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Arthur Charpentier

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DEPARTEMENT D'ECONOMIE

Route de Saclay

91128 PALAISEAU CEDEX

(33) 1 69333033

http://www.enseignement.polytechnique.fr/economie/

mailto:chantal.poujouly@polytechnique.edu

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Keywords: Heat wave, long range dependence, return period, heavy tails, GARMA processes, SARIMA processes

[1] École Polytechnique, Department of Economics & Université Rennes 7, Place Hoche, F-35000 Rennes, France, arthur.charpentier@univ-rennes1.fr. The financial support from the AXA Chair on Large Risks in Insurance (Fondation du Risque) is gratefully acknowledged.

On the return period of the 2003 heat wave

Arthur Charpentier [a]

[a] *Université Rennes 1, CREM, 7 place Hoche, 35000 Rennes, France & École Polytechnique, 91128 Palaiseau cedex, France*

Abstract

Extremal events are difficult to model since it is difficult to characterize formally those events. The 2003 heat wave in Europe was not characterized by very high temperatures, but mainly the fact that night temperature were no cool enough for a long period of time. Hence, simulation of several models (either with heavy tailed noise or long range dependence) yield different estimations for the return period of that extremal event.

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1 Introduction and motivations

In February 2005, opening the conference on *Climate change: a global, national and regional challenge*, chairman Dennis Tirpak pointed out that "*there is no longer any doubt that the Earth's climate is changing* [...] *globally, nine of the past 10 years have been the warmest since records began in* 1861". The summer of 2003 will be remembered for the extreme heat, and the approximately 30,000 heat-related deaths over western Europe (see IVS (2003) or WHO (2004)). More specifically, the period 1-15 August 2003 was the most intense heat of the summer. The report of Pirard et al. (2005) states that "*Europe experienced an unprecedented heat wave in the Summer* 2003. *In France, it was the warmest summer recorded for* 53 *years in terms of minimal, maximal and average temperature and in terms of duration*". Luterbacher et al. (2003) even claim that the "*summer of* 2003 *was by far the hottest summer*

Email address: `arthur.charpentier@univ-rennes1.fr` (Arthur Charpentier).

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since 1500". But because of global warming, their estimate of the return period of that event is 250 years. Hence, nobody was expecting such an event, and nothing had been planned in France to face it. Further discussion on the impact of that event can be found in Poumadre et al. (2005), Trigo et al. (2005) or Fink et al. (2004).

Actually, the underestimation of the probability of occurrence of such an event was already mentioned in the Third IPCC Assessment (Intergovernmental Panel on Climate Change (2001)). More specifically, it is pointed out that treatment of extremes (e.g. trends in extreme high temperature) is "*clearly inadequate*". Karl & Trenberth (2003) noticed that "*the likely outcome is more frequent heat waves*", "*more intense and longer lasting*" added Meehl & Tebaldi (2004). In this section, the goal is to get an accurate estimate of the return period of that event.

One of the characteristics of 2003's heat wave has not been the intensity, but the length during 10 to 20 days in several major cities in France. For instance, in Nîmes, there were more than 30 days with temperatures higher than 35° C (versus 4 in hot summers, and 12 in the previous heat wave, in 1947). Similarly, the average maximum (minimum) temperature in Paris peaked over 35° C (approached 20° C) for 10 consecutive days, 4-13 August. Previous records were 4 days in 1998 (8 to 11 of August), and 5 days in 1911 (8 to 12 of August). Similar conditions were found in London, where maximum temperatures peaked above 30°C during the period 4-13 August (see Burt (2004a), Burt & Eden (2004) and Burt (2004b)). Fink et al. (2004) also observed that '*night-time minimum temperature remained extremely high*'.

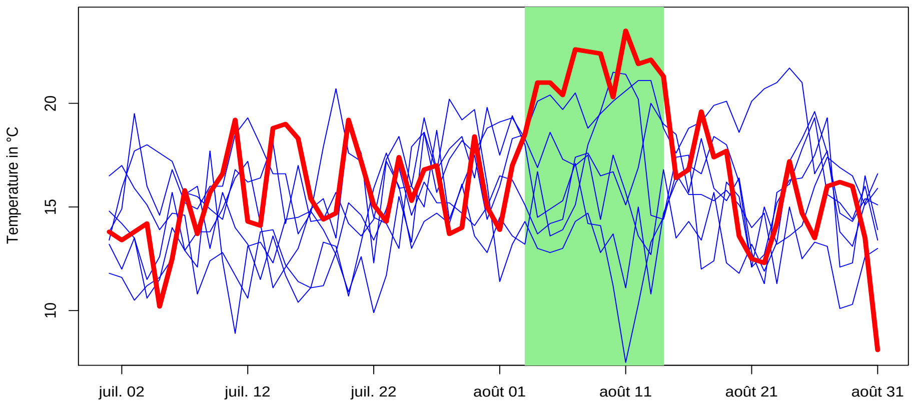

Fig. 1. Minimum daily temperature in Paris, years 1997 to 2003, 2003 being the large plain line. The plain area is the hottest 10 day period of 2003.

The use of minimal temperature was initiated by Karl & Knight (1997) when modeling 1995 heatwave in Chicago: they concentrated on the severity of an annual "*worst heat event*", and suggested that several nights with no relief from very warm nighttime minimum temperature should be most important for health impact (see also Kovats & Koppe (2005)).

In section 2 we will first try to estimate and remove the (linear) trend of the series, to obtain a stationary series that can be modeled used standard processes technique. Three models will be considered in section 3, either with long or shor range memory (to allow for persistence) and either light or heavy tails (to allow for extremal daily temperatures). A discussion on time-stability of those models will be proposed in section 4. Using those stochastic model, we will look carefully in section 5 at the return period of the 2003 heatwave.

2 Modeling trend of the daily temperature

Smith (1993) or Dempster & Liu (1995) suggested that, on a long period, the average annual temperature should be decomposed as follows:

- an increasing *linear* trend,
- a random component, with *long range dependence*.

We consider here the daily dataset provided by the European Climate Assessment & Dataset project [2]. The global warming can easily be observed using some nonparametric regression (see e.g. Figures 2 and 3, exhibiting a significant warming trend). At first sight, some linear trend can be assumed. This intuition can also be found in Beniston (2004)

In the case of the minimal temperature in Paris, the model is

$$X_t = \underset{(1.89362)}{-41.10925555} + \underset{(0.000970)}{0.02497250} \cdot t + Y_t, \tag{1}$$

where $t = 1900, \cdots, 2004$ (on a daily basis, expressed in years, from January 1900 till September 2004). Technical justifications of the validity of that estimation are given in Appendix 7.1.

In order to confirm the linear trend, on Figure 2, we consider a spline nonlinear regression on the left, and a lowess regression (locally linear regression) on the right. The linear trend is the doted line, and the plain line is the nonparametric estimation. The 95%-confidence interval is the shaded area. Not that the linear tendency is always in the 95% confidence region, which confirms the intuition of a linear tendency, as a first assumption.

[2] From `http://eca.knmi.nl/dailydata/index.php`. Temperature have been observed in Paris, Parc Montsouris, +48:49:23,+02:20:12

Another idea can be to run a stepwise model selection where a polynomial model is consider. Starting from a degree 6 polynomial tendency. When including higher degrees (2, 3 and 5) as obtained from the backward stepwise procedure, the slope of the trend (that can be seen on Figure 3) is smaller on recent years. Thus, this can justify the scenario that will be derived in Section 4, where an optimist scenario is considered (with a flat trend). On the Figure, the polynomial trend is in dotted line, and the lowess regression in plain line. The horizontal line is the slope of the linear trend. Both confirm that the slope is slower nowadays than it used to be during period 1950:1980 for instance.

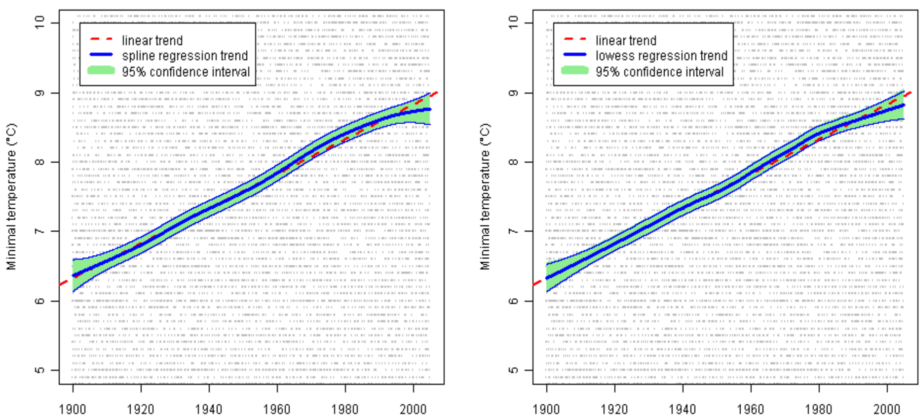

Fig. 2. Trend of the series, spline regression (on the left) and local regression (on the right).

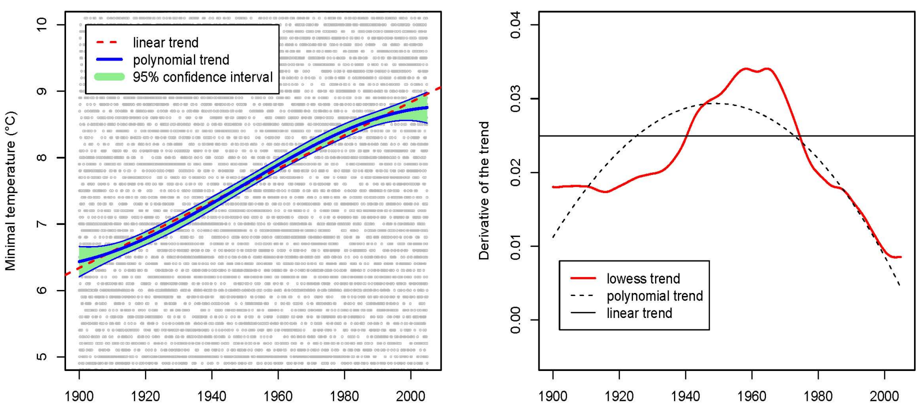

Fig. 3. Trend of the series, polynomial adjustment (on the left) and analysis of the slope (first derivative). The plain line is the slope of the linear trend.

Several authors (from Lane, Nichols & Osborn (1994) to Black et al. (2004)) have tried to explain global warming, and to find explanatory factors. As pointed out in Quereda Sala et al. (2000), the "*analysis of the trend is difficult and could be biased by non-climatic processes such as the urban effect*". In fact, "*most of the temperature rise could be due to an urban effect*": global warming can be understood as one of the consequence of "*global pollution*" (see Houghton (1997), or Braun et al. (2006) for a detailed study of the impact of transportation).

Again, the aim in this paper is not to study intensively this trend, but to focus more on the remaining series $(Y_t)_{t\in\mathbb{Z}}$ (obtained by removing the trend).

3 Long range or fat tailed distribution ?

3.1 Modeling the dynamic of $(Y_t)_{t\in\mathbb{Z}}$

The study of stationary series $(Y_t)_{t\in\mathbb{Z}}$ can be done either through its autocovariance function,

$$h \mapsto \gamma_X(h) = cov(X_t, X_{t-h}) = \mathbb{E}(X_t X_{t-h}) - \mathbb{E}(X_t) \cdot \mathbb{E}(X_{t-h}),$$

or its Fourier transform, called the spectral density of the series,

$$f_X(\omega) = \frac{1}{2\pi} \sum_{h\in\mathbb{Z}} \gamma_X(h) \exp(i\omega h)$$

for all $\omega \in [0, 2\pi]$. Those two notions are equivalent (see Brockwell & Davis (1991) for more details). Set finally $\rho_X(h)$ the autocorrelation of order h, defined as $\rho_X(h) = \gamma_X(h)/\gamma_X(0)$. For instance, a gaussian white noise is a sequence (ε_t) of independent and identically distributed gaussian variables. Hence, $\gamma_\varepsilon(h) = 0$ for any $h \neq 0$ (because of the independence assumption).

Classical models for times series are autoregressive models,

$$Y_t = \mu + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \varepsilon_t$$

where (ε_t) is a *white noise*, i.e. a series of independent observation with 0 mean and constant variance σ^2. Introduction the *lag* operator L, such a process is modeled as

$$\Phi(L)Y_t = \varepsilon_t$$

where Φ is a polynomial of order p. ARMA processes are obtained using also lag operators on the white noise,

$$\Phi(L)Y_t = \Theta(L)\varepsilon_t$$

Some processes - with interesting features in environmental sciences - have been introduced using the following formal extension of autoregressive processes

$$(1-L)^d Y_t = \varepsilon_t$$

where $d \in (-1/2, 1/2)$, where

$$(1-L)^d = 1 + \sum_{j=1}^{\infty} \frac{d(d-1)\cdots(d-j+1)}{j!}(1-)^j L^j.$$

Those processes are called *fractional* processes (see e.g. Hurst (1951) or Mandelbrot (1965)) and have long range dependence .

A stationary process $(Y_t)_{t\in\mathbb{Z}}$ is said to have long range dependence if

$$\sum_{h=1}^{\infty} |\rho_X(h)| = \infty,$$

and short range dependence if not. Recall that stationary ARMA processes have autocorrelations that are quickly decreasing, i.e. $|\rho(h)| \leq C \cdot r^h$, for all $h = 1, 2, ...$ where $r \in]0, 1[$ (see Section 3.6 in Brockwell & Davis (1991)). This is the main reason why those processes are said to have *short* range dependence: for small values of h, $corr(X_t, X_{t-h})$ can be relatively small (and non-significant).

As pointed out in Smith (1993) about temperature, "*we do not believe that the autoregressive model provides an acceptable method for assessing theses uncertainties*". Nevertheless, those models were encouraged by Nogaj et al. (2007) where it is stated that the importance when studying extremal temperature is not the dynamic, but the shape of the tails of the residuals. Hence, three models will be compared here,

- an as a benchmark, a short range dependence model for (Y_t), with a seasonal effect, where the white noise is Gaussian
- a short range dependence model for (Y_t), with a seasonal effect, where the white noise ε_t has heavy tails,
- a Gaussian model for (Y_t), with a seasonal effect, and long-range dependence (the model described in the previous section).

3.2 A short range model with light tails (benchmark model)

Consider here a SARIMA process for (Y_t), with a one-year seasonal cycle. If L stands for the *lag* operator,

$$\Phi(L)(1-L^s)(Y_t) = \Theta(L)(\varepsilon_t),$$

with $s = 365$, where Φ and Θ are two polynomial, and where (ε_t) is a Gaussian white noise, i.e. i.i.d. with $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.

From a quick look at autocorrelations of the series, we have modeled $(Y_t)_{t\in\mathbb{Z}}$ with a Gaussian SARMA(2,2) model. If $Z_t = Y_t - Y_{t-365}$

$$Z_t = \underset{(0.0419)}{1.42}\, Z_{t-1} - \underset{(0.0322)}{0.473}\, Z_{t-2} + \varepsilon_t - \underset{(0.0419)}{0.658}\, \varepsilon_{t-1} - \underset{(0.0075)}{0.103}\, \varepsilon_{t-2}$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ has variance $\widehat{\sigma^2} = 5.023$. The remaining noise $(\varepsilon_t)_{t\in\mathbb{Z}}$ satisfies the white-noise assumption, and this model has the highest AIC. Nevertheless, the distribution is far from being Gaussian, as shown on Figure 5.

3.3 A short range model with heavy tails

Here the dynamic remains unchanged, but the distribution of the noise is no longer Gaussian, and has heavier tails (allowing for more extremal events). Thus

$$\Phi(L)(1 - L^d)(Y_t) = \Theta(L)(\varepsilon_t),$$

where (ε_t) is a Student-t white noise, i.e. i.i.d. with $\varepsilon_t \sim \mathcal{S}td(0, \sigma^2, d)$, where d denotes the number of degrees of freedom.

3.4 A long range model with light tails

The shape of the autocorrelation function and the periodogram (the empirical version of the spectral density) on Figure 4 is similar to the one obtained on daily windspeed by Bouëtte et al. (2006) (in response to Haslett & Raftery (1989)). GARMA(p, d, q) processes were used to model those series. Recall that this family of stochastic processes was introduced in Hosking (1981) as

$$\Phi(L)(1 - 2uL + L^2)^d Y_t = \Theta(L)\varepsilon_t,$$

where L is the lag operator, and d is not necessarily an integer. We will consider here only GARMA(p, d) processes. Gegenbauer's frequency, defined as $\omega = \cos^{-1}(u)$, is closely related to the seasonality of the series. Here, $\widehat{u} = 2\pi/365$ (because of the annual cycle of temperature). And for the additional parameters, $\widehat{d} = 0.185$, $\widehat{\phi}_1 = 0.56$, $\widehat{\sigma^2} = 2.22^2$. A further discussion of GARMA processes can be found in Appendix 7.2.

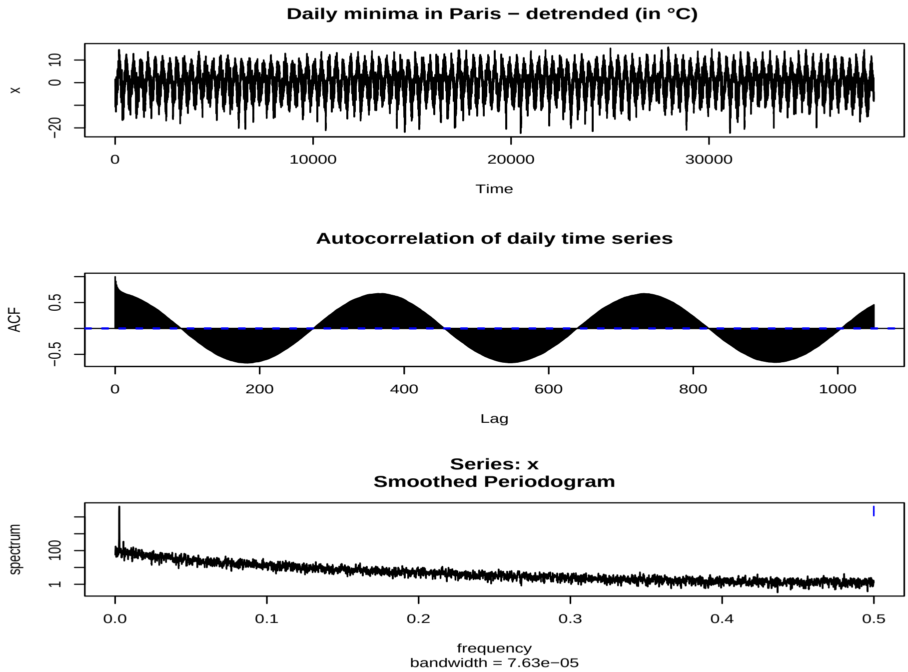

Fig. 4. Analysis of the series of residuals $(Y_t)_{t\in\mathbb{Z}}$: series, autocorrelation $(\widehat{\rho}(h))$ function and smoothed periodogram $\widehat{f}(x)$.

3.5 Discussion

Those GARMA processes have been considered on environmental applications in Bouëtte et al. (2006) and Rocha Souza & Soares (1988).

3.6 Calibration of the models and discussion

Hence, three estimations are performed and outputs are presented in Table 1. Recall that the density of the Student t distribution with ν degrees of freedom is

$$f_\nu(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)}\left(1+\frac{x^2}{2}\right)^{-(\nu+1)/2}.$$

4 Time stability of the stochastic model

A natural question that should be asked is the stationarity of the noise, i.e. the series (Z_t). In August 2006, the Washington Post headed that there were "*more frequent heat waves linked to global warming*". This was based on an international study (Alexander et al. (2006)).

	$\widehat{\phi}_1$	$\widehat{\phi}_2$	$\widehat{\theta}_1$	$\widehat{\theta}_2$	$\widehat{\sigma}^2$	$\log \mathcal{L}$	AIC
Gaussian	1.4196 (0.0419)	-0.4733 (0.0322)	-0.6581 (0.0419)	-0.1032 (0.00752)	5.023	-97578.05	195168.1
t ($\nu = 20$)	1.4191	-0.4738	-0.6571	-0.1032	5.023		
t ($\nu = 5$)	1.4134	-0.4725	-0.6551	-0.1035	5.023		

Table 1
Parameter estimation for the ARMA process, by maximum likelihood, with for the Gaussian ARMA process the log-likelihood and Akaike's AIC criteria.

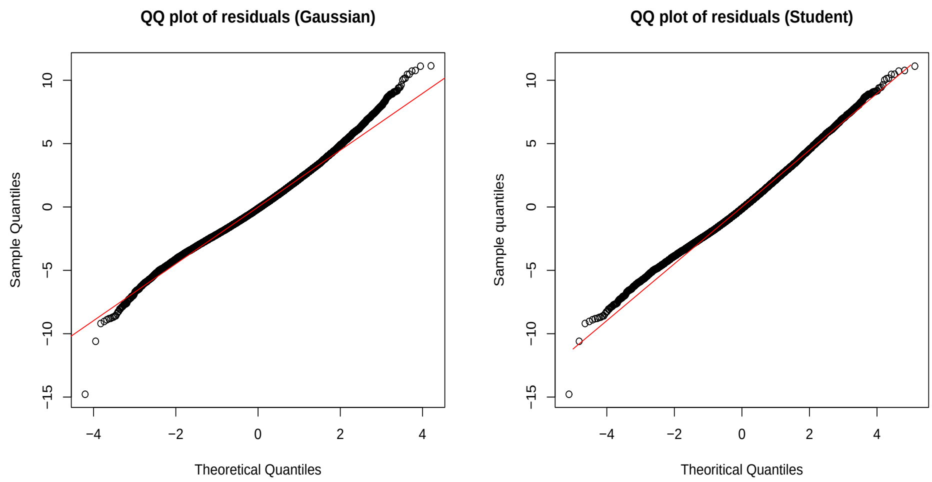

Fig. 5. QQ plot of the residuals series $(\varepsilon_t)_{t\in\mathbb{Z}}$, versus Gaussian and t-distribution (with 5 degrees of freedom).

4.1 More and more extremes ?

In order to study extremal events, instead of looking at the *average* trends as in 2, it could be interesting to focusing on a quantile regression. The slope of different quantile regressions can be visualized on Figure 6, where we to observe a *slight* change of the slope, confirming conclusions of most of the other studies (Yan et al. (2002), Meehl & Tebaldi (2004), or Alexander et al. (2006)).

To get a better understanding, we can focus on time- stability of the distribution of Y_t, the distribution of the remaining noise (considered as Gaussian or Student in the previous section) has been obtained on three period of time, on Figure 7. Overall, the distribution looks rather the same. Further, if we focus on upper tails (high nocturne temperatures) through quantiles (the temperature reached by only the 5% highest values), we can also confirm that tails are quite stable in time: we do observe more extremal event simply because the average is increasing.

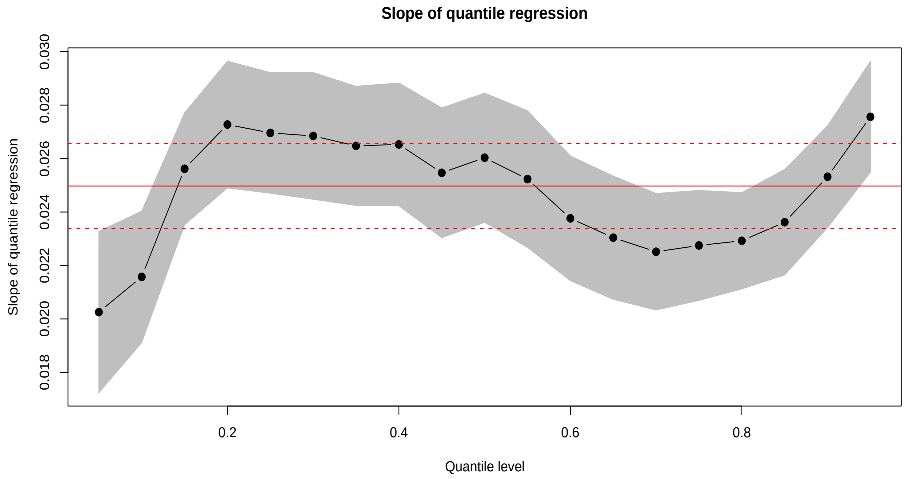

Fig. 6. Slope of the τ-quantile regression as a function of τ, on the raw temperature. The horizontal plain line is the linear trend, including a confidence interval.

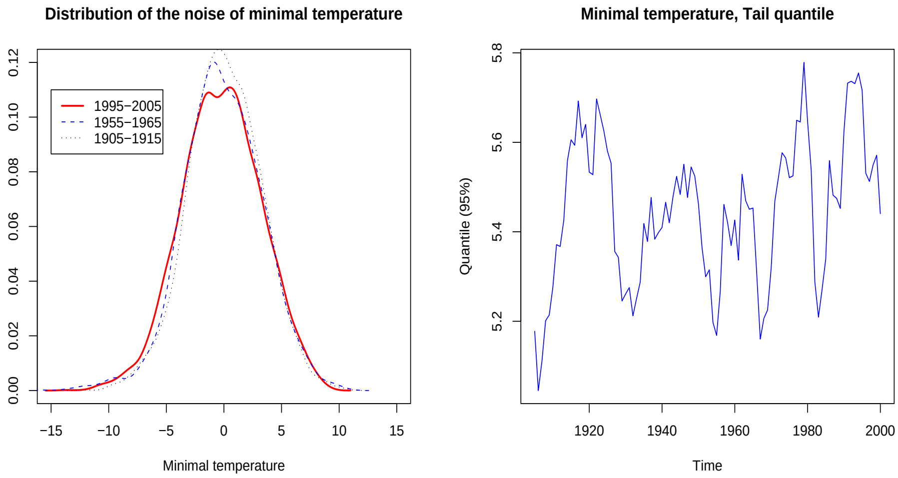

Fig. 7. Estimation of the distribution of the noise (1905-1915, 1955-1965 and 1995-2005), and estimation of the upper 95% quantile .

4.2 Stability of the dynamics

Alexander et al. (2006) does not mention only extremal temperature, but also that duration of period with extremal period has been increasing during the XXth century. Again, this can be due to the linear trend, but we can also wonder if this could also be explained by a change in persistence effects. Figure 8 compares the evolution of two quantities during the XXth century. On the left is plotted the first order autocorrelation of the noise $(r(X_t, X_{t-1}))$, and on the right, the evolution of the fractional index d in the fractional process. On the two figures, we can conclude that there is no structural change in the dynamics of the residual series.

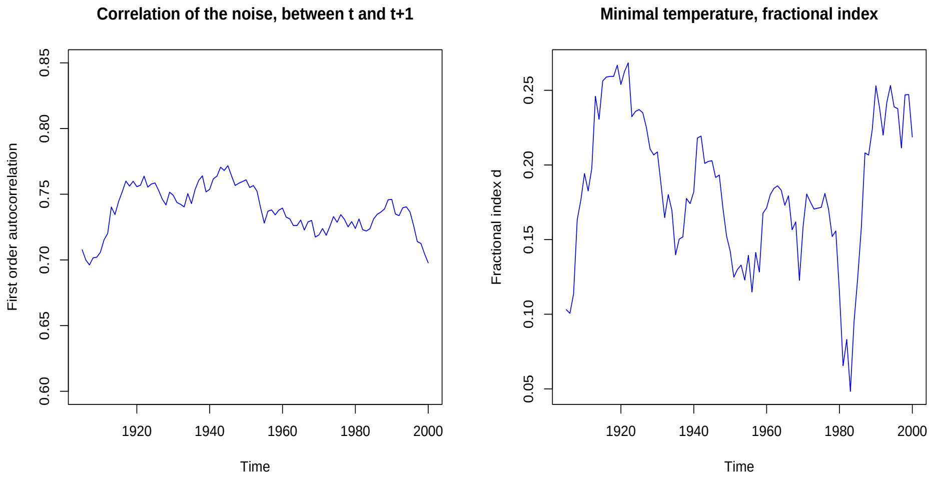

Fig. 8. First order autocorrelation of the noise ($r(X_t, X_{t-1})$) and estimation of the fractional index d.

5 Return period for two scenarios

Two scenarios on the future evolution of the linear trend will be considered in this section:

- an *optimistic* scenario, where we assume that there will be no more increasing trend in the future,
- a *pessimistic* scenario, where we assume that the trend will remain, with the same slope.

Those scenarios are obviously extremely simplistic, but not irrelevant, as mentioned in Section 2.

5.1 Definition of the heat wave

In order to compare the two models, two alternative definitions of the heat wave will be considered (both may characterize the phenomena of the beginning of August in Paris),

- during 11 consecutive days, the temperature was higher than 19° C (type (A)),
- during 3 consecutive days, the temperature was higher than 24° C (type (B)).

The first definition of the heatwave (11 days) is closely related to the common perception in France, since Météo France and most of the media have been communicating on this basis. The second one is based on Pirard et al. (2005): most of the deaths have occurred during those 3 days.

5.2 Estimation of the return period

Here $10,000$ simulations over 300 years have been used to estimate the return period. Depending on the definition of the heatwave, two different models arise. When studying extreme temperatures on a short period, one should get an accurate model for the noise (and the Gaussian standard model always underestimate the return period). But in the case of high temperature on a longer period of time, the main issue is to get an accurate model for the dynamics, in order not to underestimate persistence effects. Thus, long range dependence models should be considered.

	short memory short tail noise	short memory heavy tail noise	long memory short tail noise
optimistic	88 years	69 years	53 years
pessimistic	79 years	54 years	37 years

Table 2
Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (A)).

	short memory short tail noise	short memory heavy tail noise	long memory short tail noise
optimistic	115 years	59 years	76 years
pessimistic	102 years	51 years	64 years

Table 3
Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (B)).

Remark 1 *Recall that with a return period of* 50 *years, there is 1 chance out of 5 to have a heatwave (of type (B)) at least within the next 10 years, and 1 out of 3 within the next 20 years. With a return period of* 35 *years (type (A)), there is 1 chance out of 4 to have a heatwave at least within the next 10 years, and 1 out of 2 within the next 20 years.*

Hence, our previous study based on an estimation of the overall period in order to forecast temperature in the XXIth century is valid.

The outputs of simulations can be visualized on Figures 9 and 10. The large plain line is the result of GARMA processes simulations, i.e. long range dependence, and Gaussian noise. The dotted line is the result of ARMA processes simulations, with a Student noise (heavier tails than the Gaussian, plotted with a light gray line).

Comparing optimistic versus pessimistic scenarios, note that for pessimistic scenarios, the likelihood to have soon such an extremal event is larger. Further, we can observe that with short range dependence and heavy tailed noise, the event which is more likely to occur soon is the scenario of 3 days exceeding 24° C. On the other hand, with long range dependence, the event which is better described is the scenario of 11 consecutive days with temperature higher than 19° C.

6 Conclusion

Changes in temperature over the XXth century has been pointed out by many researchers. The global warming impact can be seen on the linear trend of the daily temperature from 1900 till 2005. But an important fact is that it had no influence on the residual series: there is no stronger temporal dependence, and tails are not heavier: the residual series is stationary, and thus, estimated return period are relevant.

Based on this first observations, we have modeled the series of minimal daily temperature in Paris (since it is the series that helps to explain the high number of casualties due to 2003' heat wave). Two models have been considered: the first one with heavy tails for the residual series (t-distribution instead of the standard Gaussian model), and the second one with persistence (and stronger temporal dependence, between date t and $t + h$). Using those two models, the return period of 2003' heat wave has been estimated. For the long period heat wave, a return period of 50 years can be expected, if urban pollution can be monitored (and the linear trend stropped), but if not, it should be shorter (35 years). In the second case (a shorter one, but with higher temperature: the scenario which caused thousands of deaths), the return period should be also 50 years. Even if those are somehow large (even it means that there is 1 chance out of 5 to have one within the next 10 years), it is much shorter than the centennial event, as claimed by medias.

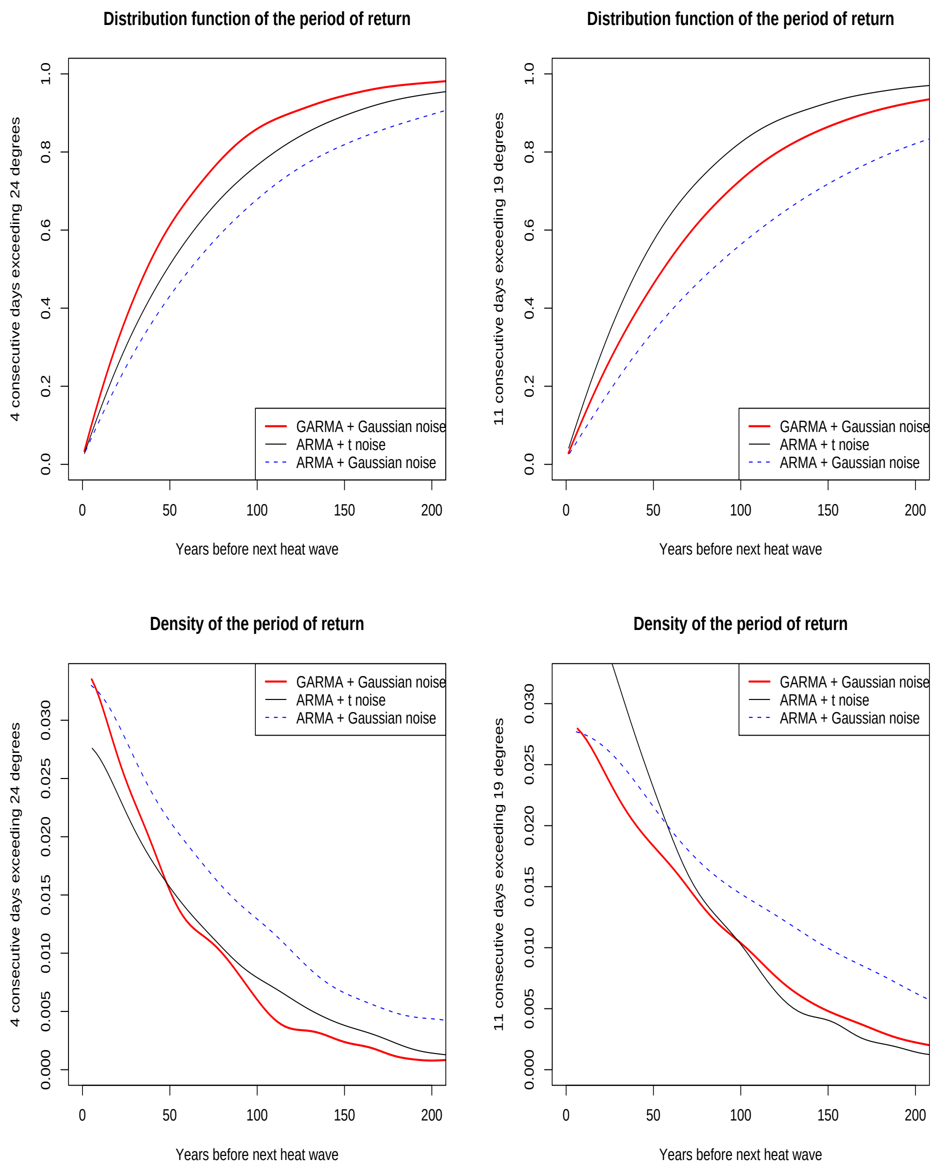

Fig. 9. Survival distributions and densities of time before the next heat wave event, when heat wave is 11 consecutive days with temperature higher than 19° C, on the left, and 3 consecutive days with temperature higher than 24° C, on the right. We assume no more linear trend in the future (optimistic scenario).

7 Technical Appendix

7.1 Estimation of the trend of a nonstationary series

Estimating the linear trend of a statistical sample is usually a trivial problem, but here, several difficulties may arise, due to the effect of (possible) long range

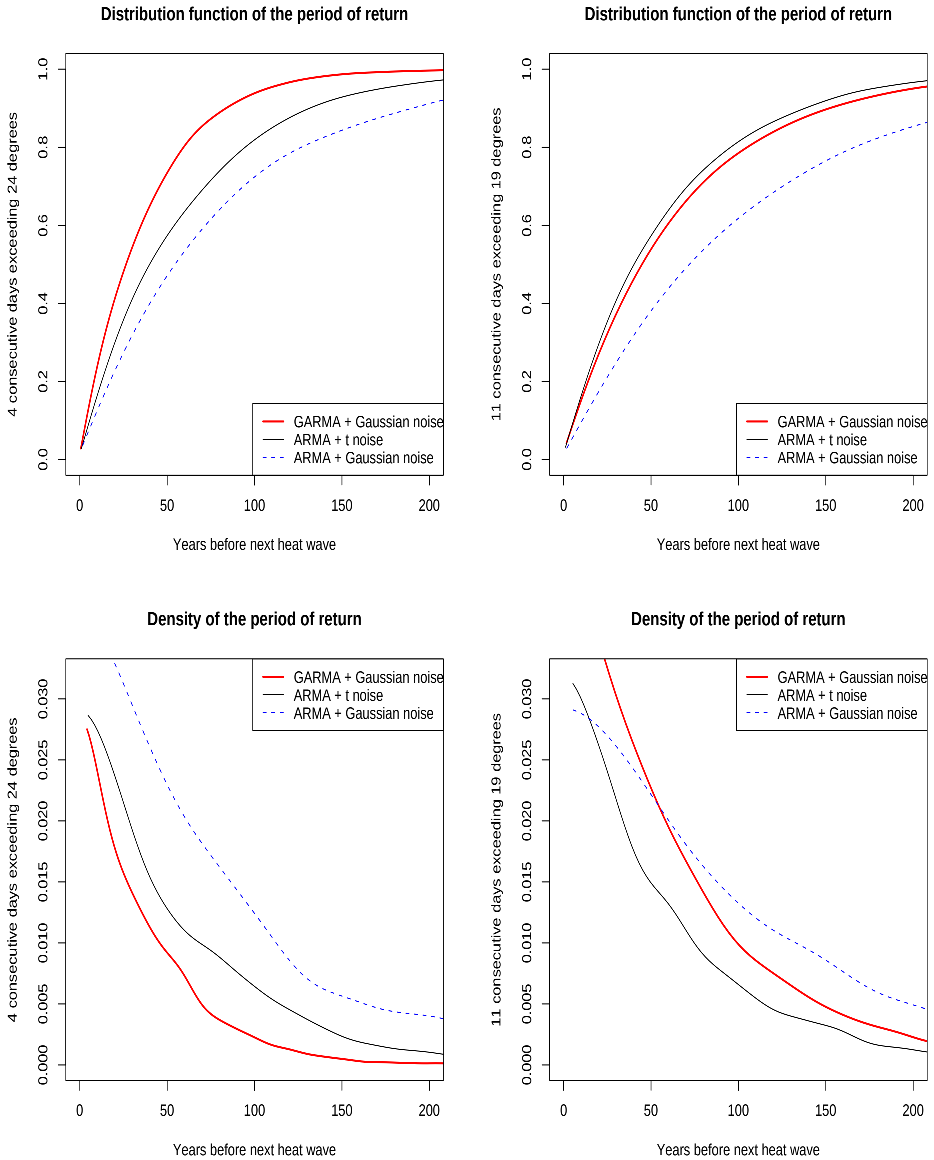

Fig. 10. Survival distributions and densities of time before the next heat wave event, when heat wave is 11 consecutive days with temperature higher than 19° C, on the left, and 3 consecutive days with temperature higher than 24° C, on the right. We keep the linear trend in the future (pessimistic scenario).

dependence .

Hence, if $\gamma_X(\cdot)$ denotes the autocovariance function of a stationary process $(X_t)_{t\in\mathbb{Z}}$,

$$Var(\overline{X}_n) = \frac{\gamma_X(0)}{n} + \frac{2}{n}\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)\gamma(k),$$

where $\overline{X}_n$ is the standard empirical mean of a sample $\{X_1, ..., X_n\}$ (see Brockwell & Davis (1991), or Smith (1993)). Furthermore, if autocovariance function satisfies $\gamma(h) \sim a \cdot h^{2d-1}$ as $h \to \infty$, then

$$Var(\overline{X}_n) \sim \frac{a}{d(2d-1)} \cdot n^{2d-2},$$

as derived in Samarov-Taqqu(1988). And further, the ordinary least squares estimator of the slope β (in the case where the X_i's are regressed on some covariate Y) is still

$$\widehat{\beta} = \frac{\sum X_i(Y_i - \overline{Y}_n)}{\sum (Y_i - \overline{Y}_n)^2}.$$

As shown in Yajima (1988), and more generally in Yajima (1991) in the case of general regressors,

$$Var(\widehat{\beta}) \sim \frac{36a(1-d)}{d(1+d)(2d+1)} \cdot n^{2d-4}.$$

7.2 Long range dependence processes: ARFIMA and GARMA

ARFIMA processes - as in introduced formally in Hosking (1981) - have the following dynamics

$$\Phi(L)(1-L)^d Y_t = \Theta(L)\varepsilon_t$$

where

$$(1-L)^d = \sum_{j=1}^{\infty} \frac{\Gamma(-d+j)}{\Gamma(-d)j!} L^j$$

where $d \in (-1/2, 1/2)$. Those processes are stationary, and further its spectral density satisfies

$$f(x) = \frac{\sigma^2}{2\pi}\left[2\sin\frac{x}{2}\right]^{-2d} \sim \frac{\sigma^2}{2\pi}\frac{1}{x^{2d}} \text{ as } x \to 0.$$

Further

$$\rho(h) \sim \frac{\Gamma(1-d)}{\Gamma(d)} h^{2d-1} \text{ as } h \to \infty.$$

Gray, Zhang & Woodward (1989) proposed an extension to model persistent *seasonal* series, using Gegenbauer's polynomial $G_n^d(\cdot)$, defined as

$$G_n^d(x) = \frac{(-2)^n}{n!}\frac{\Gamma(n+d)\Gamma(n+2d)}{\Gamma(d)\Gamma(2n+2d)}(1-x^2)^{-\alpha+1/2}\frac{d^n}{dx^n}\left[(1-x^2)^{n+d-1/2}\right].$$

and such that $G_n^d(\cdot)$ is the unique polynomial of degree n such that

$$(1-2uZ+Z^2)^d = \sum_{n=0}^{\infty} G_n^d(u) \cdot Z^n.$$

If $d \in (0, 1/2)$, and $|u| < 1$ then

$$\rho(h) \sim h^{2d-1} \sin([\pi - \arccos(u)]u) \text{ as } h \to \infty.$$

Thus, this process has a seasonal behavior (with period $u = cos\left(\frac{\pi}{365}\right)$), and persistence. See e.g. Bouëtte et al. (2006) for additional information and references therein.

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