Author manuscript, published in "Canadian Congress of Applied Mechanics, Montreal : Canada (2005)"

Comparison of Robustness Indices and Introduction of a Tolerance Synthesis Method for Mechanisms

Stéphane Caro, Fouad Bennis, Philippe Wenger
Institut de Recherche en Communications et Cybernétique de Nantes,
1, rue de la Noë, 44321, Nantes, France
e-mail: stephane.caro@irccyn.ec-nantes.fr

1 Introduction

Every engineering design is subject to variations that can arise from a variety of sources, including manufacturing operations, variations in material properties, and the operating environment. When variations are ignored, non robust designs can result, which are expensive to produce or fail in service.

The concept of robust design may be first used by Taguchi. He introduced this concept to improve the quality of a product whose manufacturing process involves significant variability or noise [1]. Robust design aims at minimizing the sensitivity of performances to variations without controlling the causes of these variations.

First, we propose a new robustness index and compare it with three robustness indices used in the literature. Then, we develop a sequential tolerance synthesis method.

2 Robust Design Problem

In a robust design problem, the distinction is made between three sets:

- Design Variables (DV): nominal values are controllable. However, they are subject to uncontrollable variations because of manufacturing errors, wear, or other uncertainties ;
- Design Environmental Parameters (DEP): cannot be adjusted by the designer, they are uncontrollable ;
- Performance Functions (PF).

DV, DEP, PF, are grouped in the l-dimensional vector $\mathbf{x} = [x_1 \ x_2 \cdots x_l]^T$, the m-dimensional vector $\mathbf{p} = [p_1 \ p_2 \cdots p_m]^T$, and the n-dimensional vector $\mathbf{f} = [f_1 \ f_2 \cdots f_n]^T$, respectively.

Let us assume a mathematical model between DV, DEP, and PF, as expressed by eq. (1).

$$\mathbf{f} = \mathbf{f}(\mathbf{x}; \mathbf{p}) \tag{1}$$

Robust design aims at rendering PF as insensitive to variations in DV and DEP as possible. Thus, if we introduce variations $\delta\mathbf{x}$ and $\delta\mathbf{p}$ in DV and DP, respectively, and use a Taylor expansion of $\mathbf{f}$ then,

$$\delta\mathbf{f} = \mathbf{J}\left[\delta\mathbf{x}^T \ \delta\mathbf{p}^T\right]^T \tag{2}$$

where $\delta\mathbf{f}$ is the variation in PF and $\mathbf{J}$ is the sensitivity Jacobian matrix of the design.

3 Optimal Robustness Index

In order to obtain a robust solution without the knowledge of the variations in DV and DEP, we need a wise a robustness index. Below, a list of three robustness indices used in the literature :

- $RI_1 = \|\mathbf{J}\|_2 \|\mathbf{J}^{-1}\|_2$, [2]
- $RI_2 = \|\mathbf{J}\|_{Frob} \|\mathbf{J}^{-1}\|_{Frob}$, [3]
- $RI_3 = \|\mathbf{J}\|_2$, [4]

where $\|.\|_2$ and $\|.\|_{Frob}$ mean the 2-norm and the Frobenius norm, respectively. Here, we suggest the use of an other robustness index :

- $RI_4 = \|\mathbf{J}\|_{Frob}$

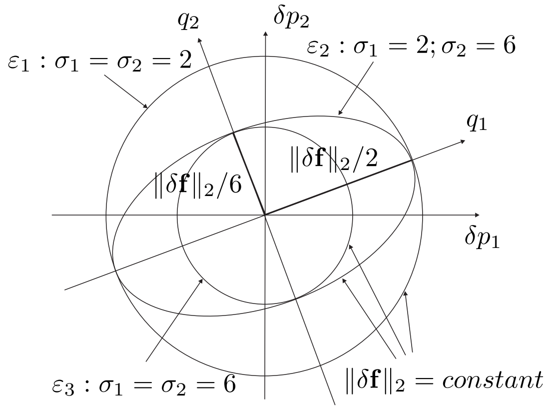

Figure 1: Design Sensitivity Ellipses

In order to illustrate the previous indices, let us compare the robustness of three designs named (1), (2), and (3), respectively. These designs have two DEP and variations in their DV are supposed to be insignificant. ε_1, ε_2, and ε_3, depicted in Fig.1, are the design sensitivity ellipses of designs (1), (2), and (3), respectively. The inclusion of ε_2 in ε_1 means that (1) is more robust than (2). Likewise, the inclusion of ε_3 in ε_2 means that (2) is more robust than (3).

Table 1 depicts the values of RI_1, RI_2, RI_3, and RI_4 corresponding to designs (1), (2), and (3), respectively. Whatever the index, the smaller it is, the more robust the design is supposed to be. However, according to RI_1 and RI_2, the robustness of designs (1) and (3) are similar, and (3) is more robust than (2). According to RI_3, (1) is more robust than (2) and (3), but the robustness of (2) and (3) are similar. Finally, RI_4 makes the difference between the robustness of all the designs accurately.

Table 1: Values of robustness indices

Robustness index	Design		
	(1)	(2)	(3)
RI_1	1	3	1
RI_2	1	1.67	1
RI_3	2	6	6
RI_4	2	3.16	6

In short, the minimization of RI_1 and RI_2 assures an homogeneity of the influence of variations in *DV* and *DEP* on *PF*, i.e.: an isotropic design, but not a minimum sensitivity of *PF* to variations in *DV* and *DEP*. Therefore, we had better use RI_3 or RI_4 in a robust design problem. Moreover, RI_4 is suitable for an optimization robust design problem because of its analytical form.

4 Tolerance Synthesis Method

The dimensional tolerances of a mechanism are usually fixed according to various parameters such as the manufacturing process, the performance tolerances, and the manufacturing cost. Here, we assume that the cost a mechanism decreases when its dimensional tolerances increase.

We suggest the use of a sequential tolerance synthesis method. First, robustness index RI_4 is used to compute the nominal values of $DV : \overline{\mathbf{x}} = [\overline{x}_1\ \overline{x}_2 \cdots\ \overline{x}_l]^T$. Then, assuming that $\|\delta\mathbf{f}\|_2$ has to be smaller than C, the optimal tolerances of DV, Δx_{iopt}, are computed by solving the following optimization problem:

$$\begin{cases} \max\limits_{\mathbf{u}} \prod\limits_{i=1}^{l} |u_i| \\ s.t.\ \ U(u_1,\ u_2, \cdots,\ u_l) \in \xi(C) \\ \qquad u_i.\text{sign}(V_i) \geq 0,\ i = 1, \cdots,\ l \\ \qquad |u_i| \geq \Delta x_{imin},\ i = 1, \cdots,\ l \end{cases}$$

where

- **V** is the eigenvector corresponding to the maximum singular value of the sensitivity Jacobian matrix of the mechanism and V_i is its i^{th} component ;
- $\xi(C)$ is the design sensitivity ellipse of the mechanism, corresponding to $\|\delta\mathbf{f}\|_2$ equal to C.

The problem aims at finding the largest tolerance box of the design of a mechanism without rejects, which is included in $\xi(C)$. Besides, it assures that each dimensional tolerance Δx_i is higher than a minimum dimensional tolerance Δx_{imin}, which depends on the manufacturing process and $\overline{x}_i$.

For instance, Fig.2 depicts all the possible positions of U when $l = 2$ and V_1, V_2 are negative and positive, respectively, and the optimal tolerance box.

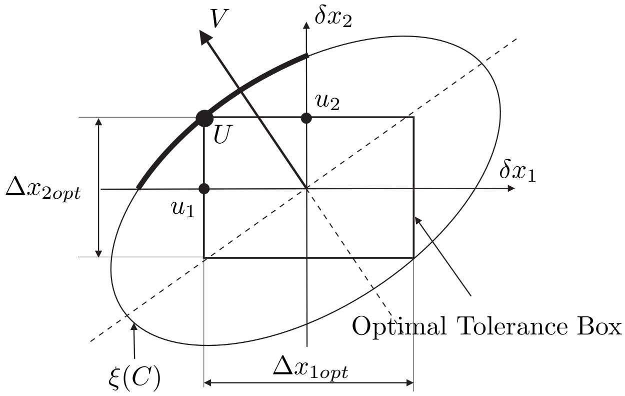

Figure 2: Tolerance Synthesis

5 Conclusions

A new robustness index was proposed. It was compared with three other robustness indices used in the literature. It turns out that the new index is an optimal criterion in a optimization robust design problem. Moreover, a sequential tolerance synthesis method was introduced.

References

[1] G. Taguchi. *On Robust Technology Development, Bringing Quality Engineering Upstream.* ASME Press, 1993.

[2] K.L. Ting and Y. Long. *Performance Quality and Tolerance Sensitivity of Mechanisms.* ASME Journal of Mechanical Design, Vol.118, march, 1996, 144-150.

[3] K. Al-Widyan and J. Angeles. *A Model-Based Formulation of Robust Design.* ASME Journal of Mechanical Design, accepted for publication, 2005.

[4] S. Caro, F. Bennis and P. Wenger. *Tolerance Synthesis of Mechanisms : A Robust Design Approach.* ASME Journal of Mechanical Design, accepted for publication, 2005.