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Defect sizing using ultrasonic plate waves

Dilbag Singh, Michel Castaings, Christophe Bacon

Laboratoire de Mécanique Physique, Université de Bordeaux,
351 Cours de la Libération, F-33405 Talence, {d.singh, m.castaing, c.bacon}@lmp.u-bordeaux1.fr

A parametric 2D Finite-Element scheme is developed for sizing strip-like defects in elastic or viscoelastic, isotropic or anisotropic material plates. The reflection and transmission coefficients produced by mode conversion phenomenon when a pure incident Lamb wave mode is sent towards the defect, are used as input data for the inversion process. The inversion process consists in quantifying one or two unknown parameters representing the geometry of the defect. A finite Element based model is used to simulate the Lamb wave scattering for various values of the aimed parameters, and specific post-processing based on the Shkerdin's orthogonality relation is applied to predict the needed reflection and transmission coefficients. The target is known reflection and transmission coefficients initially obtained from an experiment, which can be either a numerical experiment for validation purposes, or proper measurements on a sample for NDE application purposes. The inversion process is stopped when the predicted reflection and transmission coefficients fit at best those obtained from initial experiment. In this preliminary study, two geometrical parameters have successfully been quantified for two types of defects in two waveguides: the depth and width of (1) a notch at the surface of an Aluminium plate, (2) an impact damage in a composite plate.

1 Introduction

A material may be strongly affected by a defect at the surface or inside the body. Therefore, it is very important to estimate the size of a defect using non-destructive measurements to assess the structural strength and serviceability of the structure. Several authors have worked on defect detection and sizing using different approaches [1, 2, 3]. In this study, we investigate the propagation of ultrasonic guided wave modes along plate-like structures, and more specifically their scattering by a defect, to quantify the size of this defect. We use reflection and transmission coefficients of the scattered modes to estimate two geometrical parameters of the defect, considering that its location along the guide is already known. The frequency of the incident mode sent towards the defect is chosen below the A_1 mode cut-off frequency, so that only two fundamental Lamb modes can propagate. In these conditions, there can only be two waves reflected from and two waves transmitted past the defect, namely the fundamental A_0 or S_0 modes. When either A_0 or S_0 mode is made incident along a plate having a defect, then four reflection and transmission coefficients at most can be used to estimate the size of the defect.

A two-dimensional (2D) finite element –based model is used for simulating the propagation of guided waves and their scattering by a defect. Also, a specific inversion scheme is developed to quickly and efficiently estimate two geometrical parameters, which are representative of the size of a defect. Results obtained using this routine are compared to those obtained using the standard least square method. Both geometrical parameters considered in this study are the width and depth of (1) a rectangular notch in an aluminium plate and (2) an impact damage with isosceles triangle shape in a composite plate.

2 Formulation of problem

We consider the propagation of Lamb modes along the x-direction of Cartesian coordinate axis and producing nonzero strains in the Oxy plane only of a solid plate-like guide. Considering plain-strain conditions, each mode n produces displacements and stresses that can be expressed in two dimensions. The power normalized mode fields (modes with unit amplitude) of modes can be written as following:

$$\tilde{u}^n = \begin{pmatrix} \tilde{U}_x^n(y) \\ \tilde{U}_y^n(y) \end{pmatrix} e^{i(\omega t - k_n x)} \tag{1}$$

$$\tilde{\sigma}^n = \begin{pmatrix} \tilde{T}_{xx}^n & \tilde{T}_{xy}^n \\ \tilde{T}_{xy}^n & \tilde{T}_{yy}^n \end{pmatrix} e^{i(\omega t - k_n x)} \tag{2}$$

where t is the time, $\omega = 2\pi f$ is the angular frequency, f is ordinary frequency (in hertz), 'i' is the complex number such as $i^2 = -1$. $k_n = k'_n - ik''_n$ is the complex wave-number of the mode n. k'_n is the real part of the wave-number, k''_n is attenuation and

$$\tilde{U}_k^n = \frac{U_k^n}{\sqrt{|P_{x_0}|}} \text{ and } \tilde{T}_{kl}^n = \frac{T_{kl}^n}{\sqrt{|P_{x_0}|}} \quad \text{with } k,\, l = x,\, y$$

are the power-normalized displacements and stress fields of the mode, respectively and P_{x_0} is the value of $\langle P_x \rangle_n$ in the following expression of the time averaged acoustic power [4] with $x = 0$, i.e., for $e^{-2k''_n x} = 1$.

$$\langle P_x \rangle_n = e^{-2k''_n x} \operatorname{Re}\left[\frac{i\omega}{2} \int_0^h \left(T_{xx}^n U_x^n + T_{xy}^n U_y^n \right) dy \right].$$

The shape of the strip-like defect is supposed to be uniform along z-axis, as shown in Figure 1.

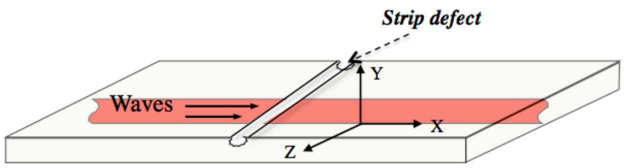

Figure 1: Schematic of a 3-D plate-like guide having strip defect normal to plane of propagation ***Oxy***.

3 FE modelling and inverse procedure

3.1 2D FE model and parametric defect

The two dimensional partial differential equations using plane strain conditions in frequency domain are

$$
\begin{aligned}
C_{11}\frac{\partial^2 u_x}{\partial x^2}+C_{66}\frac{\partial^2 u_x}{\partial y^2}+\left(C_{12}+C_{66}\right)\frac{\partial^2 u_y}{\partial x\partial y}&=-\rho\omega^2 u_x,\\
C_{22}\frac{\partial^2 u_y}{\partial y^2}+C_{66}\frac{\partial^2 u_y}{\partial x^2}+\left(C_{21}+C_{66}\right)\frac{\partial^2 u_x}{\partial x\partial y}&=-\rho\omega^2 u_y,
\end{aligned} \tag{3}
$$

where u_x and u_y are the displacement components in the Fourier domain. These equations are written in a specific COMSOL [5] formalism, and solved for a single frequency of interest. This model is written for the case of a plane of propagation ***Oxy*** coinciding with a plane of symmetry of an orthotropic material. The subscripts '*ij*' in *moduli* C_{ij} corresponds to $xx \leftrightarrow 1, yy \leftrightarrow 2, xy \leftrightarrow 6$. If the material is viscoelastic, then *moduli* C_{ij} can be defined as $C_{ij} = C'_{ij} + i\,C''_{ij}$, the real part representing the material stiffness and the imaginary part its viscoelasticity. Two efficient absorbing regions are placed at both edges of the plate. The plate is '*h*' mm thick and '*L*' mm long. Both types of defects mentioned earlier, i.e., the notch and the impact damage have been modelled using specific functions of space as parametric representations. Two parameters are considered in these functions: '*w'* as width (or extend along the guide) and *'d'* as through-thickness depth, both for the notch and the impact damage. In this latest case, the material is supposed to be strongly micro-cracked inside the defect, as to be representative of the state of a composite material after a strong impact. In the FE model, the incident wave mode is launched along the plate and towards the defect by applying its stress mode shape as a volume force through the plate thickness, over a $\lambda_{\max}/4$ mm length along *x*, just after the left-hand-side absorbing region (Figure 2). Lagrange-Quadratic triangular mesh elements with maximum size equal to $\lambda_{\min}/4$ are used. The meshing is also refined in the region supposed to be damaged (as written before, the location along the guide of the defect is supposed to be known in this study). x_l and x_r are positions along *x* selected to apply the orthogonality relation –based post-processing needed for computing reflection and transmission coefficients, respectively [6, 7]. $x_{end}^{l_{abs}}$ and $x_{begin}^{r_{abs}}$ are starting points of left and right absorbing regions, respectively. L_{abs} is the length of these absorbing regions, chosen equal to $3.5\lambda_{\max}$ for safety. The FE model is solved for a single frequency by applying Neumann boundary conditions on all four boundaries of the plate, so that all surfaces are free of stress. The schematic for such FE model is shown in Figure 2.

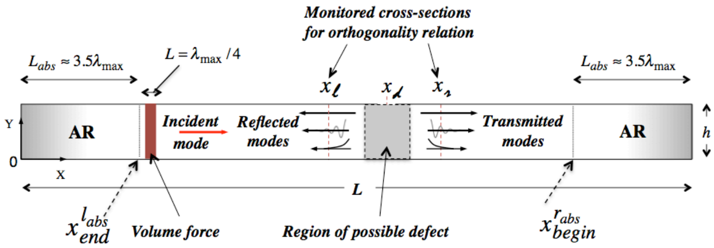

Figure 2: Inverse 2D finite element model.

Functions of space used for modelling both types of defects are defined as follows:

(a) *Rectangular notch* with two parameters i.e., depth (*d*) and width (*w*) can mathematically be modelled as

$$
\left(C_{ij},\rho\right)=\begin{cases}\left(C_{ij},\rho\right), & if\,(x,y)\notin\left[x_d-\dfrac{w}{2},\, x_d+\dfrac{w}{2}\right]\times[h-d,\,h]\\ (0,0), & if\,(x,y)\in\left[x_d-\dfrac{w}{2},\, x_d+\dfrac{w}{2}\right]\times[h-d,\,h]\end{cases}, \tag{4}
$$

where x_d is the centre position of the defect along *x*-axis. It means that inside the notch region, the material coefficients and density are zero. This assumption behaves like that the waveguide is having an actual rectangular notch for given depth (*d*) and width (*w*).

(b) *Cracked zone* is modelled as an isosceles triangle with two parameters, width as base of triangle (*w*) and through-thickness depth (*d*) as height of triangle. We assume that this cracked zone has been produced by a point impact onto the plate, which leads to a local decrease in the material stiffness. For this case, the mathematical expressions for cracked-zone like defect will be:

$$
\left(C_{ij}\right)=\begin{cases}\left(C_{ij}\right), & if\,(x,y)\notin\left[x_d\text{ - }\beta,\, x_d+\beta\right]\times[h\text{ - }d,\,h]\\ \left(20\%C_{ij}\right), & if\,(x,y)\in\left[x_d\text{ - }\beta,\, x_d+\beta\right]\times[h\text{ - }d,\,h]\end{cases} \tag{5}
$$

where $\beta = \dfrac{w(-h+y)}{2d}$.

3.2 2D Orthogonality procedure

To calculate the complex amplitude of any (incident or scattered) guided mode *n* belonging to the total fields predicted by the FE model, the Shkerdin's orthogonality relation [6] is used and applied between the power-normalized fields of that mode, and the total field (with superscript *t*), as explained in [7]. Following *formulae* is used to compute the complex amplitude α_n of the mode:

$$
\alpha_n=\frac{\int_0^h\left(\tilde{T}_{xy}^{n}U_y^{t}+T_{xy}^{t}\tilde{U}_y^{n}-\tilde{T}_{xx}^{n}U_x^{t}-T_{xx}^{t}\tilde{U}_x^{n}\right)dy}{2\int_0^h\left(\tilde{T}_{xy}^{n}\tilde{U}_y^{n}-\tilde{T}_{xx}^{n}\tilde{U}_x^{n}\right)dy} \tag{6}
$$

This post-processing technique is valid for both elastic and viscoelastic media [7]. Power normalized mode shapes for this orthogonality based processing technique are obtained by using the SAFE method [8].

3.3 Inverse procedure

When a pure Lamb mode is made incident towards a defect in waveguide, then mode conversion phenomenon occurs due to the presence of defect. The reflection and transmission coefficients obtained from scattering will provide information about the size and position of the defect. These coefficients are used as input data for this inverse process. In this present study the coefficients of reflection and transmission are computed from numerical experiment. The parametric expression representing the geometry of defect is inserted into inverse FE model in the region of possible defect and we solve this FE model for two parameters (say p_1 and p_2) for M and N different values respectively. For each pair (p_1, p_2), orthogonality relation is applied to get reflection and transmission coefficients using modal basis obtained from SAFE method [8]. Then the modulus of reflection and transmission coefficients is saved in four matrices of order $M \times N$. After this, the input data is compared with their respective reflection and transmission coefficient matrices using following two principles:

Principle 1: Let $RC_{Exp}^{A_0}$ and $RC_{Exp}^{S_0}$ be the experimental obtained absolute values for reflection coefficients for A_0 and S_0 modes at the position x_l, and $TC_{Exp}^{A_0}$ and $TC_{Exp}^{S_0}$ be the absolute experimental values for transmission coefficients for A_0 and S_0 modes at the position x_r. Let $\left[RC_{kl}^{A_0}\right]_{M\times N}$ and $\left[RC_{kl}^{S_0}\right]_{M\times N}$ be the matrices of absolute values of reflection coefficients and $\left[TC_{kl}^{A_0}\right]_{M\times N}$ and $\left[TC_{kl}^{S_0}\right]_{M\times N}$ be matrices of absolute values of transmission coefficients for A_0 and S_0 modes, respectively, at the same positions x_l and x_r, obtained from the FE model by varying two parameters (M values for w and N values for d) and post-processing using orthogonality relation. Let us assume that the relative error between input data and data obtained using the FE model is $e\%$. Then, we select the approximate solutions within this relative error as follows:

Let S_i be the sets such that

$$\begin{aligned}
S_1 &= \left\{kl : RC_{Exp}^{A_0} \times (1-e) \le \left[RC_{kl}^{A_0}\right]_{M\times N} \le RC_{Exp}^{A_0} \times (1+e)\right\} \\
S_2 &= \left\{kl : RC_{Exp}^{S_0} \times (1-e) \le \left[RC_{kl}^{S_0}\right]_{M\times N} \le RC_{Exp}^{S_0} \times (1+e)\right\} \\
S_3 &= \left\{kl : TC_{Exp}^{A_0} \times (1-e) \le \left[TC_{kl}^{A_0}\right]_{M\times N} \le TC_{Exp}^{A_0} \times (1+e)\right\} \\
S_4 &= \left\{kl : TC_{Exp}^{S_0} \times (1-e) \le \left[TC_{kl}^{S_0}\right]_{M\times N} \le TC_{Exp}^{S_0} \times (1+e)\right\}
\end{aligned} \quad (7)$$

Then the set of feasible solutions will consist of all those solutions, which satisfy all the four inequalities defined in equation (7). Let $S = \bigcap_{i=1}^{4} S_i$, then this set S shall contain all possible positions 'kl' in the matrices, which satisfy all the four inequalities in equation (7). Therefore the reflection and transmission coefficients corresponding to set S will be very close to the corresponding target values or input data. The advantage of this principle is that the result is visual and it can help to estimate the efficiency of each coefficient.

Principle 2: The second method for optimization in this study is the Least Square method. This is used as follows: Let S be a set of positions 'kl' in the matrix such that $\left|\left(RC_{kl}^{A_0} - RC_{Exp}^{A_0}\right)^2 + \left(RC_{kl}^{S_0} - RC_{Exp}^{S_0}\right)^2 + \left(TC_{kl}^{A_0} - TC_{Exp}^{A_0}\right)^2 + \left(TC_{kl}^{S_0} - TC_{Exp}^{S_0}\right)^2\right|$ is smallest. Since the set S is finite, we can easily estimate two parameters representing the geometry of the defect.

4 Validation of inverse technique

The inverse technique is validated first for a simple case, which is a rectangular notch of 1.5 mm depth (d) and 10 mm width (w) at the surface of 4 mm thick Aluminium plate, which is an isotropic and elastic material. The results obtained for both parameters w and d, using the first principle, are so that depth is estimated between 1.5 mm and 1.6 mm, and width between 10 mm and 10.1 mm. Using the second principle, these depth and width are found close to 1.5 mm and 10 mm, respectively, so equal to the true values. All results are very close to the actual width and depth of the rectangular notch. So, we will now check this inverse technique for sizing a cracked zone with isosceles triangular shape inside a Carbon epoxy composite plate, which is an anisotropic and viscoelastic material.

Cracked zone in Carbon Epoxy Composite:

(a) *Numerical experiment*: We consider a Carbon epoxy composite plate with thickness 2.36 mm. The material properties are given in Table 1. The cracked zone in this composite sample is considered as an isosceles triangle whose height (through thickness depth in the plate) is 1.86 mm and width of the base is 20 mm. The material stiffness inside this region is considered as 20% of the material stiffness of the composite sample. For FE modelling, we consider L =200 mm, h =2.36 mm, and $L_{abs} \cong 51mm\,(\approx 3.5\lambda_{s_0})$. The centre position of impact along x is x_d =100 mm. A pure S_0 mode is sent towards this cracked zone at a frequency of 220 kHz. Since at this frequency, only two modes can propagate (Figure 3), we will have four reflection and transmission coefficients. The meshing contains 9473 triangular elements with 39530 degrees of freedom. The positions for applying orthogonality relation are x_l*=80 mm* and x_r*=120* mm which are 20 mm away from the centre of the defect.

Material	Carbon-Epoxy Composite
Thickness (mm)	2.36
Density (g/cm3)	1.89
C_{11} (GPa)	24.31+0.80i
C_{22} (GPa)	15.96+0.48i
C_{33} (GPa)	45.53+0.80i
C_{12} (GPa)	8.28+0.25i
C_{13} (GPa)	7.00+0.40i
C_{23} (GPa)	9.23+0.40i
C_{44} (GPa)	5.17+0.20i
C_{55} (GPa)	4.00+0.20i
C_{66} (GPa)	3.92+0.15i

Table 1: Material properties of Carbon epoxy plate.

The absolute values of reflection and transmission coefficients obtained by numerical experiment at 20 mm away from the centre of the cracked zone are

$RC_{Exp}^{A_0} = 0.0152$, $TC_{Exp}^{A_0} = 0.1188$, $RC_{Exp}^{S_0} = 0.0519$ and $TC_{Exp}^{S_0} = 0.5023$. We now use these coefficients calculated by numerical experiment as input data to the inverse routine for quantifying both parameters w and d.

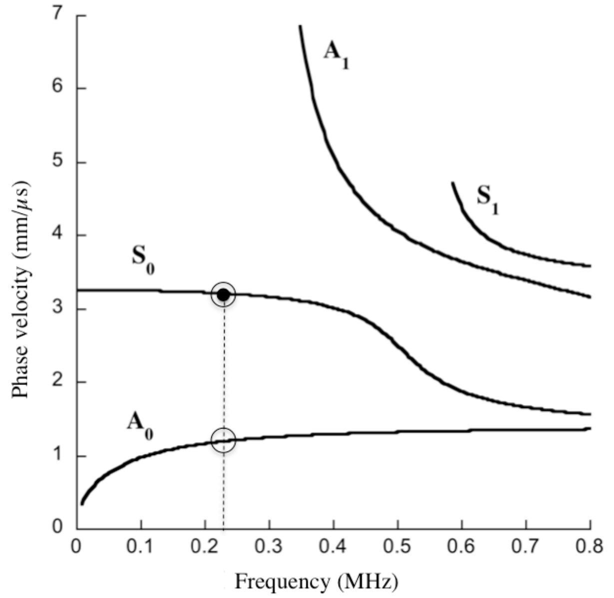

Figure 3: Dispersion curve for 2.36mm thick Carbon epoxy composite plate with operating points, incident wave (●) and scattered waves (O).

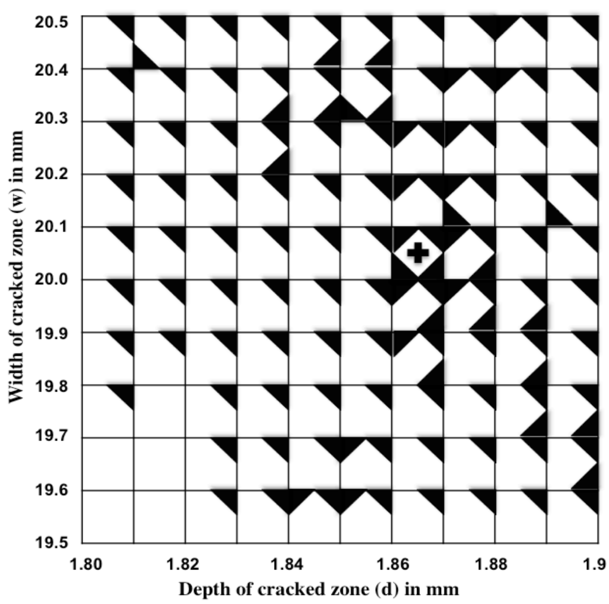

Figure 4: Positions of the elements in matrices of coefficients close to respective target values by 3% - ◤ : A_0 transmission coefficients, ◥: S_0 transmission coefficients, ◣: A_0 reflection coefficients, ◢: S_0 reflection coefficients, ✚ is the position where all four coefficients are close to the target by less than 3%.

(b) *Inverse problem*: The FE model for inverse problem is same as considered in earlier section. The parametric defect is inserted into this FE model by the expression for cracked zone discussed earlier in equation (5). The centre of cracked zone along x is x_d =100 mm. Because w and d are now going to be varied, the region of possible defect with mesh refinement is larger than in previous section. Consequently, the FE model contains 21604 triangular elements with 89166 degrees of freedom (more than for producing the initial target coefficients). It is solved with Langrage quadratic elements for through thickness depth '*d*' varying between 1.8 mm and 1.9 mm with a step size of 0.1 mm and width '*w*' of the base running between 19.5 mm and 20.5 mm with a step size of 0.1 mm. The relative error e=3% in this case.

Figure 4 represents the positions of different coefficients, which are close to their respective target coefficients by 3%. We see from Figure 4 that the width of the zone is estimated between 20.0 mm to 20.1 mm and through-thickness depth between 1.86 mm and 1.87 mm. This solution is very close to the actual initial target, which was a cracked zone with 20 mm width and 1.86 mm depth.

If we use the Least Square principle to check our above inverse routine, then the optimized values for both parameters are width = 20 mm and depth = 1.86 mm, which are exactly same as the initial target data.

5 Conclusion

In this paper, we have developed a 2D finite element - based inverse technique, which can quantify two geometrical parameters for sizing strip like defects in plate-like waveguides. This technique works for a single frequency in which only two modes can propagate in the plate. The reflection and transmission coefficients produced by defect scattering when a pure incident mode below the A_1 cut-off frequency is sent towards the defects are used as input data for inversion process to estimate the two parameters. We have used two principles to optimize these parameters, and two types of defects have been modelled by changing material properties as functions of space: a rectangular notch in an Aluminium plate and a cracked zone caused by a point impact on a Carbon epoxy plate. The mesh in the region of possible defect is strongly refined, so that the shape of the parametric defect can be properly modelled in the FE numerical simulations. The proposed inverse routine has shown to be fast and accurate for two simple cases, and has been successfully compared with the Least Square method. It presents the interest of being visual and robust, and will be further tested for more complex defect shapes.

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