Fractional Dynamics, Tiling
Equilibrium states and Riemann's zeta function

A Le Méhauté[1], L. Nivanen, A El Kaabouchi, Q. A. Wang
ISMANS, 44 Ave. F.A. Bartholdi, 72000 Le Mans, France

Summary : Il is argued that the generalisation of the mechanical principles to other variables than localisation, velocity and momentum leads to the laws of generalized dynamics under the condition of continuous and derivable space time. However, when the fractality arises, the mechanics principles may no more be extended especially because the time and space singularity appears on the boundary and creates curvature. There is no more equilibrium state, but only a horizon which might play a same role as equilibrium but does not close the problem - especially the problem of the invariance of the energy - which requires two complementary factors: a first one related to the closure in the dimensional space, and the second to scan dissymmetry stemming from the default of tilling the space time. A new discreet time arises from fractality. It leads irreversible thermodynamic properties. Space and time singularities lead to the relation between the above mentioned problematic and the Riemann zeta functions as well as its zeros.

Historical context

The relationship between the dynamics and thermodynamics was developed in a conflicting historical context in which the Newtonian mechanics, as the Queen of the sciences, has always driven the paradigms and made many eminent and unquestionable scientists, such as Boltzmann Duhem or Prigogine, expiatory victims accused to be deviationist (I. Stengers, 1997).

The mechanics was formed gradually over four centuries along successive stages dating back to the smart experience of Galilée (1564-1642) which turns out to initiate the theoretical quarrels between Leibnitz (1646-1716: mv) and Descartes (1596-1650 : mv^2) on the one hand around the vis viva (living forces), and posthumously with Newton (1642-1727) on the other hand. Although it was Galilée and Leibnitz who claimed the measurability of acceleration, it was Newton who won the first round with the expression linking the force to the mass ($F = \gamma m_1 m_2 / r^2$) for two body system which is the first foundation for Kepler laws.

The next step of the mechanics is called Lagrangian event by Stengers (1997). This step begins with the principle of d'Alembert (virtual work) who suggested the independence of external forces with respect to internal bonds (dipoles). These forces, if deriving from a potential, are independent of the kinetic global state of the system (in motion or not). This leads to the second law of Newton $F = m\gamma$ updated by Euler (1707-1783) and then by Lagrange (1736-1813). All systems and motions, reversible or irreversible, are commonly represented by an ensemble of points with neither spatial nor temporal extension. Consequently, due to the hidden permanent internal/external correlations, the simple concept of friction, depending on velocity and, in practice, on spatial extension, has been dropped out. So, there is no irreversible loss. The equilibrium of interior links (having central role) makes

[1] Corresponding author: alm@ismans.fr

the notion of cause, which is completely balanced by the effect, disappear. The universality of mechanics emerges naturally from the absence of dissipation.

This reasoning applied to motion results in the principle of virtual work. The conservation of the living force T leads to the state function called Lagrangian ($L = T - V$). It is the states associated with Lagrangian that can be related to the work (*W*) and gives sense to the potential *V*. At this step, the causality match precisely the effect, property which excludes all loss functions and gives rise to state function. Only the initial and final states count in the formulation of the dynamics. Hence there is an optimal path, or, put it in another way, there is a variational principle stuck on this optimal path.

From these principles, the mechanics is the sole theory axiomatically constructed on the paradoxical time reversibility (Albert, Coveney, Prigogine). The momentum $p = mv$ is introduced into it as a thermodynamic intensive variable which is nothing but the velocity itself. The energy of variation, or of motion, is fixed by the differential of the intensity p such that $du = vdp$. Finally the energy gets a quadratic form $u = 1/2mv^2$ which gives rise to very useful quadratic properties (Deheuvels*)* allowing analytical solution of dynamic problems. Inversely, the fine properties of the quadratic form become the intrinsic properties of this mechanical theory.

Lagrange will have two descendants whose diverging visions turns out to be conceptually formulated in very different even conflicting manners: irreversible in both time and phase space (see below) for Carnot and reversible with time symmetry for Hamilton who swear allegiance to the queen of sciences and his paradigms with his two beautiful symmetric equation $\dot{p} = -\frac{\partial H}{\partial x}$ and $\dot{x} = \frac{\partial H}{\partial p}$ where $H(x, p, t)$ is the Hamiltonian given by a sum: H=T+V. Then the conservation of energy, that is the absence of dissipation, requires $\frac{\partial H}{\partial t} = 0$.

We will come back to Carnot (1796-1832) at the end of this paper since his result on irreversibility implicitly guides our reasoning. It is Hamilton (1805-1865) who takes the role of guiding line. As shown above, Hamilton constructed a very concise formulation in establishing the equivalence between the degree of spatial freedom and the degree of dynamical freedom. The velocity is postulated as a variable independent of another one: the position, hence an *N* body system in 3 dimensional space has 6*N* degree of freedom forming the phase space and then a principle of conservation arises. This choice of Hamilton leads to the definition of a quantity called Hamiltonian ($H = T + V$) whose conservation is in the heart of mechanics[2]. Therefore mechanics has to consider non conservative properties through extensions/approximations which preserve the beauty of the quadratic properties. The system can be considered as an object on its own thanks to the principle of Poincaré $(rotdiv = 0)$ or equivalently to the absence of monopole which possibly will break, at the boundary, the symmetry imposed by the principle of d'Alembert. The Hamiltonian approach leads to the normal modes as the solution of the dynamic, that is $e^{i\omega t}$, a solution of the operator $\partial_t^2 - c^2\partial_x^2$ which is symmetrical in *t* and *x*, and invariant with respect to the inversion of *t* and *v*, meaning the absence of time arrow and space orientation. The harmonic analysis naturally becomes a central tool of the mechanics.

[2] This conservation is crucial for most of the Hamiltonian systems, although a hamiltonian system may not conserve its energy (Arnold) in some special cases where there are, for example, nonconservative forces.

In the perspective of confirmation of the emergence of irreversibility from statistics, Badiali, after Souriau, added to the above analysis some complementary elements, as for example the symplectical step (Libermann, Souriau). He extended (2008-9) the analysis of Stengers (I. Stengers, 2003) to quantum mechanics by making the different construction steps of the mechanics into hierarchical stages: the Heisenberg step, the Schrodinger step and the Feynman step, each of them giving rise to subtle additions of new concepts and not only new point of view on the existing concepts. These additional concepts (Badiali) lift the degeneracy of physical laws which remained circumstantial despite their generalities [C* algebra (Murphy) associated with Heisenberg and the algebra of von Neumann in statistics, fractal trajectories, the role of automorphism and of time with the Tomita theorem and the KMS conditions applied (or not) to the path integrals of Feynman (Feynmann)]. Following A. Connes (A. Connes) and Roveli (2007) approaches, Badiali's work is an interesting deepening into dynamical foundation of thermodynamics ($Z \propto \exp(\beta\hat{H})$). Admitting the multiplicity of paths (Feynman), it offers a more fundamental understanding of the ambiguous status of time in the absence of geodesic but also

- the exponential character of partition function in the standard case
- the status of the H theorem in statitical mechanics and the meaning of Loschmidt-Zemerlo's controversy (inversion of all velocities of all particles)
- the status of the Lagrangian action $\left[ML^2t^{-1}\right]$ as relativistic invariant which is distinct from the Euclidean action $\left[\int Hdt\right]$
- The absence of Hamiltonian invariant in General Relativity from the factual curvature of space-time

The work of Badiali sheds light for example on the question whether or not exists a time constant beyond which it is possible to introduce the thermodynamics constraints and irreversibility since inside of which the physics loses both the notion of means and its experimental attributes. One more time the key concept of time seems completely associated with the concept of velocity which appears *a posteriori as* a generic character of the mechanics (Targ). But one can ask the following question, i.e., why the reversibility of time might be a justification of the notion of velocity which imposes de facto, not de jure, the non commutativity of space-time (Connes, Roveli), and how the heuristic hypotheses might have been later changed to commutativity from the space-time homogeneity principle (Noether).

Such an assertion has been questioned by an engineering work on the dynamics of 'energy' transfer in fractal geometry (Le Méhauté 1982-1984). Contesting, from the consideration of fractal geometry, the universality of Hamilton point of view, the authors questioned the status of velocity as a dynamical state variable in fractal media. This position opposes the distribution theory and experiments against the opinions ofsome authors searching for the marginal adaptation of classical mechanics to the fractal limit conditions (Gouyet, Stauffer, Vannimenus). In such a media, there exists an intrinsic anomalous link between space and time, determined by the fractal structure, which is more general than the notion of velocity. This new variable ($L^d t^{-1}$) is needed by the consideration of the non differentiability (Tricot) of 'generalized phase' space and hence by the approach of Fourier transformation. This transformation leads to inter-scale strong correlations inside the set of Hamiltonian normal modes (states) or related coarse-graining analysis. The pertinence of this idea was initially supported by the observation that the diffusion constant represents only a particular case of that new class of generalized variables (L^2t^{-1}).

The Fourier transform, as the main tool of this distribution approach, implies a singular self-similar tiling of space-time (Bedford, Hirsch). By using adequate gauges this tiling can be

- either Euclidean (d=1) if the space is continuous, without energy in the dipole (d'Alembert) but with arbitrariness of rules for the choice of self-similar tiling (topology of torus), and with velocity as a state function of the dynamics.
- or, if d=2, hyperbolic (Toubiana) by the consideration of discreet space as an ensemble of singular points coupled with self-similar topology of punctuated torus (Bedford). We will all the same emphasize that there are two classes of possible tiling, that will upset the viewpoint which is after all orthodox.

In fact, whatever is the topological difference of self-similar space-time tiling (fundamental group and domain) between the Euclidean (d=1) and hyperbolic ($d \neq 1$) approaches (Berline), both tilings require to ground the time concept via a dynamics (Badiali, Bedford). The tiling points out the scales carried by two automorphisms: one concerning the space and one concerning the time. The last one $A \rightarrow A_{\omega}$ is based on a variable ω, where ω must be normalized from space either in the external form of velocity (Euclidean), or in an internal form of a time constant $\omega\tau = 1$ which requires an heuristic hypotheses on space time relationship. Badiali (2009) reported the same kind of problem for the Feynman's path integral of quantum Mechanics on the basis of the theorem of Tomita-Takazati – there exist a one parameter group of automorphism which leaves the algebra globally invariant. In fractal geometry this parameter is complex (Le Méhauté 1997) and involves non commutativity with respect to time (see below). Hence a specific dissipation arises.

This is easy to understand. In the Euclidean case, due to the fact that the only singular point is the, somewhat arbitrary, original point of the axis, the paving leads to the torus topology of the dynamics (Hirsch). This introduces the operator $\partial_t^2 - c^2\partial_x^2$ into a formulation associated with the existence of normal modes of the space-time manifold and of cycles, or their equivalent in quantum mechanics (Hilbert space, eigenvalues or eigenfunctions). In this approach, the reversibility is obviously the consequence of our arbitrary adjustment of the scales we use for the dynamics (coarse graining without specific correlations due to the only spatial character of the Fourier transform of a velocity). We are here at the centre of the paradox which opposes the mechanics to the thermodynamics and Hamilton to Carnot; a paradox stressed by Prigogine and his students who, after De Donder, turned out to make the irreversibility (viewed from mechanics) a metaphysical property relative to the arbitrariness of imposing *ad hoc* approximation of measure on a continuous and homogeneous space-time (Noether). This conclusion is a petitio principii since it assumes *a priori* the absence of correlations and of the pertinence of the basic Hamiltonian concepts such as velocity.

If we keep the debates at the same level of analysis, the situation may partially change if the space-time or the phase space is considered as a distribution of singularity (Schwartz) and more specifically a scaled set of singularities (Le Méhaute, Mandelbrot). As a matter of fact, if space-time become discreet, due to some mathematical constraints imposed by the singularities (Schwartz), the impossibility (unless for the case where the time is infinite) of interpreting the singularities gives rise to non commutating properties at infinity. Therefore irreversible factors emerge from collective properties of these singularities. The analytical and topological reasons are the following. The singularity exists on its own and is independent of whatever analysis scale and approach, especially for the initial or final equilibrium state with

$t = 0, \infty$, that is, for all $\omega = \infty, 0$ in Fourier space. Every test function (Schwartz) used to characterize the singularity leads an energy δ_u as a mathematical asymptote, and a limit distribution δ_r. This test function must be a coarse graining procedure characterized by renormalisation properties. It is in practice the test function which allows an access to the singularities. Like derivation it implicitly carries the variable of time through the automorphism driven by the procedure. It is just this function which couples in Fourier space the intrinsic properties with the external characteristics of the test signal. In practice, and except in a few number of cases, the convolution, which points out the correlations, breaks the Hamiltonian properties and introduces a new time factor.

The modalities for Poincaré tiling of the standard tree-hyperbolic space-time (Berline, Hirsch, Toubiana) partly mimic the exponential relaxation process in the Fourier space and clearly give rise to geodesics (analogy between arc of circle and transfer exponential function leading standard harmonic ratio, see Figure 1). The coarse graining may lead to some irreversibility factors through the discreet variable $\Delta Z(\infty) = R$ as well as through the relaxation time $\tau = 1/\omega_c = RC$ (Figure 1). Nevertheless, the above analogical viewpoint of irreversibility is still to be strengthened from the consideration of algebraic topology so that it is more robust against possible objection from mechanics. Therefore we should move farther to fractal environment starting from Poincaré tiling (Figure 1) and introducing fractal correlations.

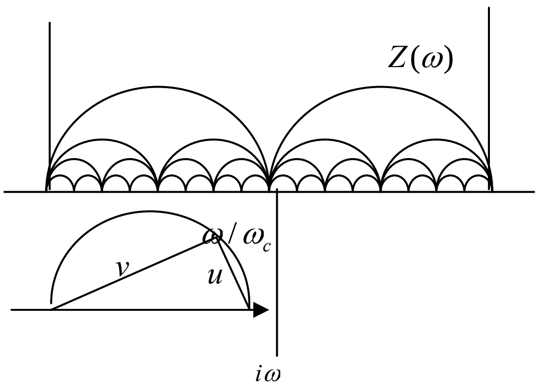

Figure 1 : Hyperbolic tiling of a complex plan using of dynamical functions on $Z(\omega) = \dfrac{R}{1 + i\omega\tau}$ (hyperbolic geodesics), in relation with the modalities of exponential relaxation. Notice the harmonic ratio $u/v = \omega/\omega_c$ where u/v is a hyperbolic distance. The boundary of the punctuated torus built from Poincaré tiling has an angle at infinity equal to zero.

Scaling correlations

As we have previously indicated (Le Méhauté 2005), the topology coming from the tiling in fractal medium can be founded neither on the periodic fundamental domain of the Poincaré type [stemming from the orthodox hyperbolicity (Bedford, Berline)] because the analogy pointed out above seems an ad doc representation, nor *a fortiori* on the simple torus, but on the punctuated torus opened on the boundary. We assert that it is just this singular punctuation, carrying the geometry correlations inside it, which leads the dynamic to intrinsic irreversibility. It is imposed by the space time parameterisation of the geometry (manifold of geodesics). Through an extended harmonic ratio $\left[(u/v) = (\omega/\omega_c)^{\alpha}\right]$ which controls its value, the angle over the punctuation (at infinity) definitely breaks the symmetry associated with the operators accessible to the normal modes and makes the physics on the torus questionable but also the physics on Poincaré punctuated torus (Bedford). In order to show all the subtleties related to the emerging punctuations opened on the boundary, we will make a detour by the analytical foundation of electrodynamics, a son of the differential geometry.

External algebra and their reductions

The orthodox physics is based on the homogeneous space-time structure (Noether). According to Kant, space-time is an *a priori* frame of human conscience but, according to the disciplines, science needs to classify the operating variables in the right space-time for the description of reality. Both points of view have to be related. Starting from the example of electrodynamic variables, we can interpret the laws of electrodynamics (Le Méhauté, 1989, 1995) as shown in Figure 2 in a 3 dimensional schematic view, with the extensive variables on a back plane, the intensive variables on a first plane separated by a certain ‘distance’ which represents the matter . The spatial gradients are given in horizontal direction and the temporal gradients in the vertical direction.

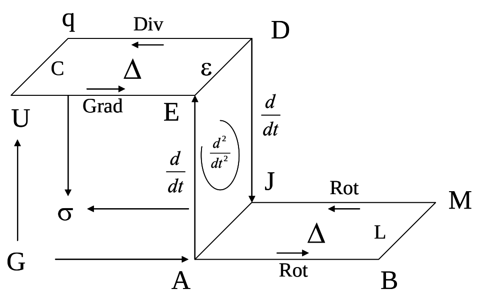

Figure 2 : 3D representation of the empirical variables for the Maxwell equations of standard electrodynamics. The time is necessary for the expression of flux and moments. However, it cannot be extended without giving a sense to the notion of velocity. The velocity appears as the key factor to give a meaning to space time differentiable manifold and even to give a meaning to external algebra, that is to rise the concept of matter (here seen through L and C)

This figure contribute to the geometrical representation of the operator Laplacian $\Delta_x u$ $((div.grad(u)+rot.rot(A)))$ as well as to d'Alembertian $(c^2\Delta_x - d_t^2)u(x,t)$ and give rise to normalizable forces $F = qE + JB$. The limit conditions impose, among other things, the continuity equations $\frac{dq}{dt} + divJ = 0$ upon the dynamics. Geometrically, this results in the laws formulated through the Maxwell equations. Hence we find $rotE + \frac{\partial B}{c\partial t} = 0$; $divB = 0$; $divE = \frac{q}{\varepsilon}$; $rotB - \frac{\partial E}{c\partial t} = \frac{\sigma}{c}$, the quadratic form of energy $U = \frac{1}{2}E^2 + \frac{1}{2}B^2$ and $S = cE \wedge B$ by analogy to the mechanics concerning the momentum density. The electromagnetic wave emerges from the diagram with the forms $\frac{\partial^2 E}{c^2\partial t^2} - \Delta E = 0; \frac{\partial^2 B}{c^2\partial t^2} - \Delta B = 0$. The suppression of the potential vector A simplifies the diagram which has only one source of extensity σ and leads to a diffusive form

$\Delta U - \frac{C\partial U}{\partial t} = 0$. The quantum mechanics is only a reduced form of the same geometrical approach even if the latter is, for this purpose, transformed into the complex space: $i\hbar \frac{\partial}{\partial t}\psi(r,t) = \left[-\frac{\hbar}{2m}\Delta + V(r) \right]\psi(r,t)$. Figure 2 is a simple representation of the geometric differential properties which support the Maxwell equations (Spivak).

This diagram can be simplified in the way of E. Vieil (Vieil, 2008-2010) according to the principle of d'Alembert, by considering that the notion of scalar and vectorial spatial gradient naturally enters into the definition of state. According to Vieil, the link defines the dipoles. Like Lagrange and d'Alembert proposal and following the expression of mechanical energy ($\delta U = Fdl$), Vieil considers two classes of coupled variables which, independently from their thermodynamic nature (active for U_q or reactive for Ξ_q), obey the conservation of energy.

$$\delta U_q = E_q dq \qquad \delta \Xi_q = I_q dp_q$$

q is the transferred extensive quantity similar to the momentum in mechanics, E_q is the related field 1, from which the transfer arises, I_q is the flux of q, that is $\dot{q}$ and p_q is the momentum (field 2 : *B*) from which the force can be derived. We then get Figure 3.

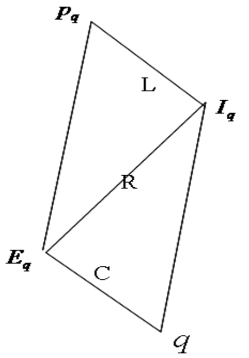

Figure 3 : Graphic representation of the variables of all physical process according to Vieil (2009). One notices the disappearance of spatial gradient with respect to the Figure 2. This diagram authorizes the extension of the mechanic concept to other field of the thermodynamics (electricity, chemistry, surface and thermal properties). Far from that point it authorized the analysis of the coupling between different fields (Energy conversion).

Although Vieil turns out to be able to find the concept of dipole, and hence the concept of discreet space, the advantage of his approach with respect to the generalized expression of the external algebra in Figure 2, that is to delete some variables in order to keep only the temporal derivation, is legitimate only if the notion of velocity is a priori taken for granted. In view of this implicit hypothesis and in the frame of an analogy with mechanics, it is possible to suggest from Vieil's proposals one further step with a homogenous version of the diagram by extending the formalism into reciprocal space, i.e., by using, to represent the temporal part, the Fourier transformation (with the normal mode of frequency $i\omega$). We get then Figure 4. This approach turns out to transform the variables of Vieil, especially $I_q(\infty), E_q(0), p_q$, into singular points of the fundamental topologic domain based on an automorphism of the Poincaré type (Figure 4), so that the variables are associated with a dynamics over q which is invariant from the viewpoint of the dipole energy $E_q(0)$.

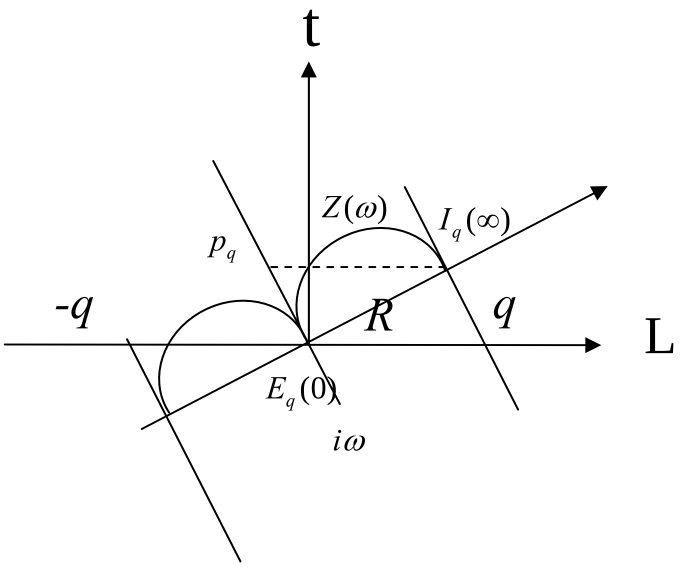

Figure 4 : Double representation in (L,t) and in $((Z(\omega), i\omega)$ of the dynamics founded on the Vieil's variables. Here the fundamental extensity q in space-time gives rise to an ensemble of fundamental variables in frequency space. These variables acquire physical sense only asymptotically in accordance with the exponential relaxation (the hyperbolic geodesics represented by cyclic arc). The reversibility in time and frequency is conserved by the factual symmetry. One notices that in order to work out this diagram, Vieil cancelled the spatial gradient (implicit he admit the principle of d'Alembert).

This figure allows us to introduce the following links:

- between the gauges of measure related to velocity (L/t). The Fourier transformation of the velocity $\eta(\omega)$ is associated with a length and $Z(\omega)$ the impedance of the dynamics is therefore related to an 'interface' for exchange. This relationship seems very conceptual but it naturally comes from the simple experience with complex medium (storage of energy, viscoelastic dynamics, thermodynamics of crystallization and percolation). In Fourier space, a flux is only a charge (capacity) proportional with the factor *Z,* to a force (impedance, see Ohm law *U=ZI* for example). In the absence of the source ϕ_∞, the flux density is equal, with a geometrical constant, to the force. Hence, in Fourier space, $1/Z$ is nothing but the measure of the interface of energy storage or exchange (length, surface, etc). From this which is the basement of the TEISI model (Le Méhauté 1982), a generalization of the concept of capacity named Fractance is authorized. Hence it is in practice possible to put in the diagram two different representations, i.e., the representation based on $[L,t]$ and the one with $[Z(\omega), i\omega]$ which take into account the concept of velocity. Assuming the presence of a temporal horizon, Z is a variable linked to $\eta(\omega)$ by a relation of Mobius type $\frac{1}{Z(\omega)} = 1 + \frac{1}{\eta(\omega)}$. The term 1 indicates the presence of a source of dissipation ϕ_∞, that is of a resistance, so a dissipative term clearly related to the behaviour at infinity (on the punctuation). This relation expresses both the pertinent transformation of displacement associated with a source which can be shifted according to our will and a pertinent inversion of the variable associated with the possible construction of reciprocal tiling (for example the diagram of Brillouin in solid state physics). Among consequences the relation $i\omega\eta(\omega) \approx 1$ which implicitly expresses the pertinence of the concept of velocity and resolves the problematic by asserting the conservation of measure in two reciprocal phase spaces $[L,t]$ and $[Z(\omega), i\omega]$. The introduction of the complex variable i is due to the arbitrariness of ω with respect to η, or to the conservation of certain velocity.

Figure 4 surprisingly keeps the Hamiltonian characteristics . The transformation of Fourier of a velocity is a length $\eta(\omega)$ a gauge of the reciprocal space. In this framework, R appears as a limit of $\Delta Z(\omega \to 0,\infty)$. The two extensities $\lfloor q, I_q(\infty) \rfloor$ have a spatial localization in the sense of $\eta(\omega \to 0)$ but the two intensities $\lfloor E_q(0), p_q \rfloor$, which are respectively the force constituting the spatial pivot, and the momentum the frequency characteristic, have singular position on the ω axis. The physical sense of the diagram is obvious. In space-time, there is no other extensity in the thermodynamic sense than the dipole $[q,-q]$ accessible at long time limit of the dynamics. The dipole $[q,-q]$ is in addition the sole source of the energy $E_q(0)$. The resistance is strictly related to the geometrical distance of the dipoles. If the space time is homogeneous, this distance is related to a length and the resistance is an ad doc distance.

The above analysis sheds light on the physical signification of the constants L and C. These constants are of spatial nature: progression and regression in the definition of the dipole through C, and cycle in the definition of the moment through L. The simplest understanding of these constants is supported by the homogeneity of the state functions in the sense of Euler $S(\lambda\{q\}) = \lambda S(\{q\})$ where S is certain measure in space-time-matter. In this way, whenever we write $q = CU_q$, the expression $\delta U_q = E_q dq$ takes place by the time derivation $\delta U_q = RCd_t U_q$ where $RC = \tau$ appears as a time constant. Obviously, this constant plays a central role whenever the dynamics is described with a series of stationary states without any correlation, hence the orthodox physic arises from differential integer analysis:

operator	$\frac{\partial}{\partial t} - \frac{1}{\tau}\Delta$ or $\frac{\partial}{\partial t} - v_0 \frac{\partial}{\partial x}$	$\frac{\partial}{\partial t} - D_0 \frac{\partial^2}{\partial^2 x}$	$\frac{\partial^2}{\partial^2 t} \pm v^2{}_0 \frac{\partial^2}{\partial^2 x}$
Geodesic	$\tau \equiv [t]$ or $v_0 \equiv [Lt^{-1}]$ Exponential response	$D_0 \equiv [L^2 t^{-1}]$ Diffusive response	$v_0 \equiv [Lt^{-1}]$ Wave response

Table 1: Differential operators of different integer order for different dynamical processes.

All this above statements are relative to an elementary examination of the extended differentiable paradigm of the mechanics. But what happens if the phase space becomes fractal and non differentiable? A non standard heuristic reasoning (Hoskins, Le Méhauté) can lead to conceptual difficulties concerning the notion of measure (Tricot) and the notion of dynamic and geodesics which can only be reached via distribution theory (Schwartz).

To summarize the advantage of the orthodox viewpoint -presented above, which nevertheless conjugates the Fourier time-space $[Z(\omega), i\omega]$ with the standard time-space$[L,t]$-, as a seed for future develoments, it naturally suggests, from geodesic geometry, the extension of the analytical form of the function $[Z(\omega)]$ to the form $[Z_\alpha(\omega)]$. This analytical extension of first order (half circle, zero angle on the boundary, standard harmonic ratio, Poincaré paving) into fractional order representation (see below) conserves the hyperbolic character of the geodesic (circle arc), expands the harmonic ratio into fractional one, restricts its Fourier space-time expansion and gives rise to the boundary singular behaviour. The related topological extension, widely justified experimentally (see reference in Le Méhauté 1990 and 1997), reinforces, in addition, a consequence of the heuristic hypothesis formulated in the TEISI model (Transfert d'Energie sur Interface Self-similaire). This model is founded on the projection of fractal phase space onto the Fourier space and on the conservation of measure in that space by taking into account the fractal property in a generalization of the concept of velocity through $i\omega[\eta(\omega)]^{1/\alpha} \approx 1$.

This detour by the fractality will shed light on the real limits of the mechanistic philosophy if one keeps the point of view of Noether, of d'Alembert and of Hamilton for answering the question of irreversibility as a simple consequence of our cognitive incapability and of approximation (Prigogine). This detour leads to the consequence of the geometry of the complex environments, that is, of the generalized metric of the space-time, on the determination of irreversibility. Let us now enter into the details of the analysis

Equilibrium and periodicity

What does the concept of resistance mean? The answer can be given indirectly by the reasoning around the differential equation of first order which describes the proportionality of the flux to the force pushing the system to equilibrium. This equation will be later expended under the form of fractional differential equation. We used here the word *equilibrium* which, as shown below, contains certain ambiguity. To understand it, the basic differential equation $\frac{d}{dt}U(t) = \frac{1}{RC}[U_\infty(t) - U(t)]$ must be used. The horizon represented by the equilibrium $U_\infty(\omega)$ imposes to the dynamics a source ϕ_∞ conducted by an external force which behaves as teleonomical characteristic. The Fourier transform of this equation is nothing but $Z(\omega) = \frac{1}{1 + iRC\omega}$, the impedance of a parallel *RC* circuit with $Z(\omega) = \frac{U(\omega)}{U_\infty(\omega)}$. The solution of that equation is an exponential function $U(t) = e^{-t/\tau}$ where $\tau = RC$. We get the notion of time constant from this particular case. The presence of R in τ is an explicit dynamical expression of the existence of an asymptotic state called equilibrium (horizon) but also it signals a thermodynamic irreversibility. In other words, the question is: is there an equilibrium thermodynamics state only under the condition of the existence *a priori* of a dynamics having exponential solution? As a matter of fact, the exponential is the only function which contains, locally, all the information carried globally by the dynamics. It is the only function that is analogous to its derivation. Put it in another way, one does not define intrinsic equilibrium just confined in its local behaviour without reference to irreversible global process characterized by the presence of *R*. Let us observe that the exponential formulation guarantees also the periodicity of the standard state through the link arising via the standard algebra (standard relation between summation and multiplication (Nivanen)). This periodicity can also be found from tiling the space. Is it associated with the presence of hyperbolic geodesic giving a sense to the least action and to the variational calculus, in both configuration space and Fourier space? Does it authorize the reduction of chaos to certain form of determinism mechanistic (Mandelbrot 2004, Prigogine)? Some (mathematically coherent) answers to these questions can be found in the fractality and fractionarity of the dynamic.

Fractality, fractionarity and associated topology

On the basis of the experience in the fields of storage of energy in fractal media (Le Méhauté, 1982), one of the authors has concluded long ago that geometry is the source of entropy. This conclusion was latter confirmed by the propagation of electromagnetic waves in heterogeneous media (Jonscher 1983, Le Méhauté 1989-91) as well as in viscoelastic media (unpublished). Conversely, many authors show that fractal born easily from flows of aggregation, percolation, deposition etc (Sapoval). If these observations does not have sufficiently robust conceptual basis to relate geometry and irreversibility they bring to the laboratory sufficient reasons to address theoretical questions based on the dynamics of non standard hyperbolic geometries.

The introduction of irreversibility into mechanics via the emblematic equation of first order is a tool to extend the mechanical analysis into Poincaré hyperbolic environment (Figure 4). Nevertheless, as shown by the TEISI model, the operator of fractional derivation in time (Poldubny, Samko, Spanier): $TF(d_t^\alpha) = (i\omega)^\alpha$ describes the fractal like dynamics. Hence the generic first order equation suggests a simple canonical extension of the dynamics using a fractional order operator. It is this operator which, making a breakthrough in the standard differential analysis, questions with respect to irreversibility, the paradigms of differential physics and then the mechanics . This can be briefly remind in table 1 without reference to the relationship between these operators and the fractal geometry established elsewhere (Le Méhauté, 1990-1997),

$\frac{\partial^\alpha}{\partial^\alpha t} - f_0 \frac{\partial}{\partial x}$	$\frac{\partial^{1/2}}{\partial^{1/2} t} - \sqrt{D_0} \frac{\partial}{\partial x}$	$\frac{\partial^\alpha}{\partial^\alpha t} \pm f_{r0} \frac{\partial^2}{\partial^2 x}$
$f_0 \equiv \left[L t^{-\alpha}\right]$ Power law response	$D_0 \equiv \left[L^2 t^{-1}\right]$ Diffusive response	$f_{r,0} \equiv \left[L^2 t^{-\alpha}\right]$ Fracton response

Table 2: Fractional differential operators of different orders. Note the difference from the corresponding operators in Table 1.

The interest of the generalization of differential analysis is to get geodesic and differential link between $q \to I_q$ and $p_q \to E_q$ controlled by a unique transfer function of the form $Z_\alpha(\omega) = \frac{1}{1+(i\tau\omega)^\alpha}$. This analytical form is associated with an α-hyperbolic geodesic in Fourier space (arc of circle). With a nonexclusive example $\alpha = 1/d$ for a α-fractionnal differential form in a fractal space of dimension *d*, the geodesic can be related to an energy transfer through a *d*-fractal interface according to the α-differential equation $\frac{d^{1/d}}{dt^{1/d}} U(t) = \left(\frac{1}{RC}\right)^{1/d} [U_\infty(t) - U(t)]$ (Le Méhauté 1982). The diagram 5 shows this particular mode of representation in the space $[L,t]$ conjugated to the space $[Z(\omega), i\omega]$.

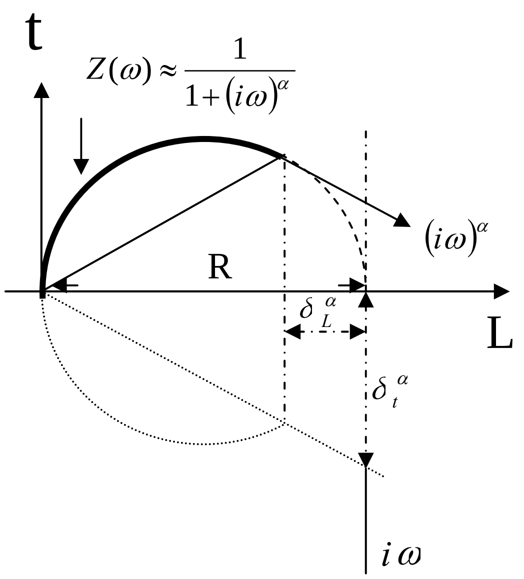

Figure 5 : Elementary form of the fractional transfer function of order α (impedance) on the basis $[L,t]$ and $[Z(\omega),i\omega]$. With the lifting of degeneracy of standard time, one sees the interesting aspect of the use of the two basis. The partial arc of the circle discloses the presence of a geodesic in a right space of representation. The spatial extension of this arc in space-time and frequency discloses the fractal character of a sub-manifold (hyperbolic distance given by $(u/v)(\omega/\omega_c)^{\alpha} \approx 1$ and the control of the dynamic by a fractional differential equation. This representation reveals in addition the singularity at the boundary required to close any quadratic form. This singularity will be associated to the opened punctuation of the torus, tiling of the manifold, which support the dynamic and therefore the geodesics. The fractal contribution leads the appearance of a new time component δ_t^{α} which does not play the same role as a new spatial contribution δ_L^{α}. This difference will play a major role in the understanding of irreversibility (Figure 8)

The figure 5 discloses the restriction of the Poincaré hyperbolic geodesic (standard semi circle) which can produce a symmetry breaking (boundary properties). This fractal consequence on the dynamics implies the disappearance of state functions and the rising of a nonextensive algebra (Nivanen) for the assessment of the thermodynamic extensity and intensity variables addressed for instance by Vieil's analysis. One of the consequences of the formal analysis related to the symmetry breaking is a generalization of the normal exponential and logarithmic functions used in an extended statistical thermodynamics developed for complex media (Nivanen). More over, in the framework of the statistical mechanics, this extension leads to the generalization of the statistical laws taking into account the incompleteness of the estimation field of the random variables (Ou,Wang) and to a more general mathematical measure on non extensive set (El Kaabouchi).

Although power laws does not lead to the convergence of the temporal series, this analysis paradoxically maintains the concept of horizon $U_{\infty}(t)$. This concept is factually extended to the notion of equilibrium as an expansion of internal scaling correlations. This equilibrium in the sense of a quadratic form, is the result of a conjugation between two complementary power laws behaviour with characteristics α et *1-*α ,taking into account the fractional order of the singularity on the boundary.

Even on waiting if possible, of the construction of the generalized exterior algebra, the algebraic and geometric formulations show a proximity, noticed long time ago, between the diffusive operators and the operator of order ½. Let us remind that the diffusive operators conjugate conventionally the equation of first order and the equation of continuity. Such a definition may probably be bypassed if we observe that, with $i\omega\eta(\omega)^2 \approx 1$, a generalized velocity exist which takes the dimensional form $\left[Lt^{-1/2}\right]$

In the absence of a dual basis $\left[L,t\right]$ and $\left[Z(\omega),i\omega\right]$ and taking into account the transfer impedance $Z_\alpha(\omega) = \dfrac{1}{1+\left(i\tau\omega\right)^\alpha}$ with $\alpha \neq 1/2$, we might believe that the introduction of the fractality expands the transformation of the dynamics solely into a double effect (i) scaling (through ω^α), on the one hand, and (ii) a rotation of axis without temporal consequence on the complex plan (through i^α), on the other. However, as shown by the representation using the double basis (Figure 1 to 5), there is an additional major effect than mentioned above. The geometric treatment of the parameterization of $Z(\omega)$ imposes a lifting of the degeneracy of the punctuated singularity on the boundary (short or long time) $\left[\delta_L^\alpha, \delta_t^\alpha\right]$. One may say that the initial Poincaré like singularity at the boundary, a point so that it has neither spatial nor temporal extension (Figure 1), acquires an emerging structure coming from the correlations related to the α-distribution. The fractality rises time-space singularity.

The splitting of the singularity on the boundary, due to the collective properties of the hyperbolic geodesic, affects the dynamics through the border properties in the space $\left[Z(\omega),i\omega\right]$. They implicitly impose a curvature of all space time $\left[L,t\right]$. Besides the scaling properties, the dynamics undergoes the effect of acceleration hidden behind the fractionality. The notion of time can not be reduced to a simple cyclic series (state functions from Fourier transformation or eigenfunctions in Hilbert space) since crucial space time correlations are partially lost in the sole geodesic. There is an irreducible space time residue which closes the energy invariance and therefore the entropy measure as a state function whenever a fractal dynamics takes place.

The additional transformation related to the splitting of singularity on the boundary plays the role of the closure of the fractal system in order to put it in contact with a thermostat. This closure returns the vocation of Noetherian invariant to the energy (Kosmann-Schwarzbach), i.e., to the relative quadratic form[3]. This quadratic closure shows another advantage: it gives a nominal equivalence to the two complementary parts of the arcs of circle, and then reduces, as shown in Figure 5, their inverse transformation with respect to the pole within two parallel straight lines, affected by the similar, but not equal, singularities on the boundary. More precisely, these two singularities denoted by ($\delta_{\omega,R}^\alpha, \delta_{\omega,R}^{1-\alpha}$) are complementary one to another in the quadratic form. This analysis turns out to modify the diagram to construct the Figure 6. This figure underlines both the role of the splitting singularity in a representation leading to

[3]Although the conceptual tools of extension of exterior algebra are not mature at this stage, one can attribute the existence of magnetic monopoles to such property (unpublished).

the tiling extension, and the two singularities $\delta_{\omega,R}^{\alpha_1}$, $\delta_{\omega,R}^{1-\alpha_2}$ sharing R as a common variable linking the fractal with the thermo-state (periodic conditions for whole boundary; quadratic global form) , that is with a certain value of temperature.

Let the arcs of circle be denoted by $Z_\alpha(\omega)$ and $Z_{1-\alpha}(\omega)$, we can generalize the periodic and non periodic hyperbolic tiling of the complex plan (Berline) using these geodesics. It is in fact possible to use the elementary tile from Figure 5 in order to cover the space. The tiling takes place in an imperfect manner with two distinct components, for the pavement: a covered and an uncovered singular zone. Despite its operational simplicity – it is in fact possible to reproduce simply the techniques of tiling of a plan and of the circle of Poincaré – this tiling is worthy to be mathematically deepened since it contains obviously open questions about the topological, and analytic formulation of the related automorphisms and associated dynamic.

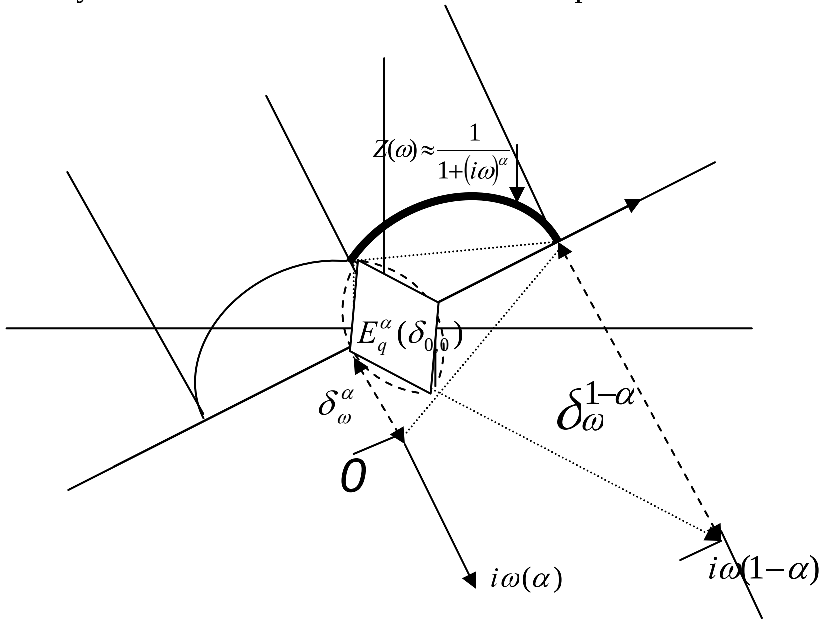

Figure 6 : Extension of the Poincaré fundamental domain required for the singular tiling of the space-time when the dynamics is controlled by a fractional differential equation. We can notice the splitting of the energy related to the appearance of the singularities at the boundary as well as the complementary metric character of these singularities. Such a diagram opens plenty of questions in topology as well as in the physics of fractal media. The appearance of the singularity at the boundary is clearly related to the concept of emergence in physics. The rising of this singularity may even sheds light upon the question of emergence in complex systems.

Let us notice that the to semi circle are related to backward and forward geodesics. What will interest us now is not only the tiling question but the question of degeneracy of the tilling which occurs when $\alpha = 1/2$. We noticed previously the specific role of the factor $\alpha = 1/2$ which returns the problematic to the case of diffusion in a field of Noetherian space-time. Extended at tiling, the singularities represented by the factor ½ return the spatial and temporal problematic to the question of the zeros of the Riemann zeta function. What happens if $\alpha = 1 - \alpha = 1/2$? The Figures 5 and 6 clearly indicate a mechanism of fusion-confusion of singularities and dynamics. The angle of phase becoming equal to $\pi/4$, the resonance of the quadratic form becomes the limit of the two fractal forms of $Z_{1/2}(\omega)$; $\delta_{\omega,R}^{1,2} = |R|$, $\delta_{\omega,R}^{\alpha_1} = \delta_{\omega,R}^{1-\alpha_1}$, $\delta_{L,R}^{1/2} = R/2$. In practice, the imperfect pavement proposed above and the related geodesics degenerates into a dual singularity. (Le Méhauté, 2009). The problem of zeros of the zeta Riemann function arises.

Zeta function

The zeta function is given by a sum of a series of power functions $\zeta(s)=\sum_{\omega=1}^{\infty}\frac{1}{\omega^{s}}$ with $s=\alpha+i\theta$. One recognizes (Le Méhauté, 2008-2009) in the real part of the zeta function the hyperbolic distance measured on a fractal arc of circle given by $Z_{\alpha}(\omega)$ which allows one to plunge the whole problematic relative to this series and also to its avatars ($\eta(s)$ function and L functions) into the complex plan. We also recognize, in each of the forward (+θ) and backward (-θ) arcs of the intersected circles $Z_{\alpha}(\omega)$, $\overline{Z}_{\alpha}(\omega)$, $Z_{1-\alpha}(\omega)$ and $\overline{Z}_{1-\alpha}(\omega)$ the real part of the four symmetrical components of the Riemann function, i.e., $\left[\zeta(s),\zeta(\bar{s}),\zeta(1-s),\zeta(1-\bar{s})\right]$ where $\bar{s}=\alpha-i\theta$.

We know in addition that the zeros of the zeta function are discreet and infinite in number since they are related to the prime numbers (Edwards, Slater). The zeros come in practice from an ultra-degeneracy over the tiling. Because the variable θ is free and used for the screening of the complex plan, what happens from the degeneracy of the tiling of chaotic zeta trajectory with ω is relative only to the functions $Z_{\alpha}(\omega)\ \overline{Z}_{\alpha}(\omega)\ Z_{1-\alpha}(\omega)\ \overline{Z}_{1-\alpha}(\omega)$? This degeneracy appears for $\alpha=1/2$. α, which is convergent with $\alpha=1/2$, will transform the intersection of the arcs into a fusion of the arcs, making the incomplete tiling disappear. There is no more difference of parameterization between backward and forward. The generalized harmonic relation $\left[(u/v)=(\omega/\omega_{c})^{1/2}\right]$ sends to the infinity the quadratic resonance factor underlining the complementary α and $1-\alpha$ transfer function $Z_{i}(\omega)$ (Figure 7) and therefore the resonance of the quadratic substructure managing the energy.

This is just the content of the Riemann conjecture which occurs as a necessity when the dynamic representation of the singularities from $Z_{\alpha}(\omega)\ \overline{Z}_{\alpha}(\omega)\ Z_{1-\alpha}(\omega)\ \overline{Z}_{1-\alpha}(\omega)$ vanishes. The time gives to the conjecture its specific representation (Figure 7 and 8) and, through a derivation of order ½, returns the problematic in the continuous field of the type $\left(\partial_{t}^{2}-v_{0}^{2}\partial_{x}^{2}\right)$ to a classical diffusive $\partial_{t}^{1/2}\pm\sqrt{D}\partial_{x}$ and to an almost symmetric gravitational problematic $\partial_{t}\pm\sqrt{(1/\gamma)}\partial_{x}^{1/2}$. The precision ‘almost’ points out the fact that to exchange L to t we must take into account a shift *R* which introduces a fundamental irreversibility; the price for the emergence of the matter.

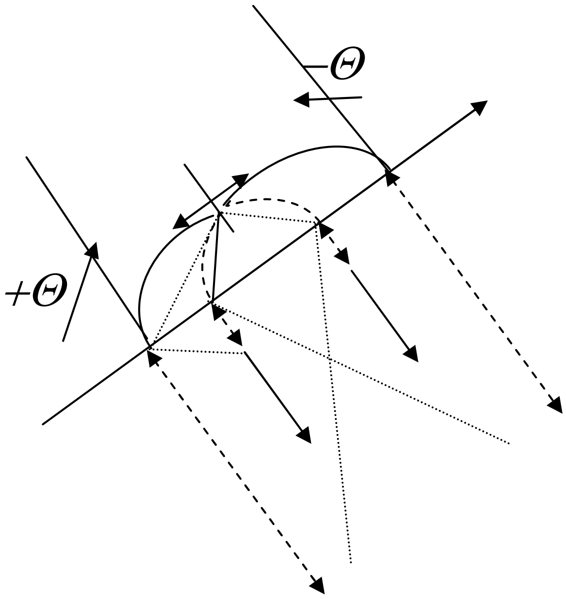

Figure 7 : A schema showing the approach leading to the zeros of the zeta function of Riemann from the expressions of different $Z_\alpha(\omega)$. The real term in the zeta function is nothing but an extension $(u/v)(\omega/\omega_c)^\alpha \approx 1$ of a hyperbolic distance (harmonic ratio) of α order measured on $Z_\alpha(\omega)$. This use of $Z_\alpha(\omega)$ allows one to plunge the real part of zeta function into complex space and hence to disclose a link between the real part in α and the complex part in θ. This double plunging relates $Z_\alpha(\omega)$ $\overline{Z}_\alpha(\omega)$ $Z_{1-\alpha}(\omega)$ $\overline{Z}_{1-\alpha}(\omega)$ to $\zeta(s), \zeta(\overline{s}), \zeta(1-s), \zeta(1-\overline{s}),$ θ being a screening variable as well as a metric variable, the degeneracy observed for the zero of the zeta function will be first related to $Z_\alpha(\omega)$ common property. The diagram, given at the neighbour of zero, points out the residue of tiling. It shows that the degeneracy is clearly obtained for $\alpha = 1/2$ (see also Figure 8). Additional properties related to prime number are related to the θ screening.

The Riemann conjecture tells us that, beyond the equilibrium in the traditional sense as a result of exponential relaxation, and for all fractal dynamics, there is no equilibrium in the strict sense of the word but a conjugated state of two complementary ‘dynamics’ in α and $1-\alpha$. This duality of the system, -thanks to the additional independent variable θ playing the role of temperature-, can make the system converge punctually to a non dissipative state (zero of zeta function) which is therefore stable state. This happens in some particular situations where the chaos becomes again the Brownian like chaos with the dissociation of the properties of the space from the dynamics (velocity) according to Hamiltonian properties i.e., integrable character. From the work of Stengers, we notice the whole cognitive difficulty in the treatment of irreversibility. It is because the Hamiltonian system is integrable that the balance between cause and effect is reached and that subsequently the notion of irreversibility disappears. On the contrary, the new vision introduced by the fractal dynamics is the paradigm of a geometrically irreversible dynamics.

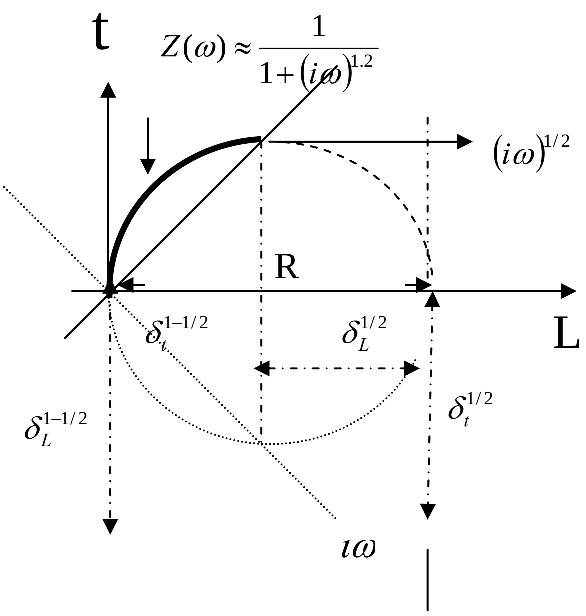

Figure 8: This is the case of α=1/2 giving rise to the degeneracy of the tiling. The geometry partly disappears here. The time constant of the sub quadratic form tends to infinity. The space time can no more be tiled as usual. More precisely, the residual discreetness is only reducible within two complementary discreet subspace-times $\left[\delta_L^{1/2}, \delta_t^{1/2}\right]$ and $\left[\delta_t^{1-1/2}, \delta_L^{1-1/2}\right]$. The complete analysis requires (i) a complex representation of the time $\left(\delta_t^{1/2} + i\tau_0\omega\right)$, (ii) a complex representation of the space $\left(\delta_L^{1/2} + il_0\omega\right)$ (iii) a quasi-commutative exchange between both 'transfer functions' Z (discreet or not), subject to the introduction of the irreversible terms *R* which belongs to a distribution of $\delta_R(\tau_0, l_0)$. *R* takes into account the shift associated with the non commutative permutation of $[L,t]$, that is, the non commutative relation between diffusive constant $D \equiv \left[L^2 t^{-1}\right]$ on the one hand which control the 2 dimensional degenerate chaos $\xi(\omega) \approx \frac{l_0}{l_0 + (i\varpi\tau_0)^{1/2}}$ and the 2 dimensional gravitation constant $\gamma : 1/\gamma \equiv \left[t^2 L^{-1}\right]$ under the control of $\xi(\omega) \approx \frac{\tau_0}{\tau_0 + (i\varpi l_0)^{1/2}}$ on the other hand. In the absence of spatial and time gauge, $\delta_R(\tau_0, l_0)$ must be the distribution of prime number. Due to the introduction of a shift into the exchange $L \leftrightarrow t$ irreversibility is therefore a factor of the discreet space time structure when α=1/2, that is at the degenerate state. There is Time/Temperature sub-relation due to incomplete algebra of space time (Nivanen).

The α-dynamics carries on itself a requirement for taking into account the fractal set of singularities and the correlations between scales (dendritic growth, aggregation, extremum phenomena, etc) hence the irreversibility appears naturally (Le Méhauté 1989). If the dynamic is controlled in the frame of chaotic environment, this environment must impose an ½ fractional dynamic to have any hope to reach a non dissipative Hamiltonian properties. The fact that the zeros of the zeta function are related to the stable quantum Hamiltonian in Hilbert's space, is just a consequence of fractal space time game of hide and seek around time and space of the dimension 1 or 2.

Facing the mechanics: the thermo-dynamics

The breaking of the symmetry, -evidenced by the presence of the singular zeros of the Riemann zeta function-, is introduced here from the fractional dynamics. This dynamics is very different from the one induced by the baker's transformation (no trajectory and no point object in phase space but only the expanding or contracting lines (Prigogine 1980)). However, the fractal chaos and the fractional dynamics related to α-differential forms do not exclude the concept of trajectory which can be found through the generalized hyperbolic geodesics underlining $Z_\alpha(\omega)$. Precisely, if one excludes the particular cases $\alpha \to 1/2$, the dynamics which interpolate the rare non dissipative signatures are in general irreversible. $\alpha = 1/2$ is characterized by an intrinsic hidden quantified irreversibility. Despite the history of the Queen of sciences which makes integrable system, optimal trajectories, variational principles, asymptotic equilibrium state, and the paradigms fixing stable and unforgettable references with plenty of symmetries, the fractional dynamics turns out to indicate the contrary, i.e., the generality of the dissipative system. Although the dynamics with chaotic character are still related to optimal trajectories, these trajectories raise dual singularities on the boundary. They are irreducible in $[Z(\omega), i\omega]$ space as well as in the space $[L, t]$. These singularities are dual also because the collective correlations of the trajectory bear additional 'elements', -we mean emerging organisation and group transformation-, on the trajectory and its boundary, which makes the system couple with the thermostat or the environment. For $\alpha = 1/2$ the dual singularity is its own 'thermostat'. That is why the idea of the relation between irreversibility and approximation keep probably a part of truth.

The respectful criticism of the works of Hamilton from fractional analysis, leads the emergence of the geometrical irreversibility. Carnot belongs to the world of the engineering, a world of action, of objective intelligence, of creativity, and of progress; a world grounded by the following question in mind « what happens if… ? » (Stengers). Like many engineers who constructed the innovation progressively without experimental facilities, Carnot, first followed by his heir Clausius, who coined the term 'entropy'[4], completed with conceptual vigour the work of Watt and then Thomson, by taking the irreversibility as an initial data and not as a by-product of mechanics. By inventing a cycle composed of isothermal and adiabatic processes, Carnot mimicked, in the field of caloric technology, the equilibrium of Lagrangian forces with the hypothesis that the state of matter can split the standard space-time (Stengers). The principle of causality does not correspond only to the laws of nature but to a will which drives the action and faces the reaction. Even if the system is opened it acquires a new kind of universality. Contrary to the ideal regular pendulum which denies the difference between the cause and effect in conserving one in another, a clock, introducing the irregularities of mechanical world and the correlations between the irregularities, is a device based on non harmonic relation between space and time $[(u/v)(\omega/\omega_c) \neq 1]$. Notice that there is no need of the presence of the observer in order that the space-time geometry gives rise to the irregularities. It suffices that the homogeneity of space time, postulated by Noether to disclose the invariance of energy, be broken in some catastrophic circumstances. Independent from the presence or not of the observer, the internal energy of a clock is not conserved. Many years will be necessary for Clausius and Thomson to shed light on this question and to introduce the concept of entropy $dS = TdQ$ as a state function. In the analysis given above, the group of transformation associated to the dual singularity plays the same role as the Carnot's cycle and therefore the dual singularity contains some quantum of entropy and negentropy. Many

[4] 'Entropy' was invented from the Greek words *en* meaning *in* and *tropē* meaning transformation or *turning towards* implying dissipation of mechanical energy into non available energy.

decades will be yet necessary for understanding this assertion as well as many others related to the singularity of the topological opened punctuation.

The geometrical irreversibility is a major characteristic at least for the systems affected by scaling laws. This is exactly what shown by the battery, a 'faitiche' of the thermodynamic counter culture (Le Méhauté 1989). The engineering of the battery, -a controllable source of dissipation associated with the fractal geometry of the electrodes , hence a seed for irreversibility-, offers a right experimental way to understand the importance of the fractional dynamic in the world of energy. The engineering problem of battery optimization for example, makes the geometrical structure of the matter a deep source of adjustable dissipation. Hence it is requiered to consider the fractionality of the dynamics of transport and of exchange. The α-dynamics contains all the appropriate singularities for the emergence of dissipation and of the associated scaling laws. Among these laws, the Peukert law (Le Méhauté 1989) which points out a scaled covariance of space-time (Nottale) and the emergence of new type of invariance with respect to the time (see also WLF laws or the relation temps/temperature). The time, as a sensor, is in practice a parameter of exploration of the Riemannian manifolds affected by singularities. These manifolds cannot be reduced to their local Euclidean projection. In this simple state of equilibrium -or of stationary- their surfaces vibrate according to the well defined normal modes even if these modes are of infinite number (Hilbert space) and controlled by the boundary conditions. This first approximation forgets the cuttings, the collages, and other overlapping of surfaces, or, in a word, the singularities which give rise to dissipative structures. The fractal based manifolds authorize the use of α-fractional equation to disclose the ‘anomalous’ behaviour. It was a chance to have to our disposable devices (batterie, tires, cables etc), but also singular matter (supraconductivity and fluidity or monopoles) to strengthen the new point of view proposed herein and to check it experimentally. In spite of the lack of mathematical studies on the topology associated with fractal geometry, these devices and this singular matter offer an opportunity to disclose the relationship between α-dynamic, geodesic and tiling and to emphasize its role in the dissipation of energy and even in some circumstances with the decrease of entropy (emergence). A lot of work must still be done with the fractional operator opening new scientific territories which have to be explored. It will be done if we just drop the umbrella of the Hamiltonian mechanics which tells us nothing about our freedom of creation and nothing but a beautiful immobile eternity of a world without life and creation (Stengers).

BIBLIOGRAPHIE

Albert D, *Time and chance*, Harvard University Press London 2000

Arnold V.L., *Mathematical methods of classical mechanics,* Springer-Verlag, New York, 1989

Badiali J-P., *Relation time thermodynamics. A path integral approach* . arXiv 0902 [V] [Cond. Mat.Stat.Mech.] 5 fevrier 2009. and *Questions about the time*, to be published

Bergé P., Pomeau Y., Vidal C., *L’ordre dans le chaos* Hermann paris 1984

Bedford T., Keane M., Series C. Edt. *Ergodic theory, Symbolic dynamics and hyperbolic spaces*, Oxford Science Publication

Berline N. & Sabbah C. Edt. Pavage journées mathématiques X-UPS 2001, Ed. Ecole Polytechnique, Palaiseau 2004

Carnot S., *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance*, Bachelier, Paris 1824

Connes A., Les géométries non-commutatives, Paris Dunod (2005)

Deveuhels R., *Formes quadratiques et groupes classiques*, Presse Universitaire de France, Paris 1981

Edwards H.M. *Riemann's zeta function*, Academic press N.Y. 1974

El Kaabouchi A., Nivanen L., Wang Q.A., Badiali J-P., Le Méhauté A., *A mathematical structure for the generalization of conventional algebra*, Cent. Eur. J. phys. **7**(3) 2009, 549-554

Coveney P. and Highfield R., *The arrow of time*, New york Fawcett Columbine 1992

Gouyet J-F., *Physique et structures fractales* Masson Paris 1992

Heudin J-C., *L'évolution au bord du chaos* Hermes 1998

Hirsch G. *Introduction à la topologie algébrique : Topologie générale I* (1964) 152-188 ; *Groupes d'homotopie II* Bulletin de la société mathématique de Belgique XVI (1966) 227-244

Holland J.H., *Emergence. From Chaos to Order*. Helix book Massachusets 1997

Hoskins R. F., *Standard and non standard analysis*, Ellis Horwood Toronto 1990

Ivic A. *The Riemann zeta function, Theory and applications* Dover publication 2003

Jonscher A.K. *Dielectric relaxation in solid*, Chelsea dielectric press 1983

Kosmann-Schwarzbach Y., Meersseman L., *Les théorèmes de Noether*, Palaiseau Ed. Ecoles Polytechnique, 2004

Lapidus M.L. and van Frankenhuysen M. *Fractal geometry and number theory: complex dimensions of fractal strings and zeros of zeta functions*, Birkhauser Boston 2000

Le Méhauté A., De Guibert A., Delaye M. and Fillipi *Introduction au transfert d'énergie en milieu fractal*. C., C.R. Acad. Sciences Paris **294** II (1982) 865-869

Le Méhauté A. and Crepy G., *Introduction to tranfer and motion in fractal media. The geometry of the kinetic.* Solid State Ionics, 9-10 (1983) 17-23

Le Méhauté A., *Transfert Process in Fractal Media*, J. Stat. Phys. **36**, 5/6, (1984) 665-669

Le Méhauté A.,and Crépy G., *Batteries as fractal object*, J of Power sources **26**, 179 (1989)

Le Méhauté A., *Geometry fractales* Hermes Paris 1990 ; *Fractal Geometries Theory and Applications* Penton Press Londre 1991.

Le Méhauté A. and H. Frédéric *Generalized electrodynamic analysis in fractal media*, in Progress in electromagnetic Research symposium Boston (1989-91); avec Cottevielle Denis dans La revue scientifique et technique de défense Paris 1992

Le Méhauté A., Héliodore F., Cottevieille Denis, Latreille Fréderic, *Introduction to wave phenomena and uncertainty in fractal space in* Quantum mechanics, diffusion and chaotic fractals Edt. El naschie M.S., Rossler O.E. Prigogine Pergamon Press Elsevier 1995.

Le Méhauté A., Nigmatullin R., Nivanen L., *Flèches du temps et géométrie fractale* Hermes paris 1997

Le Méhauté A., Nivanen L., Nigmatullin R.. *Space time with bounded curvature and non integer differential operators in* le Méhauté A; Tendiero Machado J.A., Trigeassou J.C. Sabatier J., Eds U Books 2005

Le Méhauté A., El Kaabouchi A. Nivanen L. , *Contribution of non integer differential operator NIDO, to the geometrical understanding of Riemann's conjecture (II)* Chaos solitons and fractals **35** (2008) 659-663.

Le Méhauté A., El Kaabouchi A., Nivanen L. *Riemann Conjoncture and fractional derivative,* the Special Issue on *"Fractional Differential Equations,"* in the International Journal of Differential Equations, and the deadline for submission is August 1st, 2009.

Libermann P., Marle C-M. : *Géométrie Symplectique bases théoriques de la mécanique*. Publication mathématiques de l'université Paris 7, 1979

Madelbrot B., *Géométries fractales*, Flammarion Paris 1975, *Fractal and chaos* Springer New york 2004

Nigmatulin R., le Méhauté A., *Is there a geometrical/physical meaning of the fractional integral with complex exponent?* J. of Non crystalline solid 351 (2005) 2888-2899.

Nivanen L., Wang Q.A. Le Méhauté A., *From complex system to generalized statistics and algebra*, in *Fractal differentiation and its applications* le Méhauté A; Tendiero Machado J.A., Trigeassou J.C. Sabatier J. Eds U Books 2005

NivanenL., Wang Q.A., Le Méhauté A., El Kaabouchi A., Basillais P., Donati J.D., Lacroix A., Paulet J., Perriau S., Sime Chuisse S., Simo Kamdem E. and Théry A., *Hierarchical structure of operations defined in nonextensive algebra,* Reports on Mathematical Physics Volume **63**, Issue 2, (2009), 279-288

Noether Emmy, *Oeuvres*, collectd by Jacobson N. Edt. Berlin Springer Verlag 1983

Murphy Gerard, C* algebra and operator theory Academic Press Inc. Boston 1990

Nottale Laurent, *Fractal space-time and microphysics* World Scientific Singapore 1992

Oldham K.B. and Spanier J., *The fractional calculus*. Acad. Press. New York 1974

Ou C, Huang Z., Chen J., El Kaabouchi A., Nivanen L., Le Méhauté A., *A basic problem in the correlation between statistics and thermodynamics*, Chaos, Solitons and Fractals **41** (2009) 2313-2318

Podulny I., *Fractional differential equations. An introduction to fractional derivatives fractional differential equations,to method of their solutions and some of their applications* Mathematics in Sciences and engineering. 198, Academic Press, San Diego 1999

Prigogine I. *Physique temps et devenir*, Masson Paris 1980

Roveli C., *Qu'est ce le temps ? Qu'est ce que l'espace*, B. Gilson Ed. Paris 2007

Samko S.G., Kilbas A.A., Marichev O.I. *Fractional integrals and derivatives. Theory and applications* Gordon and Breach, Yverdon et *alibi* 1993

Sapoval B., Rosso M., Gouyet J-F., *The fractal nature of a diffusive front and the relation to percolation,* J. Phys. Lett. **46**, L149 (1985)

Sapoval B., Universalité et fractales, Flammarion, Paris (1997)

Slater P.B. *Fractal fits to riemann zeros,* arXiv:math-ph/06066005 v2

Schwartz L., *Théorie des distribution*, Hermann Paris 1966

Spivak M., *A comprehensive introduction to differential geometry*, Publish or Perich Press Londre (1999)

Souriau J-M. *Géométrie symplectique et physique mathématique*, La Gazette des mathématiciens, n° **10**, pp. 90–133, 1978. Colloquium SMF.

Souriau J-M. *La structure symplectique de la mécanique décrite par Lagrange en 1811*, Mathématiques et sciences humaines, nº **94**, pp. 45–54, 1986.

Souriau J-M. *Résonances et non-résonances dans le système solaire*, communication au colloque sur la gravitation à l'Observatoire de Genève, 1989.

Souriau J-M. *Structure of Dynamical Systems: A Symplectic View of Physics*, Springer Verlag, 1997. ISBN 0817636951.

Stauffer D., Standley H.E., Lesne Annick, *Cours de physique*, Springer Paris 1999 translation of *From Newton to Mandelbrot* Springer Berlin 1990.

Stengers I., *Cosmopolitiques* Ed. La découverte, Paris 1(1997) et 2 (2003)

Tangente : *Les fractales : Art, nature et modélisation,* HS 18 Paris 2004

Targ S., *Elément de mécanique rationnelle* Ed. de Moscou (1975)

Toubiana E., Sà Earp R., *Introduction à la géométrie hyperbolique et aux surfaces de Riemann* Diderot Ed. Paris 1997

Tricot C., *Courbes et dimensions fractales*, Springer-Verlag, Paris 1993

Vannimenus J., *Proc. Summer school in theo. phys*. Les Houches 1983

Vieil E., Electrochem. Acta **53** (2008) 3877-3896 (and book to be published 2010)

Vieil E., *Representation of classical formalism without algebraic equation with formal graph*, and *Polar structure and formal graphs* to be published

Wang Q. A. Nivanen L., El Kaabouchi A., Badiali J.P., Le Méhauté A., *Incomplete normalization of probability on multifractals,* Physica A, **386** (2007) 92