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# MeV Mass Gluonic Colour Singlets in QCD

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## Abstract

It has long been textbook 'knowledge' that quarks and gluons exist exclusively inside of hadrons and glueballs because of colour confinement. However the  $SU(N)$  group contains  $N - 1$  Abelian generators. We argue that since the Abelian components of the gluon field can be meaningfully isolated by the Cho-Faddeev-Niemi-Shabanov decomposition, the corresponding gluon excitations are physical entities of neutral colour charge that can propagate freely. Their mass can be estimated on dimensional grounds and is similar to the pion mass.

*Key words:* QCD, Cho-Faddeev-Niemi-Shabanov decomposition, monopole condensate

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## 1. Introduction

Identifying the internal Abelian directions has been of interest to studies of the QCD vacuum since Savvidy's landmark paper [1] demonstrating the energetic favourability of a magnetic condensate. This led to a long-running controversy surrounding the condensate's stability [2, 3, 4, 5, 6, 7], with recent papers [7, 8, 9, 10, 11, 12] concluding in the positive.

What concerns this work is the manner in which the necessarily Abelian internal direction(s) of the condensate were identified. Two-colour studies typically assigned the Abelian direction to  $\hat{e}_3$  [1, 2, 4, 5, 13], in a blatant violation of gauge invariance that always left doubts that the calculated effects might be gauge artifacts. A further defect was that these papers were unable to prove that the magnetic background is due to monopoles.

These problems are avoided by the Cho-Faddeev-Niemi-Shabanov decomposition [14, 15, 16], which identifies the Abelian directions without choosing a special gauge. It does this by introducing the Cho connection, a topologically generated contribution to the gluon field which represents [17] a monopole potential. Thus the problems of gauge invariance and demonstrating the magnetic condensate to be of monopole origin are solved simultaneously.

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Identifying the Abelian degrees of freedom in a gauge invariant manner allows one to consider them as physical entities, and not as gauge artifacts. Furthermore, these particular physical entities are colour-neutral, and the primary claim of this paper is that they are not confined. We therefore refer to them as Free Abelian Gluons (FAGs).

Section 2 presents the CFNS decomposition for general  $SU(N)$  gauge groups. Section 3 justifies the claim that the Abelian generators are colourless and unconfined, uses the condensate coupling and dimensional arguments to estimate the FAG's mass, and then goes on to discuss other properties such as stability and decay modes. Some experimental signatures are proposed. Since the FAG mass is found to be of the same order as the pion mass, section 4 discusses the ramifications for the internucleon potential. It is suggested that several qualitative features of this potential are easier to understand in terms of a FAG contribution. The paper concludes with a discussion in section 5.

## 2. Specifying Abelian Directions

The CFNS decomposition was first presented by Cho [17], and later by Faddeev and Niemi [15] and by Shabanov [16], as a gauge-invariant means of specifying the Abelian dynamics of two-colour QCD. These authors [14, 15] also applied it to three-colour QCD. In this section we adapt it to general  $SU(N)$ , although we are not the first to do so [18, 19], and establish our notation.

The Lie group  $SU(N)$  for  $N$ -colour QCD has  $N^2 - 1$  generators  $\lambda^{(i)}$ , of which  $N - 1$  are Abelian generators  $\Lambda^{(i)}$ . For simplicity, we specify the gauge transformed Abelian directions with  $\hat{\mathbf{n}}_i = U^\dagger \Lambda^{(i)} U$ . Fluctuations in the  $\hat{\mathbf{n}}_i$  directions are described by  $c_\mu^{(i)}$ . The gauge field of the covariant derivative which leaves the  $\hat{\mathbf{n}}_i$  invariant is implicitly defined by

$$g\mathbf{V}_\mu \times \hat{\mathbf{n}}_i = -\partial_\mu \hat{\mathbf{n}}_i, \quad (1)$$

for which the general form is

$$\mathbf{V}_\mu = c_\mu^{(i)} \hat{\mathbf{n}}_i + \mathbf{B}_\mu, \quad \mathbf{B}_\mu = g^{-1} \partial_\mu \hat{\mathbf{n}}_i \times \hat{\mathbf{n}}_i, \quad (2)$$

where summation is implied over  $i$ .

We define the covariant derivative

$$\hat{\mathbf{D}}_\mu = \partial_\mu + g\mathbf{V}_\mu \times . \quad (3)$$

It is easily shown that the monopole field strength

$$\vec{H}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu, \quad (4)$$

has only  $\hat{\mathbf{n}}_i$  components, *ie.*

$$H_{\mu\nu}^{(i)} \hat{\mathbf{n}}_i = \vec{H}_{\mu\nu}, \quad (5)$$

where  $H_{\mu\nu}^{(i)}$  has the eigenvalue  $H^{(i)}$ . Since we are only concerned with magnetic backgrounds,  $H^{(i)}$  is considered the magnitude of a background magnetic field  $\mathbf{H}^{(i)}$ .

$\mathbf{X}_\mu$  contains the dynamical degrees of freedom (DOF) perpendicular to  $\hat{\mathbf{n}}_i$ , so if  $\mathbf{A}_\mu$  is the gluon field then

$$\mathbf{A}_\mu = \mathbf{V}_\mu + \mathbf{X}_\mu = c_\mu^{(i)} \hat{\mathbf{n}}_i + \mathbf{B}_\mu + \mathbf{X}_\mu, \quad (6)$$

where

$$\mathbf{X}_\mu \perp \hat{\mathbf{n}}_i. \quad (7)$$

This appears to leave the gluon field with additional DOF due to  $\hat{\mathbf{n}}_i, \mathbf{B}_\mu$ , but detailed analyses can be found in [8, 11, 20, 21] demonstrating that these fields are not fundamental, but a compound of dynamic fields. Hence  $\hat{\mathbf{n}}_i, \mathbf{B}_\mu$  are dynamic but do not constitute extra DOFs.

Substituting the CFN decomposition into the QCD field strength tensor gives

$$\begin{aligned} \vec{F}^2 &= (\partial_\mu c_\nu^{(i)} - \partial_\nu c_\mu^{(i)})^2 + (\partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g \mathbf{B}_\mu \times \mathbf{B}_\nu)^2 \\ &\quad + 2(\partial_\mu c_\nu^{(i)} - \partial_\nu c_\mu^{(i)}) \hat{\mathbf{n}}_i \cdot (\partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g \mathbf{B}_\mu \times \mathbf{B}_\nu) + (\hat{\mathbf{D}}_\mu \mathbf{X}_\nu - \hat{\mathbf{D}}_\nu \mathbf{X}_\mu)^2 \\ &\quad + 2g((\partial_\mu c_\nu^{(i)} - \partial_\nu c_\mu^{(i)}) \hat{\mathbf{n}}_i + \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g \mathbf{B}_\mu \times \mathbf{B}_\nu) \cdot (\mathbf{X}_\mu \times \mathbf{X}_\nu) \\ &\quad + g^2(\mathbf{X}_\mu \times \mathbf{X}_\nu)^2 + 2g(\hat{\mathbf{D}}_\mu \mathbf{X}_\nu - \hat{\mathbf{D}}_\nu \mathbf{X}_\mu) \cdot (\mathbf{X}_\mu \times \mathbf{X}_\nu). \end{aligned} \quad (8)$$

This expression holds for all  $N$ -colour QCD except  $N = 2$  where the last term vanishes.

The kinetic terms for  $c_\mu^{(i)}$  are unmistakably those of Abelian fields. Eq. (8) has its analogue in studies [1, 2, 7, 22] utilising the maximal Abelian gauge. However, dependence on a particular gauge casts a shadow on any analysis and makes it impossible to consider the corresponding DOFs as physically significant.

The CFN decomposition also introduces additional gauge DOFs, which a proper application must fix. This is done by imposing the condition (7) with the gauge condition

$$\hat{\mathbf{D}}_\mu \mathbf{X}_\mu = 0, \quad (9)$$

which also reduces the number of gauge degrees of freedom to that of conventional QCD [21]. These analyses were performed in two-colour QCD, but their application to  $N$  colours is straightforward [23].

The most important advantage of the CFNS decomposition for the purpose of this paper is that the Abelian dynamics can be specified in a gauge-invariant, well-defined manner that makes it physically meaningful to say that the fields  $c_\mu^{(i)}$  describe the Abelian component of the gluon field. This is in contrast to the MAG which is not gauge invariant and where the physical meaning of the Abelian direction is not well-defined. MAG depends on confinement to hide the gauge artifact. The CFNS decomposition has no gauge artifact.

### 3. The Properties of FAGs

An Abelian gluon has no colour charge, just as a photon has no electric charge. It therefore feels no confining potential, unlike quarks and the valence

gluons  $\mathbf{X}_\mu$ , which are coloured. I shall now argue on dimensional grounds that FAGs must be massive, which renders their effects short-range.

The propagation of FAGs outside of a hadron, through the monopole condensate, is like that of photons in a conventional superconductor, which are well-known to gain an effective mass from the same Cooper pairs which restrict magnetic fields to flux tubes. Indeed, eq. (8) contains the term

$$2(\partial_\mu c_\nu^{(i)} - \partial_\nu c_\mu^{(i)})\hat{\mathbf{n}}_i \cdot (\partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu), \quad (10)$$

clearly indicating that the monopole condensate does indeed act as a sink/source for Abelian gluons. Significantly, there is no corresponding sink/source term for the valence gluons, although it has been argued [11, 23] that their mass-gap term is generated from the interaction term

$$(\partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu) \cdot (\mathbf{X}_\mu \times \mathbf{X}_\nu) \quad (11)$$

The corresponding mass-gap can be estimated on dimensional grounds. The monopole condensate neutralises the magnetic component of a FAG, unless it oscillates faster than the characteristic time of the condensate. This characteristic time would of course go to infinity when the condensate vanishes at deconfinement. Hence the lower limit on a FAG's period can be estimated from the QCD critical temperature using a suitable combination of dimensional constants. Recent lattice calculations ([24] *and references therein*) of  $T_{QCD}$  vary between 151 and 195 MeV. This should be compared to the  $\pi^0$  mass of 135 MeV and the  $\pi^\pm$  mass of 140 MeV [25].

Hence an Abelian gluon can propagate outside of a hadron only if it has sufficient energy to overcome the mass-gap imposed by the QCD vacuum. It would then have properties very similar to the  $Z_0$ , except that it couples only to quarks.

This coupling to quarks is a source of instability. Any gluon can couple to a quark-antiquark pair, and from there to a photon and an  $e^+ - e^-$  pair. Another decay mode is into a  $\pi^0$  with a photon to conserve angular momentum. A more interesting process is the interception of a FAG by a virtual pion emitted by a hadron. The virtual pion could absorb the FAG and use its energy to become real while emitting a photon. Note that both neutral and charged pions can participate in this reaction, so a proton could stimulate a FAG to become a  $\pi^0$  and emit a photon, or to become a  $\pi^+$  (and emit a photon) while turning itself into a neutron.

While the energy of a FAG is certainly accessible, hadron collisions at this energy are dominated by jets. Indeed, the mass was derived from the deconfinement temperature so one should not expect the necessary energy to be available in a stable hadron. One possible source is a quark-gluon plasma. It must have sufficient energy by definition, so a quark-gluon plasma, surrounded by normal space, could lower its temperature by emitting FAGs.

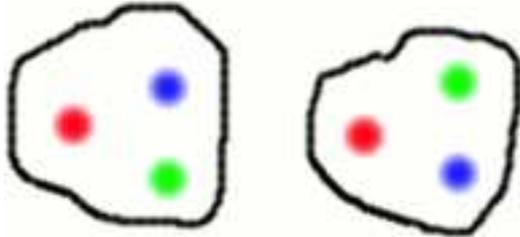


Figure 1: Simplistic representation of induced colour polarization generated by FAG exchange between two hadrons. Note that even if the colours are changed or mixed by a gauge transformation, the degree of polarization is unaffected.

#### 4. The internucleon potential

Since hadrons are colour neutral, the inter-hadron effect of FAGs is a massive, *ie.* exponentially decaying, van der Waals interaction. At the simplest level of understanding, mesons are genuine dipoles in this respect, being quark-antiquark pairs, while baryons are more complicated objects. Nonetheless their colour polarization can be crudely modelled by observing that the combination of any two colours is the opposite of the remaining colour. Hence the baryon contains three axes of polarization, of which only two are linearly independent.

Hence we expect that two baryons in close proximity to each other will attempt to align their polarization axes, as shown schematically in figure (1). This can lead to complicated behaviour for several reasons apart from the inherent difficulty of nonperturbative systems. One is that the exchange of valence gluons between quarks will alter a hadron's colour polarizations by swapping colours around. This suggests an additional mechanism for loss of nucleon mass in the atomic nucleus, in addition to sharing virtual pions. An energetically favourable configuration of nucleonic colour polarizations will suppress the valence gluon exchanges which would otherwise disturb it, thus inhibiting a significant contribution to the baryons' mass.

Another source of complicated behaviour is the difficulty of aligning multiple, coupled polarization axes when more than two hadrons are in close proximity. This would obviously contribute to the three-body effects of the internucleon force. While further analysis is needed, colour polarization may explain this naturally.

#### 5. Discussion

I have made a case for  $MeV$  mass gluonic colour singlets. It is based on the observation that two of the eight gluon generators in three-colour QCD are without colour charge, and that it is colour which is confined. The arguments require the CFNS decomposition to identify the Abelian directions in a gauge invariant way. Without this it is impossible to claim that the Abelian degrees

of freedom have real physical meaning. It has been noted that the analysis is not sensitive to the number of colours, with one Abelian degree of freedom in the two-colour case and  $N - 1$  of them for  $N$ -colour QCD.

An attractive feature of the CFNS decomposition, which makes it useful in dual-Meissner effect studies, is that it unambiguously identifies the gluon's monopole degrees of freedom [15, 16, 17]. It is easily shown furthermore, at least to one-loop order, that the corresponding monopole condensate is non-zero [1, 22]. Especially important for this work, a term describing the condensate acting as a sink/source appears. This not only allows the condensate to restrict the chromoelectric flux to flux tubes as required by the dual Meissner effect, but also provides a mass gap for unconfined gluon fluctuations.

A generic dimensional analysis then predicts a mass of the same order but slightly higher than that of the pion. Thus one expects Abelian gluon exchange to also contribute to the internucleon potential. The resulting potential is essentially a massive van der Waals interaction, although the form of its repulsive component at short distances can only be guessed at. Such a potential implies a mutual colour polarisation which is expected to be affected by the addition of a third nucleon to the immediate vicinity, which has long been observed in the internucleon potential. While it is to be expected that the pion-nucleon coupling would be complicated, the possibility remains that a FAG contribution to the internucleon potential might simplify its analysis.

While their deconfinement scale mass-gap makes FAG production by hadron scattering unlikely, a quark-gluon plasma of sufficient size could well employ FAG emission as a means of lowering its temperature.

The key signatures marking the existence of FAGs are the catalysis of pion production by hadrons in the vicinity of a quark-gluon plasma.

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## References

- [1] G.K. Savvidy. Infrared instability of the vacuum state of gauge theories and asymptotic freedom. *Phys. Lett.*, B71, 1977.
- [2] N.K. Nielsen and P. Olesen. An unstable yang-mills field mode. *Nucl. Phys.*, B144, 1978.
- [3] Kei-Ichi Kondo and Yutaro Taira. Non-abelian stokes theorem and quark confinement in  $su(n)$  yang-mills gauge theory. *Prog. Theor. Phys.*, 104, 2000.

- [4] J. Honerkamp. The question of invariant renormalizability of the massless yang-mills theory in a manifest covariant approach. *Nucl. Phys.*, B48, 1972.
- [5] Volker Schanbacher. Gluon propagator and effective lagrangian in QCD. *Phys. Rev.*, D26:489, 1982.
- [6] W. Dittrich and M. Reuter. Effective qcd lagrangian with zeta function regularization. *Phys. Lett.*, B128.
- [7] C.A. Flory. Covariant constant chromomagnetic fields and elimination of the one loop instabilities. *SLAC-PUB-3244*, 1983.
- [8] Y.M. Cho and D.G. Pak. Monopole condensation in su(2) qcd. *Phys. Rev.*, D65, 2002.
- [9] Y.M. Cho, M.L. Walker, and D.G. Pak. Monopole condensation and confinement of color in su(2) qcd. *JHEP*, 05, 2004.
- [10] Y.M. Cho and M.L. Walker. Stability of monopole condensation in su(2) qcd. *Mod. Phys. Lett.*, A19, 2004.
- [11] K.-I. Kondo. Magnetic condensation, abelian dominance and instability of savvidy vacuum. *Phys. Lett.*, B600, 2004.
- [12] D. Kay, A. Kumar, and R. Parthasarathy. Savvidy vacuum in su(2) yang-mills theory. *Mod. Phys. Lett.*, A20, 2005.
- [13] G. 't Hooft. Topology of the gauge condition and new confinement phases in nonabelian gauge theories. *Nucl. Phys.*, B190, 1981.
- [14] Y.M. Cho. Colored monopoles. *Phys. Rev. Lett.*, 44, 1980.
- [15] L.D. Faddeev and A.J. Niemi. Decomposing the yang-mills field. *Phys. Lett.*, B464, 1999.
- [16] S.V. Shabanov. An effective action for monopoles and knot solitons in yang-mills theory. *Phys. Lett.*, B458, 1999.
- [17] Y.M. Cho. A restricted gauge theory. *Phys. Rev.*, D21, 1980.
- [18] L.D. Faddeev and A.J. Niemi. Partial duality in su(n) yang-mills theory. *Phys. Lett.*, B449, 1999.
- [19] S. Li, Y. Zhang, , and Z.-Y. Zhu. Decomposition of su(n) connection and effective theory of su(n) qcd. *Phys. Lett.*, B487, 2000.
- [20] W.S. Bae, Y.M. Cho, and S.W. Kimm. Qcd versus skyrme-faddeev theory. *Phys. Rev.*, D65, 2002.
- [21] K.-I. Kondo, T. Murakami, and T. Shinohara. Brst symmetry of su(2) yang-mills theory in cho-faddeev- niemi decomposition. *Eur. Phys. J.*, C42, 2005.

- [22] H. Flyvbjerg. Improved qcd vacuum for gauge groups  $su(3)$  and  $su(4)$ . *Nucl. Phys.*, B176, 1980.
- [23] M.L. Walker. Stability of the magnetic monopole condensate in three- and four-colour qcd. *JHEP*, 01, 2007.
- [24] Y. Aoki et al. The QCD transition temperature: results with physical masses in the continuum limit II. *JHEP*, 06:088, 2009.
- [25] C. Amsler et al. Review of particle physics. *Phys. Lett.*, B667:1, 2008.