Four Comments on "The Road to Reality" by R Penrose

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Dedicated to Marie-Louise Nykamp

Abstract

Four comments are presented on the book of Roger Penrose entitled "The Road to Reality, A Complete Guide to the Laws of the Universe". The first comment answers a concern raised in the book. The last three point to important omissions in the book.

1. Preliminaries

As argued in Appendix 2 of [4], and in a version truncated by the editors in [3], the book [2] of Roger Penrose is a rather unique marvel in our times.

Here, four important issues related to it will be commented upon.

The first one, a remarkably thoughtful observation made in the book, one hardly ever heard from mathematicians, let alone physicists, is that all of the spaces, or for that matter, space-times used in present day physics have a cardinal not larger than that of the continuum.

The second one, on the other hand, is a not lesser important lapse or omission in [2]. And it regards the lack of awareness about the inevitable natural infinite branching of multiplication when it comes to singular entities such as for instance various kind of generalized functions, or in particular, distributions.

The third and fourth ones are about the omissions in [2] to mention inconsistent, respectively, self-referential mathematics, both of which are truly major relatively recent openings not only in mathematics itself, but also within widest possible ranges of its present and future applications.

2. Why Not Arbitrary Large Cardinals in Physics ?

A rather simple and immediate answer to the above question is that, ever since ancient Egypt and Babylon, we have rather tacitely accepted the Archimedean Axiom. And as a consequence, we were inevitably led to the usual field $\mathbb{R}$ of real numbers, which - as is well known - happens to be the only linearly ordered, dense and complete Archimedean field.
Consequently, the complex numbers, various finite or infinite dimensional spaces and manifolds which are used in physics have all been built upon $\mathbb{R}$, and they happen not to have a cardinal larger than that of $\mathbb{R}$ itself, that is, of the continuum.

In short, the tacit acceptance of the Archimedean Axiom led to a situation where the *geometric straight line* is coordinatised in one and only one way, namely, by the usual real numbers in $\mathbb{R}$.

And the amusing fact is the following. While in ancient times and within the limitations of the respective rather simple and primitive technologies there were certain practical reasons for accepting the Archimedean Axiom, when it comes to modern physics, and specifically, General Relativity and Quantum Theory, let alone the ongoing attempts at their unification, there is not any know practical or theoretical a priori reason why one should still hold to the Archimedean

Axiom which is typical for a macroscopic view of the world, one that is not concerned about considerably large, or on the contrary, small realms and entities.

On the other hand, it is a well known mathematical fact that the geometric straight line can be coordinatised in more than one way. Indeed, rather elementary algebraic constructions, such as ultrapowers, [1], lead to a large variety of such coordinatisations which can have arbitrary large cardinals. Consequently, if instead of using $\mathbb{R}$ as the basic ingredient in the construction of various spaces in physics one makes use of such ultrapowers, the resulting spaces as well can have arbitrarily large cardinals.

Furthermore, as seen in [6-8], it is possible to model important known physical phenomena with the use not only of the mentioned ultrapowers, but of the still more general reduced power algebras.

And then, the question can arise whether the *principle of relativity* should only be valid within one single mathematical setup, namely, the present one which is based on $\mathbb{R}$, or in fact, should also extend to the very large class of ultrapowers, or even reduced power algebras ?

In other words, should the principle of relativity be valid only with respect to various reference frames, all of them considered in the given terms of $\mathbb{R}$? Or rather, the principle of relativity should be valid in a *double sense*, namely, both with respect to arbitrary reference frames and arbitrary reduced power algebras, or at least, arbitrary ultrapowers ?

In view of [9], it may be highly tempting and appropriate to consider modelling physics in terms of reduced power algebras or ultrapowers given the resulting considerably more rich *self-similar* structure of the geometric straight line, than is the case in the usual situation when the geometric straight line is coordinatised by $\mathbb{R}$. Indeed, such a rich self-similar structure can afford the modelling of physical phenomena and processes which have so far not been considered since they could not be considered due to the simplicity of $\mathbb{R}$.
Furthermore, the mentioned rich self-similar structure allows the pres-

ence of infinitely small and infinitely large entities, as well as the legitimate - and in fact, usual - performance with them of all the customary algebraic operations. In particular, one becomes free from the difficulties brought about by the so called "infinities in physics".

Here it should be mentioned that nonstandard analysis also allows a richer coordinatisation of the geometric straight line. However, the technical complications involved are so considerable that the vast majority of mathematicians, let alone physicists, have avoided using that method.

In this regard it is, therefore, important to mention that the reduced power algebras, and in particular, the ultrapowers can be constructed using only elementary concepts and methods of algebra.

3. The Inevitable Infinite Branching of Nonlinear Operations on Singularities

In the case of the usual integers in $\mathbb{Z}$, for instance, multiplication is strongly connected to addition, and in fact, it is perfectly well defined by addition, namely, as a repeated addition.

When however, one deals with the far larger and more complex spaces of generalized functions, or in particular, distributions, there is a manifest and fundamental difference between the addition, and on the other hand, the multiplication of elements in such spaces, and the difference is more manifest, the more singular are the elements involved.
Briefly, what happens is as follows.
Addition of no matter how singular elements in such spaces extends easily and in a unique, natural, canonical manner that of nonsingular, for instance, smooth elements, that is, smooth functions.
On the other hand, and in sharp contradistinction, the same does no longer happen with multiplication.

Instead, multiplication does inevitably branch into infinitely many different ways. And such a branching is a simple consequence of basic facts relating to ideals in rings or in algebras.

Details in this regard can be found in [10,12].

As it happens, [2] misses on this important point when refers to generalized functions, and in particular, distributions.

4. No Longer "No Contradictions"

As seen in [11] and the references cited there, there is by now a development taking place in what may be called *inconsistent mathematics*.

And no matter how strange that may appear to be to many of us, one should nevertheless realize that such an essential aspect of modern life like flying on an airplane is in fact - and either we like it, or not - considerably based on such inconsistent mathematics. Indeed, in the design and construction of modern airplanes a critically important role is played by electronic digital computers. And even when reduced to operating only with integer numbers, such computers function according to the following obviously *inconsistent* set of axioms :

1) The well known Peano Axioms,

to which the following axiom is *inevitably* and always is added :

2) There exists a large positive integer M, such that $M + 1 = M$.

This M, which is usually larger than 10^{100}, is called "machine infinity". And no matter how big, expensive and state of the art one's electronic digital computer may happen to be, the presence of such a number M is simply inevitable, since each such computer can handle rigorously only a *finite* amount of integer numbers.

As it happens, although the development of inconsistent mathematics started about a decade earlier than [2] was published, no mention of it can be found in the book.
And this happens in spite of the fact that our electronic digital computers have been so busily - and inevitably - involved in inconsistent mathematics for more than half a century by now, and that so much

in our modern lives depends on such computers ...

5. No Longer "No Self-Reference"

As mentioned in [11], for about three decades by now there is a well developed mathematics which is essentially based on *self-reference*.

Amusingly, ever since ancient times, we have been afraid of self-referential thinking, since we saw in it a sure way to get into contradictions. The ancient Greek paradox of the liar, and its modern version in Russell's paradox are but well known instances which have for long kept motivating our reluctance in using self-reference in logic or in mathematics.

As it happened, however, theoretical considerations in computer science made it necessary to consider mathematics built essentially on self-referential definitions and constructions. Details in this regard can be found in [11] and the references cited there.

It is in this way not an insignificant omission in [2] that no mention whatsoever is made of the respective three decades long development of self-referential mathematics.

6. Brief Conclusion ...

Yes, the road to reality, and let alone, a complete guide to the laws of the universe may be more rich and complex than [2] managed to picture them so far ...

Hopefully, a new edition of that truly remarkable book may improve on that ...

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