TACTICS AND STIGLER FLEXIBILITY. PART I: linear differential model for a single-product firm

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ABSTRACT:

A firm which sells its product in a free competition market where the exchange price remains tationary, determines the quantity of goods to be produced equaling marginal cost to marginal revenues. In this paper we consider the more general case of markets with price instabilities.

To this aim we:

i) propose a simple mathematical model for describing the structure of industrial organizations,

ii) determine the mathematical concept which expresses the Stiglerian flexibility for an industrial organization. In this way we give a precise meaning to the classical definition of flexible industrial organizations which reads as follows: " a flexible industrial organization permit to approximate the best technology for any output, at the cost of not being able to use the best technology for any output " (see [2] on pg. 315).

In the framework of the aforementioned model it is possible to introduce the concept of tactics (time dependent production regime) chosen by a firm as a response to price instability and to prove that the presence of exchange price instability enriches the manifold of industrial organizations which are competitive. Indeed we characterize the manifolds of equally competitive industrial organizations as the level curves of the previously introduced profit function.

1. INTRODUCTION

Free Competition Markets are markets in which

i) independent industrial organizations are present

ii) the total quantity of goods exchanged is much greater than the quantity produced by any of these independent industrial organizations, so that the exchange prices cannot be modified in an appreciable way by any production and selling strategy used by any of them.

Free Competition Market, however, can present oscillations in exchange prices. These economic phenomena have been carefully investigated in the literature, starting from the work of Stigler [2]. Recently in [1] a more detailed discussion and theoretical study are attempted finally pointing out clearly the need of the development of a mathematical model for their describing.

The aim of this paper is to propose a mathematical model to describe the technology and the organization of a firm present in a market in which price oscillations (which the firm cannot at all influence) occur. In this model the concept of Stigler Flexibility of an industrial organization has to be made precise: indeed it is clear that the economic-theoretic ideas of Stigler are really fruitful in describing the behaviour of firms in presence of market instability.

For seek of simplicity we limit ourselves to deal with the case of a single-product market where the produced good G is exchanged at a price p whose variation can be describe by means of a continuously differentiable time function.

Then the problem of mathematically describing the technology available to the firm F, its organization and its choices arises.

While trying to model the effects of market instability upon a *given* industrial organization, we have to distinguish between the

i) *industrial strategy,* whose aim is that to fit (eventually changing both production technology and industrial organization) the firm to the market conditions and

ii) *tactics*, whose aim is to determine the optimal behaviour of a firm, whose means of production and organization are *fixed* , in presence of market instabilities.

The strategical choices are long term ones, and imply an investment policy whose aim is to *change* the available industrial organization and its productive technology, while tactical choices are for their own nature effective only in the short term period.

We believe that Stigler ideas imply only the consideration of the tactical choices and therefore we draw our attention to what we call *tactics* of a firm F.

A *tactics* is the function which maps the instant t belonging to the periodicity time interval [0,T] to the quantity of G produced in unit time by F.

It is obviously devisable an industrial organization in which the tactics includes the storage of a stocked quantity of the produced goods, but we delay this generalization to [9].

To model the production technology available to F, we need to underline that in the literature (see for instance [7]) the problem is completely solved in the case of stationary production regime, i.e. in the case in which the firm chooses the time-constant tactics. Indeed in this case the standard costs function found for instance in [7] is really the suitable concept. However in the literature we could not find any mathematical counterpart for those which we call *adaptation extra-costs*: i.e. those *organization and production costs* which arise because of the varying production regime. In this paper we propose a simple, preliminary mathematical form for the *adaptation extra-costs* function.

By means of the previous considerations and because of the mathematical formulation finally obtained for the theoretical-economic problems , it is now possible to determine, using the techniques of the Calculus of Variation and some Functional Analysis :

i) the *optimal* tactics

ii) an analytical expression for the *optimal profits function* and for the *Stigler Flexibility* of an industrial organization.

Moreover we can precisely formulate in the framework of quoted model and therefore rationally confront an economic problem implicitly contained in the discussion developed in [1] and explicitly proposed in [13] :

How many different industrial organizations are equally competitive in a free-competition market when the structure of this market is such that prices are subjected to periodic oscillations ?

In the literature (see [1] for a complete discussion of this point) it is generally accepted that in the markets in which no price oscillations occur just a *single kind of* industrial organization survives to the market competition. On the other hand the opinions are not unanimous when markets with relevant price oscillations are considered.

In [1] and [13] it is formulated the following conjecture:

Price instable Markets select a "manifold" of industrial organizations which are equally competitive.

The model we propose allows us to *prove* this conjecture if the assumptions we have informally listed in this introduction and formally presented in the following section 2 are accepted.

2. The Mathematical Model

In this section we introduce the mathematical concepts we need in order to model the behaviour of a large class of a single-product firms in presence of periodical variations of exchange price p of the produced good G.

We assume that

i) the exchange price p of G is not influenced at all by the quantity q of G produced in the unit of time by the firm F .

ii) the production technology of F can be characterized by means of its Costs function (see e.g. [7]).

However, in order to take into account the economic phenomena described in [1], we implement the classical approach of the microeconomic theory of the firm considering, together with the "stationary" production Costs function, another Costs function, which we call *adaptation Costs function.*

To this aim let us introduce the following definition:

2.1 Tactics (1)

We call tactics of the firm F in the period [0,T] a real valued function q:

$$q : [0,T] \longrightarrow \mathbb{R}^+$$

such that

$$q : t \in [0.T] \longmapsto q(t)$$

where q(t) is the quantity of the good G produced in unit time at the instant t.

The meaning of this definition is better understood if reformulated as follows:

$$\forall\ t \in [0,T] \text{ the quantity } \int_0^t q(\tau)\, d\tau$$

represents the quantity of G produced by F in the time interval [0,t]

(1) From the Webster's Third New International Dictionary
Tactics 1 b *The art or skill of employing available forces with an end in view.*

2.2 Stationary Production Costs Function

Let us consider stationary tactics in the time interval [0,T]. Let be q the constant value of produced quantity of G.

We assume that the technology available to the firm F is such that the production costs met during an unit time interval are determined by means of a given function of q. In other words we assume that it is possible to define the function

$$C_s: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$$

which maps q to the production costs (in stationary regime) in unit time, costs which we will denote with the symbol $C_s(q)$.

The stationary production regime is that to which in the literature (see for instance [7] or [8]) one limits his attention. In this paper we will consider those technologies which are described by the following Costs functions, which is introduced in [7] chap.I:

$$C_s: q \longmapsto C_s(q) = c_0\, q + \frac{c_1}{2}\, q^2 + c_2 \tag{2.1}$$

where the symbols c_i ($i = 0,1,2$) denote positive real constants.

In nonstationary production regimes we expect that some extra-cost due to the time variation of production quantity q have to be met by F. We are thus led to the following definitions

2.3 Adaptation Extra-Costs For Differential Industrial Organizations

We will call *differential industrial organizations* those firms that

i) are bound to choose only continuously differentiable tactics and

ii) have available a technology and a production organization which can be described by means of the following *Adaptation Extra-Costs* function:

$$C_e : (q,\dot{q}) \longmapsto C_e(q,\dot{q}) \tag{2.2}$$

which allows us to represent the extra-costs met by F in every time interval [0,t] (i.e. all the costs which have to be added to those accounted by formula (2.1) because of the time variation of the produced quantity q) by means of the following integral:

$$\int_0^t C_e(q(\tau),\dot{q}(\tau))\, d\tau$$

when the tactics chosen by F is given by the function q(τ).

The economic meaning of the assumption implicit in the previous definition is clear: we limit our consideration to those industrial organization which have a *short-range memory* of the production history represented by their tactics q(t). While a more general treatment of the problems arising when the adaptation extra-costs have to be evaluated is attempted in [9] we limit our-selves now even to a more special class of differential industrial organizations: i.e. to those which are described by the following extra costs function:

$$C_e : (q,\dot{q}) \longrightarrow \frac{c_3}{2}\dot{q}^2 \tag{2.3}$$

where c_3 is a positive real constant.

2.4 Linear Differential Single-Product Firm

To help our memory , we will call *linear* [(2)] *differential single-product firm* an industrial organization for which is available a technology which involves expenses accounted by formulas (2.1)-(2.2)-(2.3). The single firm belonging to the considered set is obviously characterized by means of the four positive real constants c_i ($i = 0,1,2,3$). We say that two firms are of the same *kind* if they are characterized by the same set of constants.
Last Definition summarizes the structure of the simple model we propose in order to account for the interesting economic phenomena which were first described in [2], and which are more completely discussed in [1].

Even if the considered model could be regarded just as a very preliminary one it will be evident from the treatment developed in the next sections that it allows a pretty careful qualitative descriptions of both the behaviour of firms and the selective market processes which are determined by the exchange price oscillations.

We conclude this section remarking that:

i) The total profits G(T) made by the linear differential single-product firm F in the time

(2) The reasons for this adjective will become evident in the next section.

interval [0,T], are obtained by means of the following formula

$$G(T) = \int_0^T e^{-r\tau} \left\{ p(\tau)q(\tau) - c_0 q(\tau) - \frac{c_1}{2} q^2(\tau) - c_2 - \frac{c_3}{2} \dot{q}^2(\tau) \right\} d\tau \tag{2.4}$$

where :

$q(\tau)$, $p(\tau)$ respectively represent the tactics chosen by F and the exchange price oscillation in the time interval [0,T],

r denotes the discount rate in the period [0,T].

In order to make the exposition smoother we will assume that the discount rate can be neglected in the considered interval [0,T]. We delay to [9] the detailed discussion of the more general case.

ii) In this paper we assume that the exchange price function p(t) is completely known in the time interval [0,T], so that the tactics q(t) can be adjusted in the whole time interval [0,T] with a completely determined optimal way. A lack of knowledge implying the possibility of forecasting only some features of the function $p(\tau)$ in the subsequent instants leads to need of developing more sophisticated approaches, following for instance the treatment found in [5].

We explicitly remark here that simple considerations could allow us to treat also in the case in which F is a monopolistic firm (for the definition of monopolistic firm see [4]). While for a complete discussion we refer to [9], here we limit ourselves to notice that also in the case of a monopolistic firm formula (2.4) is valid, however the exchange price function $p(\tau)$, instead of being determined by the market instabilities, is determined by means of the following supply and demand law with expectation which parallels that appearing in formula (15.31) on pg.489 in [10]:

$$p = k_0 - k_1 q - k_2 \dot{q} \tag{2.5}$$

where k_i ($i = 0,1,2$) are given real positive constants, which model the reactivity of market to variations of produced (and therefore exchanged) quantity q.

The optimal tactics q(t) for F can be found easily exploiting the methods used in the following section.

Market fluctuations in the considered situation are mathematically modelled by variations of the constants k_i.

3. Optimal Tactics For Linear Differential Firms.

The problem now arises: assuming completely known in the time interval [0,T] the exchange price function p(t) which is the optimal tactics to be chosen by F ?

Calculus of variations supplies us the techniques we need to settle down this question.

Indeed (see for instance [3] and [11]) the functionals (2.4) or (2.5) defined in the set of feasible tactics for F assume they optimal values when evaluated in the solutions of Euler-Lagrange Equation related to their integrand Lagrangian function.

Simple calculations lead us to the following

Proposition 3.1

Let us consider the set Σ of all tactics in [0.T] which assume the assigned boundary values

$$q(0) = q_0 \; ; \; q(T) = q_f \tag{3.1}$$

The optimal tactics in Σ with respect to the functional (2.4) are the solutions of the following ordinary linear second order differential equation:

$$c_3 \frac{d^2q}{dt^2} = c_1 q - p(t) \tag{3.2}$$

This Proposition accounts for the Definition 2.4.

We consider now the set $\Pi(T)$ of all periodic continuously differentiable functions whose period is T. As it is well known (see for instance [12]) it is possible to represent every $p \in \Pi(T)$ by means of trigonometric series. We therefore can limit ourselves to consider the case in which

$$p(t) = \langle p \rangle + \sum_{i=1}^{n} A_i \sin\left(i\omega t\right) \tag{3.3}$$

where $T\omega = 2\pi$ and $\langle p \rangle$ denotes the average of p in [0,T]..

The perturbation with respect to the average therefore can be represented by means of what we will call the *perturbation* vector

$$\mathbf{A} := \left(A_1 \, , \, ... \, , \, A_n\right)$$

The linearity of (3.2) allows us to prove the following

Proposition 3.2

The unique solution of equation (3.2), when p(t) is given by equation (3.3), which verifies the condition (3)

$$\lim_{\|A\| \longrightarrow 0} q(A,t) = \frac{<p> - c_0}{c_1} \qquad (3.4)$$

is given by the following

$$q(t) = \frac{<p> - c_0}{c_1} + \sum_{i=1}^{n} \alpha_i \sin\left(i\omega t\right) \qquad (3.5)$$

where

$$\alpha_i = \frac{A_i}{s_i c_1} \quad \text{and} \quad s_i = \frac{c_1 + c_3\left(i\omega\right)^2}{c_1} \qquad (3.6)$$

We remark that:

i) $\alpha_i \leq \frac{A_i}{c_1}$; where the quality holds if and only if $c_3 = 0$

ii) Equation (3.5) mathematically formulate the following economic requirement : when the perturbation with-respect-to-the-average term in (3,3) vanishes the optimal tactics for F is the constant one, and the constant produced quantity has to be determined (see for instance [7]) equating marginal costs MC to exchange price p.

iii) When $c_3 = 0$ the equality MC = p holds at every instant t : the industrial organization has no adaptation extra-costs, so that it can instantaneously maximize profits. In this case our model becomes exactly that proposed in [1]. The quantity s_i measures the amplitude (of oscillating production function) ratio between the "ideal" case in which c_3 is vanishing and the actual case where it cannot be neglected.

We are now ready to introduce our optimal profits function G: it is a function defined in the Cartesian product

$$\left(\mathbb{R}^+\right)^4 \times \mathbb{R}^n \times \mathbb{R}$$

whose range is $\mathbb{R}$.

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(3) Equation (3.4) represents a re-formulated conditions (3.1) when both q_0 and q_f are assumed to be equal to $\frac{<p> - c_0}{c_1}$.

G maps the (n+5)-ple

$$\left(c_i,\ A_i,\ \omega\right)$$

to the optimal profit which can be obtained by F during the time interval [0,T].

Proposition 3.3

Let us introduce the flexibility ratio

$$x = \frac{\omega^2 c_3}{c_1} \tag{3.7}$$

then for the optimal profit function G of the following formula:

$$G/T = \frac{\left(<p> - c_0\right)^2}{2c_1} - c_2 + \sum_{i=1}^{n} \left(\frac{A_i^2}{4c_1\left(1+i^2x\right)^2} \right) \tag{3.8}$$

holds.

Some considerations about this formula are immediately possible:

i) the parametre c_2 represents so called *fixed costs* for F,

ii) the parametre c_0 represents the *price threshold for F,* i.e. the price level under which the firm is surely non-competitive.

iii) we can introduce the *stationary part of profits* by means of the formula

$$G_s/T := \frac{\left(<p> - c_0\right)^2}{2c_1} - c_2 \tag{3.9}$$

Proof of Proposition 3.3

In order to obtain (3.8) it is enough to substitute the optimal tactics (3.5) in formula (2.4) and perform the integration. This last operation is simply carried on while recalling the L^2-orthogonality of the set of functions

$$\left\{ \sin\left(i\omega t\right),\ i=1,..,n \right\},\ \left\{ \cos\left(i\omega t\right),\ i=1,..,n \right\}$$

and the expression of their L^2 norms.

4. Stigler Flexibility : The Manifold of EquiCompetitive Firms.

We call, generalizing the definition introduced in [1], *Stigler Flexibility* (with respect to the price oscillation (3.3)) of a Linear Differential Firm the quantity

$$\mathcal{SF} := \sum_{i=1}^{n} \frac{A_i^2}{4c_1\left(1+i^2x\right)^2} \tag{4.1}$$

This quantity measures the capability of F of making extra-profits choosing instead of a constant tactics the optimal time variable tactics determined in the previous section.

Stigler flexibility is always positive: this means that in markets with price instability it is possible to find competitive firms which were not competitive in the absence of price oscillations.

We explicitly remark that:

i) When c_3 is vanishing then the same happens to x and formula (4.1) becomes:

$$\mathcal{SF} := \frac{1}{4c_1} \sum_{i=1}^{n} A_i^2 \tag{4.2}$$

which, when we recall that the quantity

$$\sum_{i=1}^{n} A_i^2$$

represents the variance with respect the average of price oscillation (3.3), coincides with that obtained in [1].

ii) Stigler Flexibility attains its maximum value when c_3 vanishes. Moreover it is a decreasing function of the parametre c_3 and, although it remains always positive, when $c_3 \longrightarrow \infty$ then Stigler Flexibility is vanishing.

iii) Ceteris paribus, Stigler Flexibility is a decreasing function of the pulsation ω. This result, while obvious from an economic point of view, cannot be obtained using the approach developed in [1]: indeed in the model which is proposed there no mathematical concept is available to describe in such a more detailed way the *history* of price oscillation.

iv) When a periodic price oscillation more general than (3.3) but suitably regular (for instance at least one time continuously differentiable) is considered then its trigonometric series generates an uniformly convergent trigonometric series of optimal strategies, whose optimal profits are given by the series:

$$G/T = \frac{\left(<p>-c_0\right)^2}{2c_1} - c_2 + \sum_{i=1}^{\infty} \frac{A_i^2}{4c_1\left(1+i^2x\right)^2} \qquad (4.3)$$

This series is obviously convergent, as convergent is assumed the series

$$\sum_{i=1}^{\infty} A_i^2$$

which represents the L^2-norm of price oscillation, and the sequence

$$\frac{1}{\left(1+i^2x\right)^2}$$

is bounded.

Therefore also in the general instance of C^1 price oscillations we can define Stigler Flexibility by means of the following formula:

$$\mathcal{SF} := \sum_{i=1}^{\infty} \frac{A_i^2}{4c_1\left(1+i^2x\right)^2} \qquad (4.4)$$

Let us now fix the periodic price oscillation p(t): we say that two firms, respectively determined by means of the parametres c_i^1 and c_2^2 are *equicompetitive* in presence of p(t) if and only if

$$G(c_i^1) = G(c_2^2) \qquad (4.5)$$

The relation

$$G(c_i) = k \qquad (4.6)$$

where k is a positive constant,

determines the set $\mathcal{E}$ of all equicompetitive firms in presence of price oscillation p(t) whose profits are k.

Due to Dini's Theorem equation (4.5) determines an embedded submanifold $\mathcal{V}$ of $\mathbb{R}^4$, i.e. an hypersurface of points belonging to $\mathbb{R}^4$. The dimension of such a surface is three.

Once fixed c_0 and c_2, i.e. the parametres which do not influence the Stigler Flexibility of considered firm, all equicompetitive firms lay on the level curves (in the plane (c_1,c_3)) of the so obtained partial optimal profits function.

These curves can be regarded as the intersection of $\mathcal{V}$ with the plane (c_1,c_3).

In this way we have proved the conjecture made in [1] and [13]. In a mathematical language it can be formulated as follows:

In a Free Competition Market there exists a three-dimensional manifold of equally competitive linear differential firms. Moreover once fixed the price threshold and fixed costs still there exists a onedimensional manifold of equicompetitive firms.

Indeed one can find equicompetitive firms among those whose stationary profit is lower and whose Stigler Flexibility is higher or vice versa. Moreover also among those firms whose fixed costs and price thresholds are equal, one can find equicompetitive firms whose Stigler Flexibility and stationary profits are different.

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