Experimental results of an Hinfinity-observer for an industrial semi-active suspension

Sébastien Aubouet, Luc Dugard, Olivier Sename

Experimental results of an $\mathcal{H}_\infty$-observer for an industrial semi-active suspension

S. Aubouet * **L. Dugard** ** **O. Sename** **

* *SOBEN S.A.S., Pôle Mécanique d'Alès Cévenes, Vallon de Fontanes, 30520 St-Martin de Valgalgues, FRANCE,* (sebastien.aubouet@soben.fr)
** *GIPSA-lab, Control Systems Departement, ENSE3, Domaine Universitaire, 38402 Saint-Martin d'Hères, FRANCE,* (olivier.sename@inpg.fr, luc.dugard@inpg.fr)

Abstract: In this paper, an $\mathcal{H}_\infty$-observer to be used in suspension control application is addressed to estimate a quarter car model equipped with an industrial SOBEN damper, using a reduced number of sensors. Vehicle estimation is one the main challenges in suspension control since many control strategies developped in past studies use variables that are nonmeasurable in practice. This observer is designed in the $\mathcal{H}_\infty$ framework in order to minimize the effect of the unknown road disturbance on the estimated states and uses a single, cheap and reliable acceleration sensor mounted on the unsprung mass. The estimated variables are the vertical velocities and positions of the sprung and unsprung masses of the quarter car. Some experiments done on a testing car highlight the performances of the observer. This observer could be used in a suspension control application in order to reduce the number of sensors and the cost of the control system.

Keywords: $\mathcal{H}_\infty$-observer, automotive, suspension

1. INTRODUCTION

This paper is concerned with the implementation of an embedded state observer for a quarter car equipped with a semi active suspension. Experimental results obtained by SOBEN on a testing car are presented here to highlight the performance of the synthesis method detailed in a previous paper Aubouet et al. (2006).

Suspension control based on quarter vehicles has been widely explored in the past few years to improve vertical movements. Active control laws have been developed Choia et al. (2000); Gaspar et al. (2004); Fialho and Balas (2002), and semi-active control laws Spelta (2008); Canale et al. (2006); Giorgetti et al. (2006); Sammier et al. (2003). Active suspensions provide excellent performances but are not realistic in an industrial context because of the excessive cost of the actuators and their huge energy consumption. Semi-active suspensions provide satisfying performances and can be adopted in mass-produced vehicles if the number and the cost of the sensors required by the control strategy is low, which has not always been the case in the past studies. Furthermore, many control strategies assume a full-state measurement Tseng et al. (1991); Yi et al. (1994), or require at least two sensors as in the well-known Skyhook control strategy Spelta (2008); Sammier et al. (2003). Therefore the state estimation problem is very important if we wish to reduce the number of sensors, i.e. reduce the cost and improve the reliability of the system. Unknown input observers have been studied by many authors Koenig (2006, 2005); M.Darouach (2000, 1994); Yi (1995); Tsui (1996), and also applied to automotive systems affected by road disturbances J.K. Hedrick (1994); Yi and Suk (1999). In Yi and Suk (1999), a disturbance decoupled quarter car observer is designed using the vertical accelerations of the sprung and unsprung masses, but two expensive sensors are required, and this observer is too sensible to measurement noise to be implemented in a real car, where the noise is very important due to the vibrations of the engine.

This paper deals with an observer that estimates the state of the vertical quarter car model using a single reliable accelerometer. The observer is designed in the $\mathcal{H}_\infty$ framework in order to minimize the effect of the unknown road disturbance on the estimated states. This work completes some previous results on the same project bringing experimental results obtained on a real testing car. Indeed in Aubouet et al. (2006), the synthesis equations of the observer were detailed and the observer performance was emphasized in simulation only. Here some experiments show the performances of the observer in the embedded application. The industrial SOBEN damper considered in the application has been described in detail in Aubouet et al. (2008b).

This paper is organised as follows: Section 2 presents the system to be observed, Section 3 recalls the estimation problem considered in this paper, Section 4 deals with the synthesis of the $\mathcal{H}_\infty/LPV$ observer and Section 5 gives some experimental results. This paper is finally concluded in Section 6 and some future developments are proposed.

2. VEHICLE MODEL

In this section, the system to be observed is presented. This vertical linear quarter car model represented on Figure 2 is classical and suits for vehicle estimation since its structure is simple and represents the interesting degrees of freedom of a real quarter car. Figure 2 also presents a picture of the new SOBEN damper installed on the testing car used for the experiments.

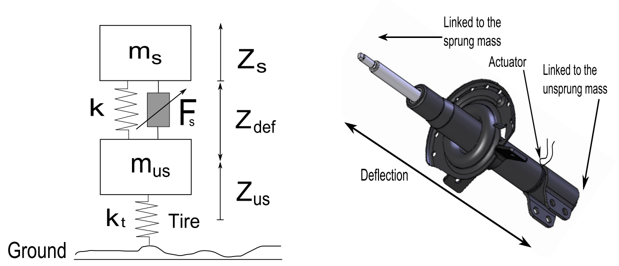

Fig. 1. Quarter car model and SOBEN damper

This simple vehicle model is made up of a sprung mass, a spring, a damper, an unsprung mass and a tire modelled by a spring. The parameters of this model are given in the Table 1.

Table 1. Quarter car parameters and variables

m_s, m_{us}	Sprung, unsprung mass
k, k_t	Suspension, tire stiffness
z_r	Ground vertical position
$\ddot{z}_s$, $\ddot{z}_{us}$	Sprung, unsprung mass acceleration
z_s, z_{us}	Sprung, unsprung mass position
$z_{def} = z_s - z_{us}$	Suspension deflection
F_s	Damping force

If the linear model of the damper is considered, the equations of this model are given by (1).

$$\begin{cases} m_s\ddot{z}_s = k(z_{us} - z_s) + c\cdot(\dot{z}_{us} - \dot{z}_s) \\ m_{us}\ddot{z}_{us} = k(z_s - z_{us}) + c\cdot(\dot{z}_s - \dot{z}_{us}) + k_t(z_r - z_{us}) \end{cases} \tag{1}$$

where c represents the linear damping rate of the suspension. This quarter car model will be used in the synthesis of the observer and can be formulated as a linear time invariant system given by (2).

$$\begin{cases} \dot{x} = A\cdot x + D\cdot v \\ y = C\cdot x \end{cases} \tag{2}$$

where $v = \dot{z}_r \in \mathcal{R}^d$ is the unknown road disturbance, $x = (z_{def}, \dot{z}_s, z_{us} - z_r, \dot{z}_{us})^T \in \mathcal{R}^n$ are the state variables of the quarter car model, $y \in \mathcal{R}^m$ is the vertical acceleration of the unsprung mass measured by an accelerometer and $A \in \mathcal{R}^{n,n}$, $D \in \mathcal{R}^{n,d}$ and $C \in \mathcal{R}^{m,n}$ are given by

$$A = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -\dfrac{k}{m_s} & -\dfrac{c}{m_s} & 0 & \dfrac{c}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{m_{us}} & \dfrac{c}{m_{us}} & -\dfrac{kt}{m_{us}} & -\dfrac{c}{m_{us}} \end{pmatrix},$$

$$C = \tfrac{1}{m_{us}}\cdot\begin{pmatrix} k \\ c \\ -k_t \\ -c \end{pmatrix}^T, \quad D = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

This model will be used in the observer synthesis in Section 4.

Remark:

In (2), no control signal u is considered. This choice will be detailed in section 5.

3. PROBLEM STATEMENT

The system to be observed is the quarter car model presented in Section 2 and given in (2). The full-order observer synthesized in this paper has the general structure given by (3).

$$\begin{cases} \dot{z} = N\cdot z + L\cdot y \\ \hat{x} = z - E\cdot y \end{cases} \tag{3}$$

Where $z \in \mathcal{R}^n$ is the state variable of the observer and $\hat{x} \in \mathcal{R}^n$ the estimated state variables. $N \in \mathcal{R}^{n,n}$, $L \in \mathcal{R}^{n,m}$, $E \in \mathcal{R}^{n,m}$ are matrices to be designed. Then considering (2) and (3), the estimation error can be expressed as

$$e = x - \hat{x} = (I_n + EC)\cdot x - z \tag{4}$$

and then the dynamics of the estimation error is:

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} \\ \dot{e} &= Ax + Dv - Nz - Ly + EC(Ax + Dv) \end{aligned} \tag{5}$$

By using (4), (5) leads to

$$\dot{e} = Ne + (A - N(I_n + EC) - LC + ECA)x + (D + ECD)v \tag{6}$$

Let us define $K = NE + L$ and $P = I_n + EC$, then (6) turns into

$$\dot{e} = Ne + (PA - (N + KC))x + PDv \tag{7}$$

The state $\hat{x}$ is an asymptotic estimate of x for any $\hat{x}(0)$ and $x(0)$ if and only if N is Hurwitz and

$$\begin{cases} N = PA - KC \\ PD = 0 \end{cases} \tag{8}$$

The design of the observer involves the calculation of $N \in \mathcal{R}^{n,n}$, $L \in \mathcal{R}^{n,m}$, $E \in \mathcal{R}^{n,m}$ satisfying (8). A method to solve this problem is proposed in Section 4.

4. OBSERVER DESIGN

In this section, a method is proposed to synthesize a road disturbance decoupled $\mathcal{H}_\infty$ full-order observer based on the unsprung mass vertical acceleration measurement. The problem formulated in Section 3 is solved. Some previous works on this topic have been used and completed, like M.Darouach (1994, 2000).

4.1 Road disturbance decoupling

The first condition of (8) is equivalent to

$$z\cdot\psi = A \tag{9}$$

where $z \in \mathcal{R}^{n,(n+2m)}$ and $\psi \in \mathcal{R}^{(n+2m),n}$ are defined by

$$\begin{cases} z = \begin{pmatrix} N & K & E \end{pmatrix} \\ \psi = \begin{pmatrix} I_n \\ C \\ -CA \end{pmatrix} \end{cases} \tag{10}$$

There exist a solution z of (9) if

$$rank(\psi) = rank\begin{pmatrix} \psi \\ A \end{pmatrix} \tag{11}$$

Since condition (11) is satisfied, the solution exists and is of the form $z = \alpha + \mathbf{Y}\beta$ where

$$\begin{cases} \alpha = A\cdot\psi^+ \\ \beta = I_{n+2m} - \psi\cdot\psi^+ \end{cases} \tag{12}$$

$\mathbf{Y}$ is any matrix with appropriate dimensions and ψ^+ is any generalized inverse matrix of ψ. The matrix $\mathbf{Y}$ will be determined later. From $z = \alpha + \mathbf{Y}\beta$, (4) turns into

$$\dot{e} = N \cdot e + (I_n + EC)D \cdot v \tag{13}$$

Let us define $\tilde{N} \in \mathcal{R}^{(n+2m),n}$ and $\tilde{E} \in \mathcal{R}^{(n+2m),m}$ such that

$$\tilde{N} = \begin{pmatrix} I_n \\ 0_{m,n} \\ 0_{m,n} \end{pmatrix} \ \tilde{E} = \begin{pmatrix} 0_{n,m} \\ 0_{m,m} \\ I_{m,m} \end{pmatrix}$$

Therefore we have

$$N = z \cdot \tilde{N}$$
$$E = z \cdot \tilde{E}$$

Finally, (13) can be expressed as

$$\dot{e} = A_0 \cdot e + B_0 \cdot v \tag{14}$$

with $A_0 = (\alpha + \mathbf{Y}\beta)\tilde{N}$ and $B_0 = (I_n + (\alpha + \mathbf{Y}\beta)\tilde{E}C)D$.

The equation (14) that rules the estimation error is affected by the unknown road disturbance v. If $PD = \mathcal{O}$, the effect of the road disturbance on the estimated states is cancelled. This condition is equivalent to $(I_n + EC)D = \mathcal{O}$, where E has to be determined. This equation is solvable if and only if $rank(CD) = rank(D)$. Otherwise the disturbance effect has to be minimized and the problem is to find $\mathbf{Y}$ such that A_0 is stable and the effect of v on e is minimized.

Proposition 1. There exists a full-order observer ensuring $\| e/v \|_\infty < \gamma_\infty$ if there exists $\mathbf{X} = \mathbf{X}^T \succ 0$, $\tilde{\mathbf{Y}}$ and a scalar γ_∞ that solve the LMI (15).

$$\begin{pmatrix} Q_1 + Q_1^T & Q_2 & I_n \\ * & -\gamma_\infty I_d & \mathcal{O}_{d,n} \\ * & * & -\gamma_\infty I_n \end{pmatrix} \prec 0 \tag{15}$$

Where $\mathbf{X} = \mathbf{X}^T \succ 0$, $\tilde{\mathbf{Y}} = \mathbf{X}\mathbf{Y}$ are the decision variables and

$$\begin{cases} Q_1 = (\mathbf{X}\alpha + \tilde{\mathbf{Y}}\beta)\tilde{N} \\ Q_2 = (\mathbf{X} + (\mathbf{X}\alpha + \tilde{\mathbf{Y}}\beta)\tilde{E}C)D \end{cases} \tag{16}$$

Remark: γ_∞ has to be minimized in order to minimize the road disturbance effect on the estimated variables.

Proof 1. This proof has already been detailed in Aubouet et al. (2006).

4.2 Filtering

In this paragraph, a weighting filter has been added to the system to focus the interesting frequency range where the disturbance effect minimization has to be done. Indeed, it is important that the bandwidth of the observer suits the bandwidth of the system to be observed. This filter allows the designer to choose the frequency range where the observer has to provide the best results. For suspension control applications, the interesting frequency range where the damper is controlled is $[0 - 20Hz]$, therefore the bandwith of the weighting filter will be chosen equal to $20Hz$. The new estimation variable to be considered in this section is the filtered estimation variable e_f. Therefore the problem is now to minimize γ_∞ such that

$$\| e_f/v \|_\infty < \gamma_\infty \tag{17}$$

Proposition 2. There exist a full-order observer ensuring (17) if there exist $\mathbf{X}_1 = \mathbf{X}_1^T \succ 0$, $\mathbf{X}_2 = \mathbf{X}_2^T \succ 0$, $\tilde{\mathbf{Y}}$ and a scalar γ_∞ that solve (18).

$$\begin{pmatrix} A_0^T\mathbf{X}_1 + \mathbf{X}_1 A_0 & \mathcal{O}_n & \mathbf{X}_1 B_0 & \mathcal{O}_n \\ \mathbf{X}_2 B_f & \mathbf{X}_2 A_f & \mathcal{O}_{n,d} & I_n \\ * & * & -\gamma_\infty I_d & \mathcal{O}_{n,d} \\ * & * & * & -\gamma_\infty I_n \end{pmatrix} \prec 0 \tag{18}$$

Where $\mathbf{X} = \mathbf{X}^T \succ 0$ is defined such that

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathcal{O}_n \\ \mathcal{O}_n & \mathbf{X}_2 \end{pmatrix} \tag{19}$$

$A_f \in \mathcal{R}^{n,n}$ and $B_f \in \mathcal{R}^{n,n}$ determine a given weighting filter.

Proof 2. This proof has already been detailed in Aubouet et al. (2006).

4.3 Pole placement

This method ensures the stability of the observer and the minimization of the disturbance effect, but the poles of the observer may be very high or comprise high imaginary parts. Such poles may render the observer oscillating and sensible to measurement noises. In order to avoid such a behavior that may lead to implementation problems and bad estimation performances, a pole placement method Chilali et al. (1999) using LMI regions has been introduced. Therefore the poles of the observer will be chosen automatically to minimize the disturbance effect, but they will be chosen in an appropriate region chosen by the designer. The poles of the observer can be placed in the intersection of a cone $\mathcal{D}_1$, given by the LMI region (20) and a half plane $\mathcal{D}_2$, given by (21). The cone is defined with apex at the origin and inner angle 2θ to ensure that the observer is stable and has poles with moderate imaginary parts compared to real parts. The half plane is delimited by a vertical straight line to ensure that the poles have real parts higher than $-p_m$.

$$\mathcal{D}_1 = \left\{ z \in \mathcal{C} : \begin{pmatrix} \sin\theta(z + \bar{z}) & \cos\theta(z - \bar{z}) \\ \cos\theta(\bar{z} - z) & \sin\theta(z + \bar{z}) \end{pmatrix} \prec 0 \right\} \tag{20}$$

$$\mathcal{D}_2 = \{ z \in \mathcal{C} : -z - \bar{z} - 2p_m \prec 0 \} \tag{21}$$

Then some LMI constraints can be added to the synthesis equations to ensure that the poles of the observer are in these regions. These LMI are given in Aubouet et al. (2006), and the pole placement method in LMI regions is detailed in Chilali et al. (1999).

Therefore to summarize 4.1, 4.2 and 4.3, the method to design the proposed LTI observer can be formulated as follows:

- Choose the weighting filter A_f and B_f appropriate to the system
- Choose $\mathcal{D}_1$ and $\mathcal{D}_2$ according to the desired poles real and imaginary parts bound
- Solve LMI (18) to find $\mathbf{X}_1$ and $\tilde{\mathbf{Y}}$
- Calculate $\mathbf{Y} = \mathbf{X}_1^{-1}\tilde{\mathbf{Y}}$, $z = \alpha + \mathbf{Y}\beta$ using (12)
- Deduce N, K, E, $L = K - NE$

Remark

The upper bound p_{max} has also to be chosen by the designer. It determines the bandwith of the observer. The observer must be faster than the system to be observed to get accurate results. However for noise filtering, this bound has to be less than the frequencies of the noise. For the application considered in this paper, the noise is located in the frequency range $[400 - 1000Hz]$. Therefore $p_{max} = 100$ allows the observer to filter the noise.

5. EXPERIMENTAL RESULTS

In this section, numerical results are given and different experimental results are presented to evaluate the observer performances.

5.1 Synthesis results

In this paragraph, the numerical values of the calculated observer are given. The chosen filter is described by the diagonal structure proposed in paragraph 4.2 where $\omega_f = 2\pi \cdot 20$ and $Gf = \omega_f$. The LMI regions (20) and (21) are respectively determined by $\theta = \frac{\pi}{4}$ and $p_m = 100$. The minimal γ_∞ obtained solving the LMI is $\gamma_\infty = 0.34$. The poles of the observer and the variables to be computed are given bellow:

$$Poles = \begin{pmatrix} -88.63 \\ -54.42 \\ -13.16 \\ -1.62 \end{pmatrix}, L = \begin{pmatrix} -0.0036 \\ 0.0880 \\ -0.0244 \\ 1.0006 \end{pmatrix},$$

$$PA-(N+KC) = 10^{-5} \cdot \begin{pmatrix} 3.41 & -2.21 & -7.15 & -0.23 \\ -78.18 & 50.72 & 163.92 & 5.29 \\ -0.27 & 0.17 & 0.57 & 0.01 \\ -6.44 & 4.18 & 13.50 & 0.43 \end{pmatrix},$$

$$PD = 10^{-2} \cdot \begin{pmatrix} 0.2100 \\ -4.2190 \\ -0.0102 \\ -0.0758 \end{pmatrix}, E = 10^{-3} \cdot \begin{pmatrix} 0.0455 \\ -0.9141 \\ 0.2144 \\ -0.0164 \end{pmatrix},$$

$$N = \begin{pmatrix} -15.6125 & 2.6223 & 31.9974 & -0.3860 \\ -149.5006 & -65.4088 & -95.8086 & 13.6066 \\ 28.6836 & 1.8587 & -76.7823 & -2.0463 \\ 1.3160 & -4.5023 & -1.0315 & -0.0384 \end{pmatrix},$$

$$cond(N) = 188$$

These numerical results show that the disturbance effect has been reduced, since PD is small and the condition $PA - (N + KC) = 0$ has been respected. The Bode diagrams of the transfer functions between the ground disturbance v and the estimation error e on each state, given on Figure 2, emphasize the attenuation of the ground disturbance effect on the estimation error.

Furthermore the pole placement constraints have been respected. Figure 3 shows that the poles have been placed in the specified cone and their real part are smaller than 100. The cone constraint corresponds to a negative real part with an imaginary part smaller than half the real part. Here the computed poles are real. Both he conditioning number *cond* of the matrix N and the pole placement are correct, therefore this observer can be implemented.

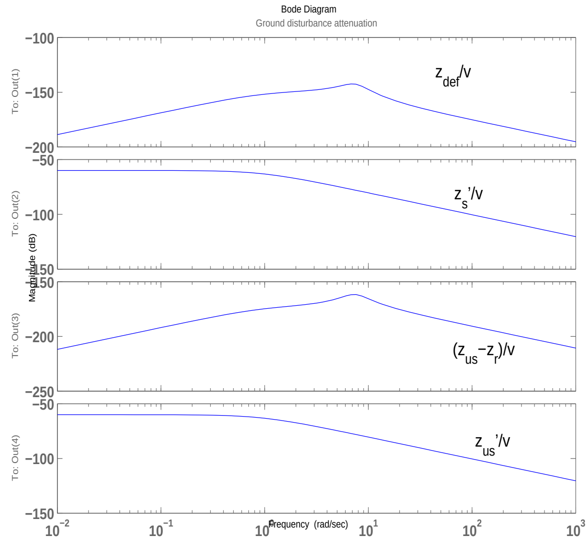

Fig. 2. Transfer $|| \, e/v \, ||$ - Bode diagrams

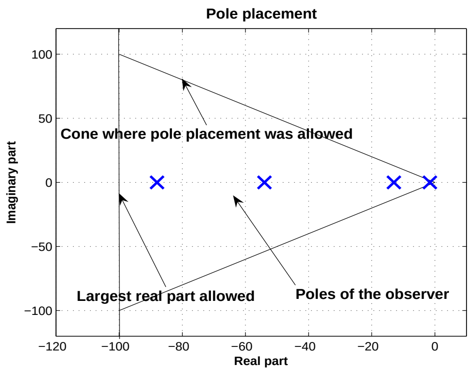

Fig. 3. Pole placement of the observer

5.2 Description and set-up of the experiment

Four semi active damper prototypes have been build by SOBEN and mounted on a testing car. These dampers can be controlled by the mean of a servomechanism installed on each damper to control the internal oil flow, and therefore, the damping properties. The Figure 4 represents the damper prototype mounted on SOBEN testing car.

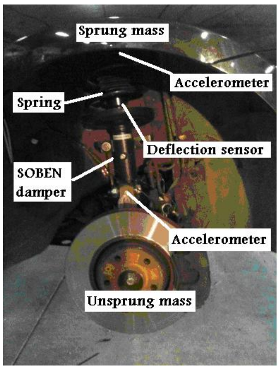

Fig. 4. Mounted semi active SOBEN damper

The vehicle is equipped with a deflection sensor, a vertical unsprung mass accelerometer and a vertical sprung mass accelerometer for each suspension. The acquisition of these data is done by a set of five electronic boards developped by SOBEN. Each damper has a small acquisition board (see Figure 5) that converts the analog measurements into CAN frames(Controller Area Network).

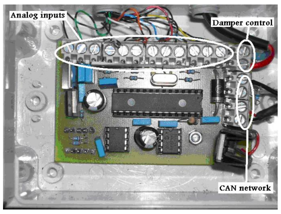

Fig. 5. Damper control board

A central board where the observer is implemented receives the frames through the CAN network and computes the estimated states for each damper, using the four unsprung masses accelerations. The other sensors (deflections and sprung masses accelerations) are not used in the observer. These sensors have only been installed to validate the observer. This architecture is described on Figure 6.

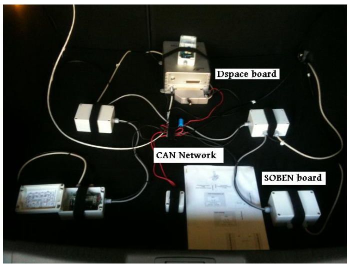

Fig. 6. Control architecture of the four dampers

The four damper boards are operational, but the central board has not been programmed yet. Therefore a Dspace card has been used for the experiments presented in this paper. The implementation of the observer is done automatically by the software provided by Dspace. However, the implementation can not be done if the observer is ill-conditionned. Therefore the poles of the observer have to be properly chosen. A PC can be connected to analyse and to record on-line the different signals.

5.3 Experimental results

This section gives the experimental results obtained with the observer designed in section 7. The estimated deflection velocity of the suspension has been compared to the measured velocity on Figure 7 (top). To validate the observer, a potentiometric sensor has been installed on the damper. The velocity measurement is very useful in a suspension control application, but this sensor can not be used in mass-produced vehicles because of its price and very small life time. Furthermore, the quality of the measured signal is very bad because of the noise. This noise could be reduced with a filter, but the introduced delay would damage the control performances if this filtered signal were used in a controller. Here the proposed observer reduces the noise witout introducing any delay.

Figure 7 also shows the sprung mass vertical acceleration. A second accelerometer has been installed on the sprung mass to validate the estimated variables. The measured sprung mass acceleration has been compared to the estimated one, wich is given on Figure 7 (middle). This variable is also well estimated, and well filtered. The unsprung mass acceleration (measured and estimated), which is the input of the observer, is represented on Figure 7 (bottom). The estimation and measurement are very similar.

Many control strategies have been developped to control the suspensions using the sprung mass acceleration. It would have been possible to install the accelerometer used by the observer on the sprung mass, but in this case, the installation of semi active dampers on an existing car would take a very long time and be very expensive. Therefore this is not possible on mass-produced vehicles. If the accelerometer is installed directly on the damper, installing semi active dampers is easier because it simply consists in changing the dampers. Then the unsprung mass acceleration is measured and the observer can be used to estimate and filter the sprung mass acceleration.

Remark:
In the observer synthesis, no control input of the damper has been taken into account. In the observer, the damper has been considered linear, with a constant damping rate, which is not realistic since SOBEN damper is nonlinear and semi active, therefore its damping rate is variable and controlled. Different experiments have been done with the observer proposed here, for different control signals, which corresponds to different damping rates. The damping rate does not influence the accuracy of the estimations, therefore taking these variations into account in the observer is not useful.

6. CONCLUSION

In this paper, a method to synthesize an observer for a suspension control application has been presented. This observer is based on a single reliable sensor providing the unsprung mass acceleration measurement. The estimation is decoupled from the unknown road disturbance through an $\mathcal{H}_\infty$ minimization and some ponderation filters are introduced to focus the accuracy of the observer on the interesting frequency range. The proposed synthesis method also includes a pole placement in LMI regions to avoid inadapted dynamics that may preclude the implementation and damage the estimation accuracy in the real embedded application. Finally, this observer has been implemented and embedded on a testing car. The experiment results emphasize the observer performance, and the industrial interest for an automotive suspension control application. Future works will consist in designing such an observer for a full car. Then this observer will be included in the static state feedback control strategy detailed in Aubouet et al. (2008a) and implemented by SOBEN on the testing car in the near future. The objective is to design a global attitude control strategy using the four suspensions with only one

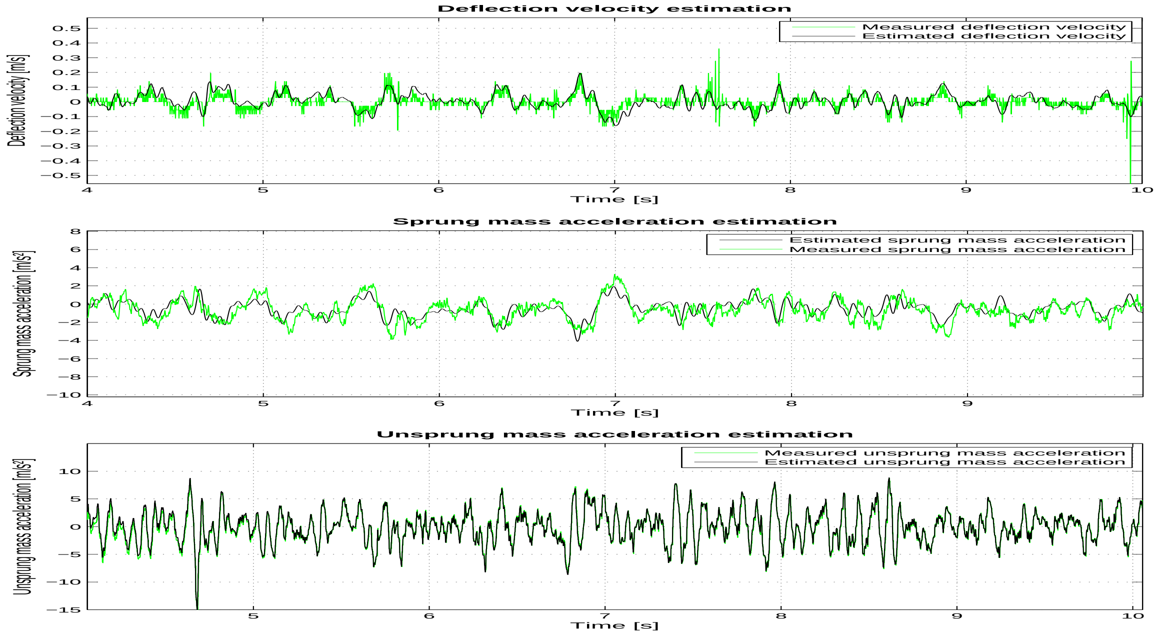

Fig. 7. Experimental results

acceleration sensor for each suspension. A reduced-order observer version of this observer could also be developped.

REFERENCES

Aubouet, S., Dugard, L., and Sename, O. (2006). $\mathcal{H}_\infty$/lpv observer for a semi active suspension. In *3rd IEEE Multi-conference on Systems and Control, Conference on Control Applications (CCA'09)*. Saint Petersburg, RUSSIA.

Aubouet, S., Dugard, L., Sename, O., Poussot-Vassal, C., and Talon, B. (2008a). Semi-active $\mathcal{H}_\infty$/lpv control for an industrial hydraulic damper. In *European control conference 2009*. Budapest, Hungary.

Aubouet, S., Sename, O., Talon, B., Poussot-Vassal, C., and Dugard, L. (2008b). Performance analysis and simulation of a new industrial semi-active damper. In *Proceedings of the* 17*th IFAC World Congress*. Seoul, Korea.

Canale, M., Milanese, M., and Novara, C. (2006). Semi-active suspension control using "fast" model-predictive techniques.

Chilali, M., Gahinet, P., and Apkarian, P. (1999). Robust pole placement in lmi regions. *IEEE Tansactionson Automatic Control*, 44(12).

Choia, S.B., Leea, H., and Chang, E. (2000). Field test results of a semi-active suspension system associated with skyhook controller. *Mechatronics*, 11, 345–353.

Fialho, I. and Balas, G. (2002). Road adaptive active suspension design using linear parameter varying gain scheduling. 10(1), 43–54.

Gaspar, P., Szaszi, I., and Bokor, J. (2004). Active suspension design using LPV control. In *Proceedings of the 1st IFAC Symposium on Advances in Automotive Control*, 584–589. Salerno, Italy.

Giorgetti, N., Bemporad, A., Tseng, H., and Hrovat, D. (2006). Hybrid model predictive control application toward optimal semi-active suspension. *International Journal of Robust and Nonlinear Control*, 79(5), 521–533.

J.K. Hedrick, R. Rajamani, K.Y. (1994). Observer design for electronic suspension applications. *Vehicle System Dynamics*, 23, 413–440.

Koenig, D. (2005). Unknown input proportional multiple-integral observer design for linear descriptor systems: Application to state and fault estimation. *IEEE Tansactions on Automatic Control*, 50(2).

Koenig, D. (2006). Observer design for unknown input nonlinear descriptor systems via convex optimization. *IEEE Tansactions on Automatic Control*, 51(6).

M.Darouach (1994). Full-order observers for linear systems with unknown inputs. *IEEE Tansactions on Automatic Control*, 39(3).

M.Darouach (2000). Existence and design of functional observers for linear systems. *IEEE Tansactions on Automatic Control*, 45(5).

Sammier, D., Sename, O., and Dugard, L. (2003). Skyhook and $\mathcal{H}_\infty$ control of active vehicle suspensions: some practical aspects. *Vehicle System Dynamics*, 39(4), 279–308.

Spelta, C. (2008). *Design and applications of semi-active suspension control systems*. Phd thesis, Politecnico di Milano, dipartimento di Elettronica e Informazione.

Tseng, H., Yi, K., and Hedrick, J. (1991). A comparison of alternative semi-active control laws. In *Proceedings of ASME WAM*. Atlanta, Georgia.

Tsui, C. (1996). A new design approach to unknown input observers. *IEEE Tansactionson Automatic Control*, 41(3).

Yi, K. (1995). Design of disturbance decoupled bilinear observers. *KSME Journal*, 344–350.

Yi, K., Oh, T., and Suh, M. (1994). A robust semi-active suspension control law to improve ride quality. In *Proceedings of AVEC'94, International Symposium on Advanced Vehicle Control*. Tsukuba, Japan.

Yi, K. and Suk, B. (1999). Observer design for semi-active suspension control. *Vehicle System Dynamics*, 23(2), 129–148.