

Toward Gb/s turbo decoding of product code onto an FPGA device.

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Abstract—This paper presents the implementation, on an FPGA device of an ultra high rate block turbo code decoder. First, a complexity analysis of the elementary decoder leads to a low complexity decoder architecture (area divided by 2) for a negligible performance degradation. The resulting turbo decoder is implemented on a Xilinx Virtex II-Pro FPGA in a communication experimental setup. Based on an innovative architecture which enables the memory blocks between all half-iterations to be removed and clocked at only 37.5 MHz the turbo decoder processes input data at 600Mb/s. The component code is an extended Bose, Ray-Chaudhuri, Hocquenghem (eBCH(16,11)) code. Ultra high-speed block turbo decoder architectures meet the demand for even higher data rates and open up new opportunities for the next generations of communication systems such as fiber optic transmissions.

I. INTRODUCTION

In telecommunications, forward error correction (FEC) is a system of error control that improves digital communication quality. A recent development in error correction is turbo coding. Block turbo codes (BTC) are an alternative solution to convolutional turbo codes (CTC) introduced in 1993 by C.Berrou [1]. BTC were proposed by R. Pyndiah [2] in 1994. The coding scheme uses a serial concatenation of systematic linear block codes (product codes introduced by P. Elias [3]) and the decoding is carried out using soft input-soft output (SISO) elementary decoders. Recently, BTC have been included in the specifications of the Wireless Metropolitan Area Networks (IEEE 802.16) standard to increase transmission rates and/or to guarantee the Quality of Service (QoS). Some processes like optical transmission or data storage require ultra high speed above 10 Gb/s data coding and decoding. The first codes used in optical transmission were the well-known Reed-Solomon codes [4] which allow respectable throughput for a Net Coding Gain (NGC) up to 6dB. In such contexts, BTC are an attractive solution since product codes structure present a high parallelism rate and a theoretical NCG of 10dB [5][6]. However, FEC decoders achieving multi-gigabit throughputs require new massively parallel architecture with reduced complexity.

In BTC, the iterative decoding algorithm of a matrix (product code) involves performing the successive decoding of all rows and all columns (two half-iterations). To increase the data rate, it is possible to perform all the rows (columns) in parallel during a half-iteration. However in classical approaches [7], a reconstruction of the data is necessary between two successive half-iterations. This requires a large

amount of memory (up to 75% of the design) and limits the implementation possibilities. In [8], an innovative architecture that enables the memory blocks between each half-iteration to be removed has been proposed. In such an architecture, the remaining complexity is in the decoding processors: the number of required SISO decoders is $DEC_n = 2 \times it \times n$, where it and n are the number of decoding iterations and the code size respectively. Therefore, implementing a full parallel BTC decoder requires to decrease the complexity of the elementary SISO decoder architecture.

This paper is organized as follows. Section II recalls the basic principles of decoding for product codes: their construction and the turbo decoding process. In section III, we propose a complexity and performance analysis of the elementary decoder which allows a rapid and efficient estimation of the decoder complexity. Finally, in section IV, an implementation of the resulting turbo decoder onto a FPGA target is presented.

II. BTC CODING AND DECODING PRINCIPLES

In this section, the concept of product codes, their construction and the principle of the decoding algorithm are presented.

A. Construction of product codes

The concept of product codes is a simple and efficient method to construct powerful codes with a large minimum Hamming distance δ using conventional linear block codes. Let us consider two identical systematic linear block codes C having parameters (n, k) , where n and k stand for code length and number of information symbols respectively. The product code $P=C \times C$ is obtained by placing k^2 information bits in a matrix of k rows and k columns, coding the k rows and k columns using code C . It is shown that all n rows are codewords of C exactly as all n columns. Furthermore, the parameters of the resulting product code P are given by $n_p=n^2$, $k_p=k^2$ and the code rate R_p is given by $R_p=R^2$. Thus, it is possible to construct powerful product codes using linear block codes. As a general rule, the more powerful a code, the more difficult the decoding process.

B. SISO decoding of product codes

Product code decoding involves sequentially decoding rows and columns using SISO decoders. Repeating this soft decoding over several iterations enables a decrease of the Bit Error Rate (BER). It is known as the block turbo decoding process. Each decoder has to compute soft information $[R']_{k+1}$ from the channel received information $[R]$ and the

previous half-iteration computed information $[R']_k$. A SISO decoder of an eBCH code based on the Chase-Pyndiah algorithm [2][9] is concisely summarized below:

- 1- Search for the L_r least reliable binary bits and compute the syndrome S_0 of $[R']_k$,
- 2- Generate T_v test vectors obtained by inverting some of the L_r least reliable binary symbols,
- 3- Binary decoding of each test vector using the syndrome computation,
- 4- For each test vector, compute the square Euclidian distance (metric) $M_i(i=0, \dots, T_v-1)$ between $[R']_k$ and the considered test vector.
- 5- Select the Decided Word (DW) having the minimal distance with $[R']_k$ and choose C_w concurrent words having the closest distance with $[R']_k$.
- 6- Compute reliability $[F]_k$ for each symbol of the DW ,
- 7- Compute extrinsic information $[W]_k=[F]_k-[R']_k$ for each symbol of the DW .
- 8- Add extrinsic information (multiplied by α_k) to the channel received word, $[R']_{k+1}=[R]_k+\alpha_k[W]_k$

An α_k coefficient allows us to damp decoding decisions during the first iterations. It should be noted that decoding parameters L_r , T_v , and C_w has a notable effect on performance.

III. COMPLEXITY AND PERFORMANCE ANALYSIS FOR eBCH SISO DECODERS

Syntheses for the complexity estimations were performed using the Synopsys tool with an STMicroelectronics 0.09 μm CMOS process target. Elementary decoders are clocked at $f=500\text{MHz}$. BER performance was simulated using C-ANSI models of a turbo decoder for the product codes eBCH(16,11)² and eBCH(32,26)² after 6 iterations.

A. Complexity analysis of BCH SISO decoders

All the soft information within the decoder is quantized and processed with Q bits (1 sign bit and $Q-1$ reliability bits). The SISO decoder architecture is structured in three pipelined stages identified as *reception*, *processing* and *emission* units. Each stage processes n symbols in n clock periods. The resulting latency is then equal to $2n$ clock periods. The *reception* unit computes the syndrome S_0 and the L_r least reliable bits of the word received $[R']_k$. The *processing* unit computes the syndrome of the T_v test vectors and their metric values. Finally, the *emission* unit calculates new reliabilities from the metrics of the decided word and the C_w concurrent words. Extrinsic information $[W]_k$ and soft output $[R']_{k+1}$ are also processed during the same clock period. A new Q -bits symbol is then transmitted at each clock period. The decoding process needs to access the $[R]_k$ and $[R']_k$ values during the three decoding phases. For this reason, these words are implemented in Random Access Memories (RAM) of size $Q*n$.

The proposed SISO decoder is composed of twelve processing parts. Running conventional decoder designs through logic synthesis showed that only four parts were critical in terms of logical gate complexity (75% of the area). As a result, our study is focused on these parts. One of these parts is the alpha multiplication unit. In classical architectures, it is implemented as a conversion table. The

input can be multiplied by $0.55 < \alpha_k < 0.75$, the value depending on the current iteration. Keeping $\alpha_k = 0.5$ for each iteration enables the unit to be removed since the multiplication becomes a simple bit shifting. Therefore, the elementary decoder area is decreased by 8%. The induced loss of performance Δ_α is very low ($0 < \Delta_\alpha < 0.1$ dB). Consequently, the complexity analysis will now be focused on the three remaining parts. By analyzing the architecture of these critical parts, five parameters appear to directly affect their complexities. Table 1 sums up the order of complexity of the three critical parts that depends on the parameters introduced in section II. B.

TABLE I. ORDER OF COMPLEXITY OF THE THREE CRITICAL PARTS

Parts	n	Q	L_r	T_v	C_w
DW-Cw sorting	$O(\log(n))$	$O(Q)$	$O(L_r)$	$O(T_v)$	$O(C_w)$
Least reliable sorting		$O(Q)$	$O(L_r)$	$O(T_v)$	
Reliability computation	$O(\log(n))$	$O(Q)$	$O(L_r)$	$O(T_v)$	$O(C_w)$

Decoding parameters L_r , T_v , C_w , as well as the code size n and the number of quantization bits Q has a direct impact on the decoder complexity (area or number of logical gates). Considering a code size n , let us define a set p_i of decoding parameters:

$$p_i = \{ Q_i, C_{w_i}, T_{v_i}, L_{r_i} \}. \quad (1)$$

Varying p_i , directly affects both the hardware complexity and the decoding algorithm performance. Increasing these parameter values improves performance while the complexity increases. The purpose of our analysis is to be able to compute easily the complexity of a SISO decoder for any set of parameters p_i with reasonable accuracy. Considering a code size n , the most favorable configuration is:

$$p_0 = \{ C_{w_0}=3, Q_0=5, T_{v_0}=16, L_{r_0}=5 \}. \quad (2)$$

Actually, increasing p_0 parameters values would increase complexity without significantly improve performances. For this reason, this configuration is the reference for our decoder architecture complexity. Synthesis showed that, in this case, the four critical parts represent 75% of the decoder area whatever the code size n . In addition, assuming that the remaining 25% of the decoder is almost not affected by the variation of a set p_i (as verified during syntheses), it can be demonstrated that the SISO decoder complexity $C_{pi}(n)$ can be expressed as:

$$C_{pi}(n) = 1/3 (C'_{p_0}(n)) + C'_{pi}(n), \quad (3)$$

where $C'_{pi}(n)$ is the cumulated complexity of the four critical parts in terms of logical gates for a parameter set p_i .

Synthesizing generic descriptions of the critical parts and carrying out a multiple linear regression analysis led to an expression of $C'_{pi}(n)$ only depending on the decoding parameters. This expression represents the cumulated

complexity $C'_{pi}(n)$ of the four critical parts in terms of logical gates for a parameter set p_i .

$$C'_{pi}(n) = 462(Cw_i-1) + 261(\log(n)-4) + 183(Q_i-4) + 55(Tv_i-4) + 46(Lr_i-2) + 1019. \quad (4)$$

The model's accuracy was measured a posteriori. The maximum and average errors (between model and synthesis results) are 8% and 2.5% respectively. Table II gives four examples of estimated complexity for code size $n=16$ and 32.

TABLE II. COMPLEXITY IN TERMS OF LOGICAL GATES OF THE CRITICAL PARTS

Parameter set p_i	n	Q	Lr	Tv	Cw	$C'_{pi}(n)$	$C_{pi}(n)$
p_0	16	6	5	16	3	3107	4143
p_1	16	4	2	4	1	1019	2055
p_0	32	6	5	16	3	3368	4491
p_1	32	4	2	4	1	1742	2865

Using this model immediately gives the complexity of an eBCH SISO decoder for any set of decoding parameters. Therefore, taking into account the implementation and performance constraints, it becomes very straightforward to select a code size n and a decoding parameter set p_i .

B. Selection of a parameter set and a code size

Simulations showed that for a small size of product code (eBCH(32,26)² and eBCH(16,11)²), the performance degradation associated with the particular set $p_1 = \{Cw=1, Q=4, Tv=4, Lr=2\}$ was only about 0.3 dB at BER=10⁻⁴. Furthermore, for $n=16$, simulations showed that the loss of performance became negligible for SNR > 4dB. It can be seen that decoding parameters in p_1 have low values. The complexity of the SISO decoder is then highly reduced for reasonable performance loss. Using (3) and (4), $C_{p_1}(16) = 2055$ logical gates and $C_{p_1}(32) = 2865$ logical gates were obtained. Compared with the decoder complexity reference $C_{p_0}(16)$ and $C_{p_0}(32)$, gains of 50% and 36% were achieved respectively.

A block turbo decoder designed according to the complexity analysis results has to be integrated. Currently, a typical hardware design approach is to use an FPGA development board to first prototype the turbo decoder design and its experimental setup. The low cost Virtex II-Pro XUP [10] development system from Digilent was selected for our experimentation. These boards contains a Xilinx Virtex II-Pro XC2VP30 FPGA device with 13696 slices. Preliminary syntheses show that only a half-iteration of the eBCH(32,26)² block turbo decoder would fit onto the Virtex II-Pro XC2VP30 device. Indeed, some elements of the experimental setup have to fit onto the same FPGA device as the block turbo decoder. In the case of BCH(16,11)², up to 3 half-iterations (48 SISO decoders) can be implemented on the same device. For this reason, the BCH(16,11)² was finally chosen for our experimentation. The parameter set $p_1 = \{Cw=1, Q=4, Tv=4, Lr=2\}$ was selected for each elementary decoder.

IV. IMPLEMENTATION OF A BLOCK TURBO DECODER IN AN ULTRA HIGH RATE COMMUNICATION SETUP

A. Experimental setup

The experimental setup is composed of two identical development boards XUP linked with a Serial ATA communication bus. BER measurement facilities are implemented in order to rapidly verify the decoding performance. Each board contains a Xilinx XC2VP30 device that can transmit data at a 2.4 Gbits/s rate. Indeed, encoded noisy data are sent from the transmitter FPGA to the receiver FPGA using the high speed Xilinx Aurora protocol. Each board has its own digital clock management system operating at 50MHz. Synchronization between the two boards is carried out by Aurora protocol control signals. The Aurora protocol is clocked at $f_1=75MHz$ and the rest of the setup is clocked at $f_0=37.5MHz$. Figure 1 shows the different components of the communication setup implemented onto the FPGA.

1) The components implemented on the transmitter FPGA device

A Pseudo Random Generator (PRG) sends out eleven pseudo random data streams at each clock period (f_0). It is composed of eleven different LFSR. An eBCH(16,11)² encoder processes the eleven data streams in parallel. This innovative architecture avoid the use of memory between row and column encoding [8]. Classical sequential encoders are cascaded with a parallel encoder. 256 data (equivalent to a matrix 16×16) are generated in 16 clock periods (f_0). The noise generator models 16 uncorrelated White Gaussian Noise (WGN) samples and adds them to the previously encoded data [11]. Each output sample is a 4 bit vector resulting in 64 bits to be sent in 1 clock period (f_0). The Signal to Noise Ratio (SNR) is controllable via on-board switches $0 < SNR < 15.75dB$ with a pitch of 0.25dB. The Aurora protocol emission module handles a set of control signals. It receives 64 data in 2 clock cycles and sends 32 data every clock cycle (f_1). The output rate is then 2.4Gb/s.

2) The components implemented on the receiver FPGA device

The Aurora protocol reception module receives data at 2.4 Gbits/s and sends out 64 bits every two clock cycles (f_1). The turbo decoder is composed of 32 SISO (16 SISO decoders per half-iteration) and two omega networks used to route data between half-iterations. More information about the decoder architecture and the omega network can be found in [8]. Data arrive at 2.4Gb/s while the working frequency is only 37.5 MHz. The same PRG is also implemented in the receiver. It generates the exact same data as in the transmitter in order to compare data before and after decoding. A BER block is finally used to measure the error rate comparing data from the PRG and the decoder output. It guarantees a minimum of 1000 errors before outputting the BER value. This value is then displayed on an LCD module. The minimum reliable BER value is 10⁻⁹.

B. Toward very high rate implementation

The purpose of this first implementation is to show that a block turbo decoder can effectively work without memories

between half-iterations at high throughput. Clocked at only 37.5MHz, the turbo decoder processes input data at 600Mb/s. This frequency is limited by the communication protocol. The turbo decoder can actually perform up to 70MHz on this target, which corresponds to 1.12 Gb/s. Using an FPGA device optimized for high-performance logic

would lead to even higher frequency. Regarding the output rate, it is defined as:

$$D_{out} = P f R. \quad (5)$$

P is the parallelism rate ($\max(P)=n$), f the decoder frequency and R the code rate.

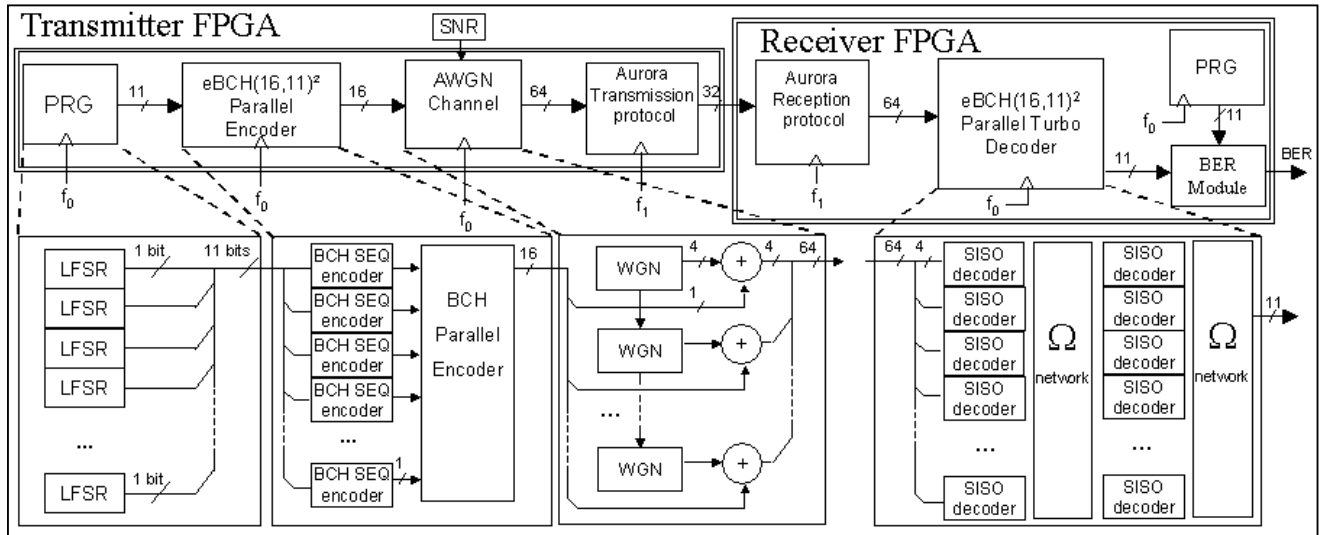


Figure 1. Multi-gigabit experimental setup

In our case, $P=16$, $f=f_0=37.5\text{MHz}$ and $R=0.473$, the resulting throughput is then 284Mb/s. Several solutions exist to increase the throughput, the more straightforward is to use a larger code (in order to increase P) with a larger rate R . For instance, assuming we are using an $e\text{BCH}(32,26)^2$ at frequency $f_0=70\text{MHz}$, the input and output data rates become 2.24Gb/s and 1.48Gb/s respectively. In our architecture, SISO decoders process data sequentially. Designing SISO decoders which decode several data in one clock period, as in [12], would again improve throughput and with a limited complexity overhead. Moreover, enhancing our study to non binary component codes like RS codes [13] can increase data rate even more. The turbo decoder was synthesized and implemented on a Virtex II Pro FPGA using Xilinx ISE 7.1i tools. The decoder occupied 7300 slices. So far, one iteration (32 SISO decoders and 2 omega networks) has been fully implemented. The available target (xc2vp30) was insufficient to implement several iterations. Duplicating the decoders simply requires a larger FPGA target. Implementing a 6-iterations full-parallel turbo decoder represents 43800 FPGA slices with a maximum throughput. Such a design can for instance, fit onto a Xilinx Virtex 4.

V. CONCLUSION

This article shows how we implemented a memory free, high-throughput, full-parallel, block turbo decoder on a FPGA target. In such parallel architectures, it is necessary to use low complexity SISO decoders. We first proposed a complexity analysis for the $e\text{BCH}(16,11)$ SISO decoder. The complexity expression gives a rapid estimation of the SISO area, for a fixed set of decoding parameters. Then, depending on the required level of performances, it becomes easy to decide on a

set of parameters to implement. This analysis led to a low complexity SISO decoder (-50%) to be duplicated in the parallel turbo decoder. Next, we describe the experimental setup designed to test the turbo decoder. Using a more efficient communication protocol the turbo decoder can process input data at 1.2Gb/s. Using a larger code, with a higher rate and parallel SISO decoders, would again, increase the data rate. Moreover non binary codes like RS codes enable even higher throughputs to be reached.

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