solutions. However, the equilibrium with the "finest" partition can be selected as the only relevant partition equilibrium, because this equilibrium partition dominates all the other partition equilibria (it always gives higher utility levels for both the CB and the PS). This is the partition which has the greater number of interval  $N^*$ . The value of  $N^*$  is given by (17) (see Crawford and Sobel (1982) p.1441) Then, using the equality  $\sigma_\eta^2 = (\eta_M - \eta_m)^2 \sigma_{\eta'}^2$  and the result for  $\sigma_{\eta'}^2$  found in Crawford and Sobel (1982) (equation 25 p.1442), we get equation (19) of Proposition 2.

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<sup>22</sup>In the case  $\xi = 0$ , our model is actually formally identical to the one of the illustrative example of Crawford and Sobel (1982). If we take  $b \equiv -\frac{\alpha^2 + \psi}{\psi} \frac{1}{\eta_M - \eta_m} \mu$ ,  $V^{CB} \equiv \frac{\alpha^2 + \psi}{\psi} \frac{1}{(\eta_M - \eta_m)^2} U^{CB}$  and  $V^{PS} \equiv \left( \frac{\alpha^2 + \psi}{\psi} \right)^2 \frac{1}{(\eta_M - \eta_m)^2} U^{PS}$ , we can write our model under the form  $V^{CB} = -(\eta'^e - \eta' - b)^2$  and  $V^{PS} = -(\eta'^e - \eta')^2$ , which is similar to the model of the illustrative example of Crawford and Sobel (1982).

Futhermore, Crawford and Sobel (1982) have shown that  $\sigma_\eta^2$  is an increasing function of  $B$ , which goes to zero when  $B$  goes to zero. This result, together with (18), implies that  $\sigma_\eta^2$  is a decreasing function of both  $\theta$  and  $\xi$ , and that  $\sigma_\eta^2$  goes to zero when  $\theta$  goes to  $\tilde{\theta}$ .

## 2. Proof of Proposition 3

*2.1 Proof when we replace  $\langle x \rangle$  by  $x$  in (17).*

Consider a partition equilibrium of bias  $\theta \in [0, \tilde{\theta}]$ . The residual variance  $\sigma_\eta^2$  is given by (17), (18) and (19). In (17) we will replace  $\langle x \rangle$ , and therefore the integer  $N^*$ , by the real variable  $x$ . As Appendix 2.2 below shows, this can be justified when  $B$  is small. From (17), we have

$$x \equiv -\frac{1}{2} + \frac{1}{2} \left(1 + \frac{2}{B}\right)^{\frac{1}{2}} \quad (24)$$

From (18) and (20), we get

$$\frac{dEU^{CB}}{d\theta} = 2\xi g(\theta) \quad (25)$$

where the function  $g(\cdot)$  of  $\theta$  is given by

$$g(\theta) = \frac{1}{2} \frac{1}{\eta_M - \eta_m} \frac{d\sigma_\eta^2}{dB} - \theta \quad (26)$$

Therefore, from (25), as we have  $\xi > 0$ , the sign (and the nullity) of  $\frac{dEU^{CB}}{d\theta}$  is the same as  $g(\theta)$ .

From (19), when we replace  $N^*$  by  $x$ , we have:

$$\frac{d\sigma_\eta^2}{dB} = \frac{1}{3} (\eta_M - \eta_m)^2 \left[ -\frac{1}{2} \frac{x'(B)}{x(B)^3} + 2B^2 x(B) x'(B) + 2B (x(B)^2 - 1) \right] \quad (27)$$

where  $x(B)$  is the function given by (24). Therefore, from (24), the derivative  $x'(B)$  is

$$x'(B) = -\frac{1}{2} B^{-2} \left(1 + \frac{2}{B}\right)^{-\frac{1}{2}} \quad (28)$$

From (17) and (28), when  $B$  is small, we have the equivalences:

$$x(B) \sim (2B)^{-\frac{1}{2}} \quad x'(B) \sim -(2B)^{-\frac{3}{2}} \quad (29)$$

From (29), we obtain  $\lim_{B \rightarrow 0} \left( -\frac{1}{2} \frac{x'(B)}{x(B)^3} \right) = \frac{1}{2}$  ;

$\lim_{B \rightarrow 0} (2B^2 x(B) x'(B)) = -\frac{1}{2}$  ; and  $\lim_{B \rightarrow 0} (2B (x(B)^2 - 1)) = 1$ . Using (27), these equalities imply  $\lim_{B \rightarrow 0} \frac{d\sigma_\eta^2}{dB} = \frac{1}{3} (\eta_M - \eta_m)^2$ . Consequently, from (26),  $g(\theta)$  and therefore also  $\frac{dEU^{CB}}{d\theta}$  have the sign of  $\frac{1}{6} (\eta_M - \eta_m) - \theta$  when  $B$  is small.

Consequently, in the case  $\frac{1}{6} (\eta_M - \eta_m) < \tilde{\theta}$ , which is equivalent to having  $\xi < \bar{\xi} \equiv \frac{1}{\eta_M - \eta_m} 6\mu$ , there is an interior solution  $\hat{\theta} = \frac{1}{6} (\eta_M - \eta_m) = \frac{\mu}{\bar{\xi}}$ . This is the equilibrium with imprecise announcements which corresponds to the partition equilibrium obtained for the optimal bias  $\hat{\theta} = \frac{1}{6} (\eta_M - \eta_m)$ . In this case, as long as we have  $\xi < \bar{\xi}$ , when  $\xi$  increases, the bias remains equal to  $\frac{1}{6} (\eta_M - \eta_m)$ . Therefore, from Proposition 2, when  $\xi$  increases, the imprecision  $\sigma_\eta^2$  decreases, and goes to zero when  $\xi$  goes to  $\bar{\xi}$ . In the case  $\xi \geq \bar{\xi}$ , or equivalently  $\frac{1}{6} (\eta_M - \eta_m) \geq \tilde{\theta}$ , we have  $\frac{dEU^{CB}}{d\theta} > 0$  for all  $\theta \in [0, \tilde{\theta}]$ , and we have  $\frac{dEU^{CB}}{d\theta} \geq 0$  for  $\theta = \tilde{\theta}$ . We therefore get the corner solution  $\hat{\theta} = \tilde{\theta}$ , which corresponds to precise announcements. This gives Proposition 3.

## 2.2 Justification to replacing $\langle x \rangle$ by $x$ in (17)

In the above proof, the function  $\langle x \rangle$  has been replaced by  $x$ , where  $x$ , given by (24), is the term inside  $\langle \cdot \rangle$  in (17). We will now argue that, when  $B$  is small, this can be justified. As, from (18), there is a one to one relationship between  $\theta$  and  $B$ , the optimization problem of the CB can be seen as finding the value of  $B$  which maximizes  $EU^{CB}$ . Then, consider the problem of maximizing  $EU^{CB}$  where  $B$ , instead of being a continuous variable, is a discrete variable obtained by only taking the values of  $B$  which, according to (24), give integer values of  $x$ . Then, for this optimization problem, where  $B$  takes these discrete values, having  $\langle x \rangle$  instead of  $x$  would make no difference. Therefore, any error due to the substitution of  $x$  to  $\langle x \rangle$  would be negligible if we can show that the difference between taking this discrete  $B$  variable instead of the continuous  $B$  variable would itself be negligible. This would be the case if the lengths of the intervals between two adjacent values of the corresponding discrete  $B$  variable are negligible.

Consider such an interval  $[\langle \bar{x} \rangle, \langle \bar{x} + 1 \rangle]$ . By definition, the length of such an interval is equal to  $\int_{\langle \bar{x} \rangle}^{\langle \bar{x} + 1 \rangle} \left| \frac{dB}{dx} \right| dx$ . This would be negligible if  $\left| \frac{dB}{dx} \right|$  is negligible. But  $\frac{dB}{dx}$  is equal to  $\frac{1}{x'(B)}$ , and, according to (29), we have  $x'(B) \sim -(2B)^{-\frac{3}{2}}$ . This implies  $\left| \frac{dB}{dx} \right| \sim (2B)^{\frac{3}{2}}$ . When  $B$  is small, this is negligible relatively to  $B$ . Therefore, by having replaced  $\langle x \rangle$  by  $x$  in the proof, any error we could have made on the optimal announcement chosen by the CB, would be negligible.

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