

Signal processing tools for geoacoustic inversion in shallow water

Julien Bonnel
 GIPSA-Lab / DIS
 Grenoble INP, France
 Email: julien.bonnel@gipsa-lab.inpg.fr

Barbara Nicolas
 Jérôme I. Mars
 GIPSA-Lab / DIS
 Grenoble INP, France

Dominique Fattaccioli
 Centre Technique des Systèmes Naval
 DGA (Toulon), France

Abstract—Low frequency propagation in shallow water is described by modal theory. One challenge has been to extract information about the modes in order to localize an acoustic source and/or characterize the propagation medium. This article presents a method for extracting modal arrival times using a single receiver. This method is illustrated on experimental small scale data recorded in an ultrasonic tank. Adaptive signal processing, based on time and frequency warpings, is applied to overcome limitations given by time-frequency overlap of modal field contributions.

I. INTRODUCTION

In this paper, we consider broadband propagation at low frequency (0- 150Hz) in shallow water (0-400m) with a single receiver. Although it is a classical configuration for underwater acoustics, it is still a challenging problem when coupled with single receiver. According to normal mode theory [1] [2], the modal components comprising the pressure field propagate dispersively, both dispersing from one another according to mode number (called intermodal dispersion) and dispersing individually according to frequency (called intramodal dispersion). Due to these effects of dispersion, highly pronounced at low frequency, arrival times of a given mode differing for each frequency are used for localization or inversion scheme. In shallow water and for a source/ receiver distance smaller than 20 km, modes are not well resolved and arrive together on the receiver. In this case, only few methods providing pseudo-automatic extraction are available in the literature and require iterative processing [3], [4]. We propose an automatic estimation of the modal arrival times that requires no a priori information about the environment and its application on experimental small scale data.

In this paper, modal propagation (section II) is briefly reviewed, and it is shown that modal information is embedded in time-frequency domain although not easily accessible. In section III, signal processing tools as warping methods in time and frequency domains are presented as a solution to adapt the received signal to classical time-frequency representations. Then, section IV presents the estimation of the modal arrival time. Finally, last section presents the application of the estimation scheme on experimental small scale data.

II. MODAL PROPAGATION

In shallow water, for low frequency source, the most suitable propagation model is the normal mode theory [1]. For a

frequency f , the transfer function $H(f)$ between a source $X(f)$ at depth z_s and a signal $Y(f)$ recorded on a receiver at a depth z_r separated by a radial distance r can be expressed as:

$$H(f) \approx Q \sum_{m=1}^N \Psi_m(f, z_s) \Psi_m(f, z_r) \frac{e^{jk_{rm}(f)r}}{\sqrt{k_{rm}(f)r}} \quad (1)$$

where N is the number of propagating modes, Ψ_m the modal depth function of mode m , $k_{rm}(f)$ the radial wavenumber of mode m , and $Q = \frac{e^{j\pi/4}}{\sqrt{8\pi\rho(z_s)}}$ (with $\rho(z_s)$ the water density at the source depth).

The spectrum $Y(f)$ of the received signal (recorded at depth z_r) is given by $Y(f) = H(f)X(f)$, with $X(f)$ the source spectrum (emitted at depth z_s). Considering an impulsive source (its spectrum is almost flat for the considered frequencies), means $Y(f) \approx H(f)$. In this case, signal in time $y(t)$ is given by the inverse Fourier Transform of $Y(f)$. As $Y(f)$ is a sum of N terms, $y(t)$ is also a sum of N terms written as:

$$y(t) = \sum_{m=1}^N A_m(t) e^{j\varphi_m(t)} \quad (2)$$

where $A_m(t)$ can be interpreted as the instantaneous amplitude of the mode m and $\Phi_m(t)$ as its instantaneous phase. As a matter of simplicity, we rewrite Eq. (1) as

$$Y(f) = \sum_{m=1}^N B_m(t) e^{j\Phi_m(f)}. \quad (3)$$

Both $Y(f)$ and $y(t)$ are multicomponent and complex functions describing the same signal. They carry exactly the same information.

For each mode m , a group velocity v_{gm} describing the propagation speed of energy is defined as:

$$v_{gm}(f) = 2\pi \frac{\partial f}{\partial k_{rm}} \quad (4)$$

As it depends both on frequency f and mode index m , each frequency of each mode travels at its own speed (dispersive propagation). To extract information of the single received signal, the different modes have to be distinguished. To extract information of the single received signal, the different modes have to be distinguished. If the distance between source and

receiver is big enough (more than 20 km in a classical shallow water waveguide), the modes are perfectly separated in time, and no further treatment is needed. In our case of short distance between source and receiver, modes overlapped in the received signal: for a given time, several modes coexist. As they also share a frequency band, it is not possible to separate them, neither in time nor in frequency domain.

To overcome this problem, the signal will be studied in the (joint) time-frequency domain. In fact, an Ideal Time-Frequency Distribution [5] of the modes *ITFD* can be given using the notion of group delay τ_m :

$$ITFD(\omega, t) = \sum_m \delta[t + \tau_m(\omega)], \quad (5a)$$

$$\tau_m(\omega) = \frac{\partial \Phi_m(\omega)}{\partial f} = -\frac{r}{v_{gm}(\omega)}, \quad (5b)$$

where v_{gm} is the group velocity of mode m . This *ITFD*, presenting the modal arrival times, carry information about the source localization (radial distance r) and about the environment (via the group velocities which are related to the wavenumbers). Equation (5b) assumes that source emission time is $t_0 = 0$. In other world, emission time and reception time are supposed to be synchronized.

Note that in an ideal waveguide (pressure release surface and rigid bottom), previous equations can be explicitly expressed. In particular, phases of the signal, both in time and frequency domain are [2]:

$$\varphi_m^{id}(t) = 2\pi f_{cm} \xi(t), \quad \xi(t) = \sqrt{t^2 - t_r^2}; \quad (6a)$$

$$\Phi_m^{id}(f) = 2\pi \frac{r}{c} \sqrt{f^2 - f_{cm}^2}, \quad (6b)$$

with $t_r = r/c$ the time of first arrival on the receiver and f_{cm} the cutoff frequency of mode m .

III. WARPING TRANSFORMATIONS

Because of time-frequency representation inherent limitations, automatic estimation of modal group delay is impossible without adapted signal processing. In our study, the received signal is transformed (warped) so that it can be represented using classical time-frequency representations. The aim of warping is to undo dispersion. To do that, warping transforms are based on a simple propagation model: the isovelocity waveguide. However, warping transformations are quite robust. Warping can be applied with canonical parameters on any shallow water received signal, even when environment is unknown. Note that by nature, warping transformations conserve energy and are invertible.

In our study, the received signal is transformed (warped) so that it can be represented using classical time-frequency representations. Mathematically, the warping transformation is mainly a substitution. Let consider a function g of the variable x . The warped version $\mathbf{W}_h g$ of g is given by:

$$\mathbf{W}_h g(x) = \sqrt{\left| \frac{dh(x)}{dx} \right|} g[h(x)] \quad (7)$$

with h the warping function. The variable x is substituted by $h(x)$, and $g[h(x)]$ is multiplied by the square root term so

that $\mathbf{W}_h g$ and g have the same energy. This allows a warping interpretation in term of unitary equivalence [6].

In our case, g is the received signal (in time or in frequency domain), and x can be time t or frequency f . Consequently, warping is equivalent to stretch one axis of the signal: t or f . To take benefits of time-frequency representations, warping is defined so that the modal structure becomes linear in the time-frequency plan. A warped mode will be a straight line (horizontal or vertical, depending of warping type) in the time-frequency plan and classical time-frequency representations will easily be used. Time warping (means time axis is stretched) makes all modes horizontal in time-frequency domain as a warped mode is now a pure frequency; frequency warping (frequency axis is stretched) makes a mode horizontal in time-frequency domain as the warped mode is now a Dirac function in time.

First, an adequate warping function has to be defined for each type of warping. In order to do so, as model of the environment is required, we choose the ideal waveguide. Indeed, it exists an analytical formulation of the warping functions in this case. Although the ideal waveguide modelisation is simplistic, the corresponding warping functions can be used on signals recorded in more realistic environments. We will prove it in next sections on experimental data.

A. Frequency warping

The aim of frequency warping is to undo intramodal dispersion. The warping function defined as:

$$w_m(f) = \sqrt{f^2 + f_{cm}^2} \quad (8)$$

depends on the mode number m , which means that modes have to be warped one by one. In an ideal waveguide, the warped spectrum (for a chosen mode m) is then:

$$\begin{aligned} \mathbf{W}_{w_m} Y^{id}(f) = & D_m(f) e^{j2\pi \frac{r}{c} f} \\ & + \sum_{i \neq m} D_i(f) e^{j2\pi \frac{r}{c} \sqrt{f^2 + f_{cm}^2 - f_{ci}^2}} \end{aligned} \quad (9)$$

where D_m is a real functions giving the spectrum amplitude of the warped mode m . For the mode m , the phase of its spectrum $2\pi \frac{r}{c} f$ is linear in frequency. This mode is a Dirac in time and all its energy is concentrated on the time $t_r = r/c$ which corresponds to the direct path propagation time. The other modes are also warped but they are still spread in time (and thus different from a Dirac). Consequently, their amplitude is considerably smaller than the amplitude of the mode m .

So after warping transformation, a given mode is transformed into a Dirac, canceling dispersion effect. Note that frequency warping operates on the spectrum of the received signal and does not require emission and reception time synchronization.

B. Time warping

The aim of time warping is to separate modes in time frequency domain [7].

For the ideal waveguide case, the warping function is:

$$h(t) = \sqrt{t^2 + (r/c)^2} = \xi^{-1}(t). \quad (10)$$

We can notice that this warping function does not depend on the mode number m , which means that all modes are warped at once. In an ideal waveguide, the result of time warping is

$$\mathbf{W}_h y^{id}(t) = \sum_{m=1}^N C_m(t) e^{j2\pi f_{cm} t} \quad (11)$$

where $C_m(t) = \sqrt{h'(t)} A_m[h(t)]$ is the instantaneous amplitude of the warped mode m . For each warped mode, the instantaneous phase $2\pi f_{cm} t$ is now linear in time as a pure frequency. Therefore, the warped signal contains only pure frequencies as every modes are transformed into sinusoids (pure frequencies). Classical time-frequency representations are adapted to represent it, and it will be very easy now to separate warped modes with classical filters. Note that time warping operates directly on the received signal in time domain, and required the reception and emission time synchronization.

IV. MODAL ARRIVAL TIME ESTIMATION

In this section, we present an algorithm to extract modal arrival times from a received signal by warping processing and then build the time-frequency pattern of the signal i.e. the ITFD presented in section 2. The modal arrival time estimation are extracted using the following tree step algorithm.

- 1) Detection of the signal using frequency warping (direct path arrival time estimation)
- 2) Modal filtering using time warping information (adaptive filtering)
- 3) Modal arrival times extraction

A. Detection of the signal using frequency warping

Frequency warping is blindly and recursively applied on the spectrum of the received signal until a mode become as a Dirac as possible. This mode is concentrated around time $t_r = r/c$ while other modes are more spread in time. This allow to estimate the time of first arrival and to synchronize emission and reception times.

B. Time warping and modal filtering

The previous estimation allows to synchronize source and receiver times. Time warping can now be applied. Warped modes ideally become pure frequencies. This is not true as warping model is based on ideal waveguide and recorded signal often comes from a more complicated environnement. At worse, warped modes are more resolved on time-frequency domain than non-processed mode. They can easily be filtered using a simple time-frequency threshold. Each filtered (warped) mode is then unwarped using the inverse time warping.

C. Modal arrival time estimation

Modal arrival time estimation is done on each filtered mode using reallocated spectrogram [8]. This is a suitable tool when the modes are separated.

V. APPLICATION TO EXPERIMENTAL DATA

To clarify the presentation, the previous algorithm is applied on small scale experimental data. These data were recorded in an ultrasonic tank at Laboratoire de Géophysique Interne et Tectonophysique in Grenoble (France). Indeed, to change scale in an underwater acoustics experiment, one only have to keep the ratio distance / wavelenght. Thus, to go from the tank scale to the oceanic scale, distance (water depth, source/receiver separation) has to be divided by a factor N , and frequency has to be divided by the same N

A. Experimental protocol

The waveguide is made up from a water column over a stainless steel bottom. The waveguide parameters are then :

- Water column : sound speed $c_1 = 1480 \text{ m.s}^{-1}$, density $\rho_1 = 1000 \text{ kg.m}^{-3}$, depth $D \simeq 230 \text{ mm}$.
- Bottom (stainless steel) : sound speed $c_2 = 5800 \text{ m.s}^{-1}$, density $\rho_2 = 7900 \text{ kg.m}^{-3}$.

Source/receiver separation is $r \simeq 112 \text{ cm}$. The source is impulsive (central frequency $f_c \simeq 0.5 \text{ MHz}$, bandwidth $B \simeq 0.4 \text{ MHz}$), and sampling frequency is $F_s = 10 \text{ MHz}$. The experimental waveguide can be considered as a Pekeris waveguide. By choosing the scaling factor $N = 1000$, this experiment corresponds to a classical low frequency at-sea experiment in shallow water. Figure 1 presents the tank used for the experiment.



Fig. 1. The tank used for the small scale experiment

Note that from now, time and frequency axis of the figures are scaled by a factor $N = 1000$ so that they correspond to the classical oceanic scale.

B. Experimental results

Data were recorded on an 10 element horizontal array of hydrophone, sampling the upper part of the water column from the surface to the depth $z \simeq 150 \text{ mm}$. The corresponding field is represented on Fig. 2. One can see that only mode 1 is slightly resolved in time. Other modes can not be detected. To apply our estimation algorithm, we select the field recorded on a single receiver (depth $z \simeq 100 \text{ mm}$). Figure 3 presents the spectrogram of the original recorded signal. In this time frequency domain, some mode can be seen but their pattern is not clear. We apply on this signal the modal arrival time estimation scheme presented in section IV.

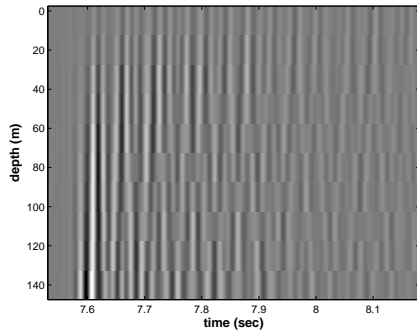


Fig. 2. Field recorded in the tank using a vertical array

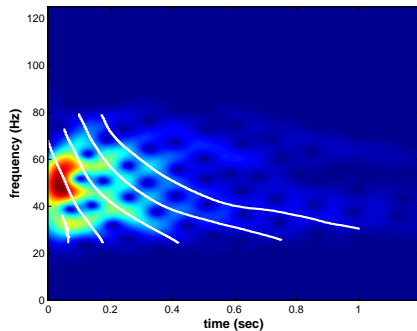


Fig. 3. Spectrogram of the signal recorded on sensor at depth 100mm and corresponding estimated modal arrival times

1-Detection of the signal using frequency warping :

figure 4 presents the spectrogram of the signal after frequency warping. One can see that mode 2 is warped. This mode is characterized and focused as a "Dirac structure" at time 0.3 s with an high level of energy, while mode 1 lays on its left and mode 3 is on its right with a lower level of energy. Estimation of the arrival time is now greatly eased.

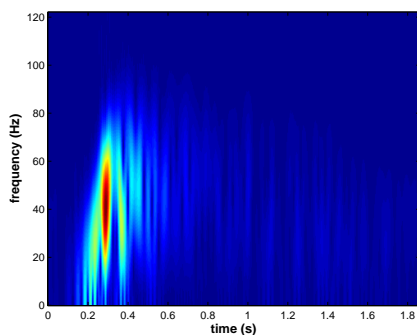


Fig. 4. Spectrogram of recorded signal after frequency warping on mode 2. Mode 2 is stretched into a Dirac structure.

2-Time warping and modal filtering :

figure 5 presents the spectrogram of the signal after time-warping. All modes are stretched into nearly pure frequency structure and can be easy filtered by classical technique.

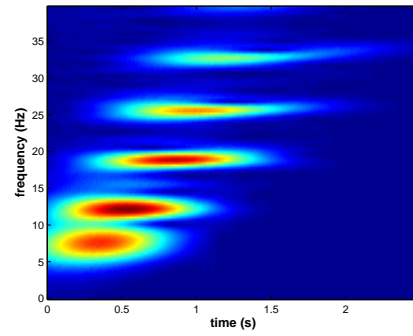


Fig. 5. Spectrogram of recorded signal after time warping. All modes can be characterized as almost pure frequency structure.

3-Modal arrival time estimation : reallocated spectrogram of each filtered mode is computed, and the final result of this estimation is superimposed on Fig. 3. One can see that although modal resolution is pretty bad in the original spectrogram, our method allows an accurate estimation of the modal arrival time. This processing has also been successfully tested on synthetic dataset to test its precision. The extracted arrival time can be used as an input for geoacoustic inversion, as presented in [9].

CONCLUSION

This paper presents propagation based signal processing tools. They allow passive estimation of modal arrival times. These arrival times carry information about environment and can be used in a single receiver inversion scheme.

REFERENCES

- [1] F.B. Jensen. *Computational ocean acoustics*. American Institute of Physics, 1994.
- [2] I. Tolstoy, C.S. Clay. *Ocean acoustics*. Published for the Acoustical Society of America by the American Institute of Physics New York, NY, 1987.
- [3] J.C. Hong, K.H. Sun, Y.Y. Kim. Dispersion-based short-time Fourier transform applied to dispersive wave analysis. *The Journal of the Acoustical Society of America*, 117:2949, 2005.
- [4] Y. Stephan, Y. Simard, C. Gevaïse, S. Vallez. Robust 2D localization of low-frequency calls in shallow waters using modal propagation modelling. *Canadian Acoustics*, 36(1):153–159, 2008.
- [5] L. Stankovic. "An analysis of some time-frequency and time-scale distributions". *Annals of Telecommunications*, Vol. 49, No. 9, pp. 505–517, 1994.
- [6] R.G. Baraniuk, D.L. Jones. Unitary equivalence: A new twist on signal processing. *IEEE transactions on signal processing*, 43(10):2269–2282, 1995.
- [7] G. Le Touzé, B. Nicolas, J.I. Mars, J.L. Lacoume. Matched representations and filters for guided waves. *IEEE Signal Processing*, , Vol. 57, pp. 1783–1795, 2009.
- [8] F. Auger, and P. Flandrin "Improving the readability of time-frequency and time-scale representations by the reassignment method". *IEEE Transactions on Signal Processing*, Vol. 43, pp. 1068–1089, 1995.
- [9] J. Bonnel, B. Nicolas, J.I. Mars, D. Fattaccioli "Rapid inversion in shallow water with a single receiver using modal time-frequency pattern extraction". *OCEANS 2009, MTS/IEEE Biloxi - Marine Technology for Our Future: Global and Local Challenges*, 26-29 Oct. 2009
- [10] B. Nicolas, J.I. Mars, J. Lacoume "Geoacoustical parameters estimation with impulsive and boat-noise sources". *IEEE Journal of Oceanic Engineering*, Vol. 28, pp. 494–501, 2003.