



# Map-Matching Integrity using Multi-Sensor Fusion and Multi-Hypothesis Road Tracking

Maged Jabbour, Philippe Bonnifait and Véronique Cherfaoui

Heudiasyc UMR CNRS 6599, Université de Technologie de Compiègne, France

{Maged.Jabbour, Philippe.Bonnifait, Veronique.Cherfaoui}@hds.utc.fr

**Abstract** – Efficient and reliable map matching algorithms are essential for *Advanced Driver Assistance Systems*. While most of the existing solutions fail to provide trustworthy outputs when the situation is ambiguous (road intersections, roundabouts, parallel roads ...), we present in this paper a new map-matching method based on a multi-hypothesis road tracking that takes advantage of the geographical database road connectedness to provide a reliable road-matching solution with a confidence indicator that can be used for integrity monitoring purposes.

*Index Terms* – GNSS-based Localization, Map-Matching, Multi-Hypothesis Tracking, Integrity.

## I. INTRODUCTION

Map-Matching (MM), using GNSS positioning and navigable maps, is a data association problem which consists in selecting the most likely road that corresponds to the current position of the mobile [17]. Unfortunately, because of large estimation errors, MM often has several solutions, i.e. several segments are declared candidates with good confidence. These segments can belong to the same road or to different roads in case of ambiguous situation. On the contrary and as a result of inaccuracies in the map or in case of a vehicle driving off-road, MM can have no solution. Therefore, MM confidence or MM integrity is a crucial issue for many Intelligent Transportation Systems (ITS) applications like Map-Aided Advanced Driver Assistance Systems that should be designed to contribute to the safety of life. In practice, any MM algorithm should be able to deliver confidence indicators. If these indicators exceed pre-defined thresholds, the end-user should be warned that the solution provided is not reliable.

To cope with this integrity problem, we propose in this paper a multi-hypothesis road tracking method that attempts to exploit data pertaining to road-connectedness. This approach belongs to the class of dynamic state observers, and therefore makes use of multisensor fusion capabilities.

Tracking techniques [12] allow a system to observe and follow the state of a mobile target by filtering noisy observations. They have very efficient implementations since they often rely on first order Markov assumption, which means that all the information can be captured in the current state estimation. Therefore, it is unnecessary to keep in memory a window of data; by using a recursive scheme, previous states can be forgotten.

For localization purposes, tracking the pose (position and attitude) of a mobile is very useful since it allows fusing sequentially redundant data, once the initial global localisation stage has been solved. Indeed, in practice model equations are non linear, and an arbitrary initialization can conduct to a wrong convergence.

The spatial road network data can be also used to improve the positioning accuracy, for instance when GPS is not available. Indeed, the road network can be used to constraint the localization space (geometry) and to predict the next future (connectivity). Therefore, a problem is to integrate such navigable map information in the localization tracker.

Map-matching induces unavoidable ambiguity situations for instance at junctions or with parallel roads, or when GPS suffers from outages. By applying a mono-hypothesis approach, the risk is to choose a wrong solution. When the system will detect this mistake, it will need time to recover the good solution and the tracking will be reset. A multi-hypotheses approach, on the contrary, will maintain all the possible solutions in case of ambiguity; each hypothesis lives in its own world ignoring the other ones. Hypotheses that become unlikely are removed as time and travelled distance evolve. Using a Bayesian framework, it is possible to quantify the probability of the hypotheses. So, at each step, the most probable hypothesis can be output. The main advantage of Multi-Hypotheses Map-Matching (MHMM) over a Mono-Hypothesis approach is that the true solution is tracked with a high probability: if the current solution is declared incorrect, the system can output immediately a new solution without any transient phase.

In general, algorithmic complexity of MHMM is exponential since each hypothesis can generate at each step new hypotheses. In this paper, we propose to use the road connectivity information of the navigable map to solve this issue in order to create new hypotheses only when necessary. We present a MHMM based on a Gaussian mixture that consists in associating an Electronic Horizon (EH) to each hypothesis that performs a Gaussian filter. The associate EH is a set of two roads that the hypothesis is supposed to follow. A weight (called also score) is associated to each hypothesis for the management of the hypotheses set. It indicates the probability of each hypothesis with respect to the others.

Finally, we propose a strategy to monitor MM integrity. Our proposal is to declare the MM confident when there is

only a credible hypothesis and when a test on the normalized residuals of this hypothesis is below a threshold.

In the paper organized in 5 sections, we present the different elements of this strategy and propose finally a new MM integrity criterion that has been tested under real conditions using a natural GPS receiver, a gyrometer, an odometer and a NavTeQ database. Experimental results illustrate the performance of this approach.

## II. LOCALIZATION WITH AN A PRIORI CARTOGRAPHICAL INFORMATION ON THE LOCALIZATION SPACE

Suppose that a map information source is available. This map provides a *a priori* information that constraints the localization space. For example, a car has a better chance of being on a road, and unlikely to go through a building. The cartographic information considered here is a set of roads described by nodes connected to each other. Each road is made of a begin node and an end node, with several intermediate points.

We formalize in this section the problem of using a *a priori* cartographic information in the localization process. We will show that the map can be used as an observation (like any exteroceptive measurement) in state observation process.

Suppose that  $s_k$  represents the mobile state vector at time  $k$ ;  $z_k$  is an exteroceptive sensor observation (a GPS for example).

$$\begin{cases} s_k = f(s_{k-1}, u_k) + \alpha_k \\ z_k = h(s_k) + \beta_k \end{cases} \quad (1)$$

The localization problem consists in estimating the probability  $p(s_k | z^k, g, u^k)$ , knowing the set of observations  $z^k = \{z_k, \dots, z_1\}$ , and the *a priori* geographical information  $g$ .

Let's see how this geographical information can be used in order to estimate this probability density.

$$p(s_k | z^k, g, u^k) = p(s_k | z_k, z^{k-1}, g, u^k) \quad (2)$$

Using Bayes theorem, eq. (2) can be also written like:

$$p(s_k | z_k, z^{k-1}, g, u^k) = \frac{p(z_k | s_k, z^{k-1}, g, u^k) \cdot p(s_k | z^{k-1}, g, u^k)}{p(z_k | z^{k-1}, g, u^k)} \quad (3)$$

The denominator  $p(z_k | z^{k-1}, g, u^k)$  is independent from  $s_k$ . it can be considered as a normalization term  $\eta$ . (3) becomes:

$$p(s_k | z_k, z^{k-1}, g, u^k) = \eta \cdot p(z_k | s_k, z^{k-1}, g, u^k) \cdot p(s_k | z^{k-1}, g, u^k) \quad (4)$$

Let's consider now each of the two expressions of this product.

The observation  $z_k$  at time  $k$  is independent of all the previous  $z^{k-1}$ , the observation noise being a white one. By remarking also that the exteroceptive sensor noise is independent of the map  $g$ , we can write:

$$p(z_k | s_k, z^{k-1}, g, u^k) = p(z_k | s_k) \quad (5)$$

Let's consider now the second term of the product and let's make the density a priori  $p(s_k | z^{k-1}, g, u^k)$  appear by using the total probabilities theorem and the Bayes theorem:

$$p(s_k | z^{k-1}, g, u^k) = \int p(s_k, s_{k-1} | z^{k-1}, g, u^k) ds_{k-1} \quad (6)$$

$$= \int p(s_k | s_{k-1}, z^{k-1}, g, u^k) \cdot p(s_{k-1} | z^{k-1}, g, u^k) ds_{k-1} \quad (7)$$

However,  $p(s_k | s_{k-1}, z^{k-1}, g, u^k)$  represents the evolution model. It is independent from the observations  $z^k$  and under the assumption of a 1<sup>st</sup> order Markov process, it only depends from the current entry  $u_k$ .

$$p(s_k | z^{k-1}, g, u^k) = \int p(s_k | s_{k-1}, g, u_k) \cdot p(s_{k-1} | z^{k-1}, g, u^{k-1}) ds_{k-1} \quad (8)$$

Let's substitute (5) and (8) into equation (4):

$$\underbrace{p(s_k | z^k, g, u^k)}_{\text{Localization at } k} = \eta \cdot p(z_k | s_k) \cdot \int p(s_k | s_{k-1}, g, u_k) \cdot \underbrace{p(s_{k-1} | z^{k-1}, g, u^{k-1}) ds_{k-1}}_{\text{Localization at } k-1} \quad (9)$$

Let's consider now the term  $p(s_k | s_{k-1}, g, u_k)$  that expresses the influence of the *a priori* information in the localization process: it can be used in the prediction step [13], [14], or considered as an observation as we proposed.

Using Bayes theorem, one can write:

$$p(s_k | s_{k-1}, g, u_k) = \frac{p(g | s_k, s_{k-1}) \cdot p(s_k | s_{k-1}, u_k)}{p(g | s_{k-1})} \quad (10)$$

By supposing that the cartographic observation does not depend on the current pose and by considering that the map is a 1<sup>st</sup> order Markov process, we can write:

$$p(g | s_k, s_{k-1}) = p(g | s_k) \quad (11)$$

$$p(g | s_{k-1}) = p(g) \quad (12)$$

To make these two assumptions valid, it is necessary that the vehicle moves relatively to the map ( $s_k \neq s_{k-1}$ ). By making substitutions in the equation (10), we obtain:

$$p(s_k | s_{k-1}, g, u_k) = \frac{p(g | s_k) \cdot p(s_k | s_{k-1}, u_k)}{p(g)} \quad (13)$$

Equation (8) becomes:

$$p(s_k | z^{k-1}, g, u^k) = \int \frac{p(g | s_k) \cdot p(s_k | s_{k-1}, u_k)}{p(g)} \cdot p(s_{k-1} | z^{k-1}, g, u^{k-1}) ds_{k-1} \quad (14)$$

$$= \frac{p(g | s_k)}{p(g)} \int p(s_k | s_{k-1}, u_k) \cdot p(s_{k-1} | z^{k-1}, g, u^{k-1}) ds_{k-1} \quad (15)$$

By introducing  $\eta' = \frac{\eta}{p(g)}$ , Eq. (4) can thus be written as:

$$p(s_k | z^k, u^k) = \eta' \cdot p(z_k | s_k) \cdot p(g | s_k) \cdot \int p(s_k | s_{k-1}, u_k) \cdot p(s_{k-1} | z^{k-1}, g, u^{k-1}) \cdot ds_{k-1} \quad (16)$$

In this expression  $p(z_k | s_k)$  and  $p(g | s_k)$  represent respectively the likelihood of the exteroceptive observation and those of the map  $g$  relatively to the predicted position  $s_k$ . The map thus is well considered here as an observation. As an example, let us consider a map made of with one segment representing a road on which the vehicle moves. This case is represented by Fig. 1 which illustrates  $p(g | s_k)$ , with  $g$  being the map.

Let us consider the line  $\Delta$  passing by  $h(s_k)$  (where  $h(s_k)$  is the projection of the state  $s_k$  in 2D map observation space) and perpendicular to the segment in question. Let us suppose that along  $\Delta$  the density of probability  $p(g | s_k)$  is Gaussian. The likelihood is obtained by calculating the innovation  $\mu$  (which is the deviation with the road here) and by using it in the density of Gaussian probability.

One can notice that the probability density function can have any shape. In this case, approximated by a Gaussian mixture of.

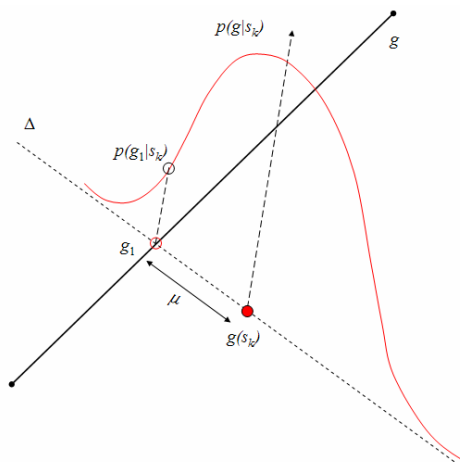


Fig. 1 Map probability with a Gaussian probability density.

### III. USING A MAP-MATCHING METHOD BASED ON A MULTI- HYPOTHESIS APPROACH

MM is a data association problem. Data association in dynamical situations is a key issue in Multi-Target Tracking (MTT) systems [3]. The stage of association consists in establishing a correlation between the predicted targets (called tracks) and the observations detected by the sensors. Multi-Hypothesis Tracking MHT is one of promising solutions of the multi targets tracking problem. It chooses at every stage the most likely solution of the tracking problem while retaining some other hypotheses for future assumptions. MHT supports the creation and destruction of tracks. By definition, a track is the state vector of a target, with a covariance and a score and updated with an

observation, while a hypothesis is a set of compatible tracks. The likelihood of a track is determined by a score maintained as a Log-Likelihood Ratio (LLR) [18]. Different implementations of MHT algorithms are described in [12]. Because the number of tracks/hypothesis can grow exponentially, ad-hoc methods can reduce the combinatory: pruning techniques allow deleting tracks/hypotheses with low probabilities and merging techniques putting together similar tracks.

In the following, MHMM tracks a single vehicle position by using multiple hypotheses. So, it is a single target tracking case, and so it is not necessary to distinguish between track and hypothesis terms. To clarify this concept, a definition of hypothesis is given and the methods for hypothesis creation and deletion are described. The track/hypothesis score (weight) is computed and maintained with the same manner as in MHT. We show in details the management policy of the hypotheses by associating with each Gaussian filter an EH made up with 2 roads.

#### A. Hypothesis definition

The EH associated with each hypothesis consists of two roads (each one being a poly-line): the current road and a next one connected to it. There are two main advantages. Firstly, there is no discontinuity when approaching the end of the current road. Secondly, the MM with this EH is extremely simple since it is a poly-line. Because of the usual length of the roads, it is unnecessary to keep in memory more than two roads: between to samples, the distance travelled by the track is limited to few meters.

A hypothesis  $F_i$  at time  $k$  is defined as being composed of the elements shown in table 1.

$F_i$ : Localization Hypothesis
★ A state: a state vector $s_{i,k}$ and its associated covariance matrix $P_{i,k}$
★ A electronic horizon $g_i$ that includes the road of the mother-hypothesis $R_{idm}$ and an upcoming one $R_{idf}$ , connected to it $g_i = \{R_{idm,i}, R_{idf,i}\}$
★ A weight (score) $w_{i,k}$ , showing (after normalization) the importance of hypotheses, one relatively to each other
★ An absolute confidence indicator $vr_{i,k}$ quantifying confidence in this hypothesis.

Table 1. Definition of a localization hypothesis

#### B. Hypothesis Creation

An important issue is to consider an efficient strategy when a hypothesis comes to the end of its road.

Let's suppose that the location of a hypothesis approaches the end its EH, and let's assume that the current road is connected to two upcoming roads. A first idea is to duplicate the actual hypothesis into two others: each one corresponding to the upcoming roads. The EH associated

with each hypothesis includes the actual road and one of the two upcoming ones. If no more likely path (precomputed route for instance) is available, please note that at the time of duplication the Gaussians have the same weight.

Another idea is to clone the current hypothesis with anticipation. This is essential in order to take into account the map and estimation errors.

More generally, let's suppose that at a moment  $k$ , a hypothesis  $i$  designated by  $F_{i,k}$  ( $s_{i,k}$ ,  $P_{i,k}$ ,  $g_i$ ,  $w_{i,k}$ ,  $vr_{i,k}$ ) arrives at the a distance  $\Delta$  from the end of its EH  $g_i$ . The hypothesis  $F_{i,k}$  is divided into a number of new hypotheses. The information on the number of roads connected to the end of the actual road  $n_c$  is stored in the map structure: the number of created hypotheses is equal to the number of roads connected  $n_c$ . For  $j = 1$  to  $n_c$ , each new hypothesis  $j$  gets the same weight as the mother-hypothesis  $i$  and the same state at the time of creation (ie state vector  $s_{i,k}$  and covariance matrix  $P_{i,k}$ ). The new EH  $g_j$  associated with each new hypothesis  $j$  contains a road from the EH  $R_{idf_j}$  (roads connected to the end of the current segment) and the road associated to the mother runway  $R_{idf_i}$ , road on which the hypothesis  $F_i$  was evolving (One could write  $R_{idm,j} = R_{idf,i}$ ). Please, note that the new hypotheses do not keep the road  $R_{idm,i}$  of their mother hypothesis because the size of each EH  $g_j$  would then increase endlessly. Moreover, as the likelihood  $vr_{i,k}$  associated with each hypothesis  $k$  is computed at each time, it makes no sense to transmit it.

After transmitting its characteristics to the new created hypotheses, the mother hypothesis  $F_{i,k}$  is eliminated. A normalizing step for the weights  $w_k$  is then carried out.

To illustrate the EH management associated with the hypotheses, consider the case of a simple intersection of three roads as shown on Fig. 2. Let's consider that the hypothesis  $F_i$ , associated with the EH  $\{ID_0, ID_1\}$  has reached the threshold distance  $\Delta$  to the end of road  $ID_1$ . Two hypotheses  $F_m$  and  $F_n$  as created from the properties of  $F_i$ . The EH associated with  $F_m$  and  $F_n$ , will be respectively composed of  $\{ID_1, ID_2\}$  and  $\{ID_1, ID_3\}$ .

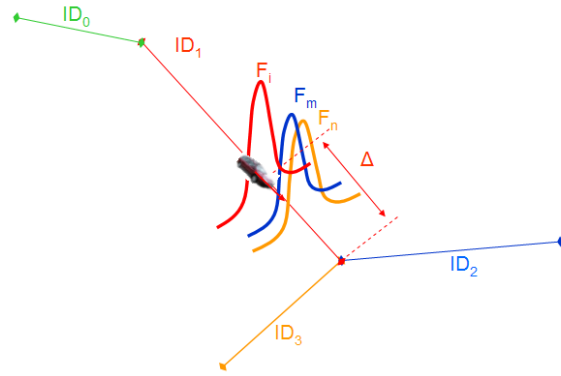


Fig. 2 Illustration of a 3 roads intersection situation

### C. Hypothesis Deletion

As soon as a hypothesis's weight falls below a fixed deletion threshold  $\zeta_{el}$ , we consider that the hypothesis is no more credible and it is eliminated. To avoid the elimination of a credible hypothesis  $F_i$  whose instant likelihood  $vr_{i,k}$  may decrease excessively at time  $k$ , because of an inappropriate observation, for example, that will make its weight  $w_{i,k}$  falls below the deletion threshold  $\zeta_{el}$ , we propose to filter the computing weight  $w_{i,k}$ :

$$w_{i,k} = vr_{i,k} \cdot w_{i,k-1} + L_{mem} \cdot w_{i,k-1} \quad (17)$$

Where  $L_{mem}$  is a forget factor that quantifies the part of the former  $w_{k-1}$  that is injected in  $w_k$ .  $L_{mem}$  must verify  $0 < L_{mem} < 1$ . Typically, one can chose  $L_{mem} = 0.1$ . Please note that threshold  $\zeta_{el}$  is a parameter that has to be tuned respectively with the map offset.

### D. Detecting the tracking divergence

The tracking divergence can occur when all hypotheses are mistaken and become far away from the observations that update these hypotheses.

In usual conditions, if a hypothesis  $F_i$  moves away from the updating observations, its instant likelihood  $vr_i$  will decrease in the update stage, and thus its weight  $w_i$  will also decrease. In the case where all available hypotheses move away from the updating observations, all their likelihoods, and then their weights will decrease, but as a normalizing step follows, the decrease of the weights will be no longer effective.

So, in order to detect the system divergence, a non-normalized sum of weights must be done, and the decision of the system detection divergence must be undertaken based on this computed sum. Besides, in order to not let intermittent outlier observations trigger the divergence detection, this sum is carried out on a time interval  $\Delta t$ . If the non-normalized computed sum stays below a fixed threshold  $\delta_{div}$  during a  $\Delta t$  interval, a re-initialization of the system is undertaken with the first valid GPS data based on the mechanism shown.

Please note that the system re-initialisation is a case that occurs rarely. It is often due to the significant offsets of the digital maps in some places but also mismanagement of the road-map cache.

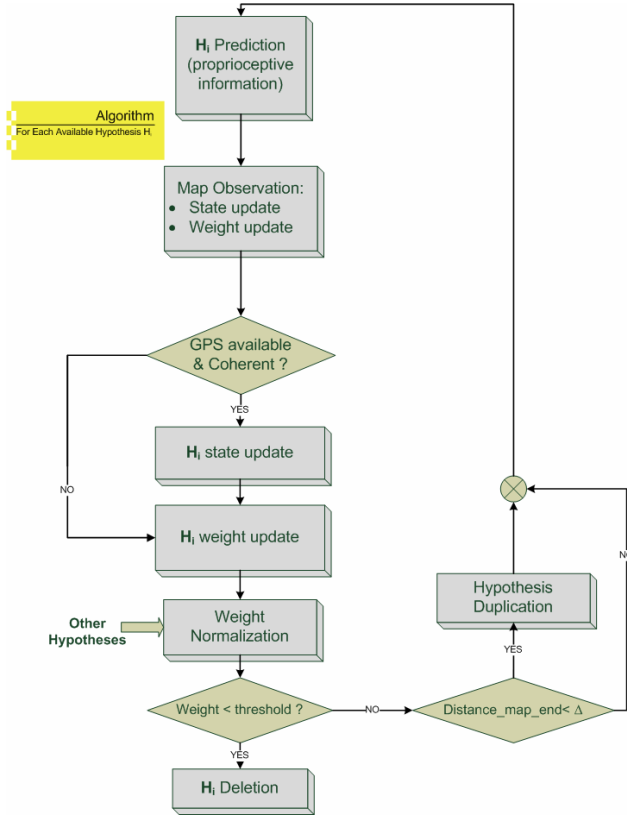


Fig. 3 Synoptic of MHMM

### E. Estimating the vehicle location from the different hypotheses

Several solutions can be proposed to achieve the estimated map-matching from the different hypotheses at time  $k$ . We propose to select a set of credible hypotheses as output: The normalization of the weights  $w_{j,k}$  with respect to the maximum of the weights is the metric that is compared to a fixed threshold  $\delta_{imp}$  to characterize the probable output hypotheses.

$$(\bar{X}_k, \bar{P}_k) = \left\{ (X_{j,k}, P_{j,k}) \mid \frac{w_{j,k}}{\max_{i=1:N} (w_{i,k})} \geq \delta_{imp} \right\} \quad (18)$$

The threshold  $\delta_{imp}$  must be chosen in some optimal way. If  $\delta_{imp}$  is too small, an important number of hypotheses (including unlikely ones) will be proposed as outputs. On the contrary, a high  $\delta_{imp}$  will reduce the number of likely hypotheses, to zero, one or two.

In the particle filters literature, the notion of “effective particles” is often used to trig a new process of particles

resampling. Let us defined the number of effective particles as:

$$N_{eff,k} = \frac{1}{\sum_{i=1}^N w_{i,k}^2} \quad (19)$$

In [5], the authors propose that if  $N_{eff}$  becomes less than two-thirds of the total number of particles  $N$ , then the particles must be resampled. We have adapted this concept to the case of the Gaussian particles. We have linked the threshold  $\delta_{imp}$  with  $N_{eff}$ :

$$\delta_{imp} = 1/(2 \cdot N_{eff}) \quad (20)$$

### F. Practical consideration

Let’s suppose that at time  $k$ , we have  $N$  hypotheses: for all the available hypotheses  $F_i$  ( $i=1:N$ ), each hypothesis has its own Kalman filter. The travelled distance and heading rotation are first obtained from the dead reckoning sensors. If the vehicle is moving, the correction of the previous prediction is computed using the previous GPS fix.

Suppose now that the system is running under normal tracking operation (after the initialization stage). If we keep all the hypotheses, their number will increase without bound, given that, at the end of each road-segment, each hypothesis will be divided into at least two. We have set a maximum number of hypotheses (denoted  $N_{pmax}$ ). Typically, the values of  $N_{pmax}$  range from 8 to 16 hypotheses. When the total number of hypotheses exceeds  $N_{pmax}$ , we keep the  $N_{pmax}$  hypotheses regarding to their score  $w_i$ .

### G. Update step

Using the result demonstrated in section II, we have two separate exteroceptive observations: GPS and map observations. To compute efficiently the weights, the update steps are serialized under the reasonable hypothesis that there is no correlation between the errors. So, in the update step, every hypothesis’ state is corrected by the two observations, and thus the weights  $w_{i,k}$  are also updated and normalized as many times as there are observations. We prefer to use a hybridized GPS instead of a standalone GPS receiver to overcome the problems of GPS jumps and especially to the low availability of GPS in urban areas. If there is a masking and thanks to the navigation using the dead-reckoning prediction, hybridized GPS continues to provide exteroceptive observation to the MHMM system and the different hypotheses continue to be updated in terms of weight and state.

It is important to remark that the map data of the EH is always coherent with its hypothesis. However, a hypothesis can rapidly become inconsistent with the GPS (if it is a wrong hypothesis). Thus, we have implemented a Chi-2 test with the hybridized GPS before making the correction of the hypothesis pose in case of any inconsistency with the

GPS. Nevertheless, the weights are always updated in order to make the confidence decrease (see Algo. 1).

The weights  $w_i$ , the likelihoods  $vr_i$ , and the filters' estimates are updated by the two exteroceptive sources. The weights of the filters are updated by the hybridized GPS location. The weight of the wrong hypotheses will decrease step after step. The likelihoods, characterizing absolute confidence in the hypotheses, will also be changing in the same way but more rapidly than the weights (which are cumulative normalized probabilities). The likelihoods can be interpreted as an indicator of the overall consistency of the system, as a normalized residual quantity.

Another important characteristic of the map-matching is its *spatial* nature. Many approaches rely on data fusion approaches that suppose independence of the errors. If the vehicle is motionless, the same map data can be used several times, violating the independence hypothesis. For these reasons, the map-matching can be formulated by a state space description space-triggered: if the travelled distance between 2 time-steps (also called abscissa curvilinear) is smaller than a threshold, then the map-updating step is not done.

#### H. Segment selection

Each hypothesis has its own map: it's a EH composed of 2 roads. The road-matching method consists in selecting the nearest segment whose direction is coherent with the vehicle's heading. The orthogonal projection is considered as the map-matched position and used as an exteroceptive observation by the corresponding hypothesis filter. This mechanism is done at each time step.

##### I. Efficient Map Update Implementation

Let see how the map correction stage is done using an observation computed from the EH.

Let's suppose first that a candidate segment has been selected. Consider the following:

- $s_{i,k} = [x_{i,k} \ y_{i,k} \ \theta_{i,k}]^T$  is the pose of the hypothesis
- $Z_{i,k} = [x_{i,k} \ y_{i,k}]^T$  is a point that corresponds to the projection of the estimated position onto the most likely segment (see eq. (21) and Fig. 4).

$$Z_{i,k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot s_{i,k} \quad (21)$$

The fusion of the estimate with the map is performed during the Kalman estimation stage. The covariance associated with the map observation is modeled by an ellipsoid around the selected segment as shown in Figure 4 [2]. The center of the ellipsoid is  $Y_m$ , the orthogonal projection on the segment of the last estimated location. In the frame associated with the segment, the longitudinal

inaccuracy is far greater than the lateral inaccuracy. Theoretically, the longitudinal inaccuracy can be chosen as large as possible, even infinite for a long segment. In practice, we consider a one-sigma value in the order of the length of the segment.

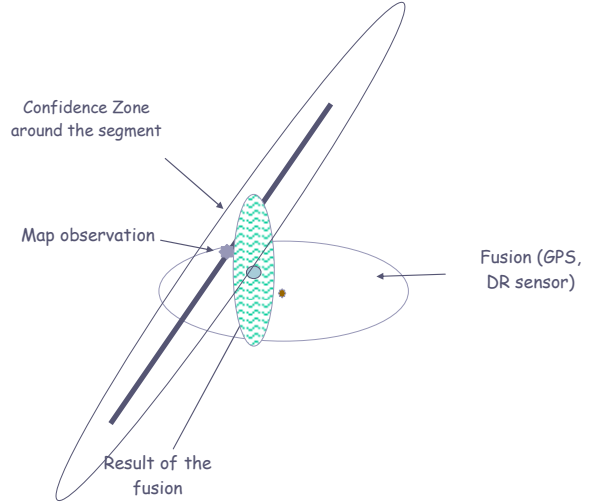


Fig. 4. Fusion of a hypothesis' estimate with the selected segment

### J. Complete Algorithm

1 For every hypothesis  $H_i$  at time  $k$ :

- 1.1. Prediction step using proprioceptive sensors
 
$$F_{i,s} = \left[ \frac{\partial f}{\partial s} (\hat{s}_{i,k-1|k-1}, u_k) \right], \quad F_u = \left[ \frac{\partial f}{\partial u} (\hat{s}_{i,k-1|k-1}, u_k) \right]$$

$$\hat{s}_{i,k|k-1} = f(\hat{s}_{i,k-1|k-1}, u_k)$$

$$P_{i,k|k-1} = F_{i,s} \cdot P_{i,k-1|k-1} \cdot F_{i,s}^T + F_u \cdot Q_u \cdot F_u^T + Q_\alpha$$
- 1.2. Map update step  
 Compute the map observation using the  $H_i$ 's EH  
 Update the state using this observation
 
$$J_{i,k} = \left[ \frac{\partial h}{\partial s} (\hat{s}_{i,k|k-1}) \right]$$

$$K_{i,k} = J_{i,k} \cdot P_{i,k|k-1} \cdot (J_{i,k} \cdot P_{i,k|k-1} \cdot J_{i,k}^T + Q_\beta)^{-1}$$

$$\hat{s}_{i,k|k} = \hat{s}_{i,k|k-1} + K_{i,k} \cdot (z_k - h(\hat{s}_{i,k|k-1}, u_k))$$

$$P_{i,k|k} = (I - K_{i,k} J_{i,k}) \cdot P_{i,k|k-1} \cdot (I - K_{i,k} J_{i,k})^T + K_{i,k} \cdot Q_\beta \cdot K_{i,k}^T$$
 Update the weight using the likelihood of the map
 
$$w'_{i,k} = \eta \cdot w_{i,k-1} \cdot N(z_k; h_k(s_{i,k|k-1}), J_k \cdot P_{i,k-1|k-1} \cdot J_k^T + R_k)$$
- 1.3. GPS update step, if a hybrid GPS fix is available  
 If the fix is coherent with the current hypothesis  $H_i$   
 Update the state using the hybrid GPS
 
$$K_{i,k} = J_{i,s} \cdot P_{i,k|k-1} \cdot (J_{i,s} \cdot P_{i,k|k-1} \cdot J_{i,s}^T + Q_\beta)^{-1}$$

$$\hat{s}_{i,k|k} = \hat{s}_{i,k|k-1} + K_{i,k} \cdot (z_k - h(\hat{s}_{i,k|k-1}, u_k))$$

$$P_{i,k|k} = (I - K_{i,k} J_{i,k}) \cdot P_{i,k|k-1} \cdot (I - K_{i,k} J_{i,k})^T + K_{i,k} \cdot Q_\beta \cdot K_{i,k}^T$$
 End If  
 Update the weight using the likelihood of the GPS
 
$$w_{i,k} = \eta \cdot w'_{i,k} \cdot N(z_k; h_k(s_{i,k|k-1}), J_k \cdot P_{i,k-1|k-1} \cdot J_k^T + R_k)$$
- 1.4. Hypotheses creation using connectivity  
 If  $H_i$ 's position  $< \Delta$  from the end of its EH  
 Duplicate  $H_i$  as many times as connected roads  
 Delete  $H_i$   
 End if

2. Normalize the weight of every hypothesis

$$w_{i,k} = \frac{w_{i,k}}{\sum_{j=1}^N w_{j,k}}$$

3. Hypotheses management  
 Sort the  $H_i$  with respect to their weight  
 Keep the  $N$   $H_i$  that have the highest weight  
 Delete the others

4 Decision stage  
 Compute the number of efficient hypotheses

**Algorithm 1.** MHMM Algorithm

$s_{i,k}$  and  $P_{i,k}$  represent respectively the state vector and the covariance matrix of the hypothesis  $i$ ,  $z_k$  is an observation,  $f$  and  $h$  are the evolution and observation equations,  $K$  is the filter gain,  $N(a,b)$  is a normal distribution of mean  $a$  and variance  $b$ .

### IV. INTEGRITY ISSUES

Nowadays, integrity is a fundamental characteristic of localization systems. For some ITS applications, integrity can be more important than precision.

By definition, integrity of a localization system is the measure of confidence that can be accorded to the exactitude of the positioning delivered by this system.

In practice, integrity means applying successive checks to ensure that the information is valid. A good example of this concept is RAIM (Receiver Autonomous Integrity Monitoring) [1], which is a technique to verify the consistency of the current GPS navigation solution when pseudorange redundancy exists (more than 5 satellites). The principle of a snapshot RAIM is to monitor normalized residuals using a threshold computed with a Chi Square distribution, under Gaussian assumptions and given selected False Alarm and Miss Detection probabilities.

Since snapshot methods are not adapted to dynamic sensor fusion, several results [4] indicate that integrity can be assured by checking the consistency of the innovation signal of a state observer. MM integrity can therefore be monitored using normalized residuals or innovations between candidate segments and the current estimated pose (position and heading). Unfortunately, as a result of inaccuracies in the map or because of large estimation errors, map-matching often has several solutions, i.e. several segments are declared candidates with good confidence. Applying a snapshot-like integrity test for MM is therefore often pessimistic, since at each sample time several candidates can be declared safe. To tackle this problem, MHMM is very useful since it is able to take benefit of the road-connectedness information, and so quantify the confidence of each hypothesis with respect to the others.

Our proposal is to declare MM confident when there is a hypothesis that is much more likely than the others and when a test on the normalized innovations of this hypothesis is below a consistency threshold. In [16], the authors propose to monitor 3 indicators to check integrity of MM (distance residuals, heading residuals, and an indicator related to uncertainty of MM). Since a mono-hypothesis scheme is used, they propose to fuse the 3 indicators using fuzzy rules. So, the integrity monitoring is done using this scalar value.

With MHMM, integrity monitoring is a different since there are two different criteria. First, we use the number of efficient hypothesis (called  $N_{eff}$  in Eq.19) and a Normalized Innovation Squared (NIS) similar to a Mahalanobis distance in position and heading. This innovation information is computed between the hybrid GPS and the predicted state of the most likely hypothesis. Algorithm 2 summarises this integrity monitoring strategy.

```

Threshold1= fct(probability ratio between the hypotheses)
Threshold2=Chi2Inv(False Alarm Probability)
Confidence=0
If Neff< Threshold1
  If NIS of the most likely hypothesis < threshold2 Then
    Confidence=1 // MM output is confident
  End If
End If
    
```

Algorithm 2. Integrity monitoring.

## V. RESULTS

Experiments have been performed in Compiègne using a KVH fibre optic gyro, an odometer input and a Trimble AgGPS 132 (L1-only receiver). The GIS used by the map-matching module is based on a Software Development Kit (SDK) provided by BeNomad [8]. The maps are size-optimized and provided in the SVS (Scalable Vector System) file format. For our prototype, we have used a geographical database converted in SVS format. The SVS format is very compact since the file size of all the town of Compiègne is only 68 KB and the one of the complete OISE department is only 3 MB. As a source digital map, we have used a NavteQ database. In this map, coordinates are expressed in the French Lambert 93 projection system.

We report hereafter results that we have obtained on a 5.7 Km test (see Fig. 5) with the car shown on Figure 6.

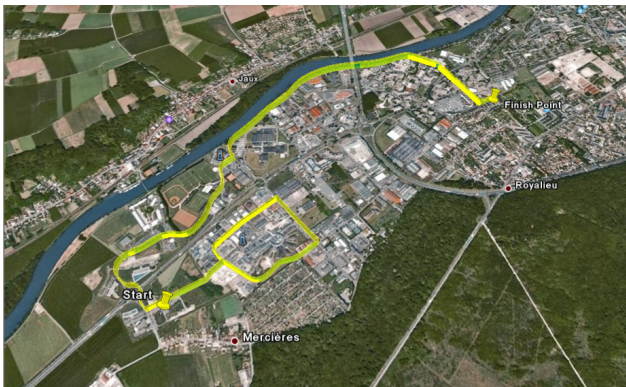


Fig. 5. Overview of the test site with the trajectory



Fig. 6. Car used in the experiments

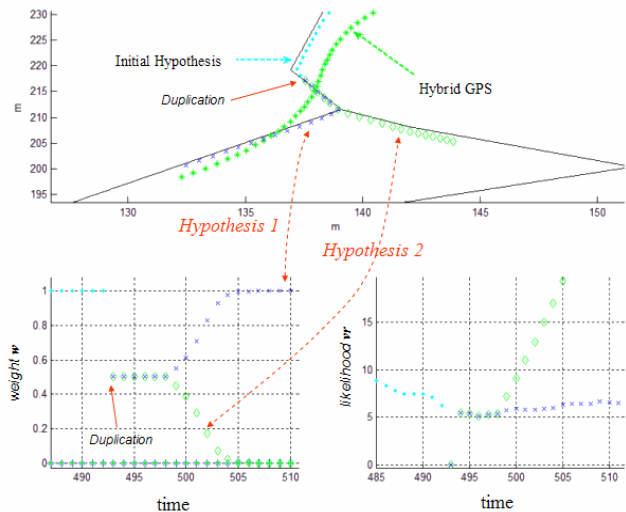


Fig. 7. Hypothesis creation at a road-intersection

To illustrate the MHMM mechanism at intersections, Figure 7 shows a real case. An initial hypothesis (shown in dotted, light blue lines) arrives at the threshold distance  $\Delta$  (here  $\Delta=7m$ ) from the end of the road. This road-end is connected to two different roads. The initial hypothesis is therefore divided into two new hypotheses (one is shown in dark blue 'x' and the other in green '◇'). The hybridized GPS is shown in green '\*'. The evolution of weight  $w$  and instantaneous absolute likelihoods of the two created hypotheses  $vr$  are shown respectively in the curves on the lower left and lower right.

Figure 8 shows, on the left part, all the hypotheses during an on-road test in Compiègne. On the right, the most likely hypothesis is shown at each moment of this trial.

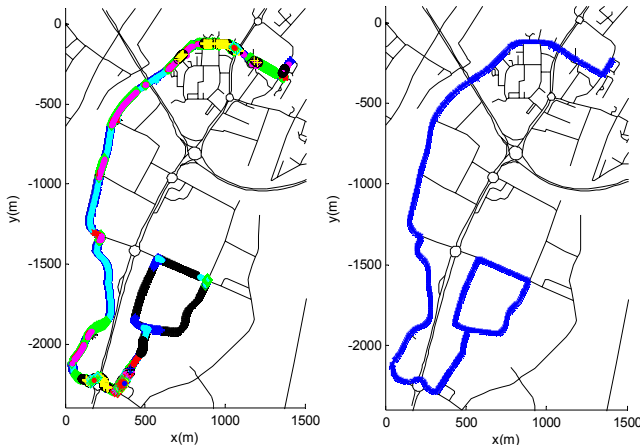


Fig. 8 hypotheses and the most likely one during an on-road trial.

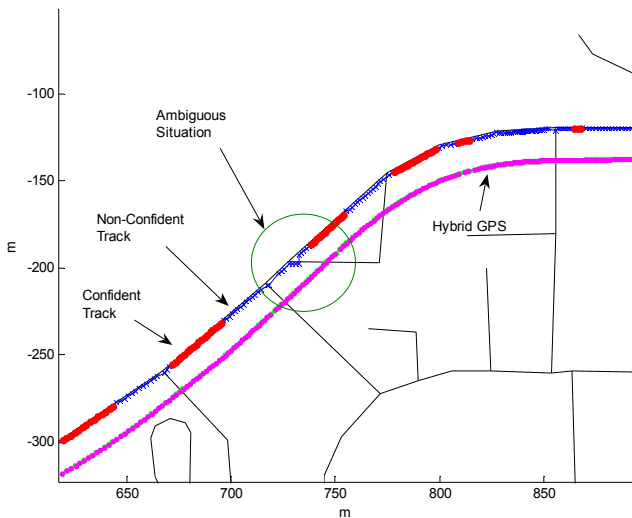


Fig. 9 Most likely hypothesis, integrity and hybrid GPS

By replaying the data slowly, we have analyzed the results provided by the MHMM, since we know exactly the roads that were travelled during the experiment.

Figure 9 shows the integrity computation result on the most difficult part of the trial (upper part of Fig. 8). The Most Likely Hypothesis (denoted by MLH) is shown in blue, the hybrid GPS in magenta. We can clearly see that the map offset with respect to the GPS. When the MLH is shown in bold red, it means that it is considered confident. The integrity indicator is here the bold red color. Please look at the ambiguous situation pointed by the circle. Because of the map offset, the MLH is not the appropriate one. Nevertheless, the confidence indicator clearly indicates that the output is not likely. This correctly corresponds to the ground truth.

Figure 10 shows the result of the number of efficient hypotheses  $N_{eff}$  during the test. Different values of  $N_{eff}$  were matched up with the following geographical cases:  $N_{eff} \cong 1$

is often obtained for the case where the vehicle is running on segment, far from an intersection, with the associated runway having a fairly large weight.  $N_{eff} \cong 2, 3, 4$  is generally obtained when approaching an intersection, with, respectively, 2, 3, 4 roads in the upcoming intersection.

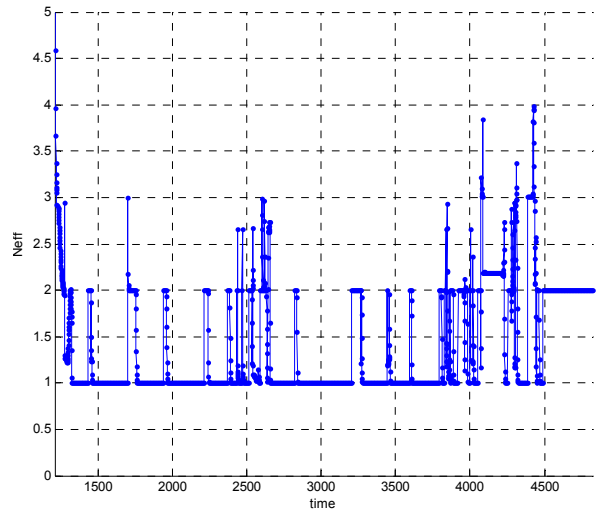


Fig. 10 Number of effective filter during the road test

With nominal settings, the percentage that the most likely hypothesis corresponds effectively to the real position of the vehicle obtained during this test is 97% of good matches. We have checked that the wrong matches correspond to ambiguous situations correctly detected by the MHMM.

We have noticed that the NIS threshold is not so sensitive. In practice, we have used a constant value corresponding to a  $Threshold_2 = 6$  (which corresponds to a False Alarm probability of 0.95 under Gaussian hypothesis).

Let us study, how the threshold on the number of effective hypotheses ( $Threshold_1$ ) has an effect on the results of the integrity monitoring. For that we consider the False Alarm (FA) and Missed Detections (MD) rates like done in [16]. We declare that there is a false alarm when the integrity computation asserts that the system is non-confident, while its output is correct. A missed detection is observed when the computation indicates that integrity is checked while it is not.

Let us define FAR the FA rate and MDR the MD rate. The Overall Correct Detection Rate is defined as  $OCDR = 1 - FAR - MDR$  [16].

Figure 11 and Table 2 shows respectively the FAR and the MDR and the FA and MD, for different  $N_{eff}$  thresholds. We clearly see that if the threshold on  $N_{eff}$  is high (more permissive), the FA number decreases, but the number of MD increases. The OCDR, which is a global indicator on the good behaviour of the integrity computation, increases also.

$N_{eff}$ Threshold	FA	MD	OCDR
1.1	536	4	85.25
1.2	527	5	85.47
1.5	467	6	87.08
1.7	403	7	88.80
1.9	395	15	88.80

Table 2. Statistics for 3661 samples.

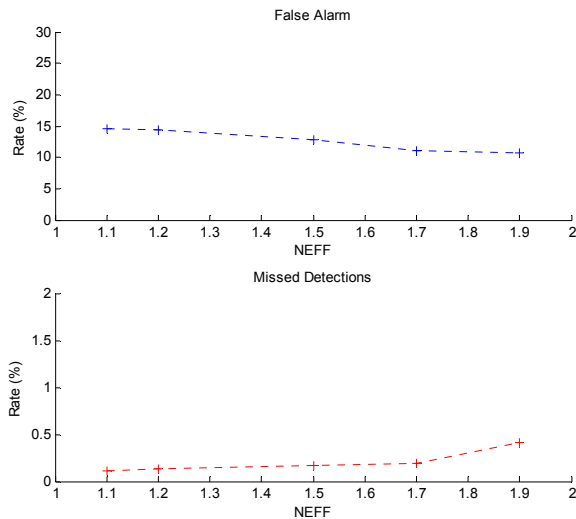


Fig. 11. False Alarms and Missed detections versus the number of effective hypotheses threshold.

We can observe that our integrity monitoring is cautious, since the FAR is high, which induces an availability of 85-88% for the MM. We can observe also that the MDR is low (<0,5%) which is a good performance. If this criterion is crucial, the  $N_{eff}$  threshold has to be tune to a low value close to 1.

## VI. CONCLUSION

This paper has presented a map-matching method that relies on multi-hypothesis tracking for on-road vehicles. This method fuses proprioceptive sensors with GPS and map information. The main idea behind this approach is to associate a hypothesis to each newly encountered road after an intersection or a roundabout. The likelihood of each available hypothesis is evaluated by computing a recursive weight or score through an instantaneous likelihood that updates the hypotheses' weight. We have proposed an integrity monitoring strategy that relies on two indicators. The decision rule we have proposed considers the estimated location consistency with the map and the probability of the hypotheses with respect to the others to handle ambiguity zones. Real tests were carried out on real road conditions and results illustrate the performance of the method.

A direct perspective of this research is to consider the approach of intersections, since we have remarked that our

strategy is too cautious in such cases. An idea is to merge the closed hypotheses before applying the decision rule.

**Acknowledgement:** This research has been carried out within the framework of the European FP6 Integrated Project CVIS, *Cooperative Vehicle Infrastructure Systems*, started in February 2006 for 4 years.

## REFERENCES

- [1] B. Belabbas and F.Gass. "Raim algorithms analysis for a combined gps/galileo constellation". ION-GNSS 2005, Long Beach, USA, 13-16 september 2005. pp 1781-1788
- [2] M. El Badaoui El Najjar, Ph. Bonnifait. "A Road-Matching Method for Precise Vehicle Localization using Kalman Filtering and Belief Theory." Journal of Autonomous Robots, Volume 19, Issue 2, September 2005, Pages 173-191.
- [3] Y. Bar-Shalom "Multitarget-Multisensor tracking: Applications and Advances", vol.III. Artech House. 2000
- [4] S. Feng, S. and W. Ochieng (2007). "Integrity of Navigation System for Road Transport". Proceedings of the 14th World Congress on Intelligent Transport Systems, 9-13, October, 2007, Beijing
- [5] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, P. Nordlund. "Particle filters for positioning, navigation, and tracking". IEEE Transactions on Signal Processing, vol. 50, pp. 425- 435, Feb. 2002.
- [6] M. Jabbour, Ph. Bonnifait, V. Cherfaoui. "Enhanced Local Maps in a GIS for a Precise Localisation in Urban Areas", 9th International IEEE Conference on Intelligent Transportation Systems (ITSC 06), Toronto, Canada, September 17-20, 2006.
- [7] See <http://www.navteq.com/> and <http://www.teleatlas.com>
- [8] See <http://www.benomad.com/>
- [9] M. Kais, Ph. Bonnifait, D. Bétaille, F. Peyret. "Development of Loosely-Coupled FOG/DGPS and FOG/RTK Systems for ADAS and a Methodology to Assess their Real-Time Performance". IEEE Intelligent Vehicles Symposium (IV 05), Las Vegas, June 2005 pp. 358-363.
- [10] P.R. Muro-Medrano, D. Infante, J. Guillo, J. Zarazaga, J.A. Banares "A CORBA infrastructure to provide distributed GPS data in real time to GIS applications". Computers, Environment and Urban Systems, n°23, 1999 pp. 271-285.
- [11] Svenzen Niklas, "Real Time Implementation of Map Aided Positioning using a Bayesian Approach". Master Thesis, Linköping University, Dec. 2002.
- [12] S. Blackman and R. Popoli, "Design and Analysis of Modern Tracking Systems, Artech House Books, 1999, ISBN: 1580530060
- [13] S.M. Oh, S. Tariq, B. N. Walker, and F. Dellaert, "Map-based Priors for Localization", in the proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2004), Sendai, Japan, 28 Sept - 02 Oct 2004.
- [14] Y. Cui and S.S. Ge, "Autonomous Vehicle Positioning With GPS in Urban Canyon Environments", in the IEEE Transactions on Robotics and Automation, Vol. 19, N. 1 February 2003.
- [15] P. Piniés, J. D. Tardós, "Fast Localization of Avalanche victims using Sum of Gaussians", Proceedings of the 2006 IEEE International Conference on Robotics and Automation. Orlando, Florida, May 2006.

- [16] M. A. Quddus , W. Y. Ochieng, R. B. Noland, "Integrity of map-matching algorithms", *Transportation Research Part C* 14 (2006) 283–302
- [17] M. A. Quddus, W. Y. Ochieng, R. B. Noland. "Current map-matching algorithms for transport applications: State-of-the art and future research directions", *Transportation Research Part C* 15 (2007) pp. 312–328.
- [18] R.W Sittler, "An optimal data association problem in surveillance theory" *IEEE transactions on military electronics*, April 1964.