Super-elastic Behavior of Shape Memory Alloys under Cyclic Loading

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Abstract

This paper concerns the super-elastic behavior of Shape Memory Alloys (SMA) under cyclic loadings. Particular attention is paid to ratchetting (i.e., evolution of residual strain with the number of cycles). First, a series of uni-axial tensile tests on Cu-Al-Be wires has been presented. In a second part, a macroscopic model is proposed and identified from experimental results.

Keywords: Cu-Al-Be Shape Memory Alloys, super-elasticity, cyclic loading, ratchetting, macroscopic approach

1. Introduction

In many engineering applications, structures and components of SMA are subjected to cyclic loads. This can create a cumulative residual deformation (i.e., ratchetting) [1,2].

Ratchetting is an aspect of the mechanical behavior of metalic alloys that has received considerable attention over the last twenty years [3,4]. Among the great number of investigations, few studies concern the super-elastic behavior of SMA.

The purpose of the present work is, on one hand, to build an experimental database under uni-axial cyclic tensile loading and, on the other hand, to propose a macroscopic model taking into account the main effects of cyclic loads on the behavior of SMA.

2. Experimental investigation under uni-axial cyclic tensile loading

The material chosen for this study is a Cu-Al-Be SMA (Cu: 87%, Al: 11%, Be: 2%, atomic percentages). The material is available as wire with a 1.4 mm diameter. Each wire was heat treated at 923 K during twenty minutes and then quenched in boiling water during one hour.

An uni-axial loading-unloading tensile test has been performed to characterize the specific behavior of our material. The tests have been performed with a Zwick electro-mechanical testing machine operating in axial strain control. A 10-mm extensometer has been used to measure the axial strain. All the tests have been performed at room temperature.

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Figure 1 shows the stress-strain curve for twenty-five loading-unloading cycles. It can be observed that for our material:

– a residual strain, ε^r, appears after the first cycle and grows with the number of cycles until a reached limit (figure 2),
– the maximum stress, σ_{max}, is not influenced by the cyclic loading (figure 3),
– the slope at reload, E_r, decreases slightly after the first cycle (figure 4).

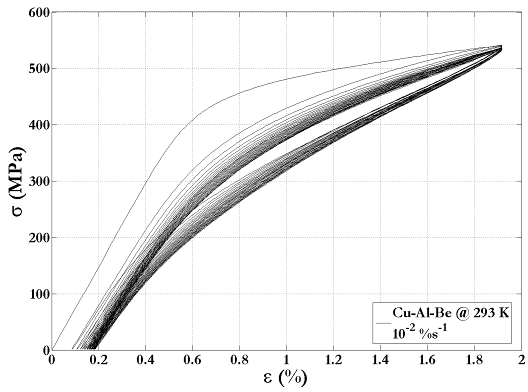

Fig. 1. Test stress-strain curves for loading-unloading cycles.

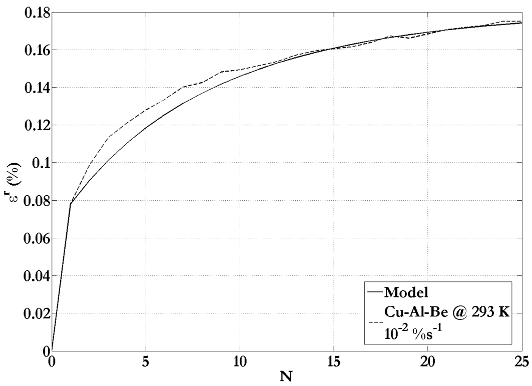

Fig. 2. Evolution according to the number of cycles of the residual strain ε^r.

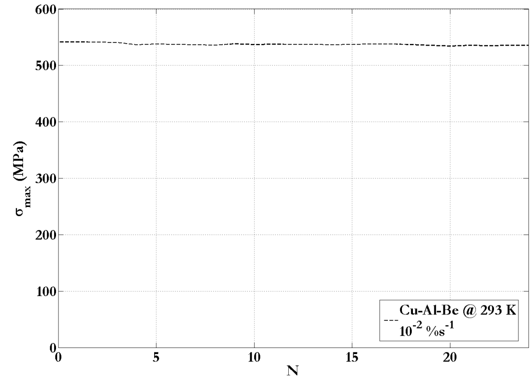

Fig. 3. Evolution according to the number of cycles of the maximum stress σ_{max}.

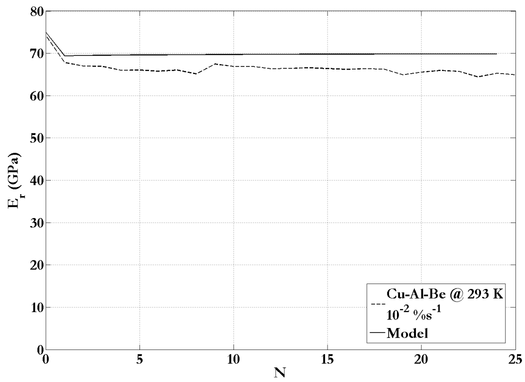

Fig. 4. Evolution according to the number of cycles of the slope at reload E_r.

3. A cyclic super-elasticity model: one-dimensional loading case

The proposed model is based on the super-elasticity model recently proposed by Bouvet et al. [5].The main originality concerns the description of the elastic domain of the material, at a two-phased state (i.e., austenite and martensite), by two different "yield" surfaces for forward and reverse transformation, respectively. In this work, a modification is proposed to take into account the effect due to cyclic loading.

The constitutive equations of the model are presented hereafter (the interested reader can find more details in [5]):

– Strain decomposition is:

$$\varepsilon^{tr} = \varepsilon - \varepsilon^{e} = \varepsilon - \frac{\sigma}{E}$$

– Relation between martensite volume fraction and transformation strain is:

$$z = \frac{\varepsilon^{tr}}{\gamma}$$

– ”Yield” function for forward transformation is:

$$f_1 = \sigma - R(z) - \sigma_0(T) \leq 0$$

– ”Yield” function for reverse transformation is:

$$f_2 = -\sigma + R(z) + \sigma_0(T) - \delta \leq 0$$

– ”Flow” rule for forward transformation is:

$$\dot{\varepsilon^{tr}} = \dot{\lambda_1}\frac{\partial f_1}{\partial \sigma} = \dot{\lambda_1} = \gamma \dot{z}$$

– ”Flow” rule for reverse transformation is:

$$\dot{\varepsilon^{tr}} = \dot{\lambda_2}\frac{\partial f_2}{\partial \sigma} = -\dot{\lambda_2} = \gamma \dot{z}$$

– Evolution of ”yield” surfaces size during forward transformation is:

$$R(z) = R_{max}\frac{\ln\left(1 + (n_1 - 1)z\right)}{\ln n_1}$$

– Evolution of ”yield” surfaces size during reverse transformation is:

$$R(z) = R_C + f\left(\frac{z}{z_B}\right)(R_B - R_C)$$

with $f(z) = z\left(1 + n_2\left(1 - z\right)^2\right)$,
$R_B = R_{max} f\left(z_B\right)$ and $R_C = \Delta R_m\left(1 - e^{-bz_c}\right)$

– Cumulated martensite volume fraction is:

$$z_c = \int |\dot{z}| dt$$

Where E, γ, σ_0, δ, R_{max}, n_1, n_2, ΔR_m and b are material parameters.

4. A cyclic super-elasticity model: multi-axial proportional loading case

The proposed model is based on the previous model. The modification is to take into account the multi-axial proportional effect (figure 5).

The constitutive equations of the model are presented hereafter:

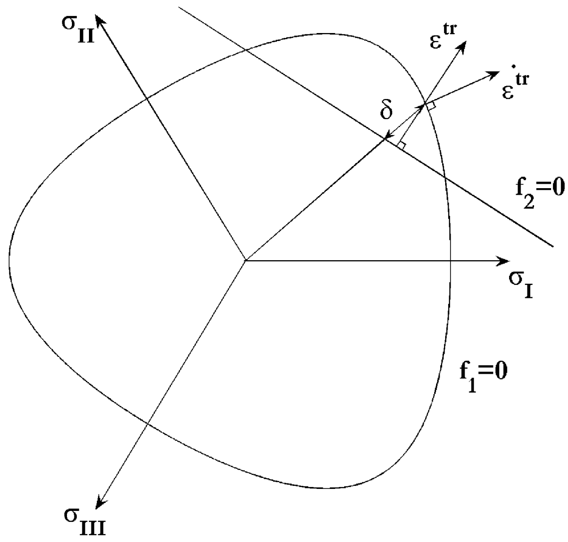

Fig. 5. Yields functions describing the behavior of shape memory alloy.

– Strain decomposition is:

$$\boldsymbol{\varepsilon}^{tr} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{e} = \boldsymbol{\varepsilon} - \mathbb{E}^{-1}\varpi$$

– Relation between martensite volume fraction and transformation strain is:

$$z = \frac{\varepsilon^{tr}_{eq}}{\gamma}$$

with $\varepsilon^{tr}_{eq} = \overline{\varepsilon^{tr}}\frac{g(-y_\varepsilon)}{g(-1)}$, $\overline{\varepsilon} = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}^{tr} : \boldsymbol{\varepsilon}^{tr}}$,
$y_\varepsilon = 4\frac{det(\varepsilon^{tr})}{\overline{\varepsilon^{tr}}^3}$

– ”Yield” function for forward transformation is:

$$f_1 = \varpi_{eq} - R(z) - \sigma_0(T) \leq 0$$

with $\varpi_{eq} = \overline{\sigma} g(y_\sigma)$, $\overline{\sigma} = \sqrt{\frac{3}{2}\varpi : \varpi}$,
$g(y) = \cos\left(\frac{\cos^{-1}(1 - a(1 - y))}{3}\right)$,
$y_\sigma = \frac{27}{2}\frac{det(dev(\sigma))}{\overline{\sigma}^3}$

– ”Yield” function for reverse transformation is:

$$f_2 = -\varpi_{eq} + R(z) + \sigma_0(T) - \delta \leq 0$$

– ”Flow” rule for forward transformation is:

$$\dot{\boldsymbol{\varepsilon}^{tr}} = \dot{\lambda_1}\frac{\partial f_1}{\partial \varpi}$$

– ”Flow” rule for reverse transformation is:

$$\dot{\boldsymbol{\varepsilon}^{tr}} = \dot{\lambda_2}\frac{\partial f_2}{\partial \varpi}$$

E	γ	σ_0	δ	R_{max}	n_1	n_2	ΔR_m	b	a
75 GPa	5.85 %	360 Mpa	10 MPa	250 MPa	81.2	0.2	3.4 GPa	0.3	0.7

Table 1
Material parameters for a Cu-Al-Be SMA at room temperature.

– Evolution of "yield" surfaces size during forward transformation is:

$$R(z) = R_{max} \frac{\ln (1 + (n_1 - 1)z)}{\ln n_1}$$

– Evolution of "yield" surfaces size during reverse transformation is:

$$R(z) = R_C + f\left(\frac{z}{z_B}\right)(R_B - R_C)$$

with $f(z) = z\left(1 + n_2 (1 - z)^2\right)$,
$R_B = R_{max} f(z_B)$ and $R_C = \Delta R_m \left(1 - e^{-bz_c}\right)$

– Cumulated martensite volume fraction is:

$$z_c = \int |\dot{z}| dt$$

Where $\mathbb{E}$, γ, σ_0, δ, R_{max}, n_1, n_2, ΔR_m, b and a are material parameters.

5. Material parameters identification

The material parameters set has been identified using the experimental results presented in the second section. The parameters values obtained for our Cu-Al-Be SMA are given in table 1.

Figure 6 shows the stress-strain evolution obtained by the model. It can be noted that the model takes into account accurately the main effect observed for a super-elastic behavior under loading-unloading cyclic test. Figures 2-4 show the comparison between experimental results and modeling concerning the residual strain, the maximum stress and the slope at reload evolutions, respectively. A good agreement can be observed.

6. Conclusion

In this work, a constitutive model has been proposed to describe the super-elastic behavior of a Cu-Al-Be SMA under cyclic loadings.

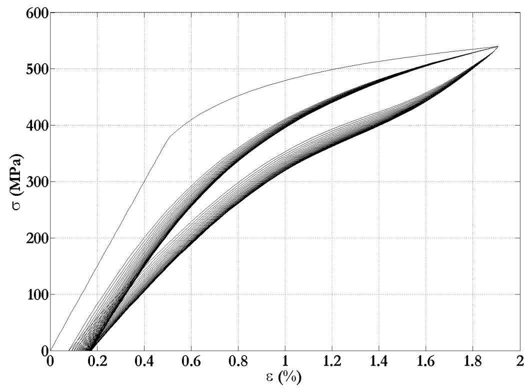

Fig. 6. Model stress-strain curves for loading-unloading cycles.

In its general version (i.e., 3D formulation), the model takes into account the super-elasticity, the tension-compression asymmetry, the return point memory effect, the martensite reorientation under non-proportional multi-axial loading and the ratcheting under cyclic loading.

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