

# The Giant Monopole Resonance in Pb isotopes

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The extraction of the nuclear incompressibility from the isoscalar giant monopole resonance (GMR) measurements is analysed. Both pairing and mutually enhanced magicity (MEM) effects play a role in the shift of the GMR energy between the doubly closed shell <sup>208</sup>Pb nucleus and other Pb isotopes. Pairing effects are microscopically predicted whereas the MEM effect is phenomenologically evaluated. Accurate measurements of the GMR in open-shell Pb isotopes are called for.

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## I. INTRODUCTION

It has been recently shown, using the Tin isotopic chain, that pairing has an effect on the nuclear incompressibility [1, 2]. In Ref. [1], including pairing effects in the description of the isoscalar giant monopole resonance (GMR) allows to explain part of the so-called Sn softness [3, 4]: pairing decreases the predicted centroid of the GMR, located at  $\sim 16$  MeV, by few hundreds of keV. In Ref. [2] an explicit decreasing correlation is found between the nuclei incompressibility, obtained from the energy of the GMR, and the magnitude of the pairing gap.

With the recent advent of accurate microscopic models in the pairing channel, such as fully self-consistent Quasi-particle Random Phase Approximation (QRPA) [1, 5], it is now possible to predict the GMR position with accuracy and to study small but non-negligible effects such as pairing. Using a microscopic approach is crucial: the GMR is mainly built from particle-hole configurations located far from the Fermi level, where pairing do not play a major role. However giant resonances are known to be very collective [6] and pairing can still have a sizable effect on the GMR properties: around 10 % on the centroid [2], which is the level of accuracy of present analysis on the extraction of  $K_\infty$  [7, 8].

Experimentally, the measurement of the GMR on an isotopic chain facilitates the study of superfluidity on the GMR properties [3], and the possibility to measure the GMR in unstable nuclei emphasizes this feature [9]. It is therefore necessary to go towards the measurement of the GMR on several nuclei, such as an isotopic chain. The overused method of precise GMR measurements in a single nucleus, such as <sup>208</sup>Pb, may not be the relevant approach. Other nuclei have been used such as <sup>90</sup>Zr and <sup>144</sup>Sm. Indeed when considering the available GMR data from which the  $K_\infty$  value has been extracted, <sup>208</sup>Pb is stiffer than the Sn, Zr and Sm nuclei:  $K_\infty$  is about 20 MeV larger, both in non-relativistic and in relativistic approaches [7, 8]. The question may not be "why Tin are so soft?" [4] but rather "why <sup>208</sup>Pb is so stiff?". Also recent results in Cadmium isotopes confirm that open-shell nuclei provide a value of  $K_\infty$  which is lower than

the one extracted from <sup>208</sup>Pb [10].

It should be underlined that it is not possible to describe the GMR both in Pb and in other open-shell nuclei with the same functional [2]. This is valid both for non relativistic and relativistic calculations. In Ref. [2], it has been shown how pairing effects play a role in the Sn isotopic chain: the energy of the GMR is increased for doubly magic <sup>132</sup>Sn, due to the vanishing of pairing in this nuclei. The aim of the present work is to look for a possible similar effect in the Pb isotopic chain. Indeed <sup>204,206</sup>Pb nuclei are stable, but almost all the experimental efforts in the past decades were devoted to the measurement of the GMR in <sup>208</sup>Pb. It would be interesting to perform accurate GMR measurement on open-shell <sup>204,206</sup>Pb nuclei by inelastic alpha scattering in direct kinematics.

Even with the inclusion of pairing effects, the SLy4 functional [11], which accurately describes the GMR in <sup>208</sup>Pb, is still overestimating the GMR in Sn isotopes (see Fig. 1 of Ref. [2]). Hence an additional shell effect is at work, to explain the discrepancy of the extracted  $K_\infty$  values between <sup>208</sup>Pb and Sn isotopes: the SLy4 ( $K_\infty=230$  MeV) [11] functional allows to describe the <sup>208</sup>Pb GMR whereas the SkM\* ( $K_\infty=215$  MeV) [12] functional is in agreement with the Sn data.

The puzzle of the stiffness of <sup>208</sup>Pb may come from its doubly magic behaviour: a possible explanation is that the experimental  $E_{GMR}$  data is especially increased in the case of doubly magic nuclei, as observed in <sup>208</sup>Pb compared to the GMR data available in other nuclei (such as in the Tin isotopic chain). This difficulty to describe with a single functional both doubly magic and other nuclei has already been observed on the masses, namely the so-called "mutually enhancement magicity" (MEM), described in Ref. [13, 14]: functionals designed to describe masses of open-shell nuclei cannot predict the masses of doubly magic nuclei such as <sup>132</sup>Sn and <sup>208</sup>Pb, which are systematically more bound than predicted. In order to evaluate the MEM effect, it may be necessary to take into account quadrupole correlation effects due to the flatness of the mean-field potential for open-shell nuclei [15]. The incompressibility being related to the second derivative of the energy with respect to the density, it would be useful to find a way to predict the GMR

beyond QRPA by taking into account quadrupole correlations. However such a microscopic approach is far beyond the scope of the present work.

Therefore the MEM effect shall be phenomenologically evaluated by using a functional which describes well the GMR in  $^{208}\text{Pb}$  (SLy4), and in the case of open-shell Pb isotopes, a functional (SkM\*) which describes well the GMR in open shell nuclei, such as Sn isotopes. The aim is to predict a value of the GMR centroid in the Pb isotopic chain which could be useful to compare with experimental data.

## II. CONSTRAINED HARTREE-FOCK-BOGOLIUBOV CALCULATIONS

In order to consider pairing effects on the GMR, it is necessary to use a fully microscopic method including an accurate pairing approach. We use the constrained Hartree-Fock method, extended to the Bogoliubov pairing treatment (CHFb) [2, 16]. It should be noted that the extension of the CHF method to the CHFb case has been recently demonstrated in Ref. [16]. The CHF(B) method has the advantage to very precisely predict the centroid of the GMR using the  $m_{-1}$  sumrule [17, 18]. The whole residual interaction (including spin-orbit and Coulomb terms) is taken into account and this method is by construction the best to predict the GMR centroid [18]. Introducing the monopole operator  $\hat{Q}$  as a constraint :

$$\hat{H}_{cons} = \hat{H} + \lambda \hat{Q} \quad (1)$$

with

$$\hat{Q} = \sum_{i=1}^A r_i^2, \quad (2)$$

the  $m_{-1}$  value is obtained from the derivative of the mean value of this operator:

$$m_{-1} = -\frac{1}{2} \left[ \frac{\partial}{\partial \lambda} \langle \hat{Q} \rangle \right]_{\lambda=0} \quad (3)$$

The  $m_1$  sumrule is extracted from the double commutator using the Thouless theorem [19]:

$$m_1 = \frac{2\hbar^2}{A} \langle r^2 \rangle \quad (4)$$

Finally the GMR centroid is given by  $E_{GMR} = \sqrt{m_1/m_{-1}}$ . All details on the CHF(B) method can be found in [7, 16, 17].

The present work uses the HFB approach in coordinate space [20] with Skyrme functionals and a zero-range surface pairing interaction:

$$V_{pair} = V_0 \left[ 1 - \left( \frac{\rho(r)}{\rho_0} \right) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (5)$$

This interaction is known to describe a large variety of pairing effects in nuclei [21]. The magnitude of the pairing interaction  $V_0$  is taken as  $-735 \text{ MeV}\cdot\text{fm}^{-1}$  for SLy4 and  $-700 \text{ MeV}\cdot\text{fm}^{-1}$  for SkM\*: it is adjusted so to describe the trend of the neutron pairing gap Pb isotopes. The single quasiparticle spectrum is considered until 60 MeV. Previous CHFb calculations in Sn isotopes are described in Ref. [2].

## III. RESULTS

Fig. 1 displays the GMR energy (times  $A^{1/3}$  to correct for the slow lowering of the GMR with the nuclear mass [6]) for  $^{204-212}\text{Pb}$  nuclei obtained from microscopic CHFb predictions using two functionals: SLy4 [11] ( $K_\infty=230 \text{ MeV}$ , which describes well the  $^{208}\text{Pb}$  GMR data), and SkM\* [12] ( $K_\infty=215 \text{ MeV}$ , which describes well the open-shell GMR data such as Tin isotopes). As expected the GMR energy is predicted higher in the SLy4 case than in the SkM\* case. For both interactions, the striking feature of Fig. 1 is the increase of the GMR centroid located at the doubly magic  $^{208}\text{Pb}$  nucleus. This indicates that pairing effects should be considered to describe the behaviour of nuclear incompressibility, and that vanishing of pairing make the nuclei stiffer to compress, confirming our previous statement on the stiffness of  $^{132}\text{Sn}$  compared to open-shell Sn isotopes [2]. Pairing effects (CHFb calculations) decrease the centroid of the GMR as observed in open-shell Pb isotopes, compared to  $^{208}\text{Pb}$ . This confirms again the results of [1, 2] in the Tin data, and show that the effect of pairing on the GMR may be universal.

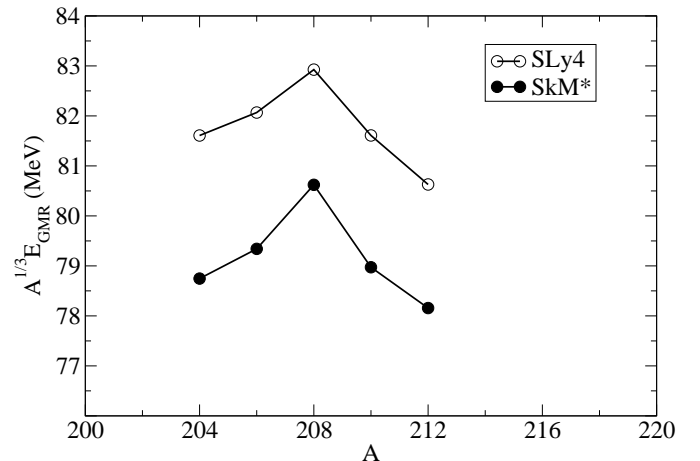


FIG. 1: Excitations energies of the GMR (times  $A^{1/3}$ ) in  $^{204-212}\text{Pb}$  isotopes calculated with constrained HFB method and the SLy4 and SkM\* interactions

We now study the hypothesis that both pairing and MEM effects are contributing to the GMR position when comparing open-shell nuclei with doubly magic ones. Pairing effects are treated microscopically, using the CHFb approach, as described above. However there is no present model to take microscopically into account the MEM effect on the GMR centroid. It should be noted that the MEM effect is not well understood yet in the case of nuclear masses. Nevertheless its effect on the GMR can be evaluated phenomenologically by calculating the predicted position of the GMR centroid with SLy4 in the  $^{208}\text{Pb}$  case, and with SkM\* in the open-shell  $^{204,206,210,212}\text{Pb}$  nuclei: SkM\* allows for a good description of the GMR in open-shell nuclei such as the Tin isotopes, where the MEM effect is at work. As stated above, the GMR position predicted with SLy4 is in agreement with the measurements on  $^{208}\text{Pb}$ , but for open-shell nuclei, a functional with lower incompressibility should be used, as showed by the previous analysis on Sn [2] and Cd isotopes [10]. We therefore use SkM\* for open-shell Pb isotopes. The aim is to provide values of the GMR in the Pb isotopes to be compared with measurements.

Fig. 2 displays the predicted energy of the GMR for  $^{204-212}\text{Pb}$ . The solid lines corresponds to CHFb calculations using the SLy4 functional: it displays the pairing effect on the GMR. The dashed line corresponds to the CHFb calculations using the SLy4 functional for  $^{208}\text{Pb}$  and the SkM\* functional for open-shell Pb isotopes: it takes into account both pairing and MEM effects. One observes that the main contribution of the increase of the GMR centroid for  $^{208}\text{Pb}$  comes from the MEM effect. However pairing effects still induce a decrease of the GMR centroid for open-shell Pb isotopes. We expect measurements on  $^{204}\text{Pb}$  and  $^{206}\text{Pb}$  to be compared with the values displayed on Fig. 2 in order to test both the pairing and MEM effects on the GMR centroid.

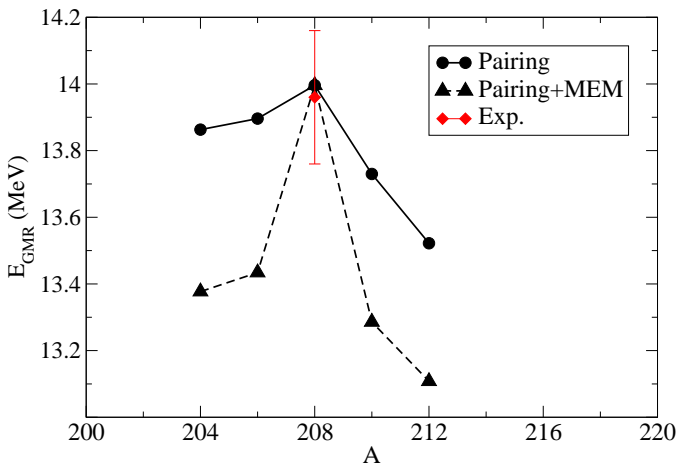


FIG. 2: Excitations energies of the GMR in  $^{204-212}\text{Pb}$  isotopes calculated with constrained HFB method, taking into account the MEM effect (see text). The experimental data is taken from Ref. [22]

It should be noted that the GMR has been measured

in  $^{206}\text{Pb}$  several decades ago [23], providing a centroid value of  $14.0 \pm 0.3$  MeV. Therefore no deviation is found with respect to  $^{208}\text{Pb}$ . However this measurement has been performed above 12 deg.: at such large angle, the GMR cross section is very weak, compared to the giant quadrupole resonance (GQR) cross section and it is delicate to extract a value of the GMR centroid since both the GQR and the high energy background are important. Hence the measurement was not optimal, especially when looking for typical 500 keV effects. Therefore it would be of particular interest to measure the GMR in  $^{206}\text{Pb}$  at 0 degree, allowing for a larger GMR cross section. Furthermore, there is no GMR data for  $^{204}\text{Pb}$ . The accuracy of future GMR measurement in  $^{204,206}\text{Pb}$  will be crucial. Moreover, it should be mentioned that a somewhat lower value for the GMR centroid of  $^{208}\text{Pb}$  has been found in the RCNP experiment ( $13.5 \pm 0.2$  [24]), compared to the Texas A&M one [22]: there is a current debate about the reason such variations. An accurate experiment on Pb isotopes, including  $^{208}\text{Pb}$  should also help to solve this issue.

Another concern is related to the MEM effect. If this effect is due to quadrupole correlations coming from the flatness of the potential, as stated in Ref. [15], it is expected smaller in the Pb case than in the Tin one:  $^{204,206}\text{Pb}$  are in the vicinity of a doubly magic nuclei and their mean-field potentials are still sharp, allowing for a small MEM effect. In the case of  $^{112-124}\text{Sn}$  nuclei, the potential is much flatter [25]: the  $^{132}\text{Sn}$  doubly magic nucleus is several neutrons away on the nuclear chart. Therefore the MEM effect is expected larger in the Sn case than in the Pb one and it may be possible that only pairing effects play a role for the Pb isotopes, which are about 100 keV to 200 keV (solid line in Fig. 2): it is crucial that future measurement are performed within this accuracy.

In the case of the interpretation of the MEM effect from Ref. [15] is correct, taking it into account microscopically would necessitate to predict the GMR with quadrupole correlations due to the flatness of the mean-field potential in the case of open-shell nuclei. We expect it to lower the predicted GMR value, as stated above. Therefore, the value  $K_\infty=230$  MeV extracted from the analysis of the  $^{208}\text{Pb}$  data should be correct, whereas the apparent softness of open-shell nuclei like Tin, may be artificial ( $K_\infty$  typically 20 MeV lower), because the MEM effect has not been included in all previous analyses.

#### IV. CONCLUSION

The GMR energy of open-shell Pb nuclei is predicted significantly lower than in the case of  $^{208}\text{Pb}$ . This is related to 2 effects: pairing and mutually enhanced magicity. These two effects may also be the explanation of the apparent softness of Sn isotopes, compared to  $^{208}\text{Pb}$ . The pairing effect is evaluated microscopically whereas the MEM effect is evaluated phenomenologically, since it is still not well characterised. A measurement of the

GMR in  $^{204}\text{Pb}$  and  $^{206}\text{Pb}$  compatible with the present predictions would solve this softness problem. It would also mean that  $K_\infty$  should not be determined by measuring the GMR in a single nuclei such as  $^{208}\text{Pb}$ , but the whole isotopic chain should be measured in order to provide a general view on the various effects on the GMR.

Additional theoretical investigations are called for in order to predict the GMR including the mutually en-

hancement magicity effect in a microscopic way. This would necessitate an ambitious microscopic approach, trying to link QRPA and GCM approaches. Experimentally, measurements of the GMR in unstable nuclei should also help to investigate this issue [9].

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