Tests for Time Reversal Violation

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Abstract

We illustrate two new theoretical conditions under which a two-body decay of a resonance violates time reversal invariance. As a consequence, we deduce two different kinds of tests of time reversal violation for such decays. The tests proposed may help detecting possible signals of physics beyond the Standard Model. In particular, they are sensitive to contributions of spontaneous CP violation. Moreover one of the two conditions found receives favourable indications from known decays and, for some strong decays, it implies selection rules to be tested experimentally.

PACS numbers: PACS Nos.: 11.30.Er, 12.39.-x, 12.39.ki, 13.30.-a, 14.20.Mr

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1 Introduction

The lively interest shown in the last years by high-energy physicists in CP violations[1-14] is mainly due to recent hints[15-26] at New Physics (NP). Indeed, such violations - and especially those involving $b \rightarrow s$ transitions[27, 28] - constitute a promising door to physics beyond the Standard Model[29-34] (SM), unsatisfactory under several aspects (for recent reviews see [35, 36]), although consistent with a wealth of data[37].

Time Reversal Violation (TRV) is commonly regarded as the counterpart of CP violation, in view of the CPT theorem, valid under very mild assumptions and not contradicted by any experiments, even supported by stringent tests[38]. However direct TRV has been observed only in the CPLEAR experiment[39], by comparing $K^0 \rightarrow \overline{K}^0$ to $\overline{K}^0 \rightarrow K^0$ transition. In fact, it is generally quite difficult to realize experimentally the inverse process of a given decay; this is why people give up showing directly such a kind of violations. Alternatively, TRV may be revealed by the presence, in a hadronic two-body weak decay amplitude, of a ”weak” phase, besides the one produced by strong Final State Interactions (FSI)[40, 41, 42, 43]. However also in this case experimental uncertainties of the ”strong” phases create serious problems in singling out the ”weak” one[40, 44]. Incidentally, in the SM the ”weak” phase is provided by the CKM scheme.

The aim of the present note is to illustrate two new theoretical conditions for realizing TRV and to suggest several tests for detecting them. Such tests are feasible in the framework of experiments like those recently suggested or realized[1-14] for CP violations and are in part suitable for two-body decays of resonances discovered in relatively recent times - like B, B_s and Λ_b -, characterized by higher masses and higher spins of the decay products, and therefore by a greater number of amplitudes and of decay modes. The tests proposed are sensitive to contributions of NP, in particular of spontaneous CP violation. Moreover one of our two conditions - the second one - may be applied not only to weak decays, but also to strong ones, for which it implies selection rules. Sections 2 and 3 are devoted to the two “new” conditions and to suggesting various tests, while in section 4 we draw some conclusions.

2 First condition for TRV

We focus on hadronic two-body decays of the type

$$R_0 \;\rightarrow\; R_1 \;\; R_2, \tag{1}$$

where R_0 is the original resonance and R_1 and R_2 the decay products, with spin J, s_1 and s_2 respectively.

The first new condition is derived by extending the standard treatment of Time Reversal Invariance (TRI) for two-body decays[40, 41] to the case where more than one non-leptonic decay mode is involved[45]. If (1) is a weak decay, the relative, rotationally invariant amplitude reads, at first order in the weak coupling constant,

$$A^J_{\lambda_1\lambda_2} = \langle f^{out}|H_w|JM\rangle, \tag{2}$$

where H_w is the weak hamiltonian, $|f^{out}\rangle$ a shorthand notation for the final two-body angular momentum eigenstate $|JM\lambda_1\lambda_2\rangle$, M the component of the spin of R_0 along the z-axis of a given frame and λ_1 and λ_2 the helicities of, respectively, R_1 and R_2 in the rest frame of R_0.

Assume H_w to be TRI, *i. e.*,

$$TH_wT^\dagger = H_w, \tag{3}$$

where T is the Time Reversal (TR) operator. Then, taking into account the antilinear character of T, we get[46]

$$A^J_{\lambda_1\lambda_2} = \langle f^{in}|H_w|JM\rangle^*. \tag{4}$$

Inserting a complete set of "out" states yields

$$A^J_{\lambda_1\lambda_2} = \sum_n \langle f^{in}|n^{out}\rangle^* \langle n^{out}|H_w|JM\rangle^*. \tag{5}$$

The only terms which survive in this sum correspond to the decay modes of R_0; furthermore the non-leptonic decay modes give the main contribution, since they involve a much greater coupling constant than the semi-leptonic decay modes. Relaxing the limitation of the state $|f^{in}\rangle$ to a two-body one, and expressing the "out" states in terms of the S-matrix - which is unitary and, under the assumption of TRI, also symmetric with respect to angular momentum eigenstates[46] -, eq. (5) can be rewritten as

$$A_m \cong \sum_n S_{mn} A_n^*. \tag{6}$$

Here, omitting spin and helicity indices,

$$A_m = \langle m^{out} | H_w | R_0 \rangle \tag{7}$$

and

$$S_{mn} = \langle m^{in} | S | n^{in} \rangle, \tag{8}$$

m, n running along all helicity states of the various hadronic decay modes of R_0. It is worth noting that eq. (6) coincides with eq. (12) of ref. [45].

We deduce from eq. (6) the most general expression of the S-matrix as a function of the amplitudes A_m:

$$S_{mn} = D^{-1} A_m A_n + K_{mn}, \qquad \text{where} \qquad D = \sum_n |A_n|^2 \tag{9}$$

and K_{mn} is a matrix such that $K_{mn} A_n^* = 0$. This fact can be verified by directly substituting formula (9) into eq. (6) and by taking into account the linear character of the latter equation. The matrix elements K_{mn} are considerably constrained, but not uniquely determined, by imposing the unitarity and symmetry condition on the S-matrix. The S-matrix involves almost exclusively[‡] strong interactions, while the amplitudes A_m, A_n, which appear at the right hand side of the first eq. (9), describe weak decays. As shown in detail in appendix, this may be accomplished only if the amplitude A_n factorizes into a strong ($A_n^{(s)}$) and a *real* weak ($a^{(w)}$) factor, *i. e.*,

$$A_n = A_n^{(s)} a^{(w)}. \tag{10}$$

As expected, eq. (10) recovers for TRI the standard condition of a zero weak phase[40, 41]. But it implies also that the factor $a^{(w)}$ is independent of the decay mode and of the helicities of the single decay products; at most it will depend on the component

[‡]Here we neglect electroweak contributions to scattering, which, however, will be taken into account in the next section

of the spin of R_0 along the momentum of one of the decay products: for example, in the case of a two-body decay, it may depend on $\lambda = \lambda_1 - \lambda_2$, but not on λ_1 or λ_2 separately. This in turn implies, for a two-body weak decay to spinning particles,

$$A_{\lambda_1\lambda_2} = A_{-\lambda_1-\lambda_2}, \tag{11}$$

owing to parity invariance of strong interactions. But this equality is forbidden by parity violation of weak interactions. Therefore, as regards non-leptonic decays where more spinning particles are involved, parity violation automatically implies TRV. The same conclusion we shall draw in the next section, by different considerations, and we shall discuss this point in the conclusions.

This result suggests some tests for weak two-body decays, by defining two asymmetry parameters for TRV. Let us consider the following decay modes of B^+ and B^0, already studied, both theoretically[10, 11, 12, 13, 14, 15, 19, 30, 31, 32, 42, 47, 48, 49, 50, 51] and experimentally[1-9]:

$$B^+ \rightarrow (J/\psi K^{*+}), \quad (D^{*0}\rho^+), \quad (K^{*0}\rho^+), \quad (K^{*+}\phi); \tag{12}$$

$$B^0 \rightarrow (J/\psi K^{*0}), \quad (K^{*0}\phi), \quad (\rho\rho); \tag{13}$$

$$B_s \rightarrow (J/\psi\phi), \quad (J/\psi\overline{K}^{*0}), \quad (\rho^0\phi). \tag{14}$$

For such decays we define the asymmetry parameters

$$\mathcal{A}_1 = \frac{|A_{11}| - |A_{-1-1}|}{|A_{11}| + |A_{-1-1}|}, \qquad \mathcal{A}_2 = \frac{arg(A_{11}A^*_{00}) - arg(A_{-1-1}A^*_{00}}{arg(A_{11}A^*_{00}) + arg(A_{-1-1}A^*_{00}}. \tag{15}$$

The moduli and the relative phases of the amplitudes $A_{\lambda\lambda}$ can be determined by means of the angular distributions, polarizations and polarization correlations of the two-body decays (12) to (14)[10, 32, 42, 48, 49, 52]. We note that a typical condition of TRV is realized in the framework of the SM, when at least two amplitudes with different weak factors contribute to a given decay, as a counterpart of direct CP violation. Such asymmetries may be helpful in detecting signals of NP, for example the contribution of two Higgs doublets[26].

3 Second condition for TRV

3.1 Spin-orbit and spin-spin FSI

The second condition for TRI is derived by considering an elastic process of the type

$$R_1 \quad R_2 \qquad \longrightarrow \qquad R_1 \quad R_2 \tag{16}$$

for energies above the inelastic threshold, so that it proceeds through different channels, among which

$$R_1 \quad R_2 \quad \rightarrow \quad R_0 \quad \rightarrow \quad R_1 \quad R_2. \tag{17}$$

This mode is especially important for overall center-of-mass energies near the mass of the resonance R_0. The rotationally invariant amplitude for such a process in the center-of-mass system is

$$F_{\lambda_1'\lambda_2'}^{\lambda_1\lambda_2} = \langle JM\lambda_1'\lambda_2'|m|JM\rangle\mathcal{D}(w)\langle JM|m^T|JM\lambda_1\lambda_2\rangle. \tag{18}$$

Here λ_1, λ_2 (λ_1', λ_2') are the initial (final) helicities of R_1 and R_2 respectively; $\mathcal{D}(w)$ is the Breit-Wigner function of the resonance R_0, m is the decay operator and m^T $= TmT^\dagger$. If the decay is weak, we approximate m by H_w, as above. Incidentally, reactions of this type, exploiting the contribution of weak interactions to scattering, have been proposed[53, 54, 55] and even realized[56]. The S-matrix element for scattering (16) may be written as

$$S_{\lambda_1'\lambda_2'}^{\lambda_1\lambda_2} = F_{\lambda_1'\lambda_2'}^{\lambda_1\lambda_2} + {\sum_c}' S_{\lambda_1'\lambda_2'}^{(c)\lambda_1\lambda_2}. \tag{19}$$

Here we have set

$$S_{\lambda_1'\lambda_2'}^{\lambda_1\lambda_2} = \langle JM\lambda_1'\lambda_2'|S|JM\lambda_1\lambda_2\rangle \tag{20}$$

and $\sum_c'$ denotes the sum over all modes except for (17). TRI implies[46]

$$S_{\lambda_1'\lambda_2'}^{\lambda_1\lambda_2} = S_{\lambda_1\lambda_2}^{\lambda_1'\lambda_2'} \qquad \text{and} \qquad m^T = m^\dagger. \tag{21}$$

Eqs. (21) yield, together with eqs. (18) and (19),

$$(B_{\lambda_1'\lambda_2'}B^*_{\lambda_1\lambda_2} - B^*_{\lambda_1'\lambda_2'}B_{\lambda_1\lambda_2})\mathcal{D}(w) + {\sum_c}'(S_{\lambda_1'\lambda_2'}^{(c)\lambda_1\lambda_2} - S_{\lambda_1\lambda_2}^{(c)\lambda_1'\lambda_2'}) = 0, \tag{22}$$

having set

$$B_{\lambda_1\lambda_2} = \langle JM\lambda_1\lambda_2|m|JM\rangle. \tag{23}$$

Incidentally, the B-amplitudes turn out to coincide with the A-amplitudes, eq. (2), in the case of weak interactions ($m \cong H_w$).

We assume, as it often occurs, that, near the resonant energy $\overline{w}$, all modes included in the sum $\sum_c'$ have a non-resonant behavior. Therefore, for $w \approx \overline{w}$, while the first term at the left-hand side of eq. (22) is rapidly varying, the other terms of the sum have a much slower variation. Since this equation has to be satisfied for *any* value of w, the first term and the sum must vanish separately. In particular TRI implies

$$\Im(B_{\lambda_1'\lambda_2'}B^*_{\lambda_1\lambda_2}) = 0. \tag{24}$$

An analogous conclusion can be drawn in the $l-s$ representation, *i. e.*,

$$\Im(B_{\ell s}B^*_{\ell' s'}) = 0, \tag{25}$$

where ℓ is the orbital angular momentum and $|s_1 - s_2| \leq s \leq s_1 + s_2$. In fact the $l-s$ representation is related to the helicity representation through the Clebsch-Gordan coefficients, which, as is well-known, are the elements of a unitary, real matrix. Therefore a nonzero value of

$$\Im(B_{\lambda_1'\lambda_2'}B^*_{\lambda_1\lambda_2}) \qquad \text{or} \quad \text{of} \qquad \Im(B_{\ell s}B^*_{\ell' s'}) \tag{26}$$

- that is, a nontrivial relative phase of two amplitudes[26] - implies necessarily TRV. Quantities of the type (26) can be determined experimentally by measuring polarization correlations of the decay products[42, 48, 19], as we have shown in the case of the Λ_b decay to a Λ and a vector meson[52].

In this connection it is worth observing that, in a decay of a spinning resonance to two spinning hadrons, spin-orbit and spin-spin FSI produce different phase shifts for different ℓ or/and s. This has two important consequences:

a) On the one hand, we expect strong decays to be characterized by only one $\ell - s$ amplitude, owing to TRI.

b) *Vice-versa* a hadronic weak two-body decay which presents more than one $\ell - s$ amplitude violates TR.

We examine in detail the two statements, starting from the former one.

3.2 Tests for strong decays

Our selection rule - which implies only one $\ell - s$ amplitude - is automatically respected in strong decays of the type

$$1/2 \quad (3/2) \quad \to \quad 1/2 \quad 0 \qquad \text{or} \qquad 1/2 \quad \to \quad 3/2 \quad 0, \tag{27}$$

owing to parity conservation. Examples of such decays are

$$N^*(1535) \ \ (J^P = 1/2^-) \quad \to \quad N\pi, \tag{28}$$
$$N^*(1440) \ \ (J^P = 1/2^+) \quad \to \quad N\pi, \quad \Delta\pi, \tag{29}$$
$$N^*(1520) \ \ (J^P = 3/2^-) \quad \to \quad N\pi. \tag{30}$$

On the contrary, other decays present more than one amplitude. For example, two amplitudes are allowed in

$$N^*(1520) \ \ (J^P = 3/2^-) \quad \to \quad \Delta\pi \tag{31}$$
$$\Delta(1620) \ \ (J^P = 1/2^-) \quad \to \quad N\rho, \tag{32}$$
$$f_0(1370) \ \ (J^P = 0^+) \quad \to \quad \rho\rho, \tag{33}$$

while three and six different amplitudes characterize the respective decays

$$N^*(1520) \ \ (J^P = 3/2^-) \quad \to \quad N\rho, \tag{34}$$
$$\pi_2(1670) \ \ (J^P = 2^-) \quad \to \quad \rho\omega. \tag{35}$$

Our selection rule can be tested in such decays, for example by analyzing the angular distributions of the decay products in terms of the rotation functions[57, 58, 52].

3.3 Tests for weak decays

Turning to weak decays, we may reasonably expect that spin-orbit or/and spin-spin FSI produce a phase difference between the amplitudes, so that TR is violated. But also in the very unlikely case that the "weak" phase depended on $\lambda = \lambda_1 - \lambda_2$ in such a way to compensate the differences between the "strong" phases, we could draw the same conclusion, since at least some "weak" phases would be different from zero. This second kind of TRV is absolutely model independent.

As an example, consider the decay $\Lambda \to \pi^- p$. Parity violation allows the presence of the s-wave as well as of the p-wave, whose strong phases are different, as results from the phase shift analysis in $\pi^- p$ elastic scattering; therefore, according to our considerations, we conclude that this decay violates TR. The relative phase of A_p to A_s can be inferred by determining the angular correlation and the various components of the polarization of the final proton[59]. This could help in setting bounds on the relative (p to s) "weak" phase, by comparison with the difference between the two "strong" phases. More generally, parity violation automatically induces TRV in those weak decays for which angular momentum conservation allows more than one partial wave, like, for instance, part of the decays studied in recent experiments[1-9].

A remark is in order. We have established that FSI may produce TRV through spin-spin and spin-orbit interactions. But this gives rise to mixed products of the type $\mathbf{p}_1 \times \mathbf{p}_2 \cdot \mathbf{s}$ or $\mathbf{s_1} \times \mathbf{s_2} \cdot \mathbf{p}$, where $\mathbf{p}$ and $\mathbf{s}$ denote respectively momenta and spins. Incidentally, we observe that the latter mixed product is necessarily associated to weak interactions, since it does not conserve parity. Such mixed products are generally associated to the so-called T-odd observables[60, 61, 62, 63, 21, 26, 33], not necessarily implying TRV. However, as we have shown, this is the case with interference between two weak decay amplitudes with different spin or orbital angular momentum. To this end, we suggest to compare, wherever possible, a given decay mode - involving more than one amplitude, as. we have considered in this section (see, for example, refs[1-9]) - with its CP-conjugate decay. This allows, in principle, to test the prediction

$$\Im(B_{\lambda_1'\lambda_2'} B^*_{\lambda_1\lambda_2} + \overline{B}_{-\lambda_1'-\lambda_2'} \overline{B}^*_{-\lambda_1-\lambda_2}) = 0, \tag{36}$$

which is a consequence of the CPT theorem; barred amplitudes refer to CP-conjugate decays.

3.4 Isospin dependent FSI

TRV may be produced also by isospin dependent FSI. To see that, consider, for example, the charge exchange scattering

$$\pi^+\pi^- \to \pi^0\pi^0. \tag{37}$$

This may occur through the channel

$$\pi^+\pi^- \to K_L^0 \to \pi^0\pi^0, \tag{38}$$

whose amplitude reads as

$$\mathcal{A} = B_{+-}^* \mathcal{D}(w) B_{00}. \tag{39}$$

Here B_{+-} and B_{00} are, respectively, the decay amplitudes of $K_L^0 \to \pi^+\pi^-$ and $K_L^0 \to \pi^0\pi^0$. Arguments similar to those shown before lead us to conclude that a nonzero

$$\mathcal{I}_k = \Im(B_{+-}^* B_{00}) \tag{40}$$

implies TRV[§] Since

$$B_{+-} = \sqrt{\frac{2}{3}}\mathcal{B}_0 + \sqrt{\frac{1}{3}}\mathcal{B}_2, \tag{41}$$

$$B_{00} = \sqrt{\frac{1}{3}}\mathcal{B}_0 - \sqrt{\frac{2}{3}}\mathcal{B}_2, \tag{42}$$

where $\mathcal{B}_0$ and $\mathcal{B}_2$ are respectively the $\Delta I = 1/2$ and $\Delta I = 3/2$ decay amplitudes, a nonzero $\mathcal{I}_k$ occurs if and only if

$$\mathcal{I}'_k = \Im(\mathcal{B}_0^* \mathcal{B}_2) \neq 0. \tag{43}$$

Owing to the CPT theorem, the quantity $\mathcal{I}'_k$ is expected to be proportional to the parameter ϵ' characterizing the direct CP violation in the decay $K_L^0 \to \pi\pi$. This can be confirmed by comparing eq. (43) with the usual expression of ϵ', which has been deduced under the assumption of the CPT symmetry, *i. e.*[64, 65, 66],

$$\epsilon' \cong \frac{i}{\sqrt{2}}\Im\left(\frac{\mathcal{B}_2}{\mathcal{B}_0}\right) e^{i(\delta_2-\delta_0)}. \tag{44}$$

[§] The $f_0(600)$ (or σ) meson has a mass very near the one of the K_L^0, but a much broader width, therefore our conclusion applies to this case.

Here δ_I are the I-isospin strong phase-shifts (I = 0,2). By comparing eq. (44) with eq. (43), we conclude that $\mathcal{I}'_k$ constitutes essentially the counterpart of the direct CP violation parameter, not accounted for by the SM, whose prediction as to ϵ' is far below the experimental value[37].

Also other recently analyzed CP violating decays, like $B \to K\pi$ and $B \to K\rho$, could be regarded as TRV effects, owing to isospin dependent FSI.

4 Conclusions

We have illustrated two new conditions for detecting TRV, which may be detected experimentally by analyzing non-leptonic decays where more than one partial wave is involved. The tests suggested may be helpful alternative ways for uncovering NP effects. Aside from that, we find that TRV in such decays follows from parity violation. Then TRV could occur even with a CP-conserving lagrangian. Owing to CPT symmetry, this is possible only under spontaneous CP violation[67, 68, 69] (for more recent cotributions see also, for example, [70, 71] and refs. therein. Then we conclude that the FSI contribute to enhance TRV, and therefore CP violations.

The former condition consists of lack of factorization between strong and weak factor, or mode dependence for the weak factor. It may help singling out contributions of NP, for example of two-doublet Higgs, in weak decays.

Our second condition implies selection rules for strong decays. In some cases, such rules are automatically fulfilled owing to parity conservation. Other, more complicate cases demand analyses of decay angular distributions and of polarizations of decay products. This condition is not in disagreement with the direct CP violation in the K^0_L decay to two pions.

Last but not least, it is worth stressing that our conditions are sufficient but not necessary for TRV; in principle, more conditions could be found.

Appendix

Here we develop in detail the arguments which lead to eq. (10) in the text. Denote by A_n the amplitudes of the non-leptonic decay modes of a given resonance,

n running from 1 to N and characterizing the angular momentum eigenstates of the decay particles of the various modes. We show that Time Reversal Invariance (TRI) implies factorization of each amplitude into a “strong” and a “weak “ factor, *i. e.*,

$$A_n = A_n^{(s)} a^{(w)}, \tag{A.1}$$

where $a^{(w)}$ is real and independent of the mode and of the helicities of the single decay particles. In the text (see eqs. (9)) we have shown that the “strong” S-matrix is related to the amplitudes A_n:

$$S_{mn} = D^{-1} A_m A_n + K_{mn}, \qquad D = \sum_m |A_m|^2. \tag{A.2}$$

Here K is a symmetric matrix such that

$$K_{mn} A_n^* = 0 \tag{A.3}$$

and

$$K_{ml} K_{nl}^* + D^{-1} A_m A_n^* = \delta_{mn}, \tag{A.4}$$

the last condition coming from unitarity of the S-matrix. Incidentally, note that eqs. (A.3) and (A.4) do not determine completely K. Since the S-matrix depends only on strong interactions, any “weak” factor must disappear from the right-hand side of the first eq. (A.2). Eq. (A.1) fulfils such a requirement, as is immediate to see from eqs. (A.2) to (refc2). We show that this is also the only possible condition for realizing that.

Set

$$A_n = A_n^{(s)} a^{(w)} + \epsilon_n, \tag{A.5}$$

where the ϵ_n are not all simultaneously vanishing and cannot be absorbed into the former term: for example, it could be $\epsilon_n = A_n^{(s)\prime} a^{(w)\prime}$, with $a^{(w)\prime} \neq a^{(w)}$. Then

$$D^{-1} A_m A_n = D_0^{-1} A_m^{(s)} A_n^{(s)} + R_{mn}^{(1)}(\vec{\epsilon}), \tag{A.6}$$

$$D^{-1} A_m A_n^* = D_0^{-1} A_m^{(s)} A_n^{(s)*} + R_{mn}^{(2)}(\vec{\epsilon}). \tag{A.7}$$

Here the $R_{mn}^{(i)}$ (i = 1,2) are well-determined functions of $\vec{\epsilon} = (\epsilon_1, \epsilon_2, ...\epsilon_N)$ and

$$D_0 = \sum_m |A_m^{(s)}|^2. \tag{A.8}$$

Since the S-matrix is independent of weak interactions, we must set, according to eqs. (A.2) and (A.6),

$$K_{mn} = K^{(0)}_{mn} - R^{(1)}_{mn}, \tag{A.9}$$

where $K^{(0)}$ is an unknown $N \times N$ complex matrix, independent of $\vec{\epsilon}$. Therefore the S-matrix reads

$$S_{mn} = D_0^{-1} A^{(s)}_m A^{(s)}_n + K^{(0)}_{mn}, \tag{A.10}$$

for which unitarity implies

$$K^{(0)}_{ml} K^{(0)*}_{nl} + D^{-1} A^{(s)}_m A^{(s)*}_n = \delta_{mn}. \tag{A.11}$$

Moreover, since $K^{(0)}$ is independent of weak interactions, eq. (A.3) splits into two equations:

$$K^{(0)}_{mn} A^*_n = 0, \tag{A.12}$$

$$K^{(0)}_{mn} \epsilon^*_n - R^{(1)}_{mn} (A^{(s)*}_n a^{(w)} + \epsilon^*_n) = 0. \tag{A.13}$$

In summary, the $1/2N(N+1)$ (complex) elements $K^{(0)}_{mn}$ are constrained by eqs. (A.4) and (A.11) to (A.13), which amounts to $6N^2 + 5N$ real equations. Therefore it is generally impossible to solve this system with respect to K_{mn}.

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