A Survey of Connectivity in Mobile Ad Hoc Networks

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1 Introduction

Mobile Ad hoc networks are networks without centralized infrastructures in which stations can move permanently in a given environment. One of the main issues in this domain is evaluation of communication algorithms which are used there. An approach to study this problem consists in studying communication graphs induced by mobile ad hoc networks. In these graphs, stations become nodes and when two stations are able to communicate, an edge is automatically created between the two corresponding nodes. Then, the study of the communication of the ad hoc networks is limited to the study of the connectivity of their connection graphs. Most studies on connectivity of ad hoc networks go in this direction. A graph is connected if there exists at least a way connecting any pair of nodes in this graph. Connectivity is the estimate of this connexity. Several studies on the connectivity of mobile ad hocnetworks have been undertaken for a few years. These studies are mainly divided into two categories.

The first category concerned the study of the K-connectivity of ad hoc networks. According to the theorem of menger, a graph is $k-connected$ if and only if, for any pair of nodes u, v, there are k internal ways of disjoined nodes connecting u to v. In other words, a graph is K-connected if this graph remains connected after the removal of any subset of $k-1$ nodes. It is possible to not consider nodes but edges of graph. In this case, a graph is K-connected by edges if this one remains connected after suppression of any subset of $k-1$ edges of the graph. K-connectivity gives an additional information about the connexity of the graph in particular the mean degree of node. When $k = 1$ the graph is quite simply connected.

The second category of study is the evaluation of the critical transmission range of the signal of stations allowing to guarantee the connexity of the ad hoc network. These studies are used in sensors networks in order to optimize the energy expenditure of the sensors whose the emitted signal power is proportional to the length of the ray of coverage of the signal.

We present in this article a survey of the various studies undertaken on these three fields whose objective is to estimate the connectivity of the ad hoc networks.

2 Preliminaries

To study the connectivity in mobile ad hoc networks, we characterize each ad hoc network according to three parameters : environment, signal transmission ray of stations and stations mobility.

The environment is the surface where stations are located. They can be various types according to the size, the form, the dimension of space considered. we classify the environment according to five criteria : the first criterion is the form (square, circle). The second criterion concern the borders quality which can be closed (any station cannot come in or goes out the considered zone) or opened (the stations can move in or move out the considered zone). The third criterion specifies if the environment is limited (i.e. if the considered zone is a torus) or not. The fourth criterion is the size of the environment. It is limited if it is not infinite and it is not limited in the contrary case. The last criterion is the obstacles. We will specify if the environment contains or not obstacles. An obstacle is a zone of the environment where stations cannot access.

The model of the signal transmission of stations is a very important parameter in the study of the connectivity of connections graphs because results obtained depend directly on the model chosen. Two models were mainly modelled by Hekmat and P. Van Mieghem in [10] in particular the pathloss model and the lognormal model. The Pathloss model simulates the behavior of the signal on large scales. It describes the existing dependence between signal power received by a receiver and distance which separates it of the transmitter. The coverage of the signal of a node is represented by a circle. The lognormal model (more realistic) captures the random variations of the radio signal on average distances to various positions. It presents the fact that the power of the radio signal received at a fixed distance from the transmitter varies considerably according to the positions around the transmitter. The coverage of the signal is deformed. The deformation of the signal can be related to interferences introduced into the model. Connectivity in this last case is evaluated in [10] [11] [15]. The majority of works study the pathloss model because it is easier to model mathematically. We will thus consider only studies using this model and we will specify if the coverage ray of the stations is homogeneous (the same one for all) or heterogeneous and if it signal crosses or not obstacles.

There exist two ways to consider the mobility of stations in order to evaluate the connectivity of the connexions graphs of modile ad hoc networks. In the first, mobility is considered implicitly. The mobility is represented as a change of topology of the graph at every moment what can involve modifications of connections between nodes. In this case the graph is static and the localization of stations in the environment is obtained by the distribution of density of probability in the environment [3] [2]. The second way to consider the mobility of stations is explicit. When stations move in the environment, the existing edges between each pairs of nodes can disappear and other edges can appear. Then time is important for the evaluation of connectivity in particular by considering periods of connexity and nonconnexity of the graph. Several models of mobility of stations were developed these last years [1] [12]. More studied being the model random waypoint. In this model the stations choose randomly a destination in the environment. They move in straight line of their position towards this destination. Arrived at destination the stations make a pause and the cycle starts again. A large majority of the studies are relate to the first case of figure.

We will specify for each study parameters used to evaluate the connectivity of the considered ad hoc networks

3 Study of Connectivity in Mobile Ad Hoc Networks

The studies on connectivity and more generally of K-connectivity on mobile ad hoc networks are especially related to the networks which mobility is supposed to be implicit. The difficulty for explicitly dynamic ad hoc networks (influence of time in the connectivity computation) resides in the fact that to study them they should be simulated before. Consequently the results obtained can to be skewed by parameters of the simulation. The approach commonly used consists in studying the connectivity of the static graphs with a discrete time which corresponds to a sequence of consecutive snapshots studied separately. This approach is divided into two main categories : on the one hand the study of the asymptotic connectivity of connection graph i.e. when the number of nodes in the environment tends to the infinity and on the other hand the study of the connectivity in finite mobile ad hoc networks (when the number of nodes in the environment is weak).

3.1 Connectivity in Asymptotic Ad Hoc Networks

3.1.1 Connectivity of a paire of nodes

In this study, an environment closed, bounded and limited is considered. The signal coverage is circular (pathloss model) and the network is homogeneous (the length of signal coverage ray is the same for all the nodes). The form of the environment can be a square (or a rectangle) or a circle in order to facilitate computing. As the network is static, a knowledge of the distribution stations in the environment must be known, i.e. the spatial node distribution in the environment. There are many spatial node distributions (uniform, Poisson, Gauss, etc). The most used is the uniform distribution where any node has the same probability to be at any position of the environment. From the spatial node distribution, we can deduce the probability density function (pdf) for a given environment. This function indicates the probability for a station to be at a precise position in the environment. Thus, at every position of the environment we have thus a probability of presence of stations. Christian Bettstetter et. al. have determined the probability density function of a homogeneous network whose stations move according to the model Random Waypoint [6] [5] and within the framework of a heterogeneous network [4]. Esa Hyytiä et. Al. have widened the field on other forms of environments (circle, polygon and triangle) in [13].

To evaluate connectivity, the probability density function is known in advance [3] [2]. Once the density probability function known, one determines the probability that two stations randomly located are connected. Connectivity between a station a and a station b is only possible if the station b is in the signal coverage area of the station a. Thus if the distance between a and b is lower than the ray of the signal transmission, these two stations are connected [2]. The probability that the station a is connected to any other station in the environment equal to the number of stations located in the coverage area of a (the whole of the positions of its coverage area) on the full number of stations of the environment (the whole of the positions of the environment). That corresponds to an integration of the probability density function of the station coverage a.

Let f the probability density function of stations in the environment, the probability of connectivity of a station with coverage a_0 at the position x in the environment is :

$$p_0(x) = \iint_{a_0(x)} f(x)dx$$

There are two manners to apprehend the computation of the connectivity of a static ad hoc network. Firstly, from a local point of view, the connectivity is evaluated on each position of the environment. That corresponds to a distribution of connectivity in each positon of the environnment. Then by an integration on the whole of environment, a global value of connectivity is obtained. Secondly, the ad hoc network can be seen as a total entity. The probability to have a isolated node is computed in the ad hoc network. An isolated node is a node which is connected to none of its neighbors. The connectivity is evaluated by computing the probability that the connection graph does not contain an isolated node.

3.1.2 Ad Hoc Networks Local Connectivity

From the local point of view, it is supposed that there are n nodes in the environment and these nodes are independent to each other. As a node can be connected or not to its neighbor, connectivity between two stations can be interpreted like a binary random variable (0 or 1). In this case the probability that the node degree at every position x of the environment (d_x) is equal to k (k is a constant) follows a binomial probability law [3] :

$$p(d_x = k) = \binom{n-1}{k}(p_0(x))^k(1-p_0(x))^{(n-1-k)}$$

The local mean degree $\mu_0(X)$ is deducted by computing the degree expectation at each position x of the environment

$$\mu_0(x) = E(d_x) = (n-1).p_0(x)$$

On the large environments, when p_0 is very small, the binömiale distribution can be approximated by a the Poisson distribution. Pasi Lassila et. Al. use Binomial and Poisson laws to estimate connectivity in a circular environment, closed, bounded, limited of a homogeneous network [14]. they are obtained :

$$p(d_x = k) \approx \frac{\mu_0(x)^k}{k!}e^{-\mu_0(x)}$$

The probability for the node to have in more k neighbors is gived by :

$$p(d_x \leq k) \approx \sum_{d_x=0}^{k} \frac{\mu_0(x)^{d_x}}{d_x!}e^{-\mu_0(x)}$$

It is possible to determine the distribution of K-connectivity in the environment. The probability that the node has at least k neighbors at the position x is :

$$p(d_x \geq k) = 1 - p(d_x \leq k-1)$$

Finally the estimate of K-connectivity (when the connection graph has at least k neighbors) on the whole of environnment is :

$$p(d \geq k) = \iint p(d_x \geq k) f(x) dx$$

3.1.3 Ad Hoc Networks Total Connectivity

From the total point of view, the ad hoc network is perceived like a single component. The connexity of the graph depends at the same time on the coverage ray r_0 of stations and the number of stations n in the environment. The study of connectivity (contrary to the local point of view) consists in evaluating the connectivity of the network according to these two parameters. Let G the connection graph resulting from the ad hoc network, the probability that there is no node isolated in the graph is estimated in order to approach the probability of connectivity of the graph G (close to 1). The probability that a node is isolated at the position x by using a Poisson distribution is :

$$p(i_x) = p(d_x = 0) \approx e^{-\mu_0(x)}$$

The probability to have a isolated node on the environment is :

$$p(i) = \iint p(i_x) f(x) dx$$

The probability that there are no isolated nodes in the graph G is not a sufficient condition to have this graph connected :

$$p(G_{connexe}) \geq p(i)$$

The probability that a node is isolated among n other nodes is :

$$p(\neg i) = (1 - p(i))^n$$

By approximating with the Poisson law we obtain :

$$p(\neg i) = e^{-n \iint e^{-\mu_0(x)} f(x) dx}$$

The estimate of K-connectivity (when the graph of connection has at least k neighbors) on the whole of environnment is :

$$p(d_{min} \geq k) \ \approx \ (p(d \geq k))^n \ \approx \ e^{-np(d \leq k-1)}$$

3.2 Connectivity in Finite Ad Hoc Networks

Results obtained of the asymptotic study of the connectivity of the ad hoc networks are not reliable when the number of nodes in the environment is finite and weak. An other method is to study the connectivity of the ad hoc networks empirically. In the Monte Carlo method in [19], n stations are randomly distributed with a density D in an square, closed, bounded and limited environment. The network is homogeneous. By using the Dijkstra algorithm, it is determined if the network is entirely connected or not . The process is repeated M time including the number of times m that the network is connected. It is deduced the probability of connectivity of the ad hoc network over the whole of the experiment period :

$$p(G_{connexe}) = \frac{m}{M}$$

In the same idea, Bettstetter approximates connectivity by the path probability computation in [3]. Time is discrete. It is computed the probability p_{path} that two stations chosen randomly in the connection graph G_i are connected.

$$p_{path}(G_i) = \frac{number\ of\ pair\ of\ connected\ nodes}{number\ of\ pair\ of\ possible\ nodes}$$

If $p_{path}(G_i) = 1$ then the graph G_i is complete and connected and if $p_{path}(G_i) = 0$ then all nodes of the graph G_i are insolated. The path probability of a graph G over one discretized period *omega* is :

$$p_{path}(G) = \lim_{\omega \to \infty} \frac{1}{\omega} \sum_{i=1}^{\omega} p_{path}(G_i)$$

Bettstetter proposes an approximation of the connectivity of the graph $G_{connected}$ compared to the path probability below :

$$p_{path}(G) \geq p(G_{connexe})$$

Ao Tang and Al propose an empirical formula in [19] to estimate the probability of connectivity of an ad hoc network whose environment is homogeneous, closed, bounded and limited. They are interested to know if the number of stations is finished (no more 125). Their formula is reliable only if the probability of connexity between two stations is closed to zero or one. The probability for the graph to be connected is given below.

$$p(G_{connexe}) \approx \frac{e^{R-Rc/E}}{1+e^{R-Rc/E}}$$

R is the stations coverage ray, Rc and E are parameters of the model and depend on the length of the environment border.

$$\begin{cases} Rc \approx (1.0362\sqrt{\frac{ln(n)}{n}} - 0.073)L \\ \\ E \approx (\frac{0.3743n-0.3331}{n.ln^2(n)})L \end{cases}$$

they propose an upper limit of the probability of connexity of two stations randomly choosen in a square, closed, limited and homogeneous ad hoc network with the stations uniform distribution. If L is the border length of the environment., n is the number of stations, r is the signal coverage ray of stations and $sigma = l/3$ then :

$$p \leq 1 - e^{-n^2r^2/4\sigma^2}$$

In the same way, Madhav Desai and D. Manjunath in [8] propose an approximation of the limit upper of the probability of connectivity of an entire ad hoc network where the environment is in two dimensions closed, limited, limited uniform distribution according to the ray of cover of the signal r, length of with dimensions of the environment and amongst stations in environnment the n.

$$p(G_{connexe}) \leq \left(\sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^k \frac{(z-kr)^n}{z^n} u(z-kr) \right)^2$$

When the network is not entirely connected, Ao Tang et. Al. propose an index of connectivity which estimate the connectivity according to the number of connected component in the environment eta and the number of nodes in each component n_i.

$$\eta = \frac{\sum_i n_i(n_i-1)}{\sum_i n_i(\sum_i n_i - 1)}$$

The stations mobility can be taken into account by considering the index of connectivity at each discretized time interval eta_i. Then, the probability of connecticity between two stations becomes :

$$p = 1 - \prod_{i=1}^{k}(1 - \eta_i)$$

Wang Hanxing, Liu Guilin and Zhao Wei compute analytically the connectivity of an ad hoc networks uniformly distributed in an environment closed, bounded, limited and homogeneous according to a F-node of a connected graph [20]. A f-node is a node of a connected graph whose the deletion returns the graph disconnexe and divides it into several components. Wang Hanxing et. Al. compute the probability of connexity of two stations in this graph according to the number of connected component and show that this graph is composed of more five components.

4 Study of Critical Transmission Range for connectivity

The second way studied is the critical transmission range (critical coverage ray) necessary to the stations to connect the whole network. Two options are possible : the case where the ad hoc network coverage is homogeneous and the case where it is heterogeneous. However the way currently studied concern the homogeneous ad hoc networks.

4.1 Critical Transmission Range of asymptotic connected Networks

P. Gupta and P.R. Kumar [9] have determined a sufficient condition on the coverage ray of stations r to connect the whole ad hoc network when the number of nodes tends to the infinity. The environment is normalized, limited, closed, bounded and without obstacles. The stations are uniformly and independently distributed in the environment. Two stations can communicate only if the distance separating them is lower or equal to the coverage ray r. If $pir^2(N) = \frac{logn+c(N)}{N}$ then the graph $G(n, r(n))$ is asymptotically connected with a probability 1 when n tends to the infinity if and only if $c(n)\ to+\ infty$. Mobility is not taken into account however it can be perceived as a change of topology of the ad hoc network in which computations are repeated when the topology change.

Paolo Santi and Douglas Mr. Blough have extended the result for square environment in 1 , 2 or 3 dimensions [18]. They suppose that if n nodes uniformly and independently distributed are in an area $R = [0, l]^d$ with $d = 2$ or $d = 3$, if $r^d n = kl^d ln(l)$ for any constant $k > 0$, r very small compared to l, n very large compared to 1, then if $k > d.k_d$ or $k = d.k_d$ with r very large compared to 1 ($k_d = 2^d d^{D/2}$), then the communication graph is connected when n tends to infinity the

According to Penrose et. al. in [16], the value of common minimal transmission such as the communication graph is connected is equivalent to the longest edge length of an Euclidean minimal spanning tree. Paolo Santi has established that this longest edge of a minimal spanning tree depends only on the minimal value of the probability density function [17]. If M is a model of mobility of nodes in a square environment standardized $R = [0.1]^2$, closed limited and without obstacles. By supposing that the probability density function of nodes in the environment f_m is continuous on borders and the minimum of f_m on R is higher than zero, then when the constant $c\ Ge1$ we have :

$$\lim_{n\to\infty} r_M = c\sqrt{\frac{\ln n}{\pi n}}$$

In the preceding formula, the problems consist in evaluating the constant c when n tends to the infinity. However it is possible to compute probability density function of the nodes moving according to random waypoint mobility model within an environment closed, bounded, limited and without obstacles [17]. The critical range of transmission depends on pause time p of the stations and their travel velocity ($v_{min} = v_{max} = v$). The minimal value of the probability density function is reached on the borders of the environment. It is equal to the pause probability of $P_p = \frac{p}{p+\frac{0.521405}{v}}$ and the critical coverage ray of stations is :

$$\lim_{n\to\infty} r_p^w = \left(\frac{p + \frac{0.521405}{v}}{p}\right)\sqrt{\frac{\ln n}{\pi n}} \qquad si\ p > 0$$

Evaluation of the probability of pause of stations moving according to the Random Waypoint mobility model was studied in [5] :

$$p_{pause} = \frac{E[t_{pause}]}{E[t_{pause}] + E[t_{deplacement}]}$$

Guanghui Zhang et Al. compute the probability of pause p_p according to the pause time t_{pause}, maximum speed v_{max} and minimal speed v_{min} in order to measure the effects of the mobility on the critical transmission range for connectivity of ad hoc networks [21].

$$p_{pause} = \frac{t_{pause}(v_{max} - v_{min})}{t_{pause}(v_{max} - v_{min}) + 0.521(\ln v_{max} - \ln v_{min})}$$

4.2 Critical transmission range of Finite Ad Hoc Networks

A fundamental result in the study of critical signal coverage of nodes in finite ad hoc networks wa presented by Penrose and Al in [16]. Indeed, the common minimal transmission value such as the communication graph is connected is equivalent to the length of the longest edge of Euclidean minimal spanning tree. Miguel Sanchez et al. in [7] use this idea in order to compute the critical transmission range of ad hoc networks by considering different types of mobility models (Random Waypoint, Random Gauss-Markov, Random direction model). Their results obtained show that there does not exist strong dependence between the mobility model and the critical coverage ray of the station.

5 Conclusion

We have presented in this article different studies undertaken these last years on connectivity for stations in the ad hoc mobile networks. Two fields were mainly studied. the probability computation of connectivity of a graph and the computation of critical transmission range of the signal of stations. The propagation model of the signal is supposed to be circular, however recent studies propose a model of the signal propagation which integrates the interferences (model lognormal). This model remains difficult to formalize mathematically as well as the interferences of environnment. Another field which was not studied in this article concern the heterogeneous ad hoc networks. However, certain articles tackled the subject. Although most these studies concern the static ad hoc networks, the ad hoc networks integrating the stations mobility explicitly start to show interest. Connectivity is very dependant of environment. however, the majority of the studies presented in this article does not integrate the obstacles in the environment (except borders).

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