A new formalism for the estimation of the CP-violation parameters

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Abstract

In this paper, we use the time super-operator formalism in the 2-level Friedrichs model [1] to obtain a phenomenological model of mesons decay. Our approach provides a fairly good estimation of the CP symmetry violation parameter in the case of K, B and D mesons. We also propose a crucial test aimed at discriminating between the standard approach and the time super-operator approach developed throughout the paper.

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1 Introduction

There have been several theoretical approaches to CP violation in kaons (see e.g, the collection of papers edited in [2]) and the question is partially open today. In this paper, we use a Hamiltonian model, describing a two-level states coupled to a continuum of degrees of freedom, that makes is possible to simulate the phenomenology of neutral kaons. Then, the time super-operator formalism for the decay probability provides new numerical estimate of the parameters of CP violation.

It is well known [3] that kaons appears in pair K^0 and $\overline{\mathrm{K}}^0$ each one being conjugated to each other. The decay processes of K^0 and $\overline{\mathrm{K}}^0$ correspond to combinations of two orthogonal decaying modes K_1 and K_2, that are distinguished by their lifetime. The discovery of the small CP-violation effect was also accompanied by the non orthonormality of the short and long lived decay modes, now denoted K_S and K_L, slightly different from K_1 and K_2 and depending on a CP-violation parameter ϵ. Lee, Oehme and Yang (LOY) [4] proposed a generalization of the Wigner-Weisskopf theory [5] in order to account the "exponential decay". Later on, L. A. Khalfin [6] has pointed that, for a quantum system with energy spectrum bounded from below, the decay could not be exponential for large times. It was also observed [7] that short-time behavior of decaying systems could not be exponential and this led to the so-called Zeno effect [8, 9]. The departure from the exponential type behavior has been experimentally observed (see references quoted in [10]). L.A. Khalfin also corrected the parameter ϵ at the lowest order

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of perturbation. His estimation has been presented and reexamined in the reference [10] and applied to other mesons.

We show that our model allows us to obtain a better estimation of the CP-violation parameter for kaon as well as B and D mesons. We also make new predictions that differ from standard predictions and that could be tested experimentally.

This paper is organized as follows. In Section 2, we introduce the time super-operator for decay probability density. Then, in Section 3 we present the 2-levels Friedrichs model. Kaon phenomenology is recalled in Section 4. In Section 5, we present the theory of CP-violation in the Hilbert space and another derivation of the intensity formula for mesons that already has been used in [11]. Finally, in Section 6, we derive the time super-operator intensity formula and we compute CP-violation parameters for K, B, and D mesons. Then, we compare our results with the experimental data.

2 Decay probability in the time super-operator (T) approach

2.1 Decay probability in the time operator (T') approach

In the Wigner-Weisskopf approximation to time evolution of quantum unstable systems, the energy spectrum of the Hamiltonian is extended from $-\infty$ to $+\infty$. In this approximation, a decay time operator T' is canonically conjugated to H. That is,

$$H\psi(\omega) = \omega\psi(\omega) \tag{2.1}$$

$$T'\psi(\omega) = -\mathrm{i}\frac{d}{d\omega}\psi(\omega) \tag{2.2}$$

so that T' satisfies to the commutation relation $[H, T'] = \mathrm{i}I$. The T'-representation is obtained by a Fourier transform

$$\widehat{\psi}(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\mathrm{i}\tau\omega}\,\psi(\omega)\,d\omega \tag{2.3}$$

and the unstable states are the those prepared such that the decay occurs in the future, that is, $\widehat{\psi}(\tau) = 0$ for $\tau < 0$. Any state of the form $\psi_{\mathrm{un}}(\omega) = A/(\omega - z_0)$, ($z_0 = a - \mathrm{i}b, b > 0$), belongs to this space, since,

$$\widehat{\psi}_{\mathrm{un}}(\tau) = \begin{cases} \mathrm{i}A\sqrt{2\pi}e^{-\mathrm{i}\tau z_0} & \tau \geq 0, \\ 0 & \tau < 0 \end{cases} \tag{2.4}$$

It is clear that these states correspond to a decay probability density:

$$|\widehat{\psi}_{\mathrm{un}}(\tau)|^2 = 2\pi|A|^2e^{-b\tau} \tag{2.5}$$

This is an exponential distribution of decay times that is very common in particle physics.

2.2 Time super-operator (T) formalism

Rigourously speaking, when the Hamiltonian has a positive spectrum, it is forbidden in principle to define a time operator that satisfies the commutation relation $[H, T'] = \mathrm{i}I$. This argument was elaborated by Pauli who showed that if one could find such an operator $\hat{T}'$ one could use it for generating arbitrary translations in the energy eigenspace so that then the spectrum of $\hat{H}$ ought to be unbounded by below, which clearly constitutes a physical impossibility.

In order to escape this contradiction one needs to go to the space of density matrices in order to obtain a time operator that is conjugated to the evolution operator (the Liouville-von

Neumann operator) because it is sufficient that the Hamiltonian is not upperly bounded so that the Liouville-von Neumann operator has a spectrum extending from $-\infty$ to ∞. In order to do so, let us consider the Liouville-von Neumann space which is the space of operators ρ on $\mathcal{H}$ equipped with the scalar product $< \rho, \rho' >= Tr(\rho^* \rho')$ for which the time evolution is given by

$$U_t \rho = e^{-itH} \rho e^{itH} \tag{2.6}$$

$U_t = e^{-itL}$ is generated by the Liouville von-Neumann operator L given by:

$$L\rho = H\rho - \rho H \tag{2.7}$$

The time super-operator T is a self-adjoint super-operator on the Liouville-von Neumann space conjugated to L, i.e. $[T, L] = \mathrm{i}I$. This definition is equivalent to the Weyl relation: $e^{itL} T e^{-itL} = T + tI$.

The average of T in the state ρ is given by [4]:

$$\langle T \rangle_\rho = \langle \rho, T\rho \rangle \tag{2.8}$$

The time of occurrence of a random event fluctuates and we speak of the probability of its occurrence in a time interval $I =]t_1, t_2]$. The observable $T' = -T$ is associated to such event. In fact, for a system in the initial state ρ_0 the average time of occurrence $\langle T' \rangle_{\rho_0}$ is to be related to the time parameter t and to the average time of occurrence in the state $\rho_t = e^{-\mathrm{it}L} \rho_0$ by:

$$\langle T' \rangle_{\rho_t} = \langle T' \rangle_{\rho_0} - t \tag{2.9}$$

This equation follows from the Weyl relation.

Let $\mathcal{P}_\tau$ denote the family of spectral projection operators of T:

$$T = \int_{\mathbb{R}} \tau d\mathcal{P}_\tau \tag{2.10}$$

and let $\mathcal{Q}_\tau$ be the family of spectral projections of T', then, in the state ρ, the probability of occurrence of the event in a time interval I is given, as in the usual formulations, by

$$\mathcal{P}(I, \rho) = \|\mathcal{Q}_{t_2}\rho\|^2 - \|\mathcal{Q}_{t_1}\rho\|^2 = \|(\mathcal{Q}_{t_2} - \mathcal{Q}_{t_1})\rho\|^2 := \|\mathcal{Q}(I)\rho\|^2 \tag{2.11}$$

The unstable "undecayed" states observed at $t_0 = 0$ are the states ρ such that $\mathcal{P}(I, \rho) = 0$ for any negative time interval I, that is:

$$\|\mathcal{Q}_\tau \rho\|^2 = 0, \quad \forall \tau \leq 0 \tag{2.12}$$

In other words, these are the states verifying $\mathcal{Q}_0 \rho = 0$. It is straightforwardly checked that the spectral projections $\mathcal{Q}_\tau$ are related to the spectral projections $\mathcal{P}_\tau$ by the following relation:

$$\mathcal{Q}_\tau = 1 - \mathcal{P}_{-\tau} \tag{2.13}$$

Let $\mathfrak{F}_\tau$ be the subspace on which $\mathcal{P}_\tau$ projects . Thus, the unstable undecayed states are those states satisfying $\rho = \mathcal{P}_0 \rho$ and they coincide with the subspace $\mathfrak{F}_0$[5]. For these states, the

[4] The linearity that usually characterizes the relation between average values of observable A and density matrix M: $\mathrm{tr}(MA)$ seems to be violated here, but one should not forget that (a) in the case of pure states the density matrix equals its square and (b) this paradox is easily solved in the case of mixtures by imposing that ρ is the square root of the density matrix $M = \rho^* \rho$, $\mathrm{tr}(MA) = \mathrm{tr}(\rho^* A \rho)$.

[5] Therefore, a subspace $\mathfrak{F}_{t_0}$ is a set of decaying states prepared at time t_0. We call it an unstable space of T.

probability that a system prepared in the undecayed state ρ is found to decay some time during the interval $I =]0,t]$ is $\|\mathcal{Q}_t\rho\|^2 = 1 - \|\mathcal{P}_{-t}\rho\|^2$ a monotonically nondecreasing quantity which converges to 1 as $t \to \infty$ while $\|\mathcal{P}_{-t}\rho\|^2$ tends monotonically to zero. As noticed by Misra and Sudarshan [9], such quantity can not exist in the usual quantum mechanical treatment of the decay processes. It should not be confused with the usual "survival probability of an unstable state χ at time t" defined by $|<\chi, e^{-itH}\chi>|^2$ where χ is an eigenstate of the free Hamiltonian. In fact, *the last quantity is interpreted as the probability, at the instant* t, for finding the system undecayed when at time 0 it was prepared in the state χ. There is no general reason for this quantity to be monotonically decreasing as should be any genuine probability distribution. This problem does not appear in the time operator approach.

Considered so, the time operator approach is non-standard. Actually, the key, non-standard, assumption that underlies the time super-operator formalism is that.

In the Liouville space, given any initial state ρ, its survival probability in the unstable space is given by:

$$p_\rho(t) = \|\mathcal{P}_0 e^{-\mathrm{i}tL}\rho\|^2 \tag{2.14}$$

This is the probability that, for a system initially in the state ρ, no decay is found during $[0,t]$. Given any initial state ρ, its survival probability in the unstable space is given by [12]

$$\begin{aligned} p_\rho(t) &= \|\mathcal{P}_0 e^{-\mathrm{i}tL}\rho\|^2 \\ &= \|U_{-t}\mathcal{P}_0 U_t\rho\|^2 \\ &= \|\mathcal{P}_{-t}\rho\|^2 \end{aligned} \tag{2.15}$$

Here we used the following relation: $\mathcal{P}_{-t} = U_{-t}\mathcal{P}_0 U_t$. *Then, the survival probability is monotonically decreasing to* 0 *as* $t \to \infty$. This survival probability and the probability of finding the system to decay some time during the interval $I =]0,t]$, $q_\rho(t) = \|\mathcal{Q}_\rho(t)\|^2$ are related by:

$$q_\rho(t) = 1 - p_\rho(t) \tag{2.16}$$

Therefore, $q_\rho(t) \to 1$ when $t \to +\infty$.

The expression of the time operator is given in a *spectral representation of* H, that is, in the representation in which H is diagonal. As shown in [13], H should have an unbounded absolutely continuous spectrum. In the simplest case, we shall suppose that H is represented as the multiplication operator on $\mathcal{H} = L^2(\mathbb{R}^+)$:

$$H\psi(\omega) = \omega\psi(\omega). \tag{2.17}$$

The Hilbert-Schmidt operators on $L^2(\mathbb{R}^+)$ correspond to the square-integrable functions $\rho(\omega,\omega') \in L^2(\mathbb{R}^+ \times \mathbb{R}^+)$ and the Liouville-von Neumann operator L is given by :

$$L\rho(\omega,\omega') = (\omega - \omega')\rho(\omega,\omega') \tag{2.18}$$

Then we obtain a spectral representation of L via the change of variables:

$$\nu = \omega - \omega' \tag{2.19}$$

and

$$E = \min(\omega,\omega') \tag{2.20}$$

This gives a spectral representation of L:

$$L\rho(\nu,E) = \nu\rho(\nu,E), \tag{2.21}$$

where $\rho(\nu, E) \in L^2(\mathbb{R} \times \mathbb{R}^+)$. In this representation, $T\rho(\nu, E) = \mathrm{i}\frac{d}{d\nu}\rho(\nu, E)$ so that the spectral representation, of T is obtained by the inverse Fourier transform:

$$\hat{\rho}(\tau, E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\mathrm{i}\tau\nu} \rho(\nu, E) d\nu = (\mathcal{F}^* \rho)(\tau, E) \tag{2.22}$$

and

$$T\hat{\rho}(\tau, E) = \tau\hat{\rho}(\tau, E). \tag{2.23}$$

The spectral projection operators $\mathcal{P}_s$ of T are given in the (τ, E)-representation by

$$\mathcal{P}_s \hat{\rho}(\tau, E) = \chi_{]-\infty,s]}(\tau)\hat{\rho}(\tau, E) \tag{2.24}$$

where $\chi_{]-\infty,s]}$ is the characteristic function of $]-\infty, s]$. So, to obtain in the (ν, E)-representation the expression of these spectral projection operators, we use the Fourier transform:

$$\begin{aligned} \mathcal{P}_s \rho(\nu, E) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{s} e^{-\mathrm{i}\nu\tau} \hat{\rho}(\tau, E)\, d\tau \\ &= e^{-\mathrm{i}\nu s} \int_{-\infty}^{0} e^{-i\nu\tau} \hat{\rho}(\tau + s, E)\, d\tau. \end{aligned} \tag{2.25}$$

Let $g \in L^2(\mathbb{R})$ and denote its Fourier transform by: $\mathcal{F}g(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\mathrm{i}\nu\tau} g(\tau)\, d\tau$. Using the Hilbert transformation:

$$\mathbf{H}g(x) = \frac{1}{\pi} \mathsf{P} \int_{-\infty}^{\infty} \frac{g(t)}{t - x}\, dt. \tag{2.26}$$

We have [14] the following formula:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\mathrm{i}\nu\tau} g(\tau)\, d\tau = \frac{1}{2}(\mathcal{F}(g) - \mathrm{i}\mathbf{H}\mathcal{F}(g)). \tag{2.27}$$

Finally, using the well-known property of the translated Fourier transform: $\sigma_s g(\tau) = g(\tau + s)$,

$$\mathcal{F}(\sigma_s g)(\nu) = e^{\mathrm{i}\nu s} \mathcal{F}.g(\nu) \tag{2.28}$$

(2.25) and (2.27) yield:

$$\mathcal{P}_s \rho(\nu, E) = \frac{1}{2} e^{-\mathrm{i}\nu s} [e^{i\nu s} \rho(\nu, E) - \mathrm{i}\mathbf{H}(e^{\mathrm{i}\nu s} \rho(\nu, E))]. \tag{2.29}$$

Thus:

$$\mathcal{P}_s \rho(\nu, E) = \frac{1}{2} [\rho(\nu, E) - \mathrm{i}e^{-\mathrm{i}\nu s} \mathbf{H}(e^{\mathrm{i}\nu s} \rho(\nu, E))]. \tag{2.30}$$

It is to be noted that $\mathcal{P}_s \rho(\nu, E)$ is in the Hardy class $\mathbb{H}^+$ (i.e. it is the limit as $y \to 0^+$ of an analytic function $\Phi(\nu + \mathrm{i}y)$ such that: $\int_{-\infty}^{\infty} \mid \Phi(\nu + \mathrm{i}y) \mid^2 \ dy < \infty$)[14].

3 The two-level Friedrichs model

The Friedrichs interaction Hamiltonian between the two discrete modes and the continuous degree of freedom is given by the operator H on the Hilbert space of the wave functions of the form $\mid \psi >= \{f_1, f_2, g(\mu)\}, f_1, f_2 \in \mathbb{C}, g \in L^2(\mathbb{R}^+)$

$$H = H_0 + \lambda_1 V_1 + \lambda_2 V_2, \tag{3.31}$$

where λ_1 and λ_2 are the complex coupling constants, and

$$H_0 \mid \psi >= \{\omega_1 f_1, \omega_2 f_2, \mu g(\mu)\}, (\omega_1 \text{ and } \omega_2 > 0). \tag{3.32}$$

The operators V_i $(i = 1, 2)$ are given by:

$$\begin{aligned} V_1\{f_1, f_2, g(\mu)\} &= \{< v(\mu), g(\mu) >, 0, f_1.v(\mu)\} \\ V_2\{f_1, f_2, g(\mu)\} &= \{0, < v(\mu), g(\mu) >, f_2.v(\mu)\} \end{aligned} \tag{3.33}$$

where

$$< v(\mu), g(\mu) >= \int d\mu v^*(\mu) g(\mu), \tag{3.34}$$

is the inner product. Thus H can be represented as a matrix :

$$H_{\text{Friedrichs}} = \begin{pmatrix} \omega_1 & 0 & \lambda_1^* v^*(\mu) \\ 0 & \omega_2 & \lambda_2^* v^*(\mu) \\ \lambda_1 v(\mu) & \lambda_2 v(\mu) & \mu \end{pmatrix} \tag{3.35}$$

$\omega_{1,2}$ represent the energies of the discrete levels, and the factors $\lambda_i v(\mu)$ $(i = 1, 2)$ represent the couplings to the continuous degree of freedom. The energies μ of the different modes of the continuum range from $-\infty$ to $+\infty$ when $v(\mu) = 1$, but we are free to tune the coupling $v(\mu)$ in order to introduce a selective cut off to extreme energy modes. Let us now solve the Schrödinger equation and trace out the continuum in order to derive the master equation for the two-level system. The two-level Friedrichs model Schrödinger equation with $\hbar = 1$ is formally written as

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1^* v^*(\mu) \\ 0 & \omega_2 & \lambda_2^* v^*(\mu) \\ \lambda_1 v(\mu) & \lambda_2 v(\mu) & \mu \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ g(\mu) \end{pmatrix} = \omega \begin{pmatrix} f_1 \\ f_2 \\ g(\mu) \end{pmatrix}. \tag{3.36}$$

That is to say:

$$\omega_1 f_1(\omega) + \lambda_1^* \int d\mu v^*(\mu) g(\mu) = \omega f_1(\omega), \tag{3.37}$$

$$\omega_2 f_2(\omega) + \lambda_2^* \int d\mu v^*(\mu) g(\mu) = \omega f_2(\omega), \tag{3.38}$$

and

$$\lambda_1 v(\omega) f_1(\omega) + \lambda_2 v(\omega) f_2(\omega) + \mu g(\omega) = \omega g(\omega). \tag{3.39}$$

The solution of (5.69), for “outgoing” wave, is:

$$g(\mu) = \delta(\mu - \omega) - \lim_{\epsilon \to 0} \frac{\lambda_1 v(\mu) f_1 + \lambda_2 v(\mu) f_2}{\omega - \mu - \mathrm{i}\epsilon}. \tag{3.40}$$

inserting the above equation in the equations(3.37) yields

$$f_1(\omega) = \frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - \left(\lambda_1^* \lambda_2 \lim_{\epsilon \to 0} \int d\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}\epsilon} \right) f_2(\omega), \tag{3.41}$$

where

$$\eta_1^+(\omega) = \omega - \omega_1 + |\lambda_1|^2 \lim_{\epsilon \to 0} \int d\mu \frac{|v(\mu)|^2}{\mu - (\omega + \mathrm{i}\epsilon)}. \tag{3.42}$$

We can also obtain the similar relations for f_2 by changing the indices 1 with 2 and vis versa as:

$$f_2(\omega) = \frac{\lambda_2^* v^*(\omega)}{\eta_2^+(\omega)} - \left(\lambda_1 \lambda_2^* \lim_{\epsilon \to 0} \int d\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}\epsilon} \right) f_1(\omega). \tag{3.43}$$

By substituting $f_2(\omega)$ from the above equation in the equation (3.41) we obtain

$$\begin{aligned} f_1(\omega) &= \frac{1}{1 - \left(\lambda_1^* \lambda_2 \int d\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}0} \right)^2} \left(\frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - \frac{\lambda_1^* |\lambda_2|^2}{\eta_2^+(\omega)} \int d\mu \frac{|v(\mu)|^2}{\mu - \omega - \mathrm{i}0} \right) \\ &= \frac{1}{1 - O(|\lambda|^4)} \left(\frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)} - O(\lambda_1^* |\lambda_2|^2) \right) \end{aligned} \tag{3.44}$$

Thus, to the order two approximation we have

$$f_1(\omega) \simeq \frac{\lambda_1^* v^*(\omega)}{\eta_1^+(\omega)}. \tag{3.45}$$

and the same formula for f_2 as:

$$f_2(\omega) \simeq \frac{\lambda_2^* v^*(\omega)}{\eta_2^+(\omega)}. \tag{3.46}$$

Also denote $\eta_i^-(\omega) = \eta_i(\omega - \mathrm{i}\epsilon)$. $\eta_i^\pm(\omega)$, $(i = 1, 2)$ are complex conjugate of each other, we can see that

$$\eta_i^\pm(\omega) = \omega - \omega_i + |\lambda_i|^2 \; \mathsf{P} \int_0^\infty \frac{|v(\omega')|^2}{\omega' - \omega} \, d\omega' \pm \mathrm{i}\pi |\lambda_i|^2 |v(\omega)|^2, \tag{3.47}$$

where P indicates the "principal value" and we used the following identity in equation (3.47)

$$\lim_{\varepsilon \to 0^+} \frac{1}{x - x_0 \pm \mathrm{i}\varepsilon} = \mathsf{P} \frac{1}{x - x_0} \mp \mathrm{i}\pi \delta(x - x_0). \tag{3.48}$$

Let $|\chi\rangle = |\epsilon_1 f_1 + \epsilon_2 f_2\rangle$ where ϵ_i, $\quad (i = 1, 2)$ is a constant complex number. The physical meaning of such a state is that it corresponds to a coherent superposition of two exponential decay processes. In the following Section we shall compute the projection of $|\chi\rangle\langle\chi|$ on the unstable spaces of time operator and then the survival probability $p_\rho(t)$ introduced in the Section 2. We compute its expression for the density matrix $|\chi\rangle\langle\chi|$ in terms of the lifetimes and energies of the (mesonic) resonances. It has been shown [15] that the average of time operator for the state $|\chi\rangle\langle\chi|$ is equal to the lifetime $1/\gamma$ in a first approximation (more precisely in the weak coupling regime that is described in the next section (equation (3.52)). We shall characterize the short time and long time behavior of this survival probability.

Let us now identify the pure state χ with the element $\rho = |\chi><\chi|$ of the Liouville space, that is the kernel operator:

$$\rho = \sum_{i=1}^{2} \sum_{j=1}^{2} \rho_{ij}(\omega, \omega') = \sum_{i=1}^{2} \sum_{j=1}^{2} \epsilon_i \epsilon_j^* f_i(\omega) \overline{f_j(\omega')} = \sum_{i=1}^{2} \sum_{j=1}^{2} \epsilon_i \epsilon_j^* \mathfrak{F}_{ij}. \tag{3.49}$$

We shall compute the survival probability $\|\mathcal{P}_{-s}\rho\|^2$ of the state ρ and show how it reaches the following limit:

$$\lim_{s \to \infty} \|\mathcal{P}_{-s}\rho\|^2 \to 0. \tag{3.50}$$

3.1 Weak coupling conditions

As explained above the Liouville operator is given by equation (2.18) and the spectral representation of L is given by the change of variables introduced in (2.19) and (2.20). Thus, we obtain for $\mathfrak{F}_{ij}(\nu, E), \quad (i,j = 1,2)$:

$$\mathfrak{F}_{ij}(\nu, E) = \begin{cases} \lambda_i \lambda_j^* \frac{v(E)}{\eta_i^-(E)} \frac{v^*(E+\nu)}{\eta_j^+(E+\nu)} & \nu > 0 \\ \\ \lambda_i^* \lambda_j \frac{v^*(E)}{\eta_j^+(E)} \frac{v(E-\nu)}{\eta_i^-(E-\nu)} & \nu < 0, \end{cases} \tag{3.51}$$

Admitting that $\eta_i^+(\omega)$ in (3.47) in the the $O(|\lambda|^2)$ has one zero in the lower half-plane [16, 17] which approaches ω_i for decreasing coupling, we can write:

$$\eta_i^+(\omega) = \omega - z_i. \tag{3.52}$$

where $z_i = \widetilde{\omega}_i - \mathrm{i}\frac{\gamma_i}{2}$ where $\gamma_i \sim |\lambda_i|^2$ is a real positive constant [17]. In this article we suppose that $\widetilde{\omega}_1 < \widetilde{\omega}_2$. Easily, we can verify that

$$\eta_i^+(\omega) - \eta_i^-(\omega) = \mathrm{i}\gamma_i. \tag{3.53}$$

From (3.47), we have

$$\frac{\mathrm{i}}{2}\left[\frac{1}{\eta_i^+(\omega)} - \frac{1}{\eta_i^-(\omega)}\right] = \frac{\pi|\lambda_i|^2|v(\omega)|^2}{|\eta_i^+(\omega)|^2}. \tag{3.54}$$

Consequently, the two above equations yield

$$\frac{\pi|\lambda_i|^2|v(\omega)|^2}{|\eta_i^+(\omega)|^2} = \frac{\frac{\gamma_i}{2}}{|\eta_i^+(\omega)|^2}. \tag{3.55}$$

Therefore, $|f_i(\omega)|^2 \sim \frac{1}{(\omega-\widetilde{\omega}_i)^2+\frac{\gamma_i^2}{4}}$ which is a Breit-Wigner like distribution. This equation will be used in the next sections.

4 Phenomenology of kaons

Kaons are bosons that were discovered in the forties during the study of cosmic rays. They are produced by collision processes in nuclear reactions during which the strong interactions dominate. They appear in pairs K^0, $\overline{\mathrm{K}}^0$ [3, 18].

The K mesons are eigenstates of the parity operator P: $P|\mathrm{K}^0\rangle = -|\mathrm{K}^0\rangle$, and $P|\overline{\mathrm{K}}^0\rangle = -|\overline{\mathrm{K}}^0\rangle$. K^0 and $\overline{\mathrm{K}}^0$ are charge conjugate to each other $C|\mathrm{K}^0\rangle = |\overline{\mathrm{K}}^0\rangle$, and $C|\overline{\mathrm{K}}^0\rangle = |\mathrm{K}^0\rangle$. We get thus

$$CP|\mathrm{K}^0\rangle = -|\overline{\mathrm{K}}^0\rangle, \qquad CP|\overline{\mathrm{K}}^0\rangle = -|\mathrm{K}^0\rangle. \tag{4.56}$$

Clearly $|\mathrm{K}^0\rangle$ and $|\overline{\mathrm{K}}^0\rangle$ are not CP-eigenstates, but the following combinations

$$|\mathrm{K}_1\rangle = \frac{1}{\sqrt{2}}\big(|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle\big), \qquad |\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}\big(|\mathrm{K}^0\rangle - |\overline{\mathrm{K}}^0\rangle\big), \tag{4.57}$$

are CP-eigenstates.

$$CP|\mathrm{K}_1\rangle = +|\mathrm{K}_1\rangle, \qquad CP|\mathrm{K}_2\rangle = -|\mathrm{K}_2\rangle. \tag{4.58}$$

In the absence of matter, kaons disintegrate through weak interactions [18]. Actually, K^0 and $\overline{\mathrm{K}}^0$ are distinguished by their mode of *production*. K_1 and K_2 are the decay modes of kaons. In absence of CP-violation, the weak disintegration process distinguishes the K_1 states which decay only into "2π" while the K_2 states decay into "$3\pi, \pi e\nu, ...$" [19]. The lifetime of the K_1 kaon is short ($\tau_S \approx 8.92 \times 10^{-11}$ s), while the lifetime of the K_2 kaon is quite longer ($\tau_L \approx 5.17 \times 10^{-8}$ s).

CP-violation was discovered by Christenson *et al.* [20]. CP-violation means that the long-lived kaon can also decay to "2π". Then, the CP symmetry is slightly violated (by a factor of the order of 10^{-3}) by weak interactions so that the CP eigenstates K_1 and K_2 are not exact eigenstates of the decay interaction. Those exact states are characterized by lifetimes that are in a ratio of the order of 10^{-3}, so that they are called the short-lived state (K_S) and long-lived state (K_L). They can be expressed as coherent superpositions of the K_1 and K_2 eigenstates through

$$|\mathrm{K}_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\big[\epsilon\ |\mathrm{K}_1\rangle + |\mathrm{K}_2\rangle\big], \quad |\mathrm{K}_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\big[|\mathrm{K}_1\rangle + \epsilon\ |\mathrm{K}_2\rangle\big], \tag{4.59}$$

where ϵ is a complex CP-violation parameter, $|\epsilon| \ll 1$ and ϵ does not have to be real. K_L and K_S are the eigenstates of the Hamiltonian for the mass-decay matrix [18, 19] which has the following form in the basis $|\mathrm{K}^0\rangle$ and $|\overline{\mathrm{K}}^0\rangle$:

$$H = M - \frac{\mathrm{i}}{2}\Gamma \equiv \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{21} - \frac{\mathrm{i}}{2}\Gamma_{21} & M_{22} - \frac{\mathrm{i}}{2}\Gamma_{22} \end{pmatrix} \tag{4.60}$$

where M and Γ are individually hermitian since they correspond to observables (mass and lifetime). The corresponding eigenvalues of the mass-decay matrix are equal to

$$m_L - \frac{\mathrm{i}}{2}\Gamma_L, \qquad m_S - \frac{\mathrm{i}}{2}\Gamma_S \tag{4.61}$$

The CP-violation was established by the observation that K_L decays not only via three-pion, which has natural CP parity, but also via the two-pion ("2π") mode with an experimentally observed violation amplitude $|\epsilon^{exp.}|$ of the order of 10^{-3}, which was truly unexpected at the time. Let us now reconsider how the simple model (4.59), (4.60) is related to the experimental data. A series of detections is performed at various distances from the source of a neutral kaon beam in order to estimate the variation of the populations of emitted pion π^+, π^- pairs in function of the proper time. This is done for times of the order of τ_S. The experiment shows that an interference term is present in the expression of the excitation rates of detectors in function of their distance to the source. It follows from (4.59) that the transition amplitude of the K_L beam is given by

$$\psi(t) = A\left(e^{-\mathrm{i}(m_S - \frac{\mathrm{i}}{2}\Gamma_S)t} + \epsilon^{\exp} e^{-\mathrm{i}(m_L - \frac{\mathrm{i}}{2}\Gamma_L)t}\right) \tag{4.62}$$

with A a global proportionality factor that remains constant in time. Then the intensity $I(t) = |\psi(t)|^2$ is given by:

$$I(t) = I_0\left(e^{-\Gamma_S t} + |\epsilon^{\exp}|^2 e^{-\Gamma_L t} + |\epsilon^{\exp}| e^{-(\frac{\Gamma_S + \Gamma_L}{2})t)} \cos(\triangle m t + \arg(\epsilon^{\exp}))\right) \tag{4.63}$$

where

$$|\epsilon^{\exp}| = \frac{\text{Amplitude}\,(K_L \to \pi^+, \pi^-)}{\text{Amplitude}\,(K_S \to \pi^+, \pi^-)} \tag{4.64}$$

By fitting the expressions (4.63) and (4.64) with the observed data one derives an estimation of the mass difference between the short and long lived state as well as the phase of $\epsilon^{\exp}$ and its amplitude.

All this leads to an experimental estimation of $\epsilon^{\exp}$ [21]

$$|\epsilon^{\exp}| = (2.232 \pm 0.007) \times 10^{-3}, \quad \arg(\epsilon^{\exp}) = (43.5 + 0.7)^{\circ}. \tag{4.65}$$

5 The Wigner-Weisskopf type theory of the CP-violation in the Hilbert space

Let us present the fundamental ideas of the theory of spontaneous emission of an atom interacting with the electromagnetic field, given by Wigner and Weisskopf . This treatment aims at obtaining an exponential time dependence for decaying states by integrating over the continuum energy. That is, we assume that the modes of the fields are closely spaced. Then, we have to assume that the variation of $v(\mu)$ over μ is negligible with $|\mu| \lesssim$ "uncertainty of the original state energy", i.e. $v(\mu) \approx v$ independent of μ or in the simple case it is taken to obey $v(\mu) = 1$. Also another assumption is that the lower limit of integration over ω is replaced by $-\infty$.

The two-level Friedrichs model time-dependent Schrödinger equation, in the Wigner-Weisskopf regime becomes:

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1^* \\ 0 & \omega_2 & \lambda_2^* \\ \lambda_1 & \lambda_2 & \mu \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\mu,t) \end{pmatrix} = \mathrm{i}\frac{\partial}{\partial t} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\mu,t) \end{pmatrix}. \tag{5.66}$$

which means:

$$\omega_1 f_1(t) + \lambda_1^* \int_{-\infty}^{\infty} d\mu g(\mu,t) = \mathrm{i}\frac{\partial f_1(t)}{\partial t}, \tag{5.67}$$

$$\omega_2 f_2(t) + \lambda_2^* \int_{-\infty}^{\infty} d\mu g(\mu,t) = \mathrm{i}\frac{\partial f_2(t)}{\partial t}, \tag{5.68}$$

and

$$\lambda_1 f_1(t) + \lambda_2 f_2(t) + \mu g(\mu,t) = \mathrm{i}\frac{\partial g(\mu,t)}{\partial t}. \tag{5.69}$$

Let us now solve the Schrödinger equation and trace out the continuum in order to derive the master equation for the two-level system. From the equation (5.69) we can obtain $g(\mu,t)$, taking $g(\mu,0) = 0$, as

$$g(\mu,t) = -\mathrm{i}e^{-\mathrm{i}\omega t} \int_0^t d\tau \big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big] e^{\mathrm{i}\omega\tau}, \tag{5.70}$$

where $t > 0$. Then, we substitute $g(\mu,t)$ in the equation (5.67) and we obtain

$$\mathrm{i}\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\lambda_1^* \int_{-\infty}^{\infty} d\mu e^{-\mathrm{i}\mu t} \int_0^t d\tau \big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big] e^{\mathrm{i}\mu\tau}, \tag{5.71}$$

we also obtain the same relation for $f_2(t)$ from equation(5.68):

$$\mathrm{i}\frac{\partial f_2(t)}{\partial t} = \omega_2 f_2(t) - \mathrm{i}\lambda_2^* \int_{-\infty}^{\infty} d\mu e^{-\mathrm{i}\mu t} \int_0^t d\tau \big[\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)\big] e^{\mathrm{i}\mu\tau}. \tag{5.72}$$

Finally, one obtains the following Markovian form of the reduced Schrödinger equation [22]

$$\mathrm{i}\frac{\partial}{\partial t} \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} \omega_1 - \mathrm{i}\pi|\lambda_1|^2 & -\mathrm{i}\pi\lambda_1^*\lambda_2 \\ -\mathrm{i}\pi\lambda_1\lambda_2^* & \omega_2 - \mathrm{i}\pi|\lambda_2|^2 \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}. \tag{5.73}$$

Thus, we obtain an effective non-Hermitian Hamiltonian evolution, $H_{\rm eff} = M - {\rm i}\frac{\gamma}{2}$. The eigenvalues of the above effective Hamiltonian under the weak coupling constant approximation are:

$$\omega_+ = \omega_1 - {\rm i}\pi|\lambda_1|^2 + O(\lambda^4), \quad \omega_- = \omega_2 - {\rm i}\pi|\lambda_2|^2 + O(\lambda^4), \tag{5.74}$$

In a first and very rough approximation, the eigenvectors of the effective Hamiltonian are the same as the postulated kaons states.

$$|f_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |K_1\rangle \quad \text{and} \quad |f_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |K_2\rangle, \tag{5.75}$$

Phenomenology imposes that the complex Friedrichs energies $\omega_\pm$ coincide with the observed complex energies. The Friedrichs energies depend on the choice of the four parameters ω_1, ω_2, λ_1 and λ_2 and the observed complex energies are directly derived from the experimental determination of four other parameters, the masses m_S and m_L and the lifetimes τ_S and τ_L. We must thus adjust the theoretical parameters in order that they fit the experimental data. This can be done by comparing the eigenvalue of the effective matrix with the eigenvalue of the mass-decay matrix which is taken in the expression (4.61). Finally, we have

$$\begin{aligned} \omega_1 = m_S, &\quad 2\pi|\lambda_1|^2 = \Gamma_S, \\ \omega_2 = m_L, &\quad 2\pi|\lambda_2|^2 = \Gamma_L. \end{aligned} \tag{5.76}$$

The above identities yield

$$\lambda_1 = \sqrt{\frac{\Gamma_S}{2\pi}}\, e^{{\rm i}\theta_S}, \quad \lambda_2 = \sqrt{\frac{\Gamma_L}{2\pi}}\, e^{{\rm i}\theta_L} \tag{5.77}$$

where θ_S and θ_L are real constants.

CPT invariance: Let us now discuss the CPT invariance in our model. As mentioned in the texts books like [18, 19], CPT invariance imposes some conditions on the mass-decay matrix, i.e.

$$M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22},\ M_{12} = M_{21}^* \ \text{ and } \ \Gamma_{12} = \Gamma_{21}^* \tag{5.78}$$

in the K^0 and $\overline{\mathrm{K}}^0$ bases. But, we note that our effective Hamiltonian is written in the K_1 and K_2 bases. Thus, we have to rewrite in the K^0 and $\overline{\mathrm{K}}^0$ bases. Thus, the transformation matrix T from the K_1 and K_2 bases to the K^0 and $\overline{\mathrm{K}}^0$ bases is obtained as

$$T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = T^{-1}. \tag{5.79}$$

Then, the effective Hamiltonian in the K^0 and $\overline{\mathrm{K}}^0$ bases, $H_{\rm eff}^{0\overline{0}}$ is obtained by

$$H_{\rm eff}^{0\overline{0}} = TH_{\rm eff}T^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \omega_1 - {\rm i}\pi|\lambda_1|^2 & -{\rm i}\pi\lambda_1^*\lambda_2 \\ -{\rm i}\pi\lambda_1\lambda_2^* & \omega_2 - {\rm i}\pi|\lambda_2|^2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{5.80}$$

we have, $H_{\rm eff}^{0\overline{0}} =$

$$\begin{pmatrix} (m_S+m_L) - \frac{\rm i}{2}\left(\Gamma_S + \Gamma_L + 2\sqrt{\Gamma_S\Gamma_L}\cos\triangle\theta\right), & (m_S-m_L) - \frac{\rm i}{2}\left(\Gamma_S - \Gamma_L + 2{\rm i}\sqrt{\Gamma_S\Gamma_L}\sin\triangle\theta\right) \\ (m_S-m_L) - \frac{\rm i}{2}\left(\Gamma_S - \Gamma_L - 2{\rm i}\sqrt{\Gamma_S\Gamma_L}\sin\triangle\theta\right), & (m_S+m_L) - \frac{\rm i}{2}\left(\Gamma_S + \Gamma_L - 2\sqrt{\Gamma_S\Gamma_L}\cos\triangle\theta\right) \end{pmatrix}. \tag{5.81}$$

where $\triangle\theta = \theta_L - \theta_S$. CPT invariance conditions in (5.78) impose that

$$\triangle\theta = k\pi + \frac{\pi}{2}, \quad (k = \cdots, -1, 0, 1, \cdots). \tag{5.82}$$

Here we choose $k = -1$, consequently, $\triangle\theta = -\frac{\pi}{2}$. Then, we have

$$\begin{aligned} M_{11} = M_{22} = (m_S + m_L), \quad \Gamma_{11} = \Gamma_{22} = \Gamma_S + \Gamma_L, \\ M_{12} = M_{21}^* = (m_S - m_L), \quad \Gamma_{12} = \Gamma_{21}^* = \Gamma_S - \Gamma_L - 2\mathrm{i}\sqrt{\Gamma_S \Gamma_L}. \end{aligned} \tag{5.83}$$

CP-violation: Let us study in this case the CP-violation. The Friedrichs model allows us to estimate the value of ϵ. For this purpose, the effective Hamiltonian (5.73) acts on the $|\mathrm{K}_S\rangle$ vector states (4.59) as an eigenstate corresponding to the eigenvalue $\omega_+ = \omega_1 - \mathrm{i}\pi|\lambda_1|^2 = m_S - \mathrm{i}\frac{\Gamma_S}{2}$, so that we must impose that $H_{\text{eff}}|(f_+ + \epsilon f_-)\rangle = H_{\text{eff}}\binom{1}{\epsilon} = \omega_+\binom{1}{\epsilon}$, from which we obtain after straightforward calculations that

$$\epsilon = \frac{\frac{1}{2}\sqrt{\Gamma_L \Gamma_S}}{(m_L - m_S) - \frac{\mathrm{i}}{2}(\Gamma_L - \Gamma_S)}. \tag{5.84}$$

By using the experimental ratio $\frac{(m_L - m_S)}{-(\Gamma_L - \Gamma_S)} \approx \triangle m \tau_S \approx 0.47$ and the above experimental values of $\Gamma_L, \Gamma_S, m_L, m_S$, we obtain the following estimated value for ϵ:

$$\epsilon = \sqrt{\frac{\Gamma_L}{\Gamma_S}}\ e^{\mathrm{i}(46.77)^\circ} = \sqrt{\frac{1.82 \times 10^{-3}}{2}}\ e^{\mathrm{i}(46.77)^\circ} = 14\,\epsilon^{\exp}. \tag{5.85}$$

We see that the ϵ argument is the same as the experimental value but the magnitude of the CP-violation parameter is quite larger than its experimental value.

The reason is that, as we have shown in a previous work [11], we did not normalize correctly the amplitudes associated to the two interfering decay processes (short and long). In that work we solved the problem by developing an analogy between the temporal density of decay and the spatial density of presence (this constitutes the so-called wave function approach).

Now we shall derive intensity formula for the meson decay [11] using the formalism of the time operator (T') sketched in Section 2. By considering the relations (3.52) and (3.55) and supposing the $v(\omega)$ is a real function, we can write the $f_1(\omega)$ and $f_2(\omega)$, the equations (3.45) and (3.46), as:

$$f_i(\omega) = \frac{\sqrt{\frac{\gamma_i}{2}}e^{-\mathrm{i}\theta_i}}{\omega - \widetilde{\omega}_i + \frac{\mathrm{i}}{2}\gamma_i}, \quad (i = 1, 2), \tag{5.86}$$

where θ_i is the phase of the possibly complex coefficients λ_i. By using the Fourier transforms, i.e. equation (2.3), for the above equation, (5.86), we obtain for $(i = 1, 2)$

$$\hat{f}_i(\tau) = \begin{cases} N\sqrt{\pi\,\gamma_i}\ e^{-(\mathrm{i}\widetilde{\omega}_i + \frac{\gamma_i}{2})\tau - \mathrm{i}\theta_i}, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases} \tag{5.87}$$

where N is the normalization constant. For $s = -\tau < 0$, we have

$$\hat{f}_i(s) = \begin{cases} \sqrt{\pi\gamma_i}\, e^{(\mathrm{i}\widetilde{\omega}_i + \frac{\gamma_i}{2})s - \mathrm{i}\theta_i}, & s \leq 0 \\ \\ 0, & s > 0. \end{cases} \tag{5.88}$$

Finally, the normalization relation, i.e.

$$\int_{-\infty}^{+\infty} ds\,|\hat{f}_i(s)|^2 = 1, \quad (i = 1, 2) \tag{5.89}$$

yields:

$$\hat{f}_i(s) = \begin{cases} \sqrt{\gamma_i}\, e^{(\mathrm{i}\widetilde{\omega}_i + \frac{\gamma_i}{2})s - \mathrm{i}\theta_i}, & s \leq 0 \\ \\ 0, & s > 0. \end{cases} \tag{5.90}$$

Here $f_i(s),\ (i = 1, 2)$ is the form of the density of the probability. Thus, the intensity is obtained by

$$\begin{aligned} I(s) &= |C|^2 |\epsilon_1 f_1(s) + \epsilon_2 f_2(s)|^2 \\ &= I_0 \left(e^{\gamma_1 s} + |\epsilon|^2 \frac{\gamma_2}{\gamma_1} e^{\gamma_2 s} + |\epsilon| \sqrt{\frac{\gamma_2}{\gamma_1}}\, e^{\frac{(\gamma_1+\gamma_2)}{2} s} \cos((\widetilde{\omega}_1 - \widetilde{\omega}_2)s + \theta_2 - \theta_1 + \arg(\epsilon)) \right) \end{aligned} \tag{5.91}$$

where $\epsilon = \epsilon_2/\epsilon_1$ and C and $I_0 = |C|^2 \epsilon_1^2 \gamma_1$ are the constants. This corresponds to an effective value for ϵ that is no longer 14 times too large as in expression (5.85) because it must be renormalized. Identifying equations (4.63) and (5.91) it is easy to show, as we have also done in [11], that, ϵ^{th}, the theoretical prediction for the experimental CP-violation parameter, obeys

$$\epsilon^{\mathrm{th}} = \epsilon \sqrt{\frac{\Gamma_L}{\Gamma_S}} = \frac{\Gamma_L}{\Gamma_S} \frac{\frac{1}{2}}{\frac{\triangle m}{\Gamma_S} - \mathrm{i}\, \frac{\triangle \gamma}{2\Gamma_S}}. \tag{5.92}$$

Substituting in the expression (5.92) the physically observed masses and lifetimes of the short and long kaon states we find that $\epsilon^{\mathrm{th}} \approx 0.6\, \epsilon^{\mathrm{exp}}$ which constitutes an improvement in comparison to the non-renormalized estimation (5.85). We shall also reconsider similar results in the case of B and D particles in a next section.

In the next coming section, we shall use the time super-operator (T) formalism as a non Wigner-Weisskopf approximation method to obtain the CP-violation parameter. This formalism also predicts a CP-violation parameter comparable to the experimental value.

6 Computation of spectral projections of T in a Friedrichs model

In this section, we will compute the survival probability and we obtain the theoretical CP-violation parameters for the mesons K, B and D. Then, we compare our results to the experimental CP-violation parameters. We shall see that our theoretical results provide a good estimation of the experimentally measured quantities. Moreover, a fine structure appears in the case of kaons, which brought us to conceive an experimental test aimed at falsifying the time super-operator approach, that we shall discuss in the conclusion.

By considering $v(\omega)$ a real test function and using the equation (3.55) we obtain $\mathfrak{F}_{ji}(\nu, E)$ in the following form:

$$\mathfrak{F}_{ji}(\nu, E) = \begin{cases} \frac{\lambda_j \lambda_i^*}{\nu_j^*(\nu + \nu_i)} & \nu > 0 \\ \\ \frac{\lambda_j^* \lambda_i}{\nu_i(\nu_j^* - \nu)} & \nu < 0. \end{cases} \tag{6.93}$$

where $i, j = 1, 2$ and

$$\nu_i := a_i + \mathrm{i} b_i := (E - \widetilde{\omega}_i) + \mathrm{i}\frac{\gamma_i}{2}. \tag{6.94}$$

For obtaining $\mathcal{P}_s \mathfrak{F}_{ij}(\nu, E)$ $(s < 0)$, we shall use the formula (2.30). First we compute:

$$G_{ji}(\nu, E) = \mathbf{H}(e^{\mathrm{i}s\nu} \mathfrak{F}_{ji})(\nu, E) = \frac{1}{\pi} \mathsf{P} \int_{-\infty}^{\infty} \frac{e^{\mathrm{i}sx} \mathfrak{F}_{ji}(x, E)}{x - \nu} dx \tag{6.95}$$

Now, we substitute (6.93) in (6.95), so we have

$$G_{ji}(\nu,E)=\frac{1}{\pi}\mathsf{P}\left[\lambda_i\lambda_j^*\int_{-\infty}^{0}\frac{e^{\mathrm{i}sx}}{\nu_i(x-\nu)(\nu_j^*-x)}\,dx+\lambda_i^*\lambda_j\int_{0}^{+\infty}\frac{e^{\mathrm{i}sx}}{\nu_j^*(x-\nu)(\nu_i+x)}\,dx\right] \tag{6.96}$$

which for the $\nu>0$ has the following form:

$$G_{ji}(\nu,E)=\frac{1}{\pi}\left[\lambda_i\lambda_j^*\int_{-\infty}^{0}\frac{e^{\mathrm{i}sx}}{\nu_i(x-\nu)(\nu_j^*-x)}\,dx+\lambda_i^*\lambda_j\mathsf{P}\int_{0}^{+\infty}\frac{e^{\mathrm{i}sx}}{\nu_j^*(x-\nu)(\nu_i+x)}\,dx\right]. \tag{6.97}$$

A complete computation of the $G_{ii}(\nu,E)$ is showed in [17]. Finally, $\mathcal{P}_s\mathfrak{F}_{ij}(\nu,E)$ is obtained as: for $i=j$

$$\begin{aligned}\mathcal{P}_s\mathfrak{F}_{ii}(\nu,E)=\mathrm{i}|\lambda_i|^2\,e^{-\mathrm{i}s\nu}&\left[\frac{-1}{2\pi\nu_i(\nu_i^*-\nu)}\left(\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu_i^*}dy-\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu}dy\right)\right.\\&\left.+\frac{1}{2\pi\nu_i^*(\nu+\nu_i)}\left(\int_{-\infty}^{0}\frac{e^{-sy}}{y-\mathrm{i}\nu_i}dy-\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu}dy\right)\right]\\&+\begin{cases}|\lambda_i|^2\,e^{-\mathrm{i}s\nu}[\frac{e^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_i^*-\nu)}-\frac{e^{-\mathrm{i}s\nu_i}}{\nu_i^*(\nu_i+\nu)}], & E<\widetilde{\omega}_1\\ 0, & E>\widetilde{\omega}_1.\end{cases}\end{aligned} \tag{6.98}$$

and by considering $\widetilde{\omega}_i<\widetilde{\omega}_j$, $\mathfrak{F}_{ij}$ for $i\neq j$ have the following form :

$$\begin{aligned}\mathcal{P}_s\mathfrak{F}_{ji}(\nu,E)=\mathrm{i}\,e^{-\mathrm{i}s\nu}&\left[\frac{-\lambda_i\lambda_j^*}{2\pi\nu_i(\nu_j^*-\nu)}\left(\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu_j^*}dy-\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu}dy\right)\right.\\&\left.+\frac{\lambda_i^*\lambda_j}{2\pi\nu_j^*(\nu+\nu_i)}\left(\int_{-\infty}^{0}\frac{e^{-sy}}{y-\mathrm{i}\nu_i}dy-\int_{-\infty}^{0}\frac{e^{-sy}}{y+\mathrm{i}\nu}dy\right)\right]\\&+\begin{cases}e^{-\mathrm{i}s\nu}[\frac{\lambda_i\lambda_j^*e^{\mathrm{i}s\nu_j^*}}{\nu_i(\nu_j^*-\nu)}-\frac{\lambda_i^*\lambda_je^{-\mathrm{i}s\nu_i}}{\nu_j^*(\nu_i+\nu)}], & E<\widetilde{\omega}_i\\ \lambda_i\lambda_j^*\,e^{-\mathrm{i}s\nu}\,\frac{e^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_j^*-\nu)}, & \widetilde{\omega}_i<E<\widetilde{\omega}_j\\ 0, & E>\widetilde{\omega}_j.\end{cases}\end{aligned} \tag{6.99}$$

In the equations (6.98) and (6.99) the non-integral terms yield the poles and lead to the resonance, and the integral terms yield an algebraical term analog to the background in the Hamiltonian theories [23]. We can also compute the same result for the case $\nu<0$. We will neglect the the background (the integrals terms). Then, the above equation is rewritten as:

$$\mathcal{P}_s\mathfrak{F}_{ii}(\nu,E)\simeq\begin{cases}|\lambda_i|^2\,e^{-\mathrm{i}s\nu}[\frac{e^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_i^*-\nu)}-\frac{e^{-\mathrm{i}s\nu_i}}{\nu_i^*(\nu_i+\nu)}], & E\leq\widetilde{\omega}_1\\ 0, & E>\widetilde{\omega}_1.\end{cases}$$

and for $i\neq j$

$$\mathcal{P}_s\mathfrak{F}_{ij}(\nu,E)\simeq\begin{cases}e^{-\mathrm{i}s\nu}[\frac{\lambda_i\lambda_j^*e^{\mathrm{i}s\nu_j^*}}{\nu_i(\nu_j^*-\nu)}-\frac{\lambda_i^*\lambda_je^{-\mathrm{i}s\nu_i}}{\nu_j^*(\nu_i+\nu)}], & E\leq\widetilde{\omega}_i\\ \lambda_i\lambda_j^*\,e^{-\mathrm{i}s\nu}\,\frac{e^{\mathrm{i}s\nu_i^*}}{\nu_i(\nu_j^*-\nu)}, & \widetilde{\omega}_i<E\leq\widetilde{\omega}_j\\ 0, & E>\widetilde{\omega}_j.\end{cases}$$

Now, we would compute the survival probability, i.e.

$$p_\rho(s) = \|\mathfrak{P}_\rho(s)\| = \| \, |\epsilon_1|^2 \mathcal{P}_s \mathfrak{F}_{11}(\nu, E) + \epsilon_1 \epsilon_2^* \mathcal{P}_s \mathfrak{F}_{12}(\nu, E) + \epsilon_2 \epsilon_1^* \mathcal{P}_s \mathfrak{F}_{21}(\nu, E) + |\epsilon_2|^2 \mathcal{P}_s \mathfrak{F}_{22}(\nu, E)\| \tag{6.100}$$

where $\| \cdot \| = \int_0^\infty dE \int_{-\infty}^\infty d\nu \, | \cdot |^2$. We see that $\mathfrak{P}_\rho(s)$ can be written as :

$$\mathfrak{P}_\rho(s) \simeq \begin{cases} e^{-\mathrm{i}s\nu} \Big[\left(\frac{\epsilon_1^* \lambda_1}{\nu_1} + \frac{\epsilon_2^* \lambda_2}{\nu_2} \right) \left(\frac{\epsilon_1 \lambda_1^* \, e^{\mathrm{i}s\nu_1^*}}{\nu_1^* - \nu} + \frac{\epsilon_2 \lambda_2^* \, e^{\mathrm{i}s\nu_2^*}}{\nu_2^* - \nu} \right) \\ \qquad - \left(\frac{\epsilon_1 \lambda_1}{\nu_1^*} + \frac{\epsilon_2 \lambda_2}{\nu_2^*} \right) \left(\frac{\epsilon_1^* \lambda_1^* \, e^{-\mathrm{i}s\nu_1}}{\nu_1 + \nu} + \frac{\epsilon_2^* \lambda_2^* \, e^{-\mathrm{i}s\nu_2}}{\nu_2 + \nu} \right) \Big] & E \leq \widetilde{\omega}_1, \\ e^{-\mathrm{i}s\nu} \Big[\left(\frac{\epsilon_1^* \lambda_1}{\nu_1} + \frac{\epsilon_2^* \lambda_2}{\nu_2} \right) \frac{\epsilon_2 \lambda_2^* \, e^{\mathrm{i}s\nu_2^*}}{(\nu_2^* - \nu)} - \left(\frac{\epsilon_1 \lambda_1}{\nu_1^*} + \frac{\epsilon_2 \lambda_2}{\nu_2^*} \right) \frac{\epsilon_2^* \lambda_2^* \, e^{-\mathrm{i}s\nu_2}}{(\nu_2 + \nu)} \Big], & \widetilde{\omega}_1 < E \leq \widetilde{\omega}_2 \\ 0, & E > \widetilde{\omega}_2 \end{cases} \tag{6.101}$$

Now, by remembering that $b_i = |\lambda_i|^2, \; (i = 1, 2)$, the square norm of $\mathfrak{P}_\rho(s)$ is obtained as:

$$|\mathfrak{P}_\rho(s)|^2 \simeq \begin{cases} \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 \Big[\frac{|\epsilon_1|^2 |\lambda_1|^2 \, e^{2b_1 s}}{|\nu_1^* - \nu|^2} + \frac{|\epsilon_2|^2 |\lambda_2|^2 \, e^{2b_2 s}}{|\nu_2^* - \nu|^2} + \frac{|\epsilon_1|^2 |\lambda_1|^2 \, e^{2b_1 s}}{|\nu_1 + \nu|^2} + \frac{|\epsilon_2|^2 |\lambda_2|^2 \, e^{2b_2 s}}{|\nu_2 + \nu|^2} \\ \qquad + e^{(b_1 + b_2)s} \left(\frac{\epsilon_1 \epsilon_2^* \lambda_1^* \lambda_2 e^{\mathrm{i}(a_1 - a_2)s}}{(\nu_1^* - \nu)(\nu_2 - \nu)} + \frac{\epsilon_1 \epsilon_2^* \lambda_1^* \lambda_2 e^{\mathrm{i}(a_1 - a_2)s}}{(\nu_1^* + \nu)(\nu_2 + \nu)} + \mathrm{C.C.} \right) \Big], & E \leq \widetilde{\omega}_1 \\ \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 \Big[\frac{|\epsilon_2|^2 |\lambda_2|^2 \, e^{2b_2 s}}{|\nu_2^* - \nu|^2} + \frac{|\epsilon_2|^2 |\lambda_2|^2 \, e^{2b_2 s}}{|\nu_2 + \nu|^2} \Big], & \widetilde{\omega}_1 < E \leq \widetilde{\omega}_2 \\ 0, & E > \widetilde{\omega}_2 \end{cases} \tag{6.102}$$

where the terms that oscillate with a frequency equal to the difference of the two masses, i.e. $(\widetilde{\omega}_2 - \widetilde{\omega}_1)$ is kept, the other decay terms oscillating with the frequency of one of the masses only are neglected since we have in the weak-coupling regime and the high-mass.

The integral over ν arrives at:

$$\int_{-\infty}^{\infty} d\nu \, |\mathfrak{P}_\rho(s)|^2 \simeq \begin{cases} 2\pi \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 \Big[|\epsilon_1|^2 e^{2b_1 s} + |\epsilon_2|^2 e^{2b_2 s} \\ \qquad + \left(\frac{2\mathrm{i}\epsilon_1^* \epsilon_2 \lambda_1^* \lambda_2 \, e^{(b_1 + b_2)s} \, e^{\mathrm{i}(\widetilde{\omega}_1 - \widetilde{\omega}_2)s}}{(\widetilde{\omega}_2 - \widetilde{\omega}_1) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.} \right) \Big], & E \leq \widetilde{\omega}_1 \\ 2\pi \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 |\epsilon_2|^2 \, e^{2b_2 s}, & \widetilde{\omega}_1 < E \leq \widetilde{\omega}_2 \\ 0, & E > \widetilde{\omega}_2 \end{cases} \tag{6.103}$$

Only the terms of the square norm are depended to E and we have

$$\left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 = \left| \frac{\epsilon_1 \lambda_1}{E - \widetilde{\omega}_1 + \mathrm{i}b_1} \right|^2 + \left| \frac{\epsilon_2 \lambda_2}{E - \widetilde{\omega}_2 + \mathrm{i}b_2} \right|^2 + \left(\frac{\epsilon_1^* \lambda_1^*}{(E - \widetilde{\omega}_1 + \mathrm{i}b_1)} \frac{\epsilon_2 \lambda_2}{(E - \widetilde{\omega}_2 - \mathrm{i}b_2)} + \mathrm{C.C.} \right) \tag{6.104}$$

The integral over E of the above expression is like the following integrals

$$\int dE \left| \frac{\sqrt{b_i}}{(E - \widetilde{\omega}_i) + \mathrm{i}b_i} \right|^2 = \arctan \left(\frac{E - \widetilde{\omega}_i}{b_i} \right) \tag{6.105}$$

and

$$\int dE \frac{\lambda_1^* \lambda_2}{(x - a_1 + \mathrm{i}b_1)(E - a_2 - \mathrm{i}b_2)} = \frac{-\lambda_1^* \lambda_2}{(\widetilde{\omega}_2 - \widetilde{\omega}_1) + \mathrm{i}(b_1 + b_2)} \Big(\mathrm{i} \arctan \frac{b_1}{E - \widetilde{\omega}_1}$$
$$+ \mathrm{i} \arctan \frac{b_2}{E - \widetilde{\omega}_2} + \log \sqrt{(E - \widetilde{\omega}_1)^2 + b_1^2} - \log \sqrt{(E - \widetilde{\omega}_2)^2 + b_2^2} \Big) \tag{6.106}$$

Now, we integrate from equation(6.104) over E from 0 to ∞. Firstly, for the interval $E \in [0, \widetilde{\omega}_1]$ we have

$$\begin{aligned}
\Im_1 &= \int_0^{\widetilde{\omega}_1} dE \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 = |\epsilon_1|^2 \arctan \frac{\widetilde{\omega}_1}{b_1} + |\epsilon_2|^2 \left(\arctan \frac{\widetilde{\omega}_2 - \widetilde{\omega}_1}{b_2} + \arctan \frac{\widetilde{\omega}_1}{b_2} \right) \\
&- \Bigg[\left(\frac{\epsilon_1^* \epsilon_2 \sqrt{b_1 b_2}}{(\widetilde{\omega}_1 - \widetilde{\omega}_2) + \mathrm{i}(b_1 + b_2)} \right) \Bigg(\mathrm{i}(\frac{\pi}{2} + \arctan \frac{b_1}{\widetilde{\omega}_1} + \arctan \frac{b_2}{\widetilde{\omega}_2} \\
&+ \arctan \frac{b_2}{\widetilde{\omega}_2 - \widetilde{\omega}_1}) + \frac{1}{2} \log \frac{b_1^2 (\widetilde{\omega}_2^2 + b_2^2)}{(\widetilde{\omega}_1^2 + b_1^2)((\widetilde{\omega}_2 - \widetilde{\omega}_1)^2 + b_2^2)} \Bigg) + \mathrm{C.C.} \Bigg].
\end{aligned} \tag{6.107}$$

For $E \in]\widetilde{\omega}_1, \widetilde{\omega}_2]$ we have

$$\begin{aligned}
\Im_2 &= \int_{\widetilde{\omega}_1}^{\widetilde{\omega}_2} dE \left| \frac{\epsilon_1 \lambda_1}{\nu_1} + \frac{\epsilon_2 \lambda_2}{\nu_2} \right|^2 = |\epsilon_1|^2 \arctan \frac{\widetilde{\omega}_2 - \widetilde{\omega}_1}{b_1} + |\epsilon_2|^2 \arctan \frac{\widetilde{\omega}_2 - \widetilde{\omega}_1}{b_2} \\
&- \Bigg[\left(\frac{\epsilon_1^* \epsilon_2 \sqrt{b_1 b_2}}{(\widetilde{\omega}_1 - \widetilde{\omega}_2) + \mathrm{i}(b_1 + b_2)} \right) \Bigg(\mathrm{i}(\arctan \frac{b_1}{\widetilde{\omega}_2 - \widetilde{\omega}_1} - \arctan \frac{b_2}{\widetilde{\omega}_2 - \widetilde{\omega}_1}) \\
&+ \frac{1}{2} \log \frac{b_1^2 b_2^2}{((\widetilde{\omega}_2 - \widetilde{\omega}_1)^2 + b_1^2)((\widetilde{\omega}_2 - \widetilde{\omega}_1)^2 + b_2^2)} \Bigg) + \mathrm{C.C.} \Bigg].
\end{aligned} \tag{6.108}$$

6.1 K-meson

For the weak-coupling constants we have $b_i \ll \widetilde{\omega}_i$, $(i = 1, 2)$ and also by supposing $\widetilde{\omega}_1 \sim \widetilde{\omega}_2$, $(\widetilde{\omega}_2 - \widetilde{\omega}_1) \sim b_1$ and $\frac{b_2}{b_1} \ll 1$, we have

$$\Im_1 \simeq \frac{\pi}{2} \left(|\epsilon_1|^2 + 2|\epsilon_2|^2 + \left(\frac{\epsilon_1^* \epsilon_2 \lambda_1^* \lambda_2}{(\widetilde{\omega}_1 - \widetilde{\omega}_2) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.} \right) \right) \approx \frac{\pi}{2} \tag{6.109}$$

$$\Im_2 \simeq \frac{\pi}{4} \left(|\epsilon_1|^2 + 2|\epsilon_2|^2 + \left(\frac{\epsilon_1^* \epsilon_2 \lambda_1^* \lambda_2}{(\widetilde{\omega}_1 - \widetilde{\omega}_2) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.} \right) \right) \approx \frac{\pi}{4} \tag{6.110}$$

where we used the normalization relation, i.e. $(|\epsilon_1|^2 + |\epsilon_2|^2) = 1$.

Finally, we obtain

$$\begin{aligned}
p_\rho(s) &\simeq \frac{\pi}{2} \left[|\epsilon_1|^2 e^{2b_1 s} + \frac{3}{2} |\epsilon_2|^2 e^{2b_2 s} + \left(\frac{\mathrm{i}\epsilon_1^* \epsilon_2 \lambda_1^* \lambda_2 \, e^{(b_1 + b_2)s} \, e^{\mathrm{i}(\widetilde{\omega}_1 - \widetilde{\omega}_2)s}}{(\widetilde{\omega}_2 - \widetilde{\omega}_1) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.} \right) \right] \\
&\simeq \frac{\pi}{2} |\epsilon_1|^2 \left[e^{2b_1 s} + \frac{3}{2} |\epsilon|^2 \, e^{2b_2 s} + \left(\frac{\mathrm{i}\epsilon \lambda_1^* \lambda_2 \, e^{(b_1 + b_2)s} \, e^{\mathrm{i}(\widetilde{\omega}_1 - \widetilde{\omega}_2)s}}{(\widetilde{\omega}_2 - \widetilde{\omega}_1) + \mathrm{i}(b_1 + b_2)} + \mathrm{C.C.} \right) \right].
\end{aligned} \tag{6.111}$$

where

$$\epsilon = |\epsilon| \, e^{\mathrm{i}\phi} := \frac{\epsilon_2}{\epsilon_1}. \tag{6.112}$$

The derivative of the equation (6.111) yields the time super-operator density of the probability or intensity:

$$I(s) := \frac{dp_\rho(s)}{ds} = I_0\left[e^{2b_1 s} + |\epsilon|^2 \frac{3}{2}\frac{b_2}{b_1}\, e^{2b_2 s} + 2|\epsilon|\sqrt{\frac{b_2}{b_1}}\, e^{(b_1+b_2)s}\, \cos((\widetilde{\omega}_1 - \widetilde{\omega}_2)s + \phi + \theta_2 - \theta_1)\right]. \tag{6.113}$$

where $I_0 = (\pi|\epsilon_1|^2 b_1)/2$ and $\lambda_i = \sqrt{b_i}\, e^{\mathrm{i}\theta_i},\ (i = 1, 2)$. This expression differs by $\frac{3}{2}$ term from the intensity derived previous by [11] from the integrated probability of decay of two exponentially decay process or relation (5.92) that we obtained in the Hilbert space which we call the time operator prediction.

Let us now evaluate the predictions related to the above equation in the different time intervals and let us compare them with the intensity introduced in the equation (4.63). Firstly, for $t = -s \sim 10 \times \tau_S$ or $t \gg \tau_S$ which that the term effective is: $|\epsilon|^2 \frac{3}{2}\frac{b_2}{b_1}\, e^{2b_2 s}$ and comparing for the same time with the equation (4.63) yields the CP-violation parameter is $|\epsilon|^2 \frac{3}{2}\frac{b_2}{b_1}$. Thus, the equations (6.113) and (4.63) for $t = -s \gg \tau_S$ can be written approximately as

$$I(s) \approx I_0 \left|\epsilon^{\mathrm{th}}\right|^2 e^{2b_2 s} \quad \text{and} \quad I(t) \approx I_0\, |\epsilon^{\mathrm{exp}}|^2\, e^{-\gamma_L t}, \quad (-s = t \gg \tau_S) \tag{6.114}$$

where

$$\epsilon^{\mathrm{th}} = \epsilon\sqrt{\frac{3}{2}\frac{b_2}{b_1}} \tag{6.115}$$

and the coefficient $\sqrt{\frac{3b_2}{2b_1}}$ in the above equation is the correction which is obtained by the time operator formalism and by using the condition $(\widetilde{\omega}_2 - \widetilde{\omega}_1) \sim b_1 \neq 0$, then $\mathfrak{I}_2 \neq 0$. Secondly, for the time of the order oft τ_S $(t < 5\tau_S)$ we have

$$I(s) \approx I_0\, e^{2b_1 s} \quad \text{and} \quad I(t) \approx I_0\, e^{-\gamma_S t}, \quad (t < 5\tau_S) \tag{6.116}$$

Finally, for intermediate times $(5\tau_S < t < 10\tau_S)$ we have

$$\begin{aligned} I(s) &\approx I_0|\epsilon|^2 \frac{b_2}{b_1}\, e^{2b_2 s}\, \cos((\widetilde{\omega}_1 - \widetilde{\omega}_2)s + \phi + \theta_2 - \theta_1) \quad \text{and} \\ I(t) &\approx I_0\, |\epsilon^{\mathrm{exp}}|^2\, e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\, \cos((m_L - m_L)s + \arg(\epsilon^{\mathrm{exp}})) \end{aligned} \tag{6.117}$$

The equations (6.114), (6.116), (6.117) and (5.77) yield

$$\begin{array}{ll} b_1 = \frac{\gamma_1}{2} = \frac{\Gamma_S}{2} = \frac{1}{2\tau_S}, & \widetilde{\omega}_1 = m_S, \\ b_2 = \frac{\gamma_2}{2} = \frac{\Gamma_L}{2} = \frac{1}{2\tau_L}, & \widetilde{\omega}_2 = m_L, \\ \theta_1 = \theta_S, & \theta_2 = \theta_L \end{array} \tag{6.118}$$

The ϵ is obtained in (5.85), thus, we have

$$\epsilon^{\mathrm{th}} = \left(\epsilon\sqrt{\frac{3}{2}\frac{\Gamma_L}{\Gamma_S}}\, e^{-\mathrm{i}\frac{\pi}{2}}\right) = \sqrt{\frac{3}{2}\frac{\Gamma_L}{\Gamma_S}}\, \frac{\frac{1}{2}}{\frac{\triangle m}{\Gamma_S} - \mathrm{i}\,\frac{\triangle\gamma}{2\Gamma_S}} \tag{6.119}$$

where $\triangle m = (m_L - m_S)$ and $\triangle\gamma = (\Gamma_L - \Gamma_S)$. Then, by replacing the experimental data we have

$$\epsilon^{\mathrm{th}} = 1.62 \times 10^{-3}\, e^{\mathrm{i}(46.77^\circ)} = 0.73\, \epsilon^{\mathrm{exp}} \tag{6.120}$$

6.2 B-meson

It easy to see that the integral $\mathfrak{I}_2$, for the B-mesons and D-mesons, is zero. So the intensity is written as:

$$I(s) = I_0 \left[e^{2b_1 s} + |\epsilon_1^{\text{th}}|^2 \, e^{2b_2 s} + 2|\epsilon_1^{\text{th}}| \, e^{(b_1+b_2)s} \, \cos((\widetilde{\omega}_1 - \widetilde{\omega}_2)s + \phi) \right] \tag{6.121}$$

where

$$\epsilon^{\text{th}} = \epsilon \sqrt{\frac{b_2}{b_1}} \, e^{-\mathrm{i}\frac{\pi}{2}} \tag{6.122}$$

This expression is the same, in the case of B and D particles in the time operator and in the super-operator approaches, and the theoretically estimated CP-violation parameter obeys the following equation

$$\epsilon^{\text{th}} = \epsilon \sqrt{\frac{\Gamma_L}{\Gamma_S}} \, e^{-\mathrm{i}\frac{\pi}{2}} = \frac{\Gamma_L}{\Gamma_S} \frac{\frac{1}{2}}{\frac{\triangle m}{\Gamma_S} - \mathrm{i}\,\frac{\triangle \gamma}{2\Gamma_S}} \tag{6.123}$$

which is not true in the case of K particles. Also for B and D particles the agreement with observations is quite good as we shall now check.

Another example is the CP-violation in the decay of B_s^0 and $\overline{\mathrm{B}}_s^0$. The experimental values are [24]

$$\frac{\triangle\Gamma_s}{2\Gamma_s} = 0.069^{+0.058}_{-0.062}, \qquad \frac{1}{\Gamma_s} = 1.470^{+0.026}_{-0.027} \text{ ps}, \tag{6.124}$$

or equivalently ($\Gamma_{L,H} = \Gamma_s \pm \triangle\Gamma_s/2$),

$$\frac{1}{\Gamma_L} = 1.419^{+0.039}_{-0.038} \text{ ps}, \qquad \frac{1}{\Gamma_H} = 1.525^{+0.062}_{-0.063} \text{ ps}, \tag{6.125}$$

and the difference of masses is

$$\triangle m = 17.7^{+6.4}_{-2.1} \text{ ps}^{-1} \tag{6.126}$$

and the experimental CP-violation parameter of the B meson is [24, 25]:

$$\mathcal{A}_{SL}^{\text{exp}} \simeq 4\mathcal{R}e(\epsilon_B^{\text{exp}}) = (-0.4 \pm 5.6) \times 10^{-3} \Rightarrow \left| \frac{q}{p} \right|^{\text{exp}} = 1.0002 \pm 0.0028. \tag{6.127}$$

where $\frac{\mathcal{A}_{SL}^{\text{exp}}}{2} \approx 1 - \left|\frac{q}{p}\right|^{\text{exp}}$. By replacing in the equation (6.123) we obtain:

$$\epsilon_B^{\text{th}} = \frac{\Gamma_L}{\Gamma_H} \frac{\frac{1}{2}}{\frac{\triangle m}{\Gamma_s} - \mathrm{i}\frac{\triangle\Gamma_s}{2\Gamma_s}} = 0.018 + 0.047 \times 10^{-3}\,\mathrm{i} \tag{6.128}$$

Thus, our theoretical $\left|\frac{q}{p}\right|^{\text{th}}$ prediction is:

$$\left| \frac{q}{p} \right|^{\text{th}} = \left| \frac{1 - \epsilon^{\text{th}}}{1 + \epsilon^{\text{th}}} \right| = 0.96 \tag{6.129}$$

which is in fairly good agreement with the experimental value.

6.3 D-meson

The other example is the CP-violation in the decay of D meson. The experimental values for CP-violation of $\mathrm{D}^0 \to \mathrm{K}^0_S\,\pi^+\,\pi^-$ as reported by Belle [26] are as follows:

$$\frac{\triangle\Gamma}{2\Gamma} = \left(0.37 \pm 0.25^{+0.07+0.07}_{-0.13-0.08}\right), \tag{6.130}$$

$$\frac{\triangle m}{\Gamma} = \left(0.81 \pm 0.30^{+0.10+0.09}_{-0.07-0.16}\right) \tag{6.131}$$

where $1/\Gamma = \tau,\ (\hbar = 1)$ is the mean life time

$$\frac{1}{\Gamma} = \tau = \frac{\tau_{\overline{\mathrm{D}}^0} + \tau_{\mathrm{D}^0}}{2} = (410.1 \pm 1.5) \times 10^{-3}\,\mathrm{ps} \tag{6.132}$$

The CP-violation parameters are experimentally denoted by $\left(\frac{q}{p}\right)$ and given by:

$$\left|\frac{q}{p}\right|^{\mathrm{exp}} = \left|\frac{1-\epsilon^{\mathrm{exp}}}{1+\epsilon^{\mathrm{exp}}}\right| = \left(0.86^{+0.30+0.06}_{-0.29-0.03}\right) \tag{6.133}$$

and

$$\phi^{\mathrm{exp}} = \arg\left(\frac{q}{p}\right)^{\mathrm{exp}} = \arg\left(\frac{1-\epsilon^{\mathrm{exp}}}{1+\epsilon^{\mathrm{exp}}}\right) = \left(-14^{+16+5+2}_{-18-3-4}\right)^{\circ}. \tag{6.134}$$

By replacing in the expression (6.123) we obtain

$$\epsilon^{\mathrm{th}} = (0.077 + 0.035\mathrm{i}). \tag{6.135}$$

Consequently,

$$\left|\frac{q}{p}\right|^{\mathrm{th}} = 0.86, \quad \phi^{\mathrm{th}} = -4.02^{\circ}. \tag{6.136}$$

which is once again in fairly good agreement with the experimental value.

7 Concluding remarks

About the relevance and novelty of our results

As we can see, the accuracy of the prediction (6.119) is comparable to the one that we derived within the Wigner-Weisskopf approach (5.92) (time operator instead of time super-operator). Now, as we said before, the present results were derived under the assumption that the spectrum of the continuous mode was not bounded by below (no cut-off). In a precedent publication [27], we considered the Friedrichs model with a Gaussian factor form and energy bounded by below (the spectrum of the continuous mode was assumed there to vary from 0 to $+\infty$). We showed that by introducing a cut-off in the coupling between discrete and continuous modes, the estimated value of ϵ slightly differs, depending on the shape that we impose to the cut-off. Therefore a fine tuning of the estimated CP-violation parameter is possible provided that the factor form is chosen conveniently. Considered so, the precision of the agreement with the measured value of the CP-violation parameter is not very convincing by it self (3 times the experimental value of the kaon CP-violation parameter [27]). What is convincing in our approach is that we obtain the right order of magnitude for the K, B and D particles altogether.

A crucial experiment for testing the validity of the Time Super-Operator (T) Formalism

The most important novelty of the time operator approach is, in our eyes, that it predicts that the distribution in time of the measured populations of pions pairs significantly differs from the predictions that could be made in the standard approach and/or in the Wigner-Weisskopf approach provided we make a fit over the full distribution (which means not only for times larger than the lifetime of the "Short" state but also for times comparable to it). Indeed, taking account of the three contributions of the distribution, which are the purely exponential, "Short" and "Long" contributions, and the oscillating contribution, one sees that the expression (6.113) radically differs from the expressions (4.63) and (5.91). This is due to the presence of the coefficient $\frac{3b_2}{2b_1}$ in the above equation which is the correction obtained by the time super-operator formalism and by using the condition $(\widetilde{\omega}_2 - \widetilde{\omega}_1) \sim b_1$, that is, $(m_L - m_s) \sim \Gamma_S$ for kaons. Since in the case of the B and D mesons, no such relation exists, the formula obtained here coincide with the one derived using the Wigner-Weisskopf time operator approach [11].

So, one can conceive crucial experiments that would allow to falsify the time operator approach and do not radically differ from the original Christenson experiment. These experiments require to measure the population of pairs of pions over a large range of times (distances to the source), and to check whether the best fit is provided by the expression (6.115) or by the expressions (4.63) and (5.91).

In principle these effects will be tested on the LHC at CERN in the coming months (years) so that the crucial experiment that we propose here is feasible in the future.

Concluding remark

The formalism of the mass-decay matrix for the kaon decay was introduced by LOY [4]. Then several other authors [8, 6, 22] improved this model. The LOY model requires the Wigner-Weisskopf approximation, i.e. it requires to assume that the energy interval varies from $-\infty$ to $+\infty$ and also that the coupling between discrete and continuous modes is not restricted by a factor form or cut-off.

In [22], we used the 2-level Friedriche model and the Wigner-Weisskopf approach to obtain a mass-decay matrix. This approach was improved by using a new concept of probability decay density for mesons in [11]. Beyond the Wigner-Weisskopf approximation, we used the Friedrichs model with a cutoff that amounts to bound from below the energy spectrum of the Hamiltonian [27]. In the present paper, we derived the decay probability density in the formalism of the time super-operator, that also goes beyond the Wigner-Weisskopf approximation.

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