

# Efficient Learning of Sparse Conditional Random Fields for Supervised Sequence Labelling

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## Abstract

Conditional Random Fields (CRFs) constitute a popular and efficient approach for supervised sequence labelling. CRFs can cope with large description spaces and can integrate some form of structural dependency between labels. In this contribution, we address the issue of efficient feature selection for CRFs based on imposing sparsity through an  $\ell^1$  penalty. We first show how sparsity of the parameter set can be exploited to significantly speed up training and labelling. We then introduce coordinate descent parameter update schemes for CRFs with  $\ell^1$  regularization. We finally provide some empirical comparisons of the proposed approach with state-of-the-art CRF training strategies. In particular, it is shown that the proposed approach is able to take profit of the sparsity to speed up processing and handle larger dimensional models.

## 1 Introduction

Conditional Random Fields (CRFs), originally introduced in [16], constitute a popular and effective approach for supervised structure learning tasks involving the mapping between complex objects such as strings and trees. An important property of CRFs is their ability to cope with large and redundant feature sets and to integrate some form of structural dependency between output labels. Directly modeling the conditional probability of the label sequence given the observation stream allows to explicitly integrate complex dependencies that can not directly be accounted for in generative models such as Hidden Markov Models (HMMs). Results presented in section 6.2 will illustrate this ability to use large sets of redundant and non-causal features.

Training a CRF amounts to solving a convex optimization problem: the maximization of the penalized conditional log-likelihood function. For lack of an analytical solution however, the CRF training task requires numerical optimization and implies to repeatedly perform inference over the entire training set during the computation of the gradient of the objective function.

This is where the modeling of structure takes its toll: for general dependencies, exact inference is intractable and approximations have to be considered. In the simpler case of linear-chain CRFs, modeling the interaction between pairs of adjacent labels makes the complexity of inference grow quadratically with the size of label set: even in this restricted setting, training a CRF remains a computational burden, especially when the number of output labels is large.

Introducing structure has another, less studied, impact on the number of potential features that can be considered. It is possible, in a linear-chain CRF, to introduce features that simultaneously test the values of adjacent labels and some property of the observation. In fact, these features often contain valuable information [24]. However, their number scales quadratically with the number of

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labels, yielding both a computational (feature functions have to be computed, parameter vectors have to be stored in memory) and an estimation problem.

The estimation problem stems from the need to estimate large parameter vectors based on sparse training data. Penalizing the objective function with the  $\ell^2$  norm of the parameter vector is an effective remedy to overfitting; yet, it does not decrease the number of feature computations that are needed. In this paper, we consider the use of an alternative penalty function, the  $\ell^1$  norm, which yields much sparser parameter vectors [28]<sup>1</sup>. As we will show, inducing a sparse vector not only reduces the number of feature functions that need to be computed, but it can also reduce the time needed to perform parameter estimation and decoding.

The main shortcoming of the  $\ell^1$  regularizer is that the objective function is no longer differentiable everywhere, challenging the use of gradient-based optimization algorithms. Proposals have been made to overcome this difficulty: for instance, the orthant-wise limited-memory quasi-Newton algorithm [1] uses the fact that the  $\ell^1$  norm remains differentiable when restricted to regions in which the sign of each coordinate is fixed (an “orthant”). Using this technique, [12] reports test performance that are on par with those obtained with a  $\ell^2$  penalty, albeit with more compact models. Our first contribution is to show that even in this situation (equivalent test performance), the  $\ell^1$  regularization may be preferable as sparsity in the parameter set can be exploited to reduce the computational cost associated with parameter training and label inference.

For parameter estimation, we consider an alternative optimization approach, which generalizes to CRFs the proposal of [10] (see also [9, 14]). In a nutshell, optimization is performed in a coordinate-wise fashion, based on an analytic solution to the unidimensional optimization problem. In order to tackle realistic problems, we propose an efficient blocking scheme in which the coordinate-wise updates are applied simultaneously to a properly selected group (or block) of parameters. Our main methodological contributions are thus twofold: (i) a fast implementation of the training and decoding algorithms that uses the sparsity of parameter vectors and (ii) a novel optimization algorithm for using  $\ell^1$  penalty with CRFs. These two ideas combined together offer the opportunity of using very large “virtual” feature sets for which only a very small number of features are effectively selected. As will be seen (in Section 2), this situation is frequent in typical natural language processing applications, particularly when the number of possible labels is large. Finally, the proposed algorithm has been implemented as C code and validated through experiments on artificial and real-world data. In particular, we provide detailed comparisons, in terms of numerical efficiency, with solutions traditionally used for  $\ell^2$  and  $\ell^1$  penalized training of CRFs in publicly available software such as CRF++ [15], CRFsuite [23] and crfsgd [3].

The rest of this paper is organized as follows. In Section 2, we introduce our notations and restate more precisely the issues we wish to address, based on the example of a simple natural language processing task. Section 3 discusses the algorithmic gains that are achievable when working with sparse parameter vectors. We then study, in Section 4, the training algorithm used to achieve sparsity, which implements a coordinate-wise descent procedure. Section 5 discusses our contributions with respect to related work. And finally, Section 6 presents our experimental results, obtained both on simulated data, a phonetization task, and a named entity recognition problem.

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<sup>1</sup>To be more precise, we consider in the following a mixed penalty which involves both  $\ell^1$  and squared  $\ell^2$  terms, also called the *elastic net penalty* [35]. The sparsity of the solution is however controlled mostly by the amount of  $\ell^1$  regularization.

## 2 Conditional Random Fields and Sparsity

Conditional Random Fields [16, 27] are based on the following discriminative probabilistic model

$$p_{\theta}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z_{\theta}(\mathbf{x})} \exp \left\{ \sum_{t=1}^T \sum_{k=1}^K \theta_k f_k(y_{t-1}, y_t, x_t) \right\} \quad (1)$$

where  $\mathbf{x} = (x_1, \dots, x_T)$  denotes the input sequence and  $\mathbf{y} = (y_1, \dots, y_T)$  is the output sequence, also referred to as the sequence of labels.  $\{f_k\}_{1 \leq k \leq K}$  is an arbitrary set of feature functions and  $\{\theta_k\}_{1 \leq k \leq K}$  are the associated real-valued parameter values<sup>2</sup>. The CRF form considered in (1) is sometimes referred to as linear-chain CRF, although we stress that it is more general, as  $y_t$  and  $x_t$  could be composed not directly of the individual sequence tokens, but on sub-sequences (e.g., trigrams) or other localized characteristics. We will denote by  $Y$ ,  $X$ , respectively, the sets in which  $y_t$  and  $x_t$  take their values. The normalization factor in (1) is defined by

$$Z_{\theta}(\mathbf{x}) = \sum_{\mathbf{y} \in Y^T} \exp \left\{ \sum_{t=1}^T \sum_{k=1}^K \theta_k f_k(y_{t-1}, y_t, x_t) \right\}. \quad (2)$$

The most common choice of feature functions is to use binary tests such that  $f_k(y_{t-1}, y_t, x_t)$  is one only when the triplet  $(y_{t-1}, y_t, x_t)$  is in a particular configuration. In this setting, the number of parameters  $K$  is equal to  $|Y|^2 \times |X|_{\text{train}}$ , where  $|\cdot|$  denotes the cardinal and  $|X|_{\text{train}}$  refers to the number of configurations of  $x_t$  observed in the training set. As discussed in Section 3 below, the bottleneck when performing inference is the computation of the pairwise conditional probabilities  $p_{\theta}(y_{t-1} = y, y_t = y' | \mathbf{x})$ , for  $t = 1, \dots, T$  and  $(y, y') \in Y^2$  for all training sequences, which involves a number of operations that scales as  $|Y|^2$  times the number of training tokens. Thus, even in moderate size applications, the number of parameters can be very large and the price to pay for the introduction of sequential dependencies in the model is rather high, explaining why it is hard to train CRFs with dependencies between more than two adjacent labels.

To motivate our contribution, we consider below a moderate-size natural language processing task, namely a word phonetization task based on the Nettek dictionary [25], where  $|Y|$  (the number of phonemes) equals 53 and  $|X|$  is 26 (one value for each English letter). For this task, we use a CRF that involves two types of features functions, which we refer to as, respectively, *unigram features*,  $\mu_{y,x}$ , and *bigram features*,  $\lambda_{y',y,x}$ . These are such that

$$\sum_{k=1}^K \theta_k f_k(y_{t-1}, y_t, x_t) = \sum_{y \in Y, x \in X} \mu_{y,x} \mathbf{1}(y_t = y, x_t = x) + \sum_{(y',y) \in Y^2, x \in X} \lambda_{y',y,x} \mathbf{1}(y_{t-1} = y', y_t = y, x_t = x) \quad (3)$$

where  $\mathbf{1}(\text{cond.})$  is equal to 1 when the condition is verified and to 0 otherwise.

The use of the sole unigram features  $\{\mu_{y,x}\}_{(y,x) \in Y \times X}$  would result in a model equivalent to a simple bag-of-tokens position-by-position logistic regression model. On the other hand, bigram

<sup>2</sup>Note that various conventions are found in the literature regarding the treatment of the initial term (with index  $t = 1$ ) in (1). Many authors simply ignore the term corresponding to the initial position  $t = 1$  for so-called (see below) bigram features. In our implementation,  $y_0$  refers to a particular (always observed) label that indicates the beginning of the sequence. In effect, this adds a few parameters that are specific to this initial position. However, as the impact on performance is usually negligible, we omit this specificity in the following for the sake of simplicity.

features  $\{\lambda_{y',y,x}\}_{(y,x)\in Y^2\times X}$  are helpful in modelling dependencies between successive labels. The motivations for using simultaneously both types of feature functions and the details of this experiment are discussed in Section 6. In the following, by analogy with the domain of constrained optimization, we refer to the subset of feature functions whose multiplier is non-zero as the “active” features.

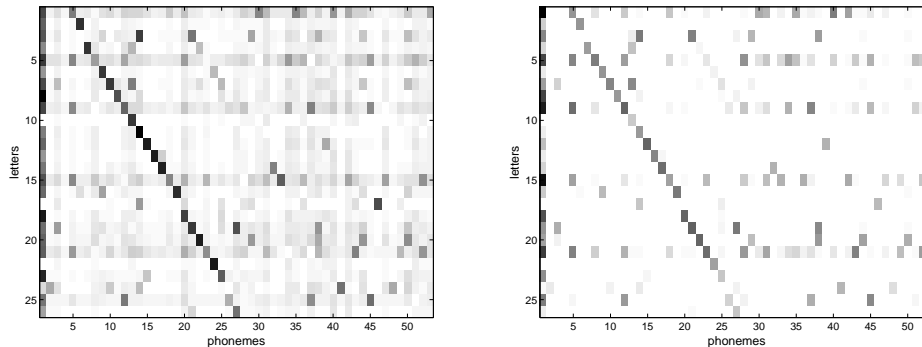


Figure 1: The  $\ell^1$  norm of the parameters estimated with standard  $\ell^2$ -regularized maximum likelihood for the Nettek task. Above:  $|\mu_{y,x}|$  for the 53 phonemes  $y$  and 26 letters  $x$ . Below:  $\sum_{y'} |\lambda_{y',y,x}|$  for the 53 phonemes  $y$  and 26 letters  $x$ .

Figure 1 allows to visualize the magnitude of the parameter vectors obtained with the  $\ell^2$ -regularized maximum likelihood approach. Sparsity is especially striking in the case of the  $\lambda_{y',y,x}$  parameters which are, by far, the most numerous ( $53^2 \times 26$ ). Another observation is that this sparsity pattern is quite correlated with the corresponding value of  $|\mu_{y,x}|$ : in other words, most sequential dependencies  $\lambda_{y',y,x}$  are only significant when the associated marginal factor  $\mu_{y,x}$  is. This suggests to take a closer look at the internal structure of the feature set.

From this picture, one would expect to attain the same classification accuracy with a much reduced set of feature functions using an appropriate feature selection approach. These preliminary considerations motivate some of the questions that we try to answer in this contribution: (1) Is it possible to take profit of the fact that a large proportion of the parameters are null to speed up computations? (2) How can we select features in a principled way during the parameter estimation phase? (3) Can sparse solutions also result in competitive test accuracy?

### 3 Fast Computations in Sparse CRFs

#### 3.1 Computation of the Objective Function and its Gradient

Given  $N$  independent labelled sequences  $\{\mathbf{x}^{(i)}, \mathbf{y}^{(i)}\}_{i=1}^N$ , where both  $\mathbf{x}^{(i)}$  and  $\mathbf{y}^{(i)}$  contain  $T^{(i)}$  symbols, conditional maximum likelihood estimation is based on the minimization, with respect to  $\theta$ , of

$$\begin{aligned}
 l(\theta) &= - \sum_{i=1}^N \log p_{\theta}(\mathbf{y}^{(i)} | \mathbf{x}^{(i)}) \\
 &= \sum_{i=1}^N \left\{ \log Z_{\theta}(\mathbf{x}^{(i)}) - \sum_{t=1}^{T^{(i)}} \sum_{k=1}^K \theta_k f_k(y_{t-1}^{(i)}, y_t^{(i)}, x_t^{(i)}) \right\} \quad (4)
 \end{aligned}$$

The function  $l(\theta)$  is recognized as the negated conditional log-likelihood of the observations and will be referred to in the following as the *logarithmic loss function*. This term is usually complemented with an additional regularization term so as to avoid overfitting (see Section 4 below). The gradient of  $l(\theta)$  is given by

$$\begin{aligned} \frac{\partial l(\theta)}{\partial \theta_k} &= \sum_{i=1}^N \sum_{t=1}^{T^{(i)}} \mathbb{E}_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} f_k(y_{t-1}, y_t, x_t^{(i)}) \\ &\quad - \sum_{i=1}^N \sum_{t=1}^{T^{(i)}} f_k(y_{t-1}^{(i)}, y_t^{(i)}, x_t^{(i)}) \end{aligned} \quad (5)$$

where  $\mathbb{E}_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})}$  denotes the conditional expectation given the observation sequence, i.e.

$$\mathbb{E}_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} f_k(y_{t-1}, y_t, x_t^{(i)}) = \sum_{(y', y) \in \mathcal{Y}^2} f_k(y, y', x_t^{(i)}) \mathbb{P}_\theta(y_{t-1} = y', y_t = y | \mathbf{x}^{(i)}) \quad (6)$$

Although  $l(\theta)$  is a smooth convex function, it has to be optimized numerically. The computation of its gradient implies to repeatedly compute the conditional expectation in (6) for all input sequences  $\mathbf{x}^{(i)}$  and all positions  $t$ .

### 3.2 Sparse Forward-Backward Algorithm

The standard approach for computing the conditional probabilities in CRFs is inspired by the forward-backward algorithm for hidden Markov models: in the case of the parameterization of (3), the algorithm implies the computation of the forward

$$\begin{cases} \alpha_1(y) = \exp(\mu_{y, x_1} + \lambda_{y_0, y, x_1}) \\ \alpha_{t+1}(y) = \sum_{y'} \alpha_t(y') \exp(\mu_{y, x_{t+1}} + \lambda_{y', y, x_{t+1}}) \end{cases} \quad (7)$$

and the backward recursion

$$\begin{cases} \beta_{T_i}(y) = 1 \\ \beta_t(y') = \sum_y \beta_{t+1}(y) \exp(\mu_{y, x_{t+1}} + \lambda_{y', y, x_{t+1}}), \end{cases} \quad (8)$$

for all indices  $1 \leq t \leq T$  and all labels  $y \in Y$ . Then,  $Z_\theta(\mathbf{x}) = \sum_y \alpha_T(y)$  and the pairwise probabilities  $\mathbb{P}_\theta(y_t = y', y_{t+1} = y | \mathbf{x})$  are given by

$$\alpha_t(y') \exp(\mu_{y, x_{t+1}} + \lambda_{y', y, x_{t+1}}) \beta_{t+1}(y) / Z_\theta(\mathbf{x})$$

These recursions require a number of operations that grows quadratically with  $|Y|$ .

Let us now consider the case where the set of bigram features  $\{\lambda_{y', y, x_{t+1}}\}_{(y', y) \in Y^2}$  is sparse with only  $r(x_{t+1}) \ll |Y|^2$  non null values and define the matrix

$$M_{t+1}(y', y) = \exp(\lambda_{y', y, x_{t+1}}) - 1$$

Observe that  $M_{t+1}(y', y)$  also is a sparse matrix and that the forward and backward equations may be rewritten as

$$\begin{aligned} \alpha_{t+1}(y) &= \exp(\mu_{y, x_{t+1}}) \left\{ \sum_{y'} \alpha_t(y') + \sum_{y'} \alpha_t(y') M_{t+1}(y', y) \right\} \\ \beta_t(y') &= \sum_y v_{t+1}(y) + \sum_y M_{t+1}(y', y) v_{t+1}(y) \end{aligned} \quad (9)$$

where  $v_{t+1}(y) = \beta_{t+1}(y) \exp(\mu_{y,x_{t+1}})$ . The resulting computational savings stem from the fact that the vector matrix products in (9) now only involve the sparse matrix  $M_{t+1}(y', y)$ . This means that they can be computed, using an appropriate sparse matrix implementation, with exactly  $r(x_{t+1})$  multiplications instead of  $|Y|^2$ . If the set  $\{\mu_{y,x_{t+1}}\}_{y \in Y}$  of unigram features is also sparse, one may use a similar idea although the computation savings will in general be less significant.

Using the implementation outlined in (9), the complexity of the forward-backward procedure for the sequence  $\mathbf{x}^{(i)}$  can be reduced from  $T^{(i)} \times |Y|^2$  to the cumulated sizes of the feature sets encountered at each position along the sequence. Thus, the complexity of the forward-backward procedure is proportional to the average number of active features per position in the parameter set rather than to the actual number of potentially active features. This observation suggests that it might even be possible to use some longer term dependencies between labels, as long as only a few of them are active simultaneously.

It should be stressed that both CRF++ [15] and crfsgd [3] use logarithmic computation in the forward-backward recursions, that is, updating  $\log \alpha_t(y)$  and  $\log \beta_t(y)$  rather than  $\alpha_t(y)$  and  $\beta_t(y)$  in (7) and (8). The advantage of logarithmic computations is that numerical over/underflows are avoided whatever the length  $T^{(i)}$  of the sequence, whereas the linear form of (7) and (8) is only suitable for sequences whose length is less than a few tens. On the other hand, logarithmic computation is not the only way of avoiding numerical issues (the ‘‘scaling’’ solution traditionally used for HMMs [5] applies as well here) and is very inefficient from an implementation point of view due to the repeated calls to the `exp` function (see Section 6.2). This being said, when logarithmic computations are used, (9) may be used in a similar fashion to reduce the complexity of the logarithmic update when  $M_{t+1}(y', y)$  is sufficiently sparse.

Note that although we focus in this paper on the complexity of the training phase, the above idea may also be used to reduce the computational burden associated with Viterbi (or optimal sequence-wise) decoding. Indeed, at position  $t + 1$ , the forward pass in the Viterbi recursion amounts to computing

$$\epsilon_{t+1}(y) = \max_{y' \in Y} \{ \epsilon_t(y') + \lambda_{y',y,x_{t+1}} \} + \mu_{y,x_{t+1}} \quad (10)$$

where  $\epsilon_t(y)$  denotes the conditional log-likelihood of the optimal labelling of the  $t$  first tokens subject to the constraint that the last label is  $y$  (omitting the constant  $-\log Z_\theta(\mathbf{x})$  which is common to all possible labellings). Assuming that  $A(y, x_{t+1}) = \{y' \in Y : \lambda_{y',y,x_{t+1}} \neq 0\}$  is limited to a few labels, it is possible to implement (10) as

$$\epsilon_{t+1}(y) = \max \left\{ \max_{y' \in \bar{A}(y, x_{t+1})} \epsilon_t(y'), \max_{y' \in A(y, x_{t+1})} (\epsilon_t(y') + \lambda_{y',y,x_{t+1}}) \right\} + \mu_{y,x_{t+1}}$$

where  $\bar{A}(y, x_{t+1})$  denotes the elements of  $Y$  that are not in the current active set  $A(y, x_{t+1})$  of bigram features. Hence the number of required additions now is of the order of the number of active features,  $|A(y, x_{t+1})|$ , rather than equal to the number of labels  $|Y|$ .

## 4 Parameter Estimation Using Blockwise Coordinate Descent

### 4.1 Regularization

The standard approach for parameter estimation in CRFs consists in minimizing the logarithmic loss  $l(\theta)$  defined by (4) with an additional squared  $\ell^2$  penalty term  $\frac{\rho_2}{2} \|\theta\|_2^2$ , where  $\rho_2$  is a regularization parameter. The objective function is then a smooth convex function to be minimized

over an unconstrained parameter space. Hence, any numerical optimization strategy may be used for this purpose and popular solutions include limited memory BFGS (L-BFGS) [18] or conjugate gradient. Note that it is important however, to avoid numerical optimizers that require the full Hessian matrix (e.g., Newton’s algorithm) or approximations of it due to the size of the parameter vector in usual applications of CRFs.

In the following, we consider the elastic net penalty [35] which combines  $\ell^1$  and  $\ell^2$  regularizers and yields an objective function

$$l(\theta) + \rho_1 \|\theta\|_1 + \frac{\rho_2}{2} \|\theta\|_2^2 \quad (11)$$

where  $\rho_1$  and  $\rho_2$  are regularization parameters. The use of both types of penalty terms seems preferable in log-linear conditional models, as it makes it possible to control both the number of non zero coefficients (through  $\rho_1$ ) and to avoid the numerical problems that might occur in large dimensional parameter settings if the magnitude of the  $\theta_k$ s is not sufficiently constrained by the penalty.

It may also be rewarding to look for some additional information in hierarchical and group structure of the data. An example is the group lasso estimator, introduced in [21] as an extension of the lasso. The motivation for the group lasso is to select not individual variables, but whole blocks of variables. Typically, the penalty takes the form of the sum of the  $\ell^2$  norms of predefined blocks of the parameter vector. This idea has been further extended in [34] under the name of Composite Absolute Penalties (CAP) for dealing with more complex a priori parameter hierarchies while still retaining an overall convex penalty term. Although our approach could be suitable for this more complex choices of the penalty function, we restrict ourselves in the following to the case of the elastic net penalty.

## 4.2 Coordinate Descent

The objective function in (11) is still convex but not differentiable everywhere due to the  $\ell^1$  penalty term. Although different algorithms have been proposed to optimize such a criterion, we believe that the coordinate-wise approach of [11] has a strong potential for CRFs as the update of the parameter  $\theta_k$  only involves carrying out the forward-backward recursions for those sequences that contain symbols  $x$  such that at least one of the values  $\{f_k(y', y, x)\}_{(y, y') \in Y^2}$  is non null, which is most often much smaller than the total number of training sequences. This algorithm operates by first considering a local quadratic approximation of the objective function around the current value  $\bar{\theta}$ :

$$l_{k, \bar{\theta}}(\theta_k) = C^{st} + \frac{\partial l(\bar{\theta})}{\partial \theta_k} (\theta_k - \bar{\theta}_k) + \frac{1}{2} \frac{\partial^2 l(\bar{\theta})}{\partial \theta_k^2} (\theta_k - \bar{\theta}_k)^2 + \rho_1 |\theta_k| + \frac{\rho_2}{2} \theta_k^2 \quad (12)$$

Then, the minimizer of the approximation (12) is easily found to be

$$\theta_k = \frac{s \left\{ \frac{\partial^2 l(\bar{\theta})}{\partial \theta_k^2} \bar{\theta}_k - \frac{\partial l(\bar{\theta})}{\partial \theta_k}, \rho_1 \right\}}{\frac{\partial^2 l(\bar{\theta})}{\partial \theta_k^2} + \rho_2} \quad (13)$$

where  $s$  is the soft-thresholding function defined by

$$s(z, \rho) = \begin{cases} z - \rho & \text{if } z > \rho \\ z + \rho & \text{if } z < -\rho \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

Interestingly, [9] originally proposed a similar idea but based on a different local approximation of the behavior of the logarithmic loss. In [9], the local behavior of the function  $l(\theta)$  is approximated under a form that is equivalent to the first-order only and leads to a closed-form coordinate-wise optimization formula as well. This approximation is however explicitly based on the fact that each parameter  $\theta_k$  is multiplied by a function that takes its values in  $\{0, 1\}$ . This property is not verified for CRF, since  $\theta_k$  is multiplied by  $\sum_{t=1}^T f_k(y_{t-1}, y_t, x_t)$ , which can be more than 1 if the corresponding feature is observed at several positions in the sequence.

### 4.3 Coordinate Descent for CRFs

To apply the algorithm described above for CRFs, one needs to be able to compute (14), which requires to evaluate the first and second order derivatives of  $l(\theta)$ . If the first order derivative is readily computable using the forward-backward recursions described in Section 3.2 and (5), the exact computation of the second derivative is harder for CRFs. In fact, standard computations show that the diagonal term of the Hessian is

$$\frac{\partial^2 l(\theta)}{\partial \theta_k^2} = \sum_{i=1}^N \left\{ E_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} \left( \sum_{t=1}^{T^{(i)}} f_k(y_{t-1}, y_t, x_t^{(i)}) \right)^2 - \left( E_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} \sum_{t=1}^{T^{(i)}} f_k(y_{t-1}, y_t, x_t^{(i)}) \right)^2 \right\} \quad (15)$$

The first term is problematic as it involves the conditional expectation of a square which cannot be computed only from the pairwise probabilities  $P_\theta(y_{t-1} = y', y_t = y | \mathbf{x}^{(i)})$  returned by the forward-backward procedure. It can be shown (see Chapter 4 of [5] and [4]) that (15) can be computed using auxiliary recursions related to the usual forward recursion with an overall complexity of order  $|Y|^2 \times T^{(i)}$  per sequence. Unfortunately, this recursion is specific for each index  $k$  and cannot be shared between parameters. As will be shown below, sharing (part of) the computations between parameters is desirable feature for handling non trivial CRFs; we thus propose to use instead the approximation

$$\frac{\partial^2 l(\theta)}{\partial \theta_k^2} \approx \sum_{i=1}^N \sum_{t=1}^{T^{(i)}} E_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} f_k^2(y_{t-1}, y_t, x_t^{(i)}) - \left( E_{p_\theta(\mathbf{y}|\mathbf{x}^{(i)})} f_k(y_{t-1}, y_t, x_t^{(i)}) \right)^2 \quad (16)$$

This approximation amounts to assuming that, *given*  $\mathbf{x}^{(i)}$ ,  $f_k(y_{t-1}, y_t, x_t^{(i)})$  and  $f_k(y_{s-1}, y_s, x_s^{(i)})$  are uncorrelated when  $s \neq t$ . Note that this approximation is exact when the feature  $f_k$  is only active at one position along the sequence. It is likely that the accuracy of this approximation is reduced when  $f_k$  is active twice, especially if the corresponding positions  $s$  and  $t$  are close.

The proposed coordinate descent algorithm applied to CRFs is summarized as Algorithm 1.

A potential issue with this algorithm is the fact that, in contrast to the logistic regression case considered in [11], we are using an approximation to  $\partial^2 l(\theta) / \partial \theta_k^2$  which could have a detrimental effect on the convergence of the coordinate descent algorithm. An important observation is that (13)–(14) used with an approximated second-order derivative still yield the correct stationary points.

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**Algorithm 1** Coordinate Descent for CRF

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```
while Convergence criterion is not met do
  for  $k = 1 : K$  do
    for Sequences for which  $f_k$  is active do
      Perform sparse forward-backward.
    end for
    Compute  $\partial l(\theta)/\partial\theta_k$  and  $\partial^2 l(\theta)/\partial\theta_k^2$  from (5)–(16).
    Update  $\theta$  according to (13)–(14).
  end for
end while
```

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To see why it is true, assume that  $\bar{\theta}$  is such that (13)–(14) leave  $\bar{\theta}_k$  unchanged (i.e.,  $\theta_k = \bar{\theta}_k$ ). If  $\bar{\theta}_k = 0$ , this can happen only if  $|\partial l(\bar{\theta})/\partial\theta_k| \leq \rho_1$ , which is indeed the first order optimality condition in 0. Now assume that  $\bar{\theta}_k > 0$ , the fact that  $\bar{\theta}_k$  is left unmodified by the recursion implies that  $\bar{\theta}_k \rho_2 + \partial l(\bar{\theta})/\partial\theta_k + \rho_1 = 0$ , which is also recognized as the first order optimality condition (note that since  $\bar{\theta}_k \neq 0$ , the criterion is differentiable at this point). The symmetric case, where  $\bar{\theta}_k < 0$ , is similar. Hence, the use of an approximated second-order derivative does not prevent the algorithm from converging to the appropriate solution. A more subtle issue is the question of stability: it is easily checked that if  $\partial^2 l(\bar{\theta})/\partial\theta_k^2$  is smaller than it should be (remember that it has to be positive as  $l(\theta)$  is strictly convex), the algorithm can fail to converge even for simple functions (e.g., if  $l(\theta)$  is a quadratic function). An elaborate solution to this issue would consist in performing a line search in the “direction”:

$$s \frac{\left( \alpha^{-1} \frac{\partial^2 l(\bar{\theta})}{\partial\theta_k^2} \bar{\theta}_k - \frac{\partial l(\bar{\theta})}{\partial\theta_k}, \rho_1 \right)}{\alpha^{-1} \frac{\partial^2 l(\bar{\theta})}{\partial\theta_k^2} + \rho_2}$$

where  $0 < \alpha \leq 1$ , is chosen as close as possible to 1 with the constraint that it indeed leads to a decrease of the objective function (note that the step size affects only the second-order term in order to preserve the convergence behavior). On the other hand, coordinate descent algorithms are only viable if each individual update can be performed very quickly, which means that using line search is not really an option. In our experiments, we found that using a fixed value of  $\alpha = 1$  was sufficient for Algorithm 1, probably due to the fact that the second-order derivative approximation in (16) is usually quite good. For the blockwise approach described below, we had to use somewhat larger values of  $\alpha$  to ensure stability (typically, in the range 2–5 near convergence and in the range 50–500 for the very few initial steps of the algorithm in cases where it is started blindly from arbitrary parameter values). Alternative updates based on uniform upper-bounds of the Hessian could also be derived in a fashion similar to the work reported in [14].

#### 4.4 Blockwise Coordinate Descent for CRFs

The algorithm described in the previous section is efficient in simple problems (see Section 6.1) but cannot be used even for moderate size applications of CRFs. For instance, the application to be considered in Section 6 involves up to millions of parameters and single component coordinate descent is definitely ruled out in this case. Following [11], we investigate the use of blockwise updating schemes, which update several parameters simultaneously trying to share as much computations as possible. It turns out that the case of CRFs is rather different from the polytomous logistic regression case considered in [11] and requires specific blocking schemes. In this discussion,

we consider the parameterization defined in (3) which makes it easier to highlight the proposed block structure.

Examining the forward-backward procedure described in Section 3.2 shows that the evaluation of the first or second order derivative of the objective function with respect to  $\mu_{y,x}$  or  $\lambda_{y',y,x}$  requires to compute the pairwise probabilities  $p_\theta(y_t = y'', y_{t+1} = y' | \mathbf{x}^{(i)})$  for all values of  $(y'', y') \in Y^2$  and for all sequences  $\mathbf{x}^{(i)}$  which contain the symbol  $x$  at any position in the sequence. Hence, the most natural grouping in this context is to simultaneously update all parameters  $\{\mu_{y,x}, \lambda_{y',y,x}\}_{(y',y) \in Y^2}$  that correspond to the same value of  $x$ . This grouping is orthogonal to the solution adopted for polytomous regression in [11], where parameters are grouped by common values of the target label.

---

**Algorithm 2** Blockwise Coordinate Descent for CRF

---

```

while Convergence criterion is not met do
  for  $x \in X$  do
    for Sequences which contain the symbol  $x$  do
      Perform sparse forward-backward on relevant indices.
    end for
    Compute

```

$$\begin{aligned} & \{\partial l(\mu, \lambda) / \partial \mu_{y,x}, \partial^2 l(\mu, \lambda) / \partial \mu_{y,x}^2\}_{y \in Y} \\ & \{\partial l(\mu, \lambda) / \partial \lambda_{y',y,x}, \partial^2 l(\mu, \lambda) / \partial \lambda_{y',y,x}^2\}_{(y',y) \in Y^2} \end{aligned}$$

using (5) and (16).

Update  $\{\mu_{y,x}\}_{y \in Y}$  and then  $\{\lambda_{y',y,x}\}_{(y',y) \in Y^2}$  according to (13)–(14).

```

  end for
end while

```

---

Different variants of this algorithm are possible, including updating only one of the two types of blocks ( $\{\mu_{y,x}\}_{y \in Y}$  or  $\{\lambda_{y',y,x}\}_{(y',y) \in Y^2}$ ) at a time. Although the block coordinate-wise algorithm requires scanning all the  $|X|$  possible symbols  $x$  at each iteration, it is usually relatively fast due to the fact that only those sequences that contain  $x$  are considered. In addition, careful examination reveals that for each sequence that contains the token  $x$ , it is only required to carry the forward recursion up to the index of last occurrence of  $x$  in the sequence (and likewise to perform the backward recursion down to the first occurrence of  $x$ ). The exact computational saving will however depend on the target application as discussed in Section 6.2 below.

## 5 Discussion

As mentioned above, the standard approach for CRFs is based on the use of the  $\ell^2$  penalty term and the objective function is optimized using L-BFGS [18], conjugate gradient [22] or Stochastic Gradient Descent (SGD) [2]. The CRF training softwares CRF++ [15] and CRFsuite [23] use L-BFGS while crfsgd [3] is, as the name suggests, based on SGD. The latter approach differs from the others in that it processes the training sequences one by one: thus each iteration of the algorithm is very fast and it is generally observed that SGD converges faster to the solution, especially for large training corpora. On the other hand, as the algorithm approaches convergence, SGD becomes slower than global quasi-Newton algorithms such as L-BFGS. [31] discusses improvements of the SGD algorithm based on the use of an adaptive step size whose computation necessitates second-order information. However, these approaches based on the  $\ell^2$  penalty term do not perform feature

selection.

To our knowledge, [20] made the first attempt to perform model selection for conditional random fields. The approach was mainly motivated by [7] and is based on a greedy algorithm which selects features with respect to their impact on the log-likelihood function. Related ideas also appear in [8]. These greedy approaches are different from our proposed algorithm in that they do not rely on a convex optimization formulation of the learning objective.

To deal with  $\ell^1$  penalties, the simplest idea is that of [13] which was introduced for maximum entropy models but can be directly applied to conditional random fields. The main idea of [13] is to split every parameter  $\theta$  into two positive constrained parameters,  $\theta^+$  and  $\theta^-$ , such that  $\theta = \theta^+ - \theta^-$ . The penalty takes the form  $\rho(\theta^+ - \theta^-)$ . The optimization procedure is quite simple, but the number of parameters is doubled and the method is reported to have a slow convergence [1]. A more efficient approach is the already mentioned orthant-wise quasi-Newton algorithm introduced in [1]. [12] shows that the orthant-wise optimization procedure is faster than the algorithm proposed by [13] and performs model selection even in very high-dimensional problems with no loss of performance compared to the use of the  $\ell^2$  penalty term. Orthant-wise optimization is available in the CRFsuite [23] package. Recently, [30] proposed an adapted version of SGD with  $\ell^1$  penalization, which is claimed to be much faster than the orthant-wise algorithm.

As observed in [24, 12],  $\ell^1$  regularization per se does not in general warrant improved test set performance. We believe that the real challenge is to come up with methods for CRFs that can take profit of the parameter sparsity to either speed up processing or, more importantly, make it possible to handle larger “virtual” sets of parameters (i.e., a number of parameters that is potentially very large but only a very limited fraction of them being selected). The combined contributions of Sections 3 and 4 are a first step in that direction. Related ideas may be found in [6] who considers “generalized” feature functions. Rather than making each feature function depend on a specific value of the label (or on specific values of label pairs), the author introduces functions that only depend on subsets of (pairs of) labels. This amounts to introducing tying between some parameter values, a property that can also be used to speed-up the forward-backward procedure during training. This technique allows to considerably reduce the training time, with virtually no loss in accuracy. From an algorithmic perspective, this work is closest to our approach, since it relies on a decomposition of the clique potential into two terms, the first has a linear complexity (w.r.t. the number of labels), and the other is sparse; this idea was already present in [26]. This method however requires to specify a priori the tying pattern, a requirement that is not needed here. The important dependencies emerge from the data, rather than being heuristically selected a priori. A somewhat extreme position is finally advocated in [17], where the authors propose to trade the explicit modeling of dependencies between labels for an increase in the number of features testing the local neighborhood of the current observation token. Our proposal explores the opposite choice: reducing the number of features to allow for a better modeling of dependencies.

## 6 Experiments

### 6.1 Simulated data

The experiments on an artificial dataset reported here are meant to illustrate two aspects of the proposed approach. Firstly, we wish to show that considering unnecessary dependencies in a model can really hurt the performance, and that using  $\ell^2$  and  $\ell^1$  regularization terms can help solve this problem. Second, we wish to demonstrate that the blockwise algorithm (Algorithm 2) enables to achieve accuracy results that are very close to those obtained with the coordinate descent approach

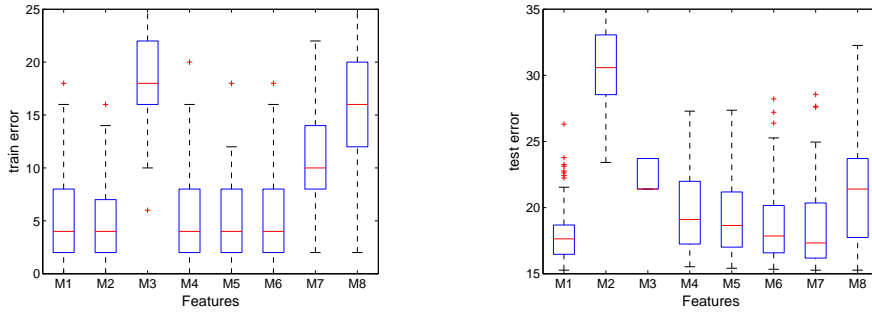


Figure 2: Performance of the models on artificial data. Models  $M1 - M3$  are trained with  $\ell^2$  penalty (L-BFGS), models  $M4 - M8$  with the  $\ell^1$  penalty term (block coordinate-wise descent). Above: performance on training set. Below: performance on testing set.

(Algorithm 1).

The data we use for these experiments are generated with a first-order hidden Markov model. Each observation and label sequence has a length of 5, the observation alphabet contains 5 values, and the label alphabet contains 6 symbols. This HMM is designed in such a way that the transition probabilities are uniform for all label pairs, except for two. The emission probability matrix on the other hand has six distinctively dominant entries such that most labels are well identified from the observations, except for two of them which are very ambiguous. The minimal (Bayes) error for this model is 15.4%.

Figure 2 compares several models:  $M1$  contains both  $(y_{t-1}, y_t, x_t)$  and  $(y_t, x_t)$  features,  $M2$  and  $M3$  are simpler, with  $M2$  containing only the bigram features, and  $M3$  only the unigram features. The models  $M1-M3$  are penalized with the  $\ell^2$  norm. Models  $M4-M8$  contain both features, bigram and unigram, but are penalized by the elastic net penalty term. For the  $\ell^2$ -penalized models ( $M1-M3$ ), the regularization factor  $\rho_2$  is set to its optimal value (obtained by cross validation). For  $M4-M8$  however, the value of  $\rho_2$  does not influence much the performance and is set to 0.001 while  $M4-M8$  correspond to different choices of  $\rho_1$ , as shown in Table 1.

For this experiment, we used only  $N = 10$  sequences for training, so as to reproduce the situation, which is prevalent in practical uses of CRFs, where the number of training tokens (here  $10 \times 5 = 50$ ) is of the same order as the number of parameters, which ranges from  $6 \times 5 = 30$  for  $M3$  to  $6 \times 5 + 6^2 \times 5 = 210$  for  $M1$  and  $M4-M8$ . Figure 2 displays box-and-whiskers plots summarizing 100 independent replications of the experiment.

	M4	M5	M6	M7	M8
$\rho_1$	0.001	0.01	0.1	1	2.5
Number of active $\mu$	28.5	15.0	10.9	6.2	5.8
Number of active $\lambda$	50.6	26	17.2	4.9	1.3

Table 1: Impact of  $\rho_1$  on the number of active features ( $\rho_2 = 0.001$ ).

Unsurprisingly,  $M1$  and  $M2$ , which contain more parameters, perform very well on the training set, much better than  $M3$ . The test performance tells a different story:  $M2$  performs in fact much worse than the simple unigram model  $M3$ , which is all the more remarkable that we know from the simulation model that the observed tokens are indeed not independent and that the models are nested (i.e. any model of type  $M3$  corresponds to a model of type  $M2$ ). Thus, even with regularization, richer models are not necessarily the best, hence the need for feature selection

techniques. Interestingly, M1 which embarks both unigram and bigram features, achieves the lowest test error, highlighting the interest of using simultaneously both feature types to achieve some sort of smoothing effect. With proper choice of the regularization (here, M7),  $\ell^1$ -penalized models achieve comparable test set performance. As a side effect of model selection, notice that M7 is somewhat better than M1 at predicting the test performance at training time: for M1, the average train error is 6.4% vs. 18.5% for the test error while for M7, the corresponding figures are 10.3% and 17.9%, respectively. Finally, closer inspection of the sparsity pattern determined by M7 shows that it is most often closely related to the structure of the simulation model which is also encouraging.

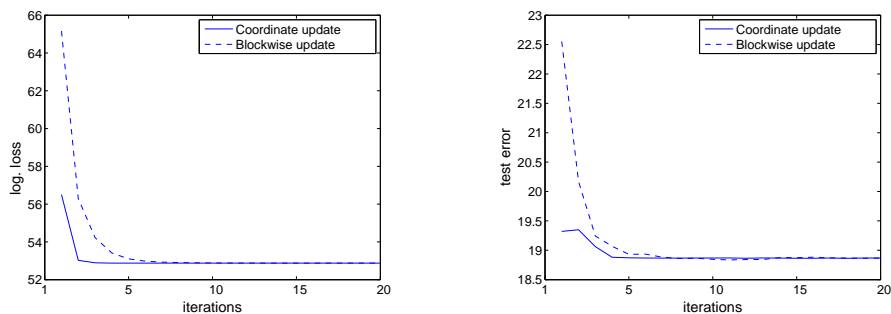


Figure 3: Comparison of the coordinate-wise update with the block update on simulated data. Above: values of logarithmic loss being minimized. Below: performance on test data.

Figure 3 compares the behavior of the coordinate-wise update policy with the blockwise approach, where one iteration refers to a complete round where all model parameters are updated exactly once. As can be seen on these graphs, the convergence behavior is comparable for both approaches, both in terms of objective function (top plot) and test error (bottom plot). Each iteration of the blockwise algorithm is however about 50 times faster than the coordinate-wise update, that roughly correspond to the size of each block. Clearly, the blockwise approach is the only viable strategy when tackling more realistic higher-dimensional tasks such as those considered in the next two sections.

## 6.2 Experiments with Nettetalk

This section presents results obtained on a word phonetization task, using the Nettetalk dictionary [25]. This dataset contains approximately 18000 English words and their pronunciations. Graphemes and phonemes are aligned at the character level thanks to the insertion of a “null sound” in the latter string when it is shorter than the former. The set of graphemes  $X$  thus includes 26 letters, the alphabet of phonemes  $Y$  contains 53 symbols. In our experiments, we consider that each phoneme is a target label and we consider two different settings. The first only uses features that test the value of one single letter, and is intended to allow for a detailed analysis of the features that are extracted. The second setting is more oriented towards performance and uses features that also test the neighboring letters. The training set comprises 16452 sequences and the test set contains 1628 sequences. The results reported here are obtained using the blockwise version of the coordinate descent approach (Algorithm 2).

Figure 4 displays the estimated parameter values when the  $\ell^1$  penalty is set to its optimal value of 0.2 (see Table 2 below). Comparing this figure with Figure 1 shows that the proposed algorithm correctly identifies those parameters that are important for the task while setting the

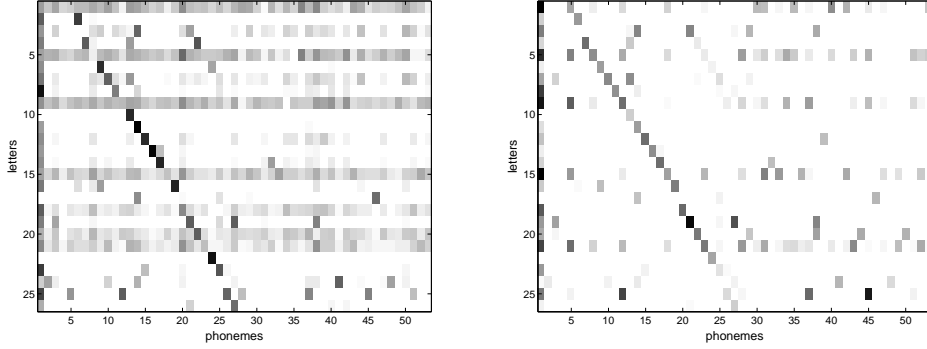


Figure 4: Nettalk task,  $\ell^1$  norm of the parameters estimated with elastic net penalty,  $\rho_1 = 0.2$ ,  $\rho_2 = 0.05$  Above:  $|\mu_{y,x}|$  for the 53 phonemes  $y$  and 26 letters  $x$ . Below:  $\sum_{y'} |\lambda_{y',y,x}|$  for the 53 phonemes  $y$  and 26 letters  $x$ .

other to zero. It confirms the impression conveyed by Figure 1 that only a very limited number of features is actually used for predicting the labels. The first column in this figure corresponds to the null sound and is unsurprisingly associated with almost all letters. One can also directly visualize the ambiguity of the vocalic graphemes which correspond to the first ('a'), fifth ('e'), ninth ('i'), etc. gray rows; this contrasts with the much more deterministic association of a typical consonant grapheme with a single consonant phoneme.

Method	Iter.	Time (min.)	Train (%)	Test (%)	$K_\mu$	$K_\lambda$
SBCD, $\rho_1 = 0$	30	125	13.3	14.0	1378	73034
SBCD, $\rho_1 = 0.1$	30	76	13.5	14.2	1155	4171
SBCD, $\rho_1 = 0.2$	30	70	14	14.2	1089	3598
SBCD, $\rho_1 = 0.5$	30	63	13.7	14.3	957	3077
SBCD, $\rho_1 = 1$	30	55	16.3	16.8	858	3111
SBCD, $\rho_1 = 2$	30	43	16.4	16.9	760	2275
SBCD, $\rho_1 = 10$	30	25	17.3	17.7	267	997
OWL-QN, $\rho_1 = 0.1$	50	165	13.5	14.2	1864	4079
L-BFGS	90	302	13.5	14.1	74412	
SGD	30	17	18.5	19.1	74412	

Table 2: Upper part: summary of results for various values of  $\rho_1$  for the proposed Sparse Blockwise Coordinate Descent (SBCD) algorithm (with  $\rho_2 = 0.001$ ) and orthant-wise L-BFGS (OWL-QN). Lower part: results obtained with  $\ell^2$  regularization only, for L-BFGS and stochastic gradient descent (SGD).

Table 2 gives the per phoneme accuracy with varying level of sparsity, both for the proposed algorithm (SBCD) and the orthant-wise L-BFGS (OWL-QN) strategy of [1]. For comparison purposes the lower part of the table also reports performance obtained with  $\ell^2$  regularization only. For  $\ell^2$ -based methods (L-BFGS and SGD) the regularization constant was set to its optimal value determined by cross validation as  $\rho_2 = 0.02$ . The proposed algorithm (SBCD) is coded in C while OWL-QN and L-BFGS use the CRF++ package [15] modified to use the `liblbfgs` library provided with CRFsuite [23] that implements the standard and orthant-wise modified versions of L-BFGS. Finally, SGD use the software of [3]. All running times were measured on a computer

with an Intel Pentium 4 3.00GHz CPU and 2G of RAM memory. Measuring running times is a difficult issue as each iteration of the various algorithm does not achieve the same improvement in term of performance. For our method, 30 iterations were found necessary to reach reasonable performance in the sense that further iterations did not significantly reduce the error rates (with variations smaller than 0.3%). Proceeding similarly for the other methods showed that OWL-QN and L-BFGS usually require more iterations to reach stable performance, which is reflected in Table 2. Finally, SGD requires few iterations (where an iteration is defined as a complete scan of all the training sequences) although we obtained disappointing performance on this dataset with SGD.

First, Table 2 shows that for  $\rho_1 = 0.1$  or  $0.2$  our method reaches an accuracy that is comparable with that of non-sparse trainers (SBCD with  $\rho_1 = 0$  or L-BFGS) but with only about 5000 active features. Note in particular the dramatic reduction achieved for the bigram features  $\lambda_{y',y',x}$  as the best accuracy/sparsity compromise ( $\rho_2 = 0.2$ ) nullifies about 95% of these parameters. We observe that the performance of SBCD (for  $\rho_1 = 0.1$ ) is comparable to that of OWL-QN, which is reassuring as they optimize related criterions, except for the fact that OWL-QN is based on the use of the sole  $\ell^1$  penalty. There are however minor differences in the number of selected features for both methods. In addition to the slight difference in the penalties used by SBCD and OWL-QN, it was constantly observed in all our experiments that for  $\ell^1$ -regularized methods the performance stabilizes much faster than the pattern of selected features which may require as much as a few hundreds of iterations to fully stabilize. This effect was particularly noticeable with the OWL-QN algorithm. We have not found satisfactory explanation regarding the poor performance of SGD on this dataset: further iterations do not significantly improve the situation and this failure has not been observed on the CoNLL 2003 data considered below. In general, SGD is initially very fast to converge and no other algorithm is able to obtain similar performance with such small running time. The fact that SGD fails to reach satisfactory performance in this example is probably related to an incorrect decrease of the step size. In this regard, an important difference between the Nettealk data and the CoNLL 2003 example considered below is the number of possible labels which is quite high here (53). A final remark regarding timings is that all methods except SBCD use logarithmic computation in the forward-backward recursions. As discussed in Section 3.2, this option is slower by a factor which, in our implementation, was measured to be about 2.4. Still, the SBCD algorithm compares favorably with other algorithms, especially with OWL-QN which optimizes the same objective function.

Table 2 also shows that the running time of SBCD depends on the sparsity of the estimated model, which is fully attributable to the sparse version of the forward-backward recursion introduced in Section 3.2. To make this connection clearer, Figure 5 displays the running time as a function of the number of active features (rather than  $\rho_1$ ). When the number of active feature is less than 10000, the curve shows a decrease that is proportional to the number of active features (beware that the x-axis is drawn on a logarithmic scale). The behavior observed for larger numbers of active features, where the sparse implementation becomes worse than the baseline (horizontal blue line) can be attributed to the overhead generated by the use of sparse matrix-vector multiplications for matrices that are not sparse. Hence the ideas exposed in Section 3.2 have a strong potential for reducing the computational burden in situations where the active parameter set is very small compared to the total number of available features. Note also that the OWL-QN optimizer could benefit from this idea as well.

The simple feature set used in the above experiments is too restricted to achieve state-of-the-art performance for this task. We therefore conducted another series of experiments with much larger feature sets, including bias terms and tests on the neighbors of the letter under focus. For

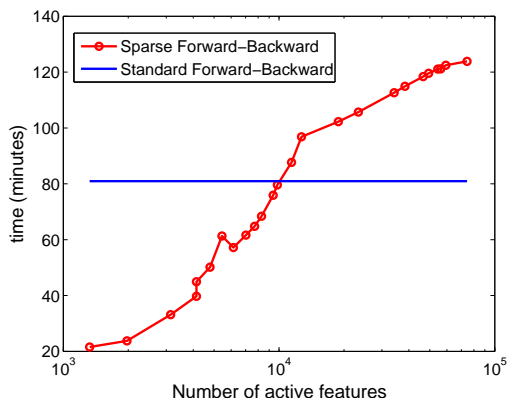


Figure 5: Running time as a function of the number of active features for the SBCD algorithm on the Nettetalk corpus. The blue line correspond to the running time when using non-sparse forward-backward.

these experiments, we keep the same baseline feature set and add the bias terms  $\mathbb{1}(y_t = y)$  and  $\mathbb{1}(y_{t-1} = y', y_t = y)$  for all possible values of  $(y, y')$ . For the context, we consider two variants: in the first, termed *pseudo n-gram*, we also add  $\mathbb{1}(y_t = y, x_{t\pm i} = x)$  and  $\mathbb{1}(y_{t-1} = y', y_t = y, x_{t\pm i} = x)$  for all values of  $(x, y, y')$  and of the offset  $1 \leq i \leq n - 1/2$ . In other terms, in this variant, we test separately the values of the letters that occur before and after the current position. In the second variant, termed *n-gram*, we add features  $\mathbb{1}(y_t = y, z_t = z)$  and  $\mathbb{1}(y_{t-1} = y', y_t = y, z_t = z)$  where  $z_t$  denotes the letter k-gram centered on  $x_t$ , and  $z$  ranges over all observed k-grams (with  $k \leq n$ ). This second variant seems of course much more computationally demanding [17] as it yields a much higher number of features (see the top line of Table 3, where the total number of features is given in millions).

	pseudo n-gram		n-gram	
	$n = 3$	$n = 5$	$n = 3$	$n = 5$
M feat.	0.236	0.399	14.2	121
20 iter.	8.98 (12.5)	6.77 (19.8)	8.22 (12.8)	6.51 (21.7)
30 iter.	8.67 (10.9)	6.65 (17.1)	8.04 (11.7)	6.50 (20.1)

Table 3: Experiments with contextual features: performance of the SBCD algorithm after 20 and 30 iterations in terms of error rate and, between parentheses, number of selected features (in thousands).

As can be seen in Table 3, these extended feature sets yield results that compare favorably with those reported in [33, 19], with a phoneme-error-rate of 6.5% for the 5-gram system. We also find that even though both variants extract comparable numbers of features, the results achieved with “true” n-grams are systematically better than for the pseudo n-grams. More interestingly, the n-gram variants are also faster: this paradoxical observation is due to the fact that for the n-gram features, each block update only visits a very small number of observation sequences, and further, that for each position, a much smaller number of features are active as compared to the pseudo n-gram case. Finally, the analysis of the performance achieved after 20 and 30 iterations suggests that the n-gram systems are quicker to reach their optimal performance. This is because a very large proportion of the n-gram features are zeroed in the first few iterations. For

instance, the 5-gram model, after only 9 iterations, has an error rate of 6.56% and selects only 27.3 thousand features out of 121 millions. These results clearly demonstrate the computational reward of exploiting the sparsity of the parameter set as described in Section 3: in fact, training our largest model takes less than 5 hours (for 20 iterations), which is quite remarkable given the very high number of features.

### 6.3 Experiments with CoNLL 2003

Named entity recognition consists in extracting groups of syntagms that correspond to named entities (e.g., names of persons, organizations, places, etc.). The data used for our experiments are taken from the CoNLL 2003 challenge [29] and implies four distinct types of named entities, and 8 labels. Labels have the form B-X or I-X, that is begin or inside of a named entity X (however, the label B-PER is not present in the corpus). Words that are not included in any named entity, are labeled with O (outside). The train set contains 14987 sequences, and the test set 3684 sequences. At each position in the text, the input consists of three separate components corresponding respectively to the word (with 30290 distinct words in the corpus), part-of-speech (46), and syntactic (18) tags. To accommodate this multidimensional input the standard practice consists in superposing unigram or bigram features corresponding to each of those three dimensions considered separately. The parameters we use in the model are of the form  $\{\mu_{y,x^d}, \lambda_{y',y,x^d}\}$  for  $d \in \{1, 2, 3\}$ , which corresponds to a little more than 9 million parameters. Hence, the necessity to perform model selection is acute.

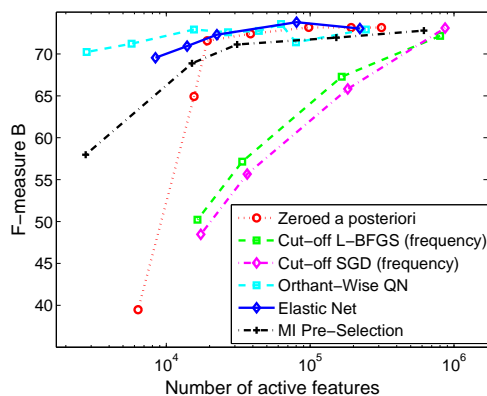


Figure 6: Performance comparison of different model selection approaches on CoNLL 2003 (English): test set B.

To illustrate the efficiency of  $\ell^1$ -based feature selection, we compare it with three simple-minded approaches to feature selection, which are often used in practice. The first one, termed “cut-off”, consists in incorporating only those features that have been observed sufficiently often in the training corpus. This amounts to deleting *a priori* all the rare dependencies. The second strategy, termed *MI preselection*, selects features based on their Mutual Information (MI) with the label, as in [32] (where the MI is referred to as the *information gain*). The third option consists in training a model that is not sparse (e.g., with an  $\ell^2$  penalty term) and in removing *a posteriori* all features whose values are not of sufficient magnitude. Figure 6 compares the error rates obtained with these strategies to those achieved by the SBCD or OWL-QN algorithms. Obviously, a priori cut-off strategies imply some performance degradation, although MI preselection clearly

dominates frequency-based selection. A posteriori thresholding is more efficient but cannot be used to obtain well-performing models that are very sparse (here, with less than 10000 features). From a computational point of view, a posteriori thresholding is also penalized by the need to estimate a very large model that contains all available features.

In this experiment, SBCD proves computationally less efficient, for at least two reasons. First, many POS or chunk tags appear in all training sequences; the same is true for very frequent words: this yields many very large blocks containing almost all training sentences. Second, the sparse forward-backward implementation is less efficient than in the case of the phonetization task as the number of labels is much smaller: SBCD needs 42 minutes (with  $\rho_1 = 1$ , corresponding to 6656 active features) to achieve a reasonable performance while OWL-QN is faster, taking about 5 minutes to converge. If sparsity is not needed, SGD appears to be the most efficient method for this corpus as it converges in less than 4 minutes. L-BFGS in contrast requires about 25 minutes to reach a similar performance.

## 7 Conclusions

In this paper, we have proposed an algorithm that performs feature selection for CRFs. The benefits of working with sparse feature vectors are twofold: obviously, less features need to be computed and stored; more importantly, sparsity can be used to speed up the forward-backward and the Viterbi algorithms. Our method combines the  $\ell^1$  and  $\ell^2$  penalty terms and can thus be viewed as an extrapolation of the elastic net concept to CRFs. To make the method tractable, we have developed a sparse version of the forward-backward recursions; we have also introduced and validated two novel ideas: an approximation of the second order derivative of the CRF logarithmic loss as well as an efficient parameter blocking scheme. This method has been tested on artificial and real-world data, using large to very large feature sets containing more than one hundred million features, and yielding accuracy that is comparable with conventional training algorithms, and much sparser parameter vectors.

The results obtained in this study open several avenues that we wish to explore in the future. A first extension of this work is related to finding the optimal weight for the penalization term, a task that is usually achieved through heuristic search for the value(s) that will deliver the best performance on a development set. Based on our experiments, this search can be performed efficiently using pseudo regularization-path techniques, which amount here to start the training with a very constrained model, and to progressively reduce the weight of the  $\ell^1$  term so as to increase the number of active features. This can be performed effectively at very little cost by restarting the blockwise optimization from the parameter values obtained with the previous weights setting (so called “warm-starts”), thereby greatly reducing the number of iterations needed to reach convergence. A second interesting perspective, aiming at improving the training speed, is based on the observation that after a dozen iterations or so, the number of active features is decreasing steadily. This suggests that those features that are inactive at that stage will remain inactive till the convergence of the procedure. Hence, in some situations, limiting the updates to the features that are currently active can be an efficient way of improving the training speed. Finally, the sparse forward-backward implementation appears to be most attractive when the number of labels is very large. Hence, extensions of this idea to cases where the features include, for instance, conjunctions of tests that operate on more than two successive labels are certainly feasible. The perspective here consists in taking profit of the sparsity to allow for inclusion of longer range label dependencies in CRFs.

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