POTENTIAL DISTRIBUTION SIMULATION IN HIGH VOLTAGE MEASUREMENT SYSTEM

N. BURAS, L. KRAHENBUHL, A. NICOLAS

Abstract - Very high voltage can generally be measured with a capacitor divider. In order to design such a sensor on one hand and to standardize it on the other, a calculation of the field distribution is necessary to determine the different capacitances of the system. A large number of numerical methods are very efficient for this. However, the constant improvement of dielectric materials allows us to use thinner and thinner insulations to the point where traditional methods seem inadequate. This is why the authors propose and analyse the different approaches that they have used to treat this problem. This is how first an extended finite elements method with the calculation of floating potential and of analytical elements to simulate thin layers, then a method of two potentials Boundary Integral Equation; on one hand using a direct approach, on the other introducing external global value like capacitances. In conclusion, we shall analyse the efficiency of these methods.

INTRODUCTION

The use of capacitive dividers is often the only possible technique to measure very high potentials. The principle is very simple: a sensor, conducting medium, is placed between the medium whose potential must be measured and another with reference potential, generally the earth. This sensor will, by capacitive influence, be reaching a potential which will change according to capacitances it presents with regard to the two potentials - the one measured and the earth. Figure 1 represents the real structure of such a sensor.

Figure 1: capacitor divider

The authors are with Ecole Centrale de Lyon, Departement d'Electrotechnique, UA CNRS 829, 36 avenue Guy de Collongues, BP 163, 69131 ECULLY Cedex, FRANCE.

We are dealing with a circuit-breaker with SF6. Conductor 1 has a potential that must be measured; the sensor (conductor 3) is insulated from the body of the circuit-breaker by a 10 μ m thick plastic film. A numerical method should allow us to calculate either the repartition of potential or the capacitances which intervene in this system. From the numerical methods generally used in electrical engineering, we have used two:

- the finite element method
- the Boundary Integral Equation method

For these two methods, two non traditional phenomena must be taken into account:

- the floating potential of conductor 3
- the major influence of the very thin insulating film.

Although this method is obviously not the best suited to this type of problem, we have developed a specific technique using finite elements to solve this problem: this method is in fact very well developed and often applied from existing packages.

The Boundary Integral Equation (BIE) method is ideal to deal this problem. However, in a case like this, this technique can be used in different ways and we shall see how it is possible to combine local values calculations (potential and electric fields) with external global values like capacitances.

APPROACH BY FINITE ELEMENT METHOD

The detailed diagram of the sensor showing the specific nature of the problem to be solved is on figure 2.

Figure 2 : Diagram of the problem to be solved

We are therefore dealing with the calculation of conductor 3's potential (floating potential) whereas it is insulated from the body of the circuit breaker by a very thin layer of dielectric.

Thin mediums :

The use of the finite element method (necessitating a meshing of the field of study implies the transformation of the equations of the electrostatic problem into a problem of finding the minimum integral value (functional). It is

unnecessary to develop it more before hand. The thin mediums, whose planar representation, in planar or axisymmetric problems, in fact quasimonodimensional domains, cannot be meshed identically to the field of study. We must therefore develop a particular method. Two possibilities : linear or bilinear elements.

Linear elements :

This technique, initially developed in magnetodynamics, consists of introducing linear elements, sandwiched between traditional bidimensional elements (figure 3).

Figure 3 : Linear and bidimensional elements position in Magnetodynamics

This process is worthwhile if we allow the hypothesis that the nodes I, I', I'' are at the same potential, which is the case in magnetodynamics. Unfortunately, in electrostatics, the existence of a thin layer causes the appearance of the difference in potential and the hypothesis can no longer be accepted.

Bilinear elements

To solve this problem, we have developed bilinear elements with the following principles (figure 4) :

- doubling of the nodes situated on the thin medium,
- meshing of the medium in quadrilaterals.

Figure 4 : Bilinear and bidimensional elements position in Electrostatics

These quadrilaterals are of course fictitious and do not appear on the meshing level in finite elements. Only their energy contribution is taken into account.

Energy functional :

To the classic functional of the system associated with the Laplace equation, we add the contribution of the thin medium :

$$\mathcal{E} = \iint_{\Omega} \frac{1}{2} \epsilon |\text{grad } V|^2 ds + \iint_{\Omega_m} \frac{1}{2} (\epsilon_m - \epsilon) |\text{grad } V|^2 ds \quad (1)$$

Ω = field of study

Ω_m = thin medium with permittivity ϵ_m .

The resolution of this problem is not developed, a classic technique.

Insulated conductor :

The resolution of an electrostatic problem leads to the resolution of a matrix system :

$$[S_{11}][V_1] = -[S_{12}][V_2] \quad (2)$$

if there is no volume charges. For an insulated conductor, we may note that :

- inside the conductor, the field is zero, so no meshing is necessary.
- all the nodes situated on the contour of the conductor, are at the same potential but all appear in vector V_1 .

Of all these nodes, only one is necessary to calculate (the degree of freedom can therefore decrease). If $[V_1']$ indicates the new vector, the unknown quantities :

$$[V_1] = [R][V_1'] \quad (3)$$

where $[R]$ is rectangular matrix.

The system to be resolved is :

$${}^t[R][S_{11}][R][V_1'] = -{}^t[R][S_{12}][V_2] \quad (4)$$

This mathematical transformation expresses physically the conservation of the insulated conductor's charge. This approach has a major disadvantage : it considerably increases the bandwidth of the resolution matrix.

APPROACH WITH THE AID OF A BIE METHOD

We shall see below the need to develop another technique to resolve the concrete problem that we came across and that we presented in the introduction. The structure of the sensor as well as the nature of the problem to be resolved naturally led us to the use of the Boundary Integral Equation method in axisymmetric geometry ; a method that we have developed.

Principle of division :

The principle of the division is on figure 2. The division ratio can be expressed by :

$$\alpha = \frac{v_1}{v_3} \Big|_{v_2=0} = - \frac{C_{33}}{C_{13}} = - \frac{q_3}{q_1} \Big|_{v_1=v_2=0} \quad (5)$$

This ratio obviously depends greatly on the position of conductor 3. The capacitance of conductor 3 can be

divided into 3 parallel capacitances as shown in figure 5.

Figure 5 : Capacitances of conductor 3

Capacitance C can be calculated analytically as it is a cylindrical capacitance. In order to determine the capacitances C_A and C_B , it is necessary to solve the Laplace problem in the regions A + B. Two processes are possible : either, the use of BIE method to determine the partial capacitances, or the direct resolution of the BIE problem.

Determination of partial capacities by BIE method :

In order to determine the capacitances C_{13} and C_A , we must resolve the Laplace problem in the region A by applying $V_1 = V_2 = 0$ and V_3 . The solution of the BIE method gives the value of $\partial V / \partial n$ on the equipotential surfaces. The capacitances are therefore calculated in the following way :

$$C_A = \frac{2 \pi \epsilon_0}{V_3} \int_{S_2, A} r \frac{\partial V}{\partial n} ds \quad (6)$$

For the back part (calculation of C_B), we proceed in the same way, taking account of the dielectric while resolving a problem of two linked regions by using the conditions at the interface. The relationship between capacitive divider is thus expressed as :

$$\alpha = \frac{C_A + C + C_B}{C_{13}} \quad (7)$$

Direct solution by BIE method :

The BIE method also allows the direct resolution of the problem such that it resembles the real case:

- V_1 fixed at a non zero value
- V_2 zero
- V_3 unknown

It is the problem of a conductor at a floating (non-fixed) potential. It should be noted that such a problem does not cause any complications at the matrix level. Contrary to the variational formulation, the BIE method does not automatically fix a zero value for the charge on the conductor 3. This necessary supplementary condition, is written :

$$2\pi\epsilon_0 \int_{S_3} \epsilon_r \frac{\partial V}{\partial n} ds = 0 \quad (8)$$

relationship in which we can introduce the cylindric capacitance C :

$$C \cdot (V_3 - V_2) + 2\pi\epsilon_0 \int_{S_3, A \cup S_3, B} \epsilon_r \frac{\partial V}{\partial n} ds = 0 \quad (9)$$

Conclusion :

The method that we have just described, can be used for no matter what value of charge of the conductor 3 (not only zero charge) and more generally allows the use of the charge of the conductors in the place of their potential.

Two important characteristics of the BIE method are shown clearly by the example that we have just given: there is the possibility of treating the geometries with a large difference between the dimensions (1 to 500 in this example) and of introducing in a very simple fashion, a partial analytic solution. The capacitance C can even describe an external capacitance. For example, the capacitance of the instrument measurement V3.

COMPARATIVE RESULTS

A certain number of tests have been made which provide interesting informations. These tests allow the comparison of the results obtained :

- with a BIE method,
- with a finite element method,
- experimentally

Matching results :

The comparative process is always identical with test configurations that become more and more complex with the possibilities of analytic calculation. One of first configurations was the following (Figure 6):

Figure 6 : Insulated conductor in a parallel plate capacitor

The calculations made in 2 configurations - planar and axisymmetric - give exactly the same results analytically by the BIE method and by the use of finite elements.

These perfectly matching results, justify the use of this process for the two methods.

Non matching results :

We used our two techniques in order to determine the division ratio of the capacitor divider. It has been shown that these present large gaps as are shown in figure 9.

Figure 7 : field lines in planar geometry

Figure 8 : field lines in axisymmetric geometry

Figure 9 : The evolution of the division ratio as a function of the distance :

- a) calculation by BIE method and experimental results.
b) calculation by Finite Elements.

This difference can be explained by the importance of the corner effects, which are crucial in electrostatics, and also by the important influence on the transformation ratio of the distance x . The BIE method takes into account the distance x in its kernells and by the fact is more precise for this parameter.

CONCLUSION

We have tried to show that for the same problem with the following two difficulties - thin medium and floating potential, two methods could be used :

The finite elements method, which is widely used and very successful. After an inexpensive adaptation, one would hope for good results. Our experience has shown that we must be prudent : If the results are acceptable in simple geometry (as they would probably be in magnetostatics), they become less acceptable in electrostatics with complex geometry. That proves that, more generally speaking, electrostatic remain a domain which is best adapted for :

The BIE method, greatly limited in the nature of the materials used, is particularly well suited for this kind of problems : it takes into account exactly the effects of tips and corners, the effects of distance, and above all the possibility of introducing an external value, here the capacitance of the measuring system, in the resolution of field equation. It is perhaps also necessary to search for the mediocrity of the results given by a finite elements method.

REFERENCES

- [1] P. Dumarquez, "Etude de l'influence des couches ou inclusions de dielectrique minces, Diplome d'Etudes Approfondies, ECL - INPG, sept. 1984.
- [2] N. Burais, A. Nicolas, A. Foggia, J.C. Sabonnadiere, "Electromagnetic field formulation for eddy current calculations in non destructive testing systems", IEEE Transactions on Magnetics, vol MAG 18, n 6, November 1982, p. 1058.
- [3] L. Krahenbuhl, A. Nicolas, "Efficient techniques for Boundary Integral Equations methods", IEEE Transactions on Magnetics, vol MAG 19, n 6, November 1983, p. 2364.
- [4] L. Krahenbuhl, A. Nicolas, "Methode des equations integrales de frontieres, developpements des techniques et formulation axisymetrique", Revue Generale de l'Electricite, n 3, Mars 1985, p 222.
- [5] L. Krahenbuhl, "La methode des equations integrales de frontiere pour la resolution des problemes de potentiel en electrotechnique, et sa formulation axisymetrique", These Docteur Ingenieur, Ecole Centrale de Lyon, Decembre 1983.
- [6] N. Burais, A. Nicolas, "Physical and geometric singularities modelling techniques for electrostatic and electromagnetism problems, Conference COMPUMAG, Fort Collins, Colorado, USA, June 1985.