INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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Hierarchical Optimization in Isogeometric Analysis : Application to Thermal Conduction

L. Blanchard*, R. Duvigneau†, G. Xu ‡, B. Mourrain §

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Abstract: A multi-level parametrization approach for a shape optimization problem is developed in the framework of Isogeometric Analysis for a thermal conduction problem. The Isogeometric Analysis approach proposed by T. Hughes, consists in integrating Finite Element Analysis (FEA) into conventional Computational Aided Design (CAD) tools using a single software entity. We propose two multi-levels parametrization strategies to improve the convergence rate of a thermal conduction optimization problem. Some numerical experiments are carried out using a two dimensional test-case, to demonstrate the efficiency of the proposed strategies.

Key-words: Hierarchical optimization, isogeometric analysis, multi-level approach, thermal conduction.

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Optimisation hiérarchique en analyse isogéométrique : application à la conduction thermique

Résumé : Une méthode de paramétrisation multi-niveau est développée pour un problème d'optimisation de forme en conduction thermique, dans le cadre d'une analyse isogéométrique. L'analyse isogéométrique, proposée par T. Hughes consiste à integrer l'analyse par éléments-finis (EF) dans les outils de Conception Assistée par Ordinateur (CAO) en utilisant une seule entité logicielle. Nous proposons deux statégies de paramétrisation multi-niveaux afin d'améliorer le taux de convergence d'un problème d'optimisation en conduction thermique. Des expériences numériques sont réalisées en utilisant un cas test de dimensions deux pour démontrer l'efficacité des stratégies proposées.

Mots-clés : Optimisation hiérarchique, analyse isogéométrique, approche multiniveau, conduction thermique.

Contents

Introduction

In Physical problems governed by Partial Differential Equations (PDEs), the use of a shape optimization method implies, in several cases, multimodal cost functionals and large computational costs related to the evaluations. Hence the use of an efficient optimization algorithm is required in order to avoid possible local minima and reduce the computational cost through a lower number of evaluations.

Recent works demonstrate the efficiency of a multi level parametrization strategy applied to a gradient-based method [1] [7][6] or applied to stochastic optimization methods [5], to reduce the computational cost through an acceleration of the convergence rate. We propose in this study to apply such a hierarchical optimization to isogeometric analysis framework and demonstrate its capability for a two dimensional thermal conduction problem.

1 Hierarchical parametrization algorithm

A hierarchical parametrization approach consists in using different shape representation levels during the optimization process, ranging from a coarse level parametrization which implies a small number of design variables to a fine level parametrization which implies a large number of design variables.

Hence,the search for the optimal shape is carried out in several embedded design spaces of variable dimension. Several strategies can be considered, ranging from simple level increase to V-cycle or Full Multi-Grid (FMG) approaches inspired from the multi-grid theory.

The present study is focused on the simple level increase approach which consists in using first a coarse level parametrization and increasing progressively the number of design parameters. Thus, we propose two possible strategies, presented in the next sections. In the isogeometric framework, this refinement can be applied to optimization variables only, or to both optimization and simulation variables.

1.1 Multi-level optimization

The multi-level optimization consists in doing a h-refinement of the design space for a fixed fine solution space as depicted in Figure 1. Some iterations are first carried out during a first optimization phase step with a coarse level parametrization (low number of optimization variables) but with a fine solution space (large number of simulation variables). Then the best shape found is projected into the space corresponding to a finer level parametrization and a second optimization phase can be performed from this new starting shape. For this strategy, the simulation is always carried out with the same accuracy. Only the optimization process is hierarchical.

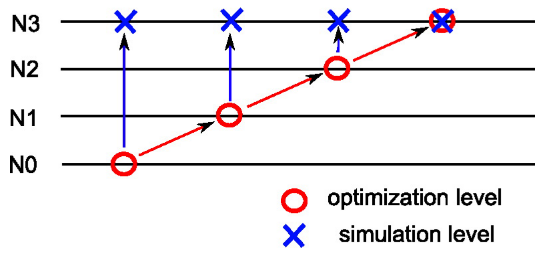


Figure 1: Multi-level optimization algorithm.

1.2 Multi-level optimization and modeling

The multi-level optimization and modeling consists in doing a h-refinement of both the design and solution spaces which are the same as depicted in Figure 2. Some iterations are first carried out during a first optimization phase with a coarse level parametrization and a coarse level solution space (low number of optimization and simulation variables). Then the best shape found is projected into the space corresponding to a finer level parametrization and a second optimization step can be performed from this new starting shape. Contrary to the previous approach, both optimization and simulation are carried out with an increasing accuracy.

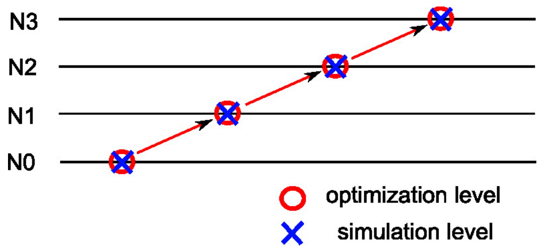


Figure 2: Multilevel optimization and modeling algorithm.

2 Application to thermal conduction

Let us consider a domain Ω closed by the boundary $\Gamma = \Gamma_D \cup \Gamma_N$ with $\Gamma_D \cap \Gamma_N = \emptyset$. A general thermal conduction problem is :

$$\begin{cases} -\nabla.\big(\,\kappa(x)\,\nabla T(x)\,\big) & = & f(x) & \text{in } \Omega \\ T(x) & = & g(x) & \text{on } \Gamma_D \\ \kappa(x)\,\partial_n T(x) & = & \phi_0(x) & \text{on } \Gamma_N \end{cases} \tag{1}$$

where x are Cartesian coordinates, T represents the temperature field and κ is the thermal conductivity, $g \in H^{1/2}(\Gamma_D)$ is the imposed temperature on the Dirichlet boundary, and $\phi_0(x) \in H^{1/2}(\Gamma_N)$ is the imposed thermal flux on the Neumann boundary.

In this study, we consider a rectangular plate of homogeneous material with uniform thermal conductivity κ, two uniform Dirichlet conditions are imposed at left and right boundaries with respectively $T = 3$ and $T = 0$, and finally two zero Neumann conditions are imposed at the top and at the bottom boundaries corresponding to adiabatic conditions as depicted in Figure 3.

Then the Dirichlet boundary can be split into the following : $\Gamma_D = \Gamma_{D_l} \cup \Gamma_{D_r}$ where $g(x) = \big\{\, 3 \text{ if } x \in \Gamma_{D_l}\,,\ 0 \text{ if } x \in \Gamma_{D_r}\,\big\}$, and consequently the thermal conduction problem (1) turns into the following laplace equation

$$\begin{cases} -\Delta T(x) & = & 0 & \text{in } \Omega \\ T(x) & = & g(x) & \text{on } \Gamma_D \\ \partial_n T(x) & = & 0 & \text{on } \Gamma_N \end{cases} \tag{2}$$

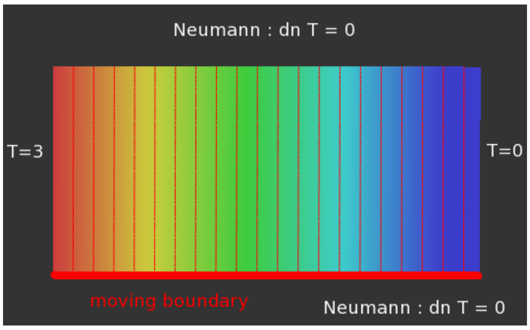


Figure 3: Thermal conduction for the rectangular plate test case.

2.1 Variational formulation

Let us consider the function $T_g(x) = \big\{\, g(x) \text{ if } x \in \Gamma_D\,,\ 0 \text{ if } x \in \Omega\,\big\}$, then using the notation $\bar{T}(x) = T(x) - T_g$, the equation (2) turns into the following equation

$$\begin{cases} -\Delta \bar{T}(x) & = & 0 & \text{in } \Omega \\ \bar{T}(x) & = & 0 & \text{on } \Gamma_D \\ \partial_n \bar{T}(x) & = & 0 & \text{on } \Gamma_N \end{cases} \tag{3}$$

According to the classical variational formulation, we seek for a solution $\bar{T} \in H^1_{\Gamma_D}(\Omega)$ where : $H^1_{\Gamma_D}(\Omega) = \{\phi \in H^1(\Omega) \,|\, \phi = 0 \text{ on } \Gamma_D\}$ and we consider a test function $\psi \in H^1_{\Gamma_D}(\Omega)$ such that :

$$\int_\Omega \nabla \bar{T}(x) \,.\, \nabla \psi(x) \, d\Omega \;=\; 0 \quad , \quad \forall \psi \in H^1_{\Gamma_D}(\Omega) \tag{4}$$

2.2 Isogeometric analysis

Isogeometric Analysis is a recently developed computational approach, proposed by T. Hughes [2], that integrates Finite Element Analysis (FEA) into conventional Computational Aided Design (CAD) tools using a single representation for the geometry and solution field. Currently, it is necessary to generate a mesh according to the CAD data, to simulate a new design using a FEA package. Isogeometric Analysis employs directly complex NURBS-based geometry in the FEA application to define the integration elements used in the variational formulation. This approach allows to define a computational domain that matches exactly the geometry of the problem whatever the number of degrees of freedom.

Let us consider the following transformation from the parametric space into the physical space :

$$\begin{array}{rccc} \mathcal{T}: & \hat{\mathcal{P}} & \rightarrow & \mathcal{P} \\ & \boldsymbol{\xi} = (\xi, \eta) & \rightarrow & \mathbf{x} = (x, y) \end{array} \tag{5}$$

We define this a tensor product of the B-spline basis functions in the parametric space $\hat{N}_{ij}(\xi, \eta) = \hat{N}_j^{p_j}(\eta)\hat{N}_i^{p_i}(\xi)$: see [8] or [4] for description of B-spline functions. Then, the computational domain is defined as a B-splines surface. A similar parametric description is used for the temperature field:

$$\bar{T}(\xi, \eta) = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \hat{N}_i^{p_i}(\xi)\, \hat{N}_j^{p_j}(\eta)\, \bar{T}_{ij} \tag{6}$$

Where we define the test function in the physical domain such as:

$$N_{ij}(\mathbf{x}) = N_{ij}(x, y) = N_{ij}(\mathcal{T}(\xi, \eta)) = \hat{N}_{ij}(\xi, \eta) = \hat{N}_{ij}(\boldsymbol{\xi}) \tag{7}$$

and the gradient of the functions turns into the following equation :

$$\nabla N_{ij}(\mathbf{x}) = B(\boldsymbol{\xi})\, \nabla_{\boldsymbol{\xi}} \hat{N}_{ij}(\boldsymbol{\xi}) \tag{8}$$

where B is the transposed of the inverse of the Jacobian matrix. Consequently, the variational formulation (4) reads, for each knot span $\mathcal{P}$:

$$\sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \bar{T}_{kl} \int_{\mathcal{P}} \nabla N_{kl}(\mathbf{x}) \,.\, \nabla N_{ij}(\mathbf{x}) \; d\Omega = 0 \quad , \quad \forall (i, j) \in [1, n_i][1, n_j] \tag{9}$$

Finally, by integrating in the parametric space, we obtain for the variational formulation (9), the linear system $M\,\bar{T} = 0$, where the matrix M is defined by the following equation :

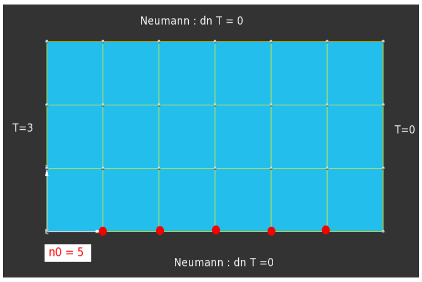
$$M_{ij,kl} = \int_{\hat{\mathcal{P}}} B(\boldsymbol{\xi})^\top\, B(\boldsymbol{\xi})\; \nabla_{\boldsymbol{\xi}}\, \hat{N}_{kl}(\boldsymbol{\xi}) \,.\, \nabla_{\boldsymbol{\xi}}\, \hat{N}_{ij}(\boldsymbol{\xi})\; |det(B(\boldsymbol{\xi}))|\; d\hat{\Omega} \tag{10}$$

3 Hierarchical optimization

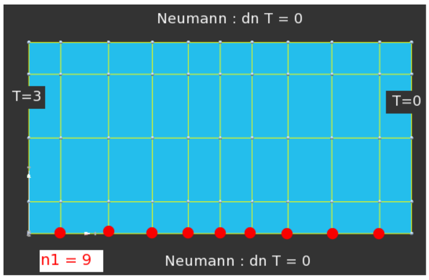
The optimization problem is an inverse problem, for which we try to reach a given target temperature denoted by T_c on the Neumann bottom boundary Γ_{N_b}, consequently the cost function turns into :

$$J(\Gamma_{N_b}, T) \;=\; \int_{\Gamma_{N_b}} |\, T(x) - T_c(x) \,|^2 \, d\Gamma \tag{11}$$

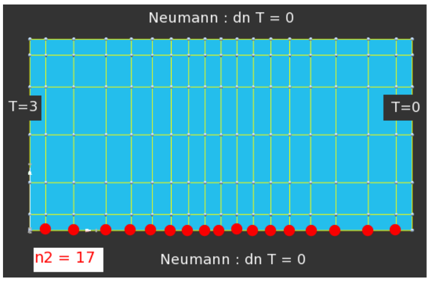
The shape optimization problem is : $\min\; J(\Gamma_{N_b}, T)$ where T is solution of the state equation (3). In order to solve the optimization problem we use a deterministic approach with the steepest descent method based on centered finite-difference approximation of gradient and a stochastic approach with a evolution strategy method. Then for the two proposed strategies described in 1.1 and 1.2, we use four parametrization levels in the hierarchical algorithm as depicted in figure 4.



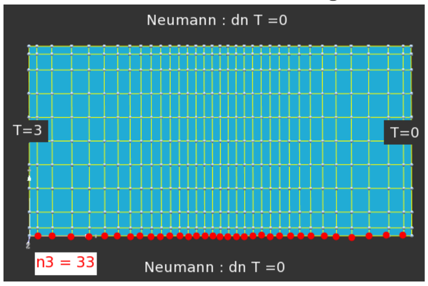
(a) coarse level : $n0 = 5$ design variables



(b) medium level : $n1 = 9$ design variables



(c) medium level : $n2 = 17$ design variables



(d) fine level : $n3 = 33$ design variables

Figure 4: Representation of the different parametrization levels.

3.1 First hierarchical optimization test case

The first test case consists in choosing a continuous target temperature field as a linear distribution perturbed by a sine function as depicted in figure 5(c).

3.1.1 Steepest descent method

Figures 5 and 7 represent the optimal design obtained respectively using coarse and fine parametrization levels (single level optimization) and hierarchical parametrization strategies. The figure 6 represents the convergence plots for the various single level optimizations and the hierarchical parametrization strategies. As expected, the increase of the number of design variables leads to an improvement of the solution accuracy, but decreases the convergence speed. On the coarse level parametrization the temperature field is very far from the target due to the low number of design variables (see figure 5(c)), but the convergence is fast. Contrary, on the fine level parametrization the target temperature field is approximated with a high accuracy due to the large number of design variables (see figure 5(d)), but the convergence is slow.

Concerning the multi-levels strategies, we can notice that on one side, the multi-level optimization algorithm converges towards the same solution as the fine parametrization level with no CPU time gain (see figures 7(a)(e) and 6). On the other side, the multi-level optimization and modeling algorithm converges towards the same solution as the fine parametrization level with CPU time gain during the first phase (see figures 7(b)(f) and 6).

Conclusion

The inefficiency of the hierarchical multi-level strategies combined with the steepest descent method can be justified by the fact that descent methods are efficient for the reduction of low frequency error [3]. Comparing the results obtain with the two different strategies, we can conclude that in this example, a low level parametrization provides suitable information for the steepest descent method.

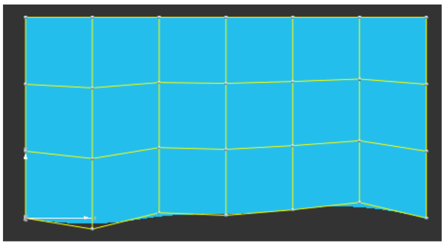
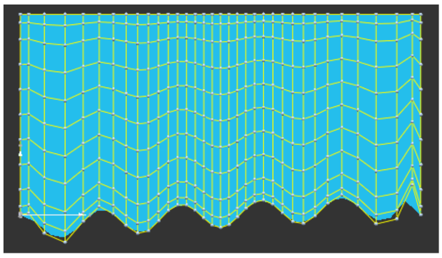
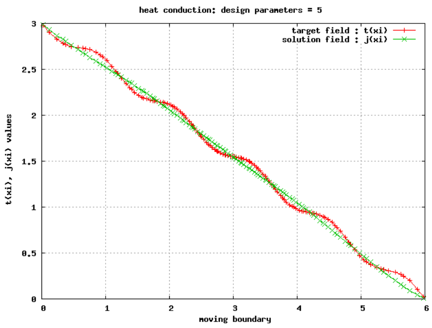
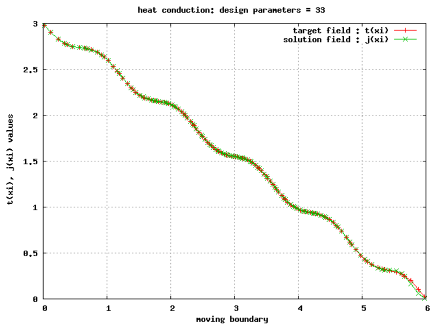
coarse level : $n0 = 5$	fine level : $n3 = 33$
 (a) optimal design	 (b) optimal design
 (c) optimal temperature on Γ_{N_b}	 (d) optimal temperature on Γ_{N_b}

Figure 5: Optimal design for coarse and fine parametrization levels (Steepest descent).

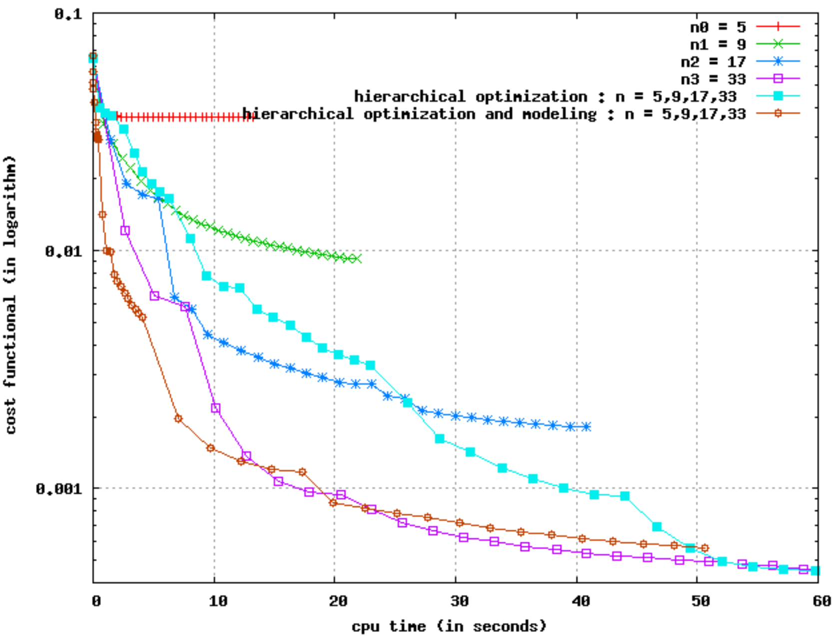
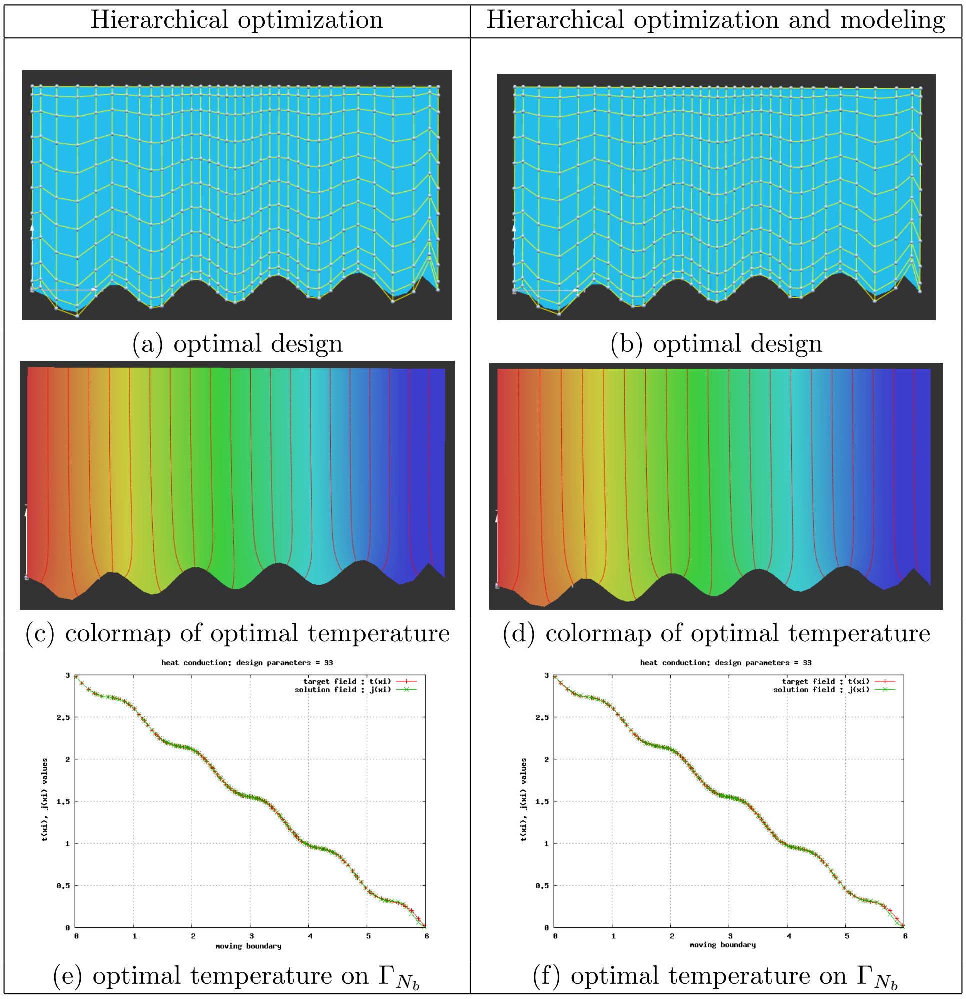


Figure 6: Convergence of the different parametrization levels (Steepest descent).



(a) optimal design (b) optimal design

(c) colormap of optimal temperature (d) colormap of optimal temperature

(e) optimal temperature on Γ_{N_b} (f) optimal temperature on Γ_{N_b}

Figure 7: Optimal design for hierarchical optimization (Steepest descent).

3.1.2 Evolution strategy method

Figures 8 and 10 represent the optimal design obtained respectively using coarse and fine parametrization levels and hierarchical parametrization strategies. The figure 9 represents the convergence plots for the single level optimizations and for the hierarchical parametrization strategies. As previously, the increase of the number of design variables leads to an improvement of the solution accuracy, but decreases strongly the convergence speed.

Concerning the multi-levels strategies, we can notice that on one side, the multi-level optimization algorithm converges towards a better solution than all level optimizations with CPU time gain with respect to the finest level parametrization (see figures10(a)(c) and 9). On the other side, the multi-level optimization and modeling algorithm converges towards a similar solution with significant CPU time gain (see figures10(b)(d) and 9).

Conclusion

The efficiency of the hierarchical multi-levels strategies combined with the evolution strategy method is observed yielding a significant CPU time gain. Comparing the results obtain with the two different strategies, we can conclude that in this example, a low level parametrization provides suitable information for the evolution strategy method, for a lower cost.

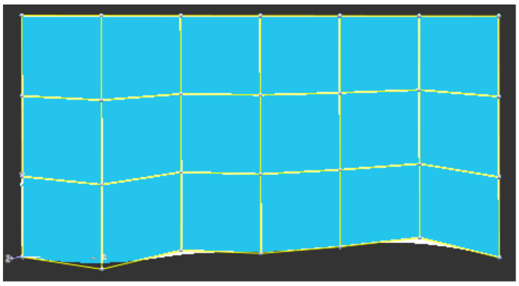
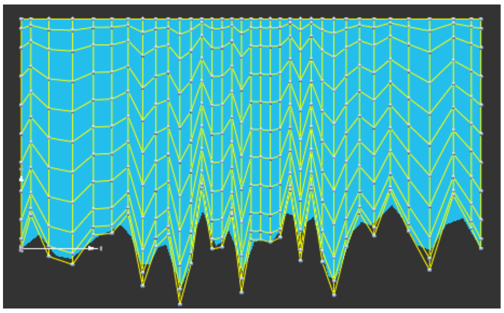
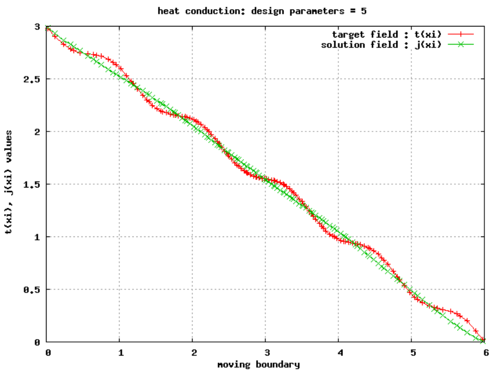
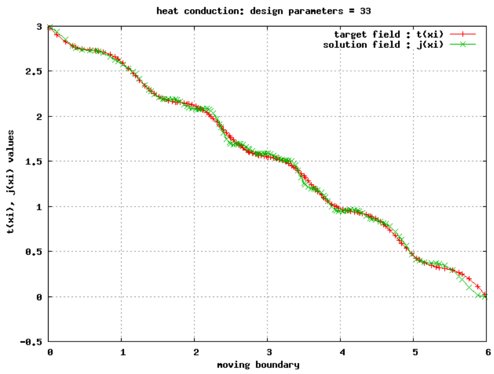
coarse level : $n0 = 5$	fine level : $n3 = 33$
 (a) optimal design	 (b) optimal design
 (c) optimal temperature on Γ_{N_b}	 (d) optimal temperature on Γ_{N_b}

Figure 8: Optimal design for coarse and fine parametrization (Evolution strategy).

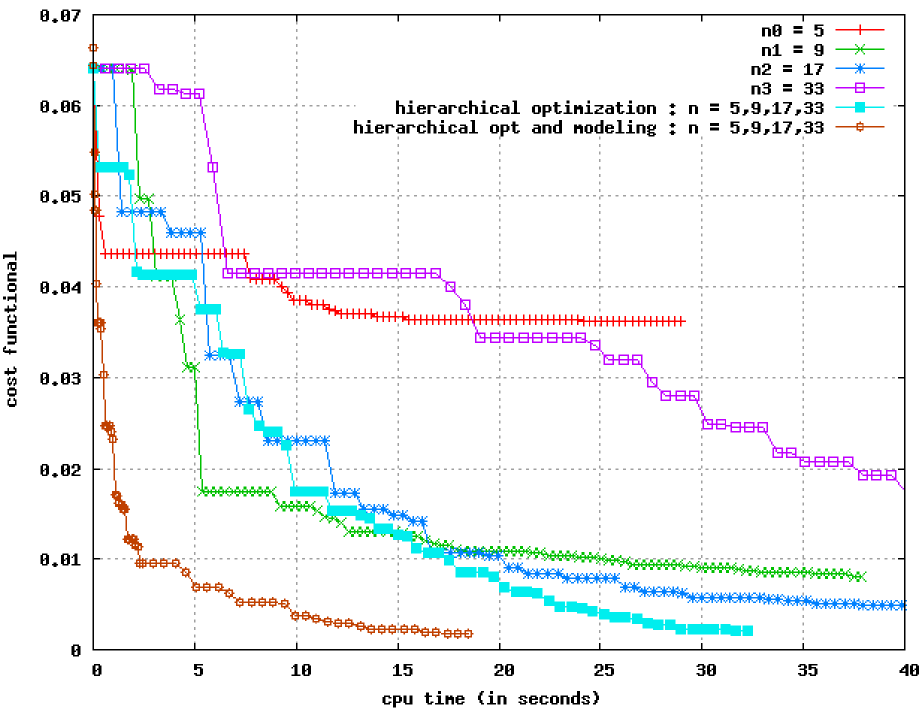
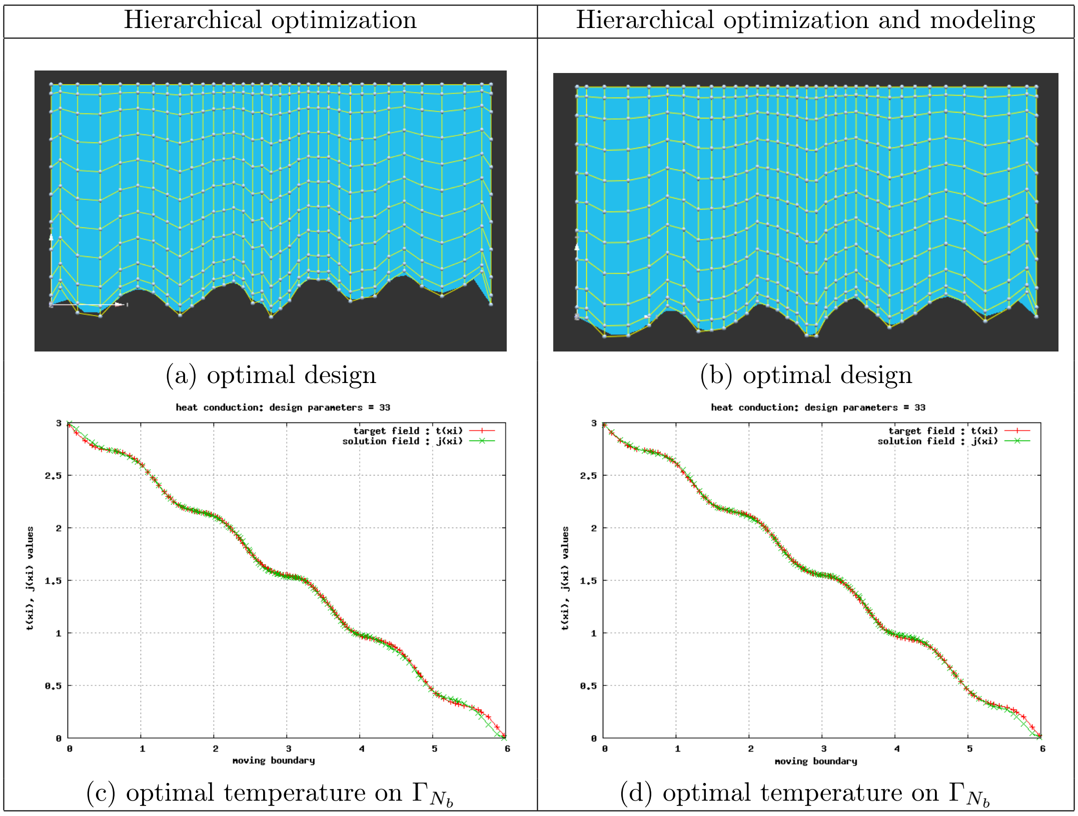


Figure 9: Convergence of the different parametrization levels (Evolution strategy).



(a) optimal design

(b) optimal design

(c) optimal temperature on Γ_{N_b}

(d) optimal temperature on Γ_{N_b}

Figure 10: Optimal design for hierarchical optimization (Evolution strategy).

3.2 Second hierarchical optimization test case

The second test case consists in choosing a continuous target temperature field composed of three linear distributions as depicted in figure 11(c).

3.2.1 Steepest descent method

Figures 11 and 13 represent the optimal design obtained respectively using coarse and fine parametrization levels (single level optimization) and hierarchical parametrization strategies. The figure 12 represents the convergence plots for the various single level optimizations and the hierarchical parametrization strategies.

On the coarse level parametrization the temperature field is very closed to the target despite the low number of design variables and the convergence is fast (see figures 11(c) and 12). Contrary, on the fine level parametrization the target temperature field is not approximated with a high accuracy despite the large number of design variables and the method seems to converge toward a local minimum (see figures 11(d) and 12).

Concerning the multi-levels strategies, we can notice that on one side, the multi-level optimization algorithm converges towards a better solution than all single level optimizations with CPU time gain (see figures 13(a)(e) and 12). On the other side, the multi-level optimization and modeling algorithm converges towards a similar solution with significant CPU time gain (see figures 13(b)(f) and 12).

Conclusion

The efficiency of the hierarchical multi-levels strategies combined with the steepest descent method is observed yielding a CPU time gain. In this particular case, we can suppose that the multilevel stategy permits to avoid local minima. Comparing the results obtain with the two different strategies, we can conclude that in this example, a low level parametrization provides suitable information for the steepest descent method, for a lower cost.

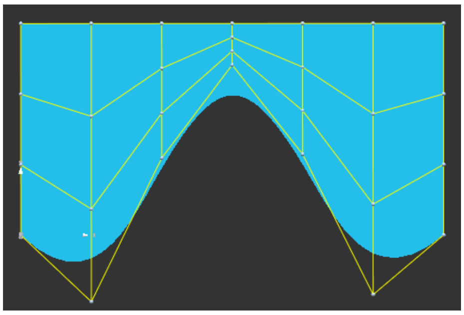
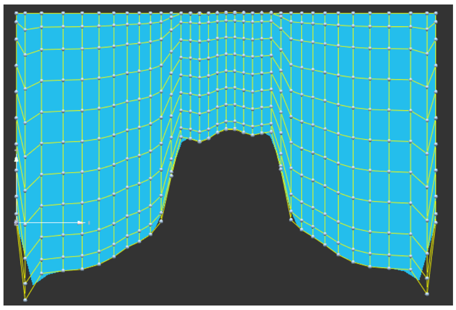
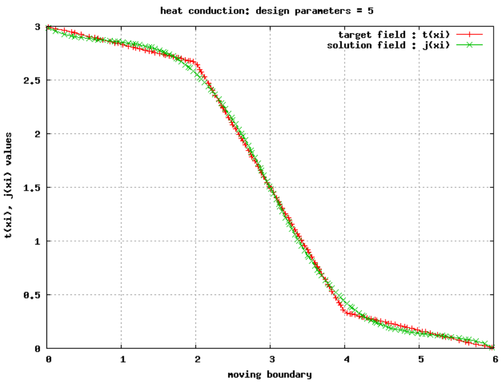
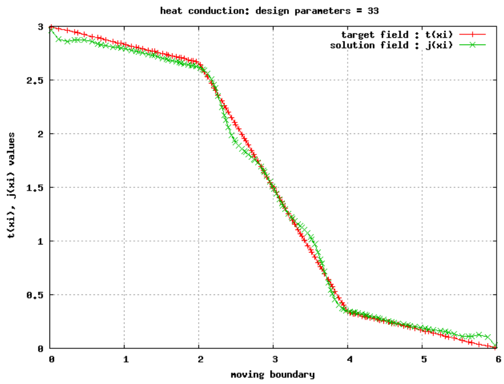
coarse level : $n0 = 5$	fine level : $n3 = 33$
	
(a) optimal design	(b) optimal design
	
(c) optimal temperature on Γ_{N_b}	(d) optimal temperature on Γ_{N_b}

Figure 11: Optimal design for coarse and fine parametrization levels (Steepest descent).

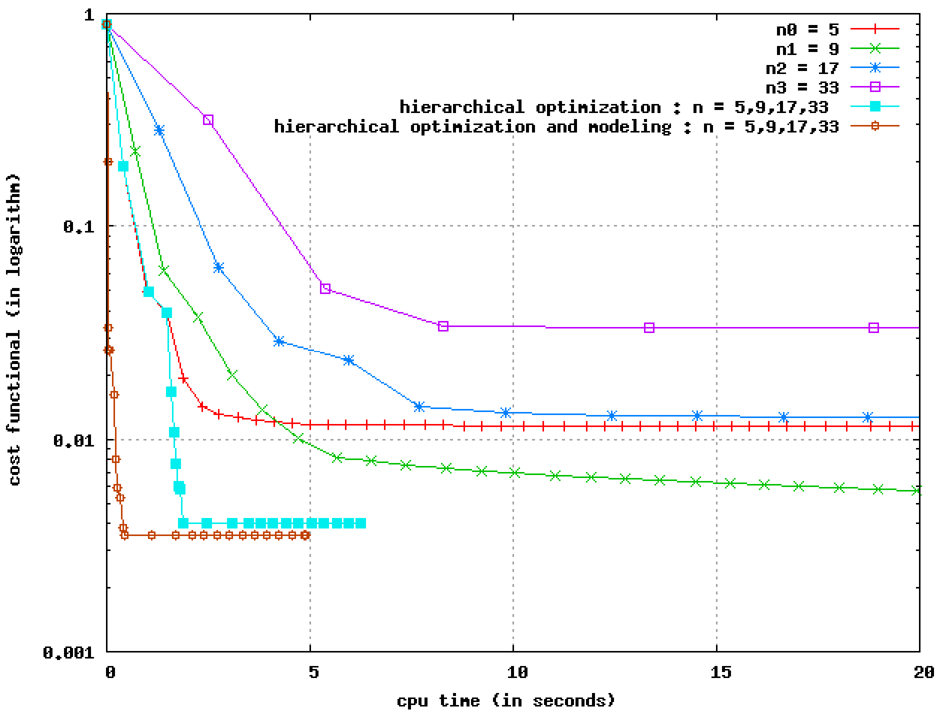
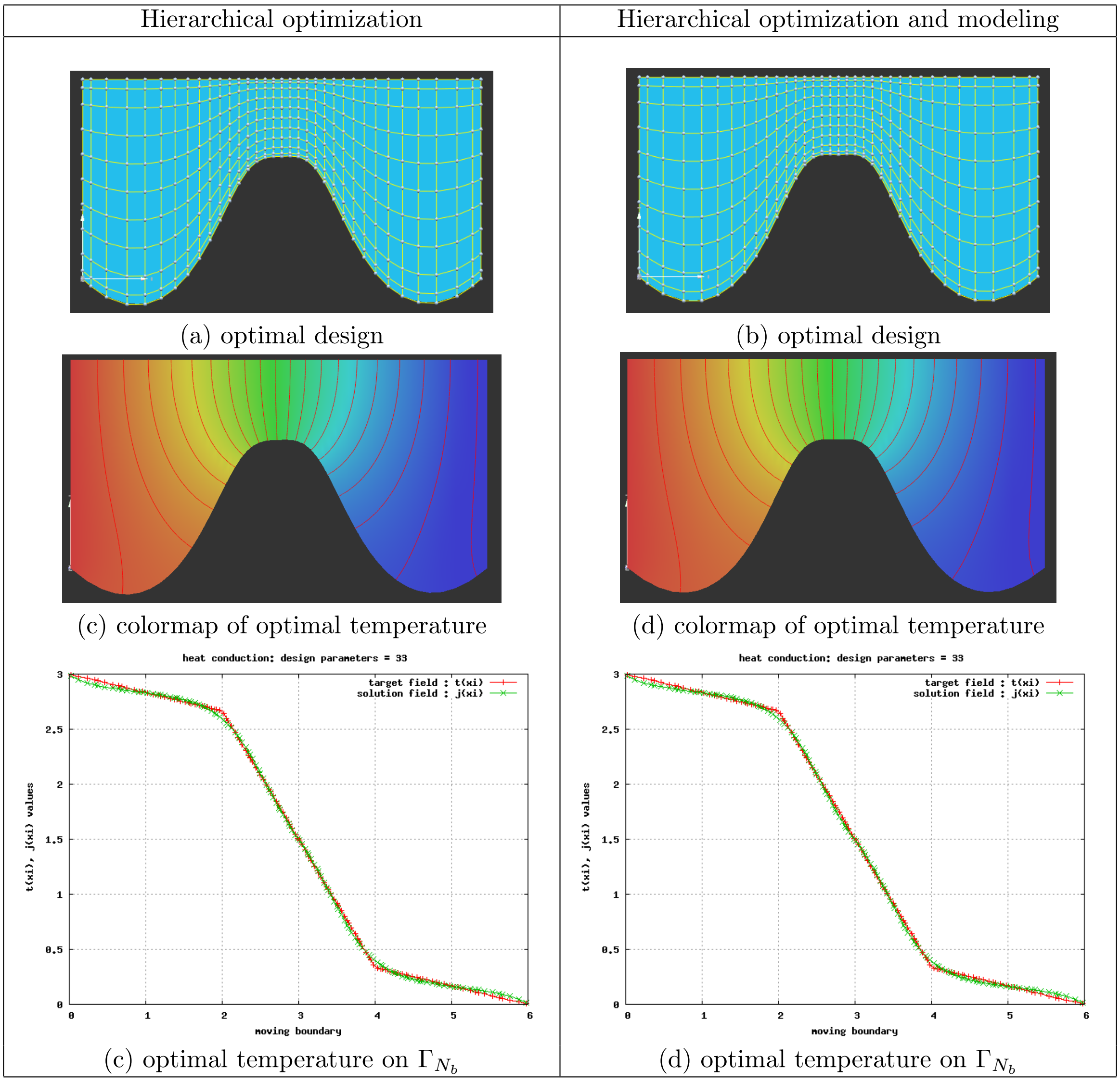


Figure 12: Convergence of the different parametrization levels (Steepest descent).



(a) optimal design (b) optimal design

(c) colormap of optimal temperature (d) colormap of optimal temperature

(c) optimal temperature on Γ_{N_b} (d) optimal temperature on Γ_{N_b}

Figure 13: Optimal design for hierarchical optimization (Steepest descent).

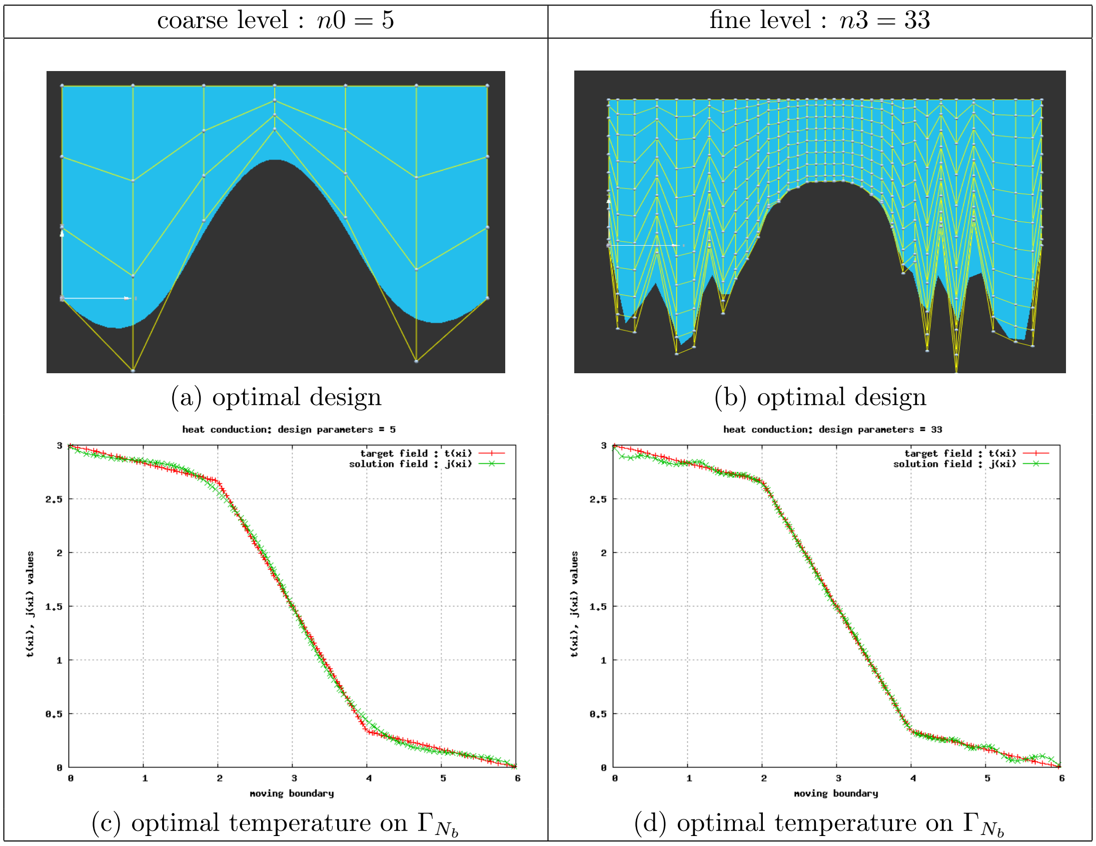
3.2.2 Evolution strategy method

Figures 14 and 16 represent the optimal design obtained respectively using coarse and fine parametrization levels and hierarchical parametrization strategies. The figure 15 represents the convergence plots for the single level optimizations and for the hierarchical parametrization strategies. As previously, the increase of the number of design variables leads to a reduction of the solution accuracy, and decreases strongly the convergence speed.

Concerning the multi-levels strategies, we can notice that on one side, the multi-level optimization algorithm converges towards a better solution than all single level optimizations with CPU time gain (see figures 16(a)(e) and 15). On the other side, the multi-level optimization and modeling algorithm converges towards a similar solution with significant CPU time gain (see figures 16(b)(f) and 15).

Conclusion

The efficiency of the hierarchical multi-levels strategies combined with the evolution strategy method is observed yielding a significant CPU time gain. Comparing the results obtain with the two different strategies, we can conclude that in this example, a low level parametrization provides suitable information for the evolution strategy method, for a lower cost.



(a) optimal design (b) optimal design

(c) optimal temperature on Γ_{N_b} (d) optimal temperature on Γ_{N_b}

Figure 14: Optimal design for coarse and fine parametrization (Evolution strategy).

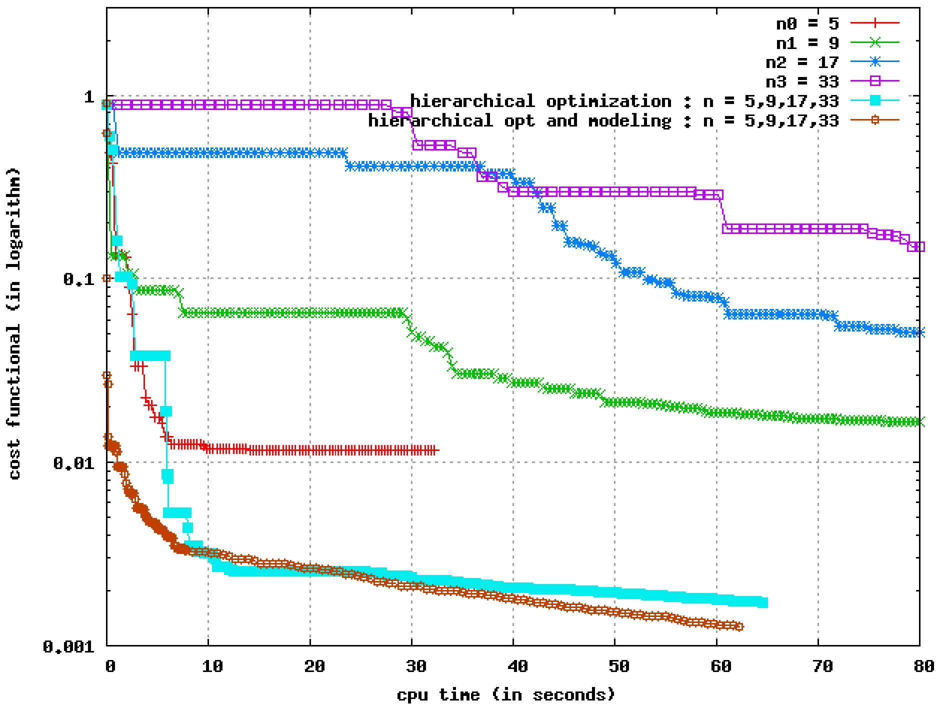


Figure 15: Convergence of the different parametrization levels (Evolution strategy).

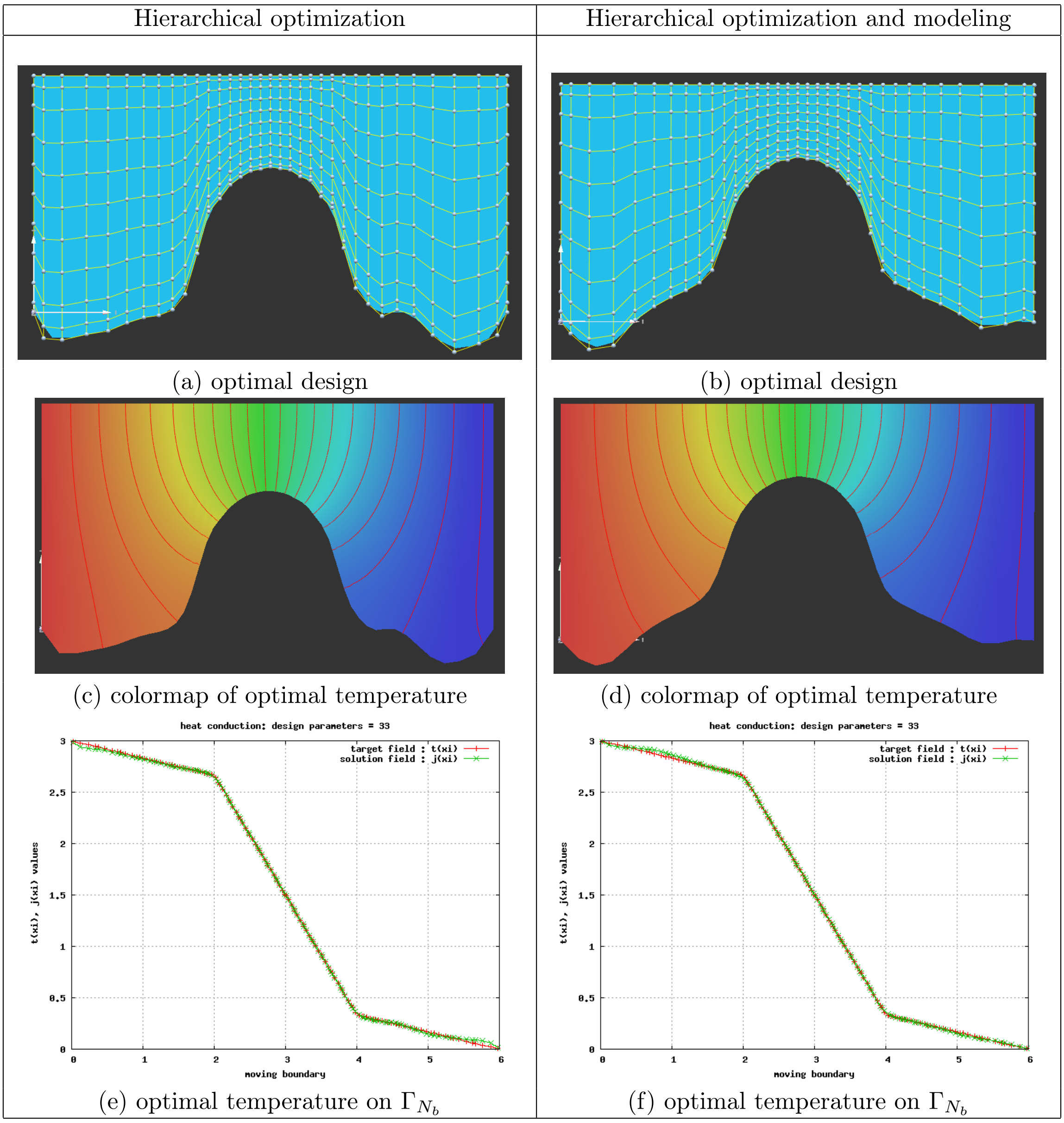


Figure 16: Optimal design for hierarchical optimization (Evolution strategy).

4 Conclusion

In this study, we demonstrate the efficiency of hierarchical strategies approach applied to a two dimensional thermal conduction problem in the framework of Isogeometric Analysis. Especially, the multi-level optimization and modeling approach in the case of a evolution strategy method, yields a significant decrease of the computational cost, since it allows the use of several parametrization level for the design and solution spaces without altering the final result obtained on a fine parametrization level.

We obtain promising results for simple h-refinement strategies. Future works will be focused on the possibility to use more sophisticated strategies, such as V-cycles methods, applied on more realistic problems.

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