Comments on "Direct Calculation of Minimum Set of Inertial Parameters of Serial Robots"

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Abstract: The paper presented by Gautier and Khalil [1] gives a direct and efficient method to calculate most of the minimum inertial parameters of serial robots. Some parameters concerning the translational links between the first rotational joints which are not parallel need particular calculation, partial results concerning the case where these links are either perpendicular or parallel are given in the paper [1]. This correspondence presents a direct solution to this particular case, such that all the minimum inertial parameters can be obtained directly without calculating the energy or the dynamic model of any link.

I- Calculation of the minimum inertial parameters

The minimum inertial parameters can be calculated from the standard inertial parameters by eliminating the parameters which have no effect on the dynamic model and those which can be regrouped to some others. The expression of the total energy (potential +kinetic) which is linear in the inertial parameters can be used to determine these parameters, it can be seen that the energy of the robot is given as:

$$H = \sum_{j=1}^{n} \mathbf{h}^j \, \mathbf{X}^j \qquad (1)$$

where: $\mathbf{X}^j$ represent the standard inertial parameters of link j, and $\mathbf{h}^j$ contains the coefficients of the inertial parameters in the energy expression.

It can be seen that [1]:

$$\mathbf{X}^j = [\ XX_j \quad XY_j \quad XZ_j \quad YY_j \quad YZ_j \quad ZZj \quad mX_j \quad mY_j \quad mZ_j \quad m_j\ ]$$

$$\mathbf{h}^j = [\ h_{XXj} \quad h_{XYj} \quad h_{XZj} \quad h_{YYj} \quad h_{YZj} \quad h_{ZZj} \quad h_{mXj} \quad h_{mYj} \quad h_{mZj} \quad h_{mj}\ ]$$

The expressions of the elements of $\mathbf{h}^j$, can be found in [2].

Based on relation (1) the following results can be given:

a- An inertial parameter X_i has no effect on the dynamic model if:

$$h_i = \text{constant} \qquad (2)$$

with h_i is coefficient of X_i in the total energy (potential and kinetic) of the robot.

b-An inertial parameter X_i can be regrouped to some others $X_{i1},\ldots, X_{ir}$ if :

$$h_i = \sum_{p=i_1}^{i_r} \alpha_p \, h_p + \text{constant} \qquad (3)$$

where α_p is constant.

In this case the parameter X_i can be eliminated while the parameters X_{i1}, X_{i2}, ... X_{ir}, will be replaced by $XR_p = X_p + \alpha_p X_i$. In this case we say that X_i has been regrouped to $X_{i1},\ldots, X_{ir}$.

Conditions (2) and (3) are equivalent to conditions (4) and (5) in [1].

It has been pointed out in [1] that the inertial parameters satisfying condition (2) belong to the links near the base side, some results which permit to calculate most of these parameters without the calculation of the energy are also given in [1]. These results will be completed in this correspondence.

Relations (15) and (16) of the paper of Gautier and Khalil [1] permit to regroup the parameters YY_j, mZ_j and m_j if joint j is rotational and XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j if joint j is translational. This result gives most of the minimum inertial parameters.

Supposing that r_1 is the first rotational joint and r_2 is the first rotational joint not parallel to r_1 more parameters may be regrouped. Partial results are given in [1] if the joint axes between r_1 and r_2 are either parallel or perpendicular. In this comment the general results will be given.

II- Particular regrouping of the inertial parameters between r1 and r2

Two cases are to be considered:

<u>a- the axis of a translational link j is not parallel to the axis of r1 for (r1 < j < r2)</u>

Noting that the projection of $\mathbf{h}_{mSj} = [\; h_{mXj} \quad h_{mYj} \quad h_{mZj} \;]$ on the axis of joint r_1 is constant, the following linear relation is obtained :

$$\boxed{{}^{j}a_{xr1}\; h_{mXj} + {}^{j}a_{yr1}\; h_{mYj} + {}^{j}a_{zr1}\; h_{mZj} = \text{constant}} \qquad (4)$$

where ${}^{j}\mathbf{a}_{r1} = [\,{}^{j}a_{xr1} \quad {}^{j}a_{yr1} \quad {}^{j}a_{zr}]^T$ is the unit vector of link r_1 axis referred to frame j.

The regrouping relation or elimination are given in table 1 .

${}^{j}a_{zr1} \neq 0$	${}^{j}a_{zr1} = 0, {}^{j}a_{xr1}\, {}^{j}a_{yr1} \neq 0$	${}^{j}a_{zr1} = 0, {}^{j}a_{xr1} = 0$	${}^{j}a_{zr1} = 0, {}^{j}a_{yr1} = 0$
$mXR_j = mX_j - \frac{{}^{j}a_{xr1}}{{}^{j}a_{zr1}} mZ_j$ $mYR_j = mY_j - \frac{{}^{j}a_{yr1}}{{}^{j}a_{zr1}} mZ_j$	$mXR_j = mX_j - \frac{{}^{j}a_{xr1}}{{}^{j}a_{yr1}} mY_j$	$mY_j = 0$	$mX_j = 0$

Table 1

This mean that a parameter will be always regrouped or has no effect on the dynamic model in this case.

b) the axis of a translational link j is parallel to the axis of r1 for (r1 < j < r2)

The following two results are given:
i- As the x and y rotational velocity componentes of link j are equal to zero then mZ_j has no effect on the dynamic model.
ii- Assuming the nearest rotational joint for j to the base side is i, then the following relation between $\mathbf{h}_{msj}$ and $\mathbf{h}_{msj-1}$ is obtained:

$$\mathbf{h}_{mSj}^{T} = {}^{j}\mathbf{A}_{j-1}\, \mathbf{h}_{mSj-1}^{T} - [\ 2P_x\, h_{ZZi} \quad 2P_y\, h_{ZZi} \quad 0\ ]^T \tag{5}$$

with:

$${}^{j}\mathbf{P}_{j-1} = [\ P_x \quad P_y \quad P_z\ ]^T = [\ -d_j\ C\theta_j \quad d_j\, S\theta_j \quad -r_j\ ]^T$$

thus, the parameters mX_j and mY_j can be regrouped using the following relations:

$$\begin{aligned}
mXR_{j-1} &= mX_{j-1} + C\theta_j\, mX_j - S\theta_j\, mY_j \\
mYR_{j-1} &= mY_{j-1} + S\theta_j\, C\alpha_j\, mX_j + C\theta_j\, C\alpha_j\, mY_j \\
mZR_{j-1} &= mZ_{j-1} + S\theta_j\, S\alpha_j\, mX_j + C\theta_j\, S\alpha_j\, mY_j \\
ZZR_i &= ZZ_i + 2\, d_j\, C\theta_j\, mX_j - 2\, d_j\, S\theta_j\, mY_j
\end{aligned} \tag{6}$$

where α_j, d_j, r_j, θ_j are the geometric parameters defining frame j with respect to frame j-1 according to the modified Denavit and Hartenberg notations [3].

III.Practical Calculation of the minimum parameters

The following rules permit to define all the parameters which will be regrouped or eliminated, the rest of the parameters constitute the minimum inertial parameters (base or identifiable parameters) of the dynamic model.

1- Use the general regrouping relations (15) and (16) in [1] to eliminate the following parameters:

a- YY_j , mZ_j , m_j if joint j is rotational for j =n,...1,

b- XX_j , XY_j , XZ_j , YY_j , YZ_j , ZZ_j , if joint j is translational for j =n,...1,

2- Eliminate mZ_j and regroup mX_j and mY_j using (6) if j is translational and $\mathbf{a}_j$ // $\mathbf{a}_{r1}$ for $r_1 < j < r_2$

3- Regroup or eliminate one of the parameters mX_j , mY_j , mZ_j, if $\mathbf{a}_{r1}$ is not parallel to $\mathbf{a}_j$, and j is translational and $r_1 < j < r_2$, using table 1.

4- Eliminate XX_j , XY_j , XZ_j , YZ_j if j is rotational and $r_1 \leq j < r_2$ (the axes of these joints are parallel to r_1). It is to be noted that YY_1 has also been eliminated in rule 1.

5- Eliminate mX_j , mY_j, they have no effect when j is rotational for ($r_1 \leq j < r_2$) and ($\mathbf{a}_j$ is along $\mathbf{a}_{r1}$) and ($\mathbf{a}_{r1}$ // $\mathbf{a}_i$ // ${}^0\mathbf{g}$ for all $i < j$). With ${}^0\mathbf{g}$ denotes the acceleration of gravity with respect to frame 0. It is to be noted that mZ_j has also been eliminated in rule 1.

6- Eliminate mX_j, mY_j, mZ_j they have no effect, when $j < r_1$ (they represent the translational links before r_1, where the angular velocity is equal to zero) .

The main advantage of this method with respect to recently proposed method [4], is that the given minimum parameters can be used directly in classical Newton-Euler algorithm to calculate the dynamic model. The given results complete also the algorithm calculating the minimum inertial parameters of tree structure robots given in [5] such that all the minimum parameters can be obtained directly.

References

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