

# Time-varying non linear modeling of electrodynamic loudspeakers

R. Ravaud <sup>a</sup>, G. Lemarquand <sup>a,\*</sup> and T. Roussel

<sup>a</sup>*Laboratoire d'Acoustique de l'Universite du Maine, UMR CNRS 6613, Avenue  
Olivier Messiaen, 72085 Le Mans Cedex 9, France*

---

## Abstract

This paper deals with the time-varying nonlinear analytical modeling of the electrodynamic loudspeaker. We propose a model which takes into account the variations of Small signal parameters. The six Small signal parameters ( $R_e$ ,  $L_e$ ,  $Bl$ ,  $R_{ms}$ ,  $M_{ms}$ ,  $C_{ms}$ ) depend on both time and input current. The electrodynamic loudspeaker is characterized by the electrical impedance which, precisely measured, allows us to construct polynomial functions for each Small signal parameter. By using this analytical model, we propose to compare two identical electrodynamic loudspeakers. One of them is supposed to be run in and the other one is not. The experimental methodology is based on a precise measurement. In all the paper, the time scale is assumed to be much longer than one period of the harmonic excitation.

*Key words:* Loudspeaker, Electrodynamic, Electrical Impedance

*PACS:* 43-38 Ja

---

\* Corresponding author.

*Email address:* [guy.lemarquand@univ-lemans.fr](mailto:guy.lemarquand@univ-lemans.fr) (G. Lemarquand).

## 1 Introduction

2 The reference model describing the electrodynamic loudspeaker designed by  
3 Thiele and Small [1] predicts that the electrodynamic loudspeaker is both a  
4 linear system and a stationary one. This analytical model is very useful since  
5 it is simple to use. However, an electrodynamic loudspeaker exhibits nonlin-  
6 earities which depend on time. Some authors, such as A.J.M Kaiser [2] and  
7 W.Klippel [3] [4], have studied the nonlinearities of electrodynamic loudspeak-  
8 ers. These nonlinearities have become better and better known [5] [6] and some  
9 authors have proposed a new structure of loudspeaker with an ironless motor  
10 and without any outer rims and spider [7] in order to eliminate these nonlin-  
11 earities.

12 The other drawback of an electrodynamic loudspeaker is that it is an time-  
13 varying system [8]. Indeed, the electrical resistance  $R_e$  increases in time due  
14 to the heat produced by the voice coil. Then, the compliance  $C_{ms}$  depends  
15 on time since the outer rim and the spider become more elastic because of  
16 the heat produced by the resistance. The Small signal model using lumped  
17 parameters does not forecast these time-varying phenomena, and such an  
18 time-varying analytic model taking into account these properties does not  
19 exist. In this paper, we put forward a way of characterizing experimentally  
20 the time dependence and the level dependence of the Small signal parame-  
21 ters. This experimental characterization allows us to compare two identical  
22 electrodynamic loudspeakers. One of them is supposed to be run in and the  
23 other one is not. The knowledge of the time necessary to break-in an electro-  
24 dynamic loudspeaker is very important because this element of information  
25 gives indications about the physical properties of both the mechanical stiffness

26  $k$  and the mechanical damping parameter  $R_{ms}$ . The first section presents the  
27 Small signal model using lumped parameters and the main nonlinearities of  
28 an electrodynamic loudspeaker. The second section presents the experimental  
29 methodology to identify the variations of the Small signal parameters. In the  
30 third section, the time dependence of the Small signal parameters and its con-  
31 sequences are discussed. The last section presents an analytical model which  
32 takes into account the variations of the Small signal parameters in time and  
33 according to the input current.

## 34 **2 The Small signal model using lumped parameters and its limits**

### 35 *2.1 The Small signal model using lumped parameters*

36 According to the Small signal model using lumped parameters, two cou-  
37 pled differential equations are necessary to describe the electrodynamic loud-  
38 speaker. One of them is called the electrical differential equation and is given  
39 by:

$$40 \quad u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + Bl \frac{dx(t)}{dt} \quad (1)$$

41 The other one is called the mechanical differential equation and is given by:

$$42 \quad M_{ms} \frac{d^2 x(t)}{dt^2} = Bl i(t) - R_{ms} \frac{dx(t)}{dt} - \frac{1}{C_{ms}} x(t) \quad (2)$$

43 The parameters used in Eqs.(1) and (2) are the following:

44  $i(t)$ =coil current [A]

45  $u(t)$ =input voltage [V]

46  $x(t)$ =position of voice coil [ $m$ ]

47  $Bl$ =electrodynamic driving parameter [ $T.m$ ]

48  $R_{ms}$ =mechanical damping parameter and drag force [ $N.s.m^{-1}$ ]

49  $C_{ms}$ =mechanical compliance of suspension(spider, outer rim)[ $m.N^{-1}$ ]

50  $M_{ms}$ =equivalent mass of moving voice coil, cone, air[ $Kg$ ]

51  $R_e$ =electrical resistance of voice coil[ $\Omega$ ]

52  $L_e$ =inductance of voice coil [ $H$ ]

53 Eqs. (1) and (2) allow us to define the electrodynamic loudspeaker electrical  
54 impedance  $Z_e$  which is expressed as follows:

$$55 \quad Z_e = R_e + jL_e w + \frac{Bl^2}{R_{ms} + jM_{ms}w + \frac{1}{jC_{ms}w}} \quad (3)$$

56 Eq. (3) is well known and is often used to describe the electrodynamic loud-  
57 speaker. However, Eq.(3) does not forecast the distortions created by an elec-  
58 trodynamic loudspeaker and the time dependence of the Small signal parame-  
59 ters. Moreover, if we take into account the eddy currents [9] which occur when  
60 the input frequency increases, the electrical impedance  $Z_e$  should be written  
61 as follows:

$$Z_e(i, t) = R_e(i, t) + \frac{jR_\mu(i, t)L_e(i, t)w}{jL_e(i, t)w + R_\mu(i, t)} \\ + \frac{Bl(i, t)^2}{R_{ms}(i, t) + jM_{ms}(i, t)w + \frac{1}{jC_{ms}(i, t)w}} \quad (4)$$

62 where  $R_\mu(i, t)$  is the eddy current resistance.  $Z_e(i, t)$  is a time-varying nonlin-  
63 ear transfer function; at each time and for different input currents, its value  
64 changes. In Eq. (4), we assume all the parameters depend on both time and  
65 input current. Strictly speaking, these dependences exist but it is very diffi-  
66 cult to find them experimentally and to predict them analytically. All these

67 parameters have not the same sensitivity both to input current and to time.  
68 Moreover, some parameters vary a lot with the input current but do not create  
69 important distortions

## 70 *2.2 Nonlinearities of electrodynamic loudspeakers*

71 The nonlinearities that produce distortion phenomena can be classified into  
72 three categories. The first type corresponds to the motor nonlinearities and is  
73 described in section (2.2.1). The second type corresponds to the suspension  
74 nonlinearities and is described in section (2.2.4). The third type corresponds  
75 to the acoustical nonlinearities [10] and is not described here since these non-  
76 linearities are not directly produced by the electrodynamic loudspeaker.

### 77 *2.2.1 The motor structure*

78 The force factor  $Bl$  is not uniform in the air gap. First, the force factor de-  
79 pends on the voice coil position. Indeed, the magnetic field induction  $B$  is the  
80 superposition of two fields. One of them is created by the permanent magnet  
81 and is time independent. This field crosses through the yoke pieces but only  
82 thirty per cents serves to move the coil. The other one is created by the coil  
83 and is time dependent. Klippel [3] proposed to model the force factor by using  
84 a polynomial writing.

$$85 \quad Bl(x) = Bl_0 + Bl_1x + Bl_2x^2 \quad (5)$$

86 *2.2.2 The voice coil inductance*

87 The coil self inductance depends on the moving part position. This dependence  
88 generates a reluctant force. This reluctant force is given by:

89 
$$F_{rel}(t) = \frac{1}{2}i(t)^2 \frac{dL_e(x)}{dx} \quad (6)$$

90 We see that when  $L_e$  does not depend on the voice coil position  $x$ , the reluctant  
91 force  $F_{rel}(t)$  equals zero, it is one of the assumptions of the Small signal model  
92 using lumped parameters.

93 *2.2.3 Eddy currents*

94 The electrical conductivity of the iron is high enough to let the eddy currents  
95 appear in the iron yoke pieces of the motor. Vanderkooy [9] proposed a model  
96 which takes this phenomenon into account, the electrical impedance varies like  
97  $L_e\sqrt{w}$ . The interaction between the eddy currents and the current in the coil  
98 generates a drag force  $F_{drag}$  which can be written as follows:

99 
$$F_{drag} = \eta(i, x) \frac{dx(t)}{dt}^{1,7} \quad (7)$$

100 where  $\eta(i, x)$  can be defined as the sensitivity of the drag force according to  
101 the eddy currents ; this one depends on input current and the position of the  
102 voice coil.

103 *2.2.4 The suspension*

104 A classical suspension is mostly made of rubber, impregnated fabric or molded  
105 plastic. The Small signal model using lumped parameters describes a suspen-

106 sion as an ideal spring but an actual suspension shows non linear behaviour.  
107 In consequence, its compliance  $C_{ms}$  depends on the movement amplitude and  
108 the induced damping parameter  $R_{ms}$  depends greatly on both the amplitude  
109 and frequency. More generally, many authors use the mechanical stiffness  $k$   
110 which is defined by:

$$111 \quad k = \frac{1}{C_{ms}} \quad (8)$$

112 Like the force factor  $Bl$ ,  $k$  can be written in terms of a polynomial function.

$$113 \quad k(x) = k_0 + k_1x + k_2x^2 \quad (9)$$

114 Such a model has been used by Klippel [3] to model the non linear behaviour  
115 of both the outer rim and the spider. However, such a model cannot take into  
116 account the effect of the hysteretic response of elastomers.

## 117 *2.3 Time varying properties of the electrodynamic loudspeakers*

### 118 *2.3.1 The electrical resistance $R_e$*

119 Many authors studied the non stationnarities of electrodynamic loudspeakers  
120 as M.Gander [11],[8] and showed that the Small signal parameters depend  
121 on time. The parameter which seems to be the most sensitive to time is the  
122 electrical resistance  $R_e$ . The electrical resistance  $R_e$  increases in time due to  
123 the heat produced by the coil:

$$124 \quad R_e(t) = \rho \frac{l}{S} (1 + \alpha \Delta T + \dots) \quad (10)$$

125 where  $\alpha = 4.10^{-2}K^{-1}$  for the copper,  $l$  is the electric wire length,  $S$  is the  
126 electric wire cross section area and  $\Delta T$  is the temperature elevation due to  
127 the heat produced by the coil. The electrical resistance variation can mod-  
128 ify both the outer rim and the spider properties. The heat produced by the  
129 electrical resistance due to the heat produced by the coil passes through to  
130 both the outer rim and the spider. Consequently, their temperature increases.  
131 The increase in the temperature of the spider and the outer rim generates a  
132 modification of their mechanical behaviour.

### 133 *2.3.2 Time dependence of the mechanical stiffness $k$*

134 Although analytical models taking into account the time dependence of the  
135 mechanical stiffness  $k$  do not exist, the properties of the outer rim change  
136 in time on account of the heat produced by the electrical resistance due to  
137 the Joule effect. Experimentally, this dependence is visible on the electrical  
138 impedance and this phenomenon is discussed in this paper. The outer rim and  
139 the spider exhibit both viscous and elastic characteristics. The type of vis-  
140 coelasticity which occurs in the case of an actual electrodynamic loudspeaker  
141 is non linear. In consequence, a volterra equation cannot be used to connect  
142 stress and strain and a simple model to describe such a behaviour does not ex-  
143 ist. Indeed, the outer rim deformations are large and the outer rim properties  
144 change under deformations.

### 145 **3 Improvement of the Small signal model using lumped param-** 146 **eters: experimental methodology**

#### 147 *3.1 Introduction*

148 This section presents a way of deriving the time dependence and the input  
149 current dependence of the Small signal parameters. For this purpose, an ex-  
150 perimental way based on a measurement algorithm is described. The electro-  
151 dynamic loudspeaker is characterized by the electrical impedance which, pre-  
152 cisely measured, allows us to construct polynomial functions for each Small  
153 signal parameter. The knowledge of the Small signal parameter variations al-  
154 lows us to derive analytically the distortions created by the electrodynamic  
155 loudspeaker.

#### 156 *3.2 Principle of the measurement*

157 In order to measure the electrical impedance of a loudspeaker, it must be  
158 placed in an anechoic chamber in a normalized plane. By varying the fre-  
159 quency and the input current, we can measure the electrical impedance. So  
160 as to increase the measurement precision when impedance variation is impor-  
161 tant, different measurement algorithms have been developed. Basically, the  
162 aim is to acquire more points when impedance variation is important and less  
163 information when impedance tends to be constant with frequency.

164 *3.3 Measurement equipment and devices*

165 The electrical impedance is measured by a Wayne Kerr wedge that has an ex-  
166 cellent precision ( $10^{-4}\Omega$ ). Different algorithms are used to determine at which  
167 frequencies impedance must be measured. Basically, points must be measured  
168 when electrical impedance reaches a maximum or when impedance variation  
169 with frequency is important. To do so, a dichotomic search of the maximum  
170 impedance is used first to measure accurately the impedance near the reso-  
171 nance frequency. The second algorithm is called in order to detect important  
172 variation of impedance while the first algorithm is called to refine measurement  
173 near impedance maxima.

174 *3.4 Determination of the Small signal parameters*

175 The Small signal parameters vary both in time and with the input current.  
176 As it is very difficult to find the two dependences for each parameter, the  
177 measurement algorithm is first used to derive the time dependence and after-  
178 wards to derive the input current dependence. On the one hand, the input  
179 current level is fixed and the electrical impedance is measured each time. On  
180 the other hand, Thiele and Small variations in time are neglected and the  
181 electrical impedance is measured for many input currents. In each case, we  
182 work with three degrees of freedom. These three degrees of freedom are the  
183 time  $t$ , the input current  $i$  and the frequency  $f = \frac{\omega}{2\pi}$ . The measured value is  
184 always the electrical impedance  $Z_e$ .

186 To determine the nonlinear parameter variations, two impedance layers are  
 187 used. One of them can be called the experimental impedance layer  $Z_e^{(exp)}$  and  
 188 is determined by using the measurement algorithm described in section (3).  
 189 The other one can be called the theoretical impedance layer  $Z_e^{(theo)}$  and is  
 190 determined as follows: the Small signal parameters are assumed to vary with  
 191 either the input current or time. In a first approximation, a polynomial writing  
 192 is used to represent the dependence on the parameters with either the input  
 193 current or time. The expansion is truncated after the 2nd term. Therefore, in  
 194 the case of the input current dependence, we assume the electrical resistance  
 195  $R_e$  and  $R_\mu$  to be constant; the Small signal parameters are expressed as follows:

$$196 \quad Bl(i) = Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2) \quad (11)$$

$$197 \quad R_{ms}(i) = R_{ms}(1 + \mu_{R_{ms}}i + \mu_{R_{ms}}^2 i^2) \quad (12)$$

$$198 \quad k(i) = k(1 + \mu_k i + \mu_k^2 i^2) \quad (13)$$

$$199 \quad M_{ms}(i) = M_{ms}(1 + \mu_{M_{ms}}i + \mu_{M_{ms}}^2 i^2) \quad (14)$$

$$200 \quad L_e(i) = L_e(1 + \mu_{L_e}i + \mu_{L_e}^2 i^2) \quad (15)$$

201 and the electrical impedance becomes:

$$Z_{e1}^{(theo)}(i) = R_e + \frac{jR_\mu L_e(i)w}{jL_e(i)w + R_\mu} + \frac{Bl(i)^2}{R_{ms}(i) + jM_{ms}(i)w + \frac{k(i)}{jw}} \quad (16)$$

202 Again, in the case of the time dependence, we assume that  $R_\mu$  is constant.  
 203 The Small signal parameters are expressed as follows:

$$204 \quad R_e(t) = R_e(1 + \nu_{R_e}t + \nu_{R_e}^2 t^2) \quad (17)$$

205  $Bl(t) = Bl(1 + \nu_{Bl}t + \nu_{Bl}^2t^2)$  (18)

206  $R_{ms}(t) = R_{ms}(1 + \nu_{R_{ms}}t + \nu_{R_{ms}}^2t^2)$  (19)

207  $k(t) = k(1 + \nu_k t + \nu_k^2 t^2)$  (20)

208  $M_{ms}(t) = M_{ms}(1 + \nu_{M_{ms}}t + \nu_{M_{ms}}^2t^2)$  (21)

209  $L_e(t) = L_e(1 + \nu_{L_e}t + \nu_{L_e}^2t^2)$  (22)

210 and the electrical impedance becomes:

$$Z_{e2}^{(theo)}(t) = R_e(t) + \frac{jR_\mu L_e(t)\omega}{jL_e(t)\omega + R_\mu} + \frac{Bl(t)}{R_{ms}(t) + jM_{ms}(t)\omega + \frac{k(t)}{j\omega}} \quad (23)$$

211 A least square method is used to identify all the parameters in the both cases ;  
 212 this method is based on the Symplex algorithm. The principle of this algorithm  
 213 is to minimize the difference  $\Delta Z_e$  between the experimental impedance and  
 214 the theoretical impedance. In the case of the time dependence of the Small  
 215 signal parameters, this difference is expressed as follows:

216  $\Delta Z_e^1(t) = \sum_{n=0}^{n=2} \left| \left| Z_e^{(exp)}(t) - Z_{e1}^{(theo)}(t) \right| \right|^2$  (24)

217 In the case of the input current dependence of the Small signal parameters,  
 218 this difference is expressed as follows:

219  $\Delta Z_e^2(i) = \sum_{n=0}^{n=2} \left| \left| Z_e^{(exp)}(i) - Z_{e2}^{(theo)}(i) \right| \right|^2$  (25)

220 When the algorithm converges, all the values describing the nonlinear param-  
 221 eters are obtained and allow us to predict analytically the distortions created  
 222 by the electrodynamic loudspeaker by solving the time-varying nonlinear dif-  
 223 ferential equation.

## 224 4 Time dependence of the Small signal parameters

225 This section describes a temporal study of two electrodynamic loudspeakers.  
226 The electrodynamic loudspeakers used are two woofers (Eminence Alpha). One  
227 of them is run in and the other one is not. First, the measurement algorithm  
228 presented in the previous section is used in order to derive all the non-linear  
229 parameters. Then, time-varying effects experimentally observed are discussed  
230 and physically interpreted.

### 231 4.1 Obtaining the experimental impedance

232 The first step to derive the time dependence of the Small signal parameters  
233 is to use the experimental impedance layer. As explained previously, the cur-  
234 rent input current is fixed. A current which equals  $i = 100mA$  is injected in  
235 the electrodynamic loudspeaker. The electrodynamic loudspeaker used is sup-  
236 posed to be run in. The lower measurement frequency equals  $50Hz$  and the  
237 upper measurement frequency equals  $250Hz$ . The experimental impedance is  
238 measured for eight hours. Such an experimental impedance layer is represented  
239 in Fig.(1). It can be noted that the time-varying effects are not visible in this  
240 impedance layer but they are clearly shown in Figs.(2),(6) and (7).

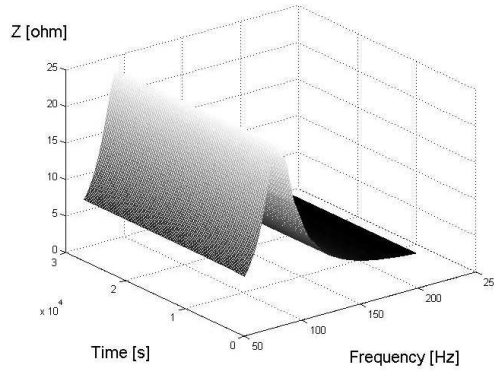


Fig. 1. Experimental three-dimensional representation of the electrical impedance modulus of the electrodynamic loudspeaker (x: time 0s to  $3 \cdot 10^4$ s) (y: 0Hz to 200Hz) (z:  $0\Omega$  to  $25\Omega$ )

241 *4.2 Obtaining the parameters sensitive to time*

242 *4.2.1 Error sheet between the experimental impedance and the theoretical*  
 243 *impedance*

244 In the previous section, the experimental impedance layer is determined with  
 245 the measurement algorithm presented in section (3). In this section, the experi-  
 246 mental impedance is compared to the theoretical one calculated with the Small  
 247 signal model using lumped parameters. For this purpose, the difference  $\Delta Z_e^1(t)$   
 248 between the experimental impedance modulus and the theoretical impedance  
 249 modulus is calculated for each frequency and at each time. This difference  
 250  $\Delta Z_e^1(t)$  is represented in Fig.(2). The mean difference  $\overline{\Delta Z_e}$  is defined as the  
 251 difference  $\Delta Z_e^1(t)$  divided by the number of points necessary to plot the ex-  
 252 perimental impedance layer. By using the Small signal model using lumped  
 253 parameters with constant parameters, the mean difference  $\overline{\Delta Z_e}$  equals  $0, 20\Omega$ .

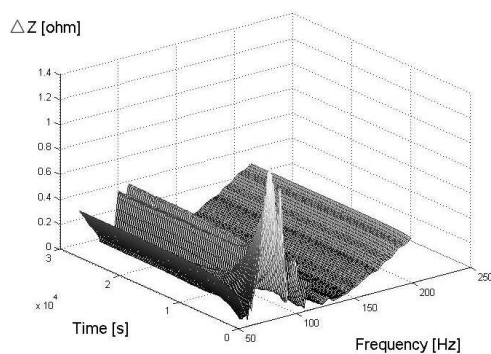


Fig. 2. Three-dimensional representation of the difference  $\Delta Z_e^1(t)$  between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with constant parameters (x: time 0s to  $3 \cdot 10^4$ s) (y: 0Hz to 200Hz) (z:  $0\Omega$  to  $25\Omega$ )

#### 254 4.2.2 Parameter sensitive to time

255 To reduce  $\overline{\Delta Z_e}$ , we use the Symplex algorithm and the parameter which is the  
 256 most sensitive to time is the equivalent mechanical stiffness  $k$ . As a remark,  
 257 although the electrical resistance of the voice coil  $R_e$  increases in time, its time  
 258 variation is less important than the mechanical stiffness one. Moreover, the  
 259 variations of the other Small signal parameters are not so important as the  
 260 mechanical stiffness variation. In Fig.(3), we represent the difference  $\Delta Z_e^1(t)$   
 261 between the experimental impedance modulus and the theoretical impedance  
 262 modulus which takes into account the time variation of the mechanical stiffness  
 263  $k$ . This difference is a function of both time and frequency. The impedance  
 264 layer is zoomed for more legibility. The temporal axe varies from 0s to 200s.  
 265 The mean difference  $\overline{\Delta Z_e}$  equals  $0,19\Omega$ . The figure (4) shows the relative  
 266 mechanical stiffness as a function of time ( $k_0 = 3714N/m$ ). The mechanical  
 267 stiffness  $k$  decreases in time since heat produced by the electrical resistance  
 268 passes through to the outer rim and modifies its properties. The increasing  
 269 temperature is one factor contributing to the deformation of the outer rim,

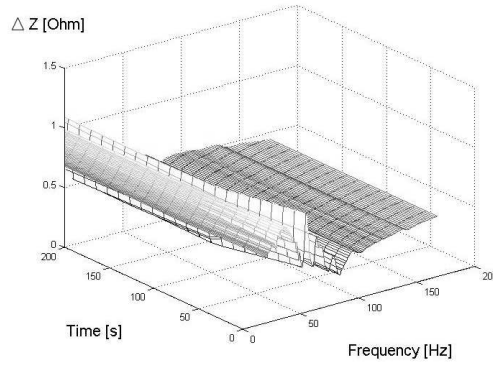


Fig. 3. Three-dimensional representation of the difference between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on Small signal model using lumped parameters with variable mechanical stiffness (x: time 0s to  $3.10^4$ s) (y: 0Hz to 200Hz) (z:  $0\Omega$  to  $25\Omega$ )

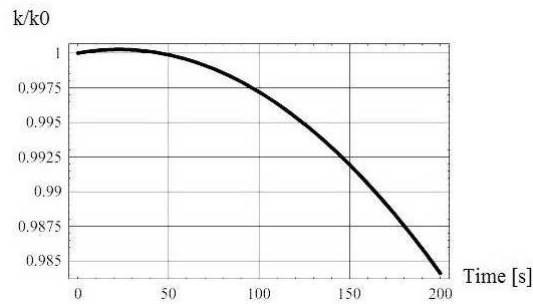


Fig. 4. The relative mechanical stiffness is a function of time [s]

270 and viscoelastic properties change with decreasing or increasing temperature.

### 271 4.3 Resonance frequency variation

272 Another interesting temporal effect is the resonance frequency variation. It is  
 273 quite difficult to obtain the resonance frequency experimental measurement  
 274 in time because its variation is very fast and the time necessary to get the  
 275 measurement points by the algorithm is only about half a second. Fig.(5)  
 276 shows the resonance frequency  $f_{res}$  as a function of time. We see in this figure

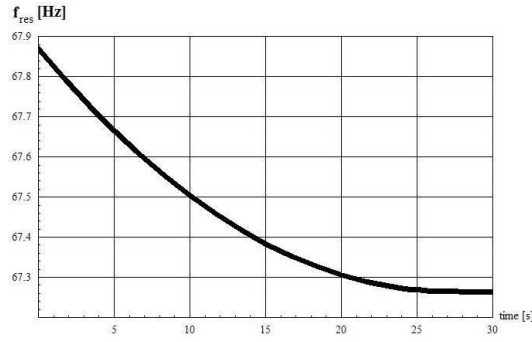


Fig. 5. The resonance frequency [Hz] is a function of time [s]

277 that the resonance frequency decreases in time. This effect can be explained  
 278 since the mechanical stiffness of suspension ( spider, outer rim) depends on  
 279 time. In consequence, the resonance frequency is not constant and depends also  
 280 on time. In short, the decrease in mechanical stiffness generates the decrease  
 281 in the resonance frequency.

$$282 \quad f_{res}(t) = \frac{1}{2\pi} \sqrt{\frac{k - k_3t - k_4t^2}{Mms}} \quad (26)$$

283 *4.4 Comparison between two loudspeakers: one of them is not run in and the*  
 284 *other one is*

285 This section presents an experimental comparison between two electrodynamic  
 286 loudspeakers. One of them is supposed to be run in and the other one is not.  
 287 The electrodynamic loudspeaker which is run in has been used for one year.  
 288 In consequence, its mechanical properties have changed, particularly for the  
 289 outer rim and the spider which have become both more elastic and worn.  
 290 For five hours, we measured continually the electrical impedance of the two  
 291 electrodynamic loudspeakers. The experimental electrical impedance modulus  
 292  $Z_e(t)$  of the electrodynamic loudspeaker which is not run in is represented in  
 293 Fig.(6). As said previously,  $Z_e(t)$  is plotted at different instants and is a func-

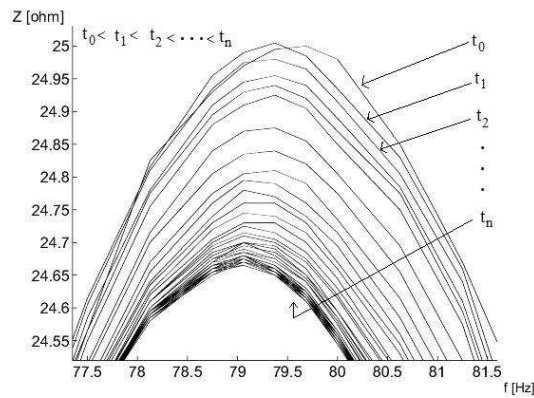


Fig. 6. Electrical impedance modulus of the woofer which is not run in. The electrical impedance modulus  $[\Omega]$  is a function of frequency  $[\text{Hz}]$  and is plotted at different instants around the resonance frequency.

294 tion of frequency. In this figure, we see that the electrical impedance decreases  
 295 in time and it is mainly due to the change of the mechanical properties. An-  
 296 other interesting point is that the resonance frequency varies quickly in time  
 297 between  $t_0$  and  $t_1$  which corresponds to 8 seconds. This variation is probably  
 298 due to the dry friction behaviour of the outer rim.

299 Fig. (7) represents the electrical impedance modulus of the electrodynamic  
 300 loudspeaker which is supposed to be run in. As in the previous case,  $Z_e(t)$  is  
 301 plotted at different instants and is a function of frequency. This figure shows  
 302 that the decrease in electrical impedance modulus is less important for the  
 303 woofer which is run in than the one which is not. This diminution is about  
 304  $0,4\Omega$  for the woofer which is not run in, whereas this diminution is  $0,05\Omega$  for  
 305 the woofer which is run in. Moreover, the resonance frequency variation is less  
 306 important for the woofer which is run in than the one which is not. This reso-  
 307 nance frequency variation is about  $1\text{Hz}$  for the woofer which is not, whereas  
 308 this variation is  $0,4\text{Hz}$  for the woofer which is run in. Furthermore, the reso-  
 309 nance frequency is very different between the two loudspeakers although they

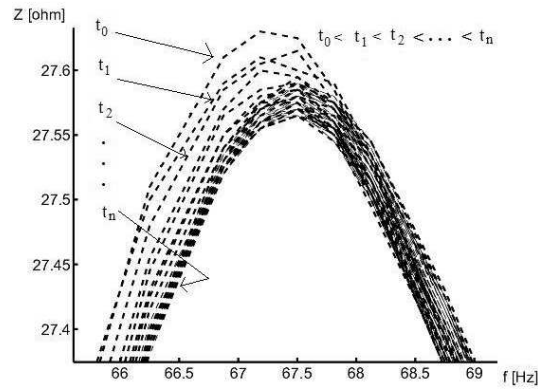


Fig. 7. Electrical impedance modulus of the woofer which is run in. The electrical impedance modulus  $[\Omega]$  is a function of frequency  $[\text{Hz}]$  and is plotted at different instants around the resonance frequency.

310 are both the same. The resonance frequency of the woofer which is run in is  
 311 about  $67\text{Hz}$  whereas the resonance frequency of the woofer which is not run  
 312 in is about  $79\text{Hz}$ . This resonance frequency discrepancy is probably due to  
 313 the fabrication scattering and the change in time of the membrane mechanical  
 314 properties.

#### 315 4.5 Electrical impedance variation in time

316 The previous section shows that the electrical impedance varies in time. The  
 317 aim of this section is to show that the electrical impedance does not vary in the  
 318 same way according to the frequency measurement. For this purpose, we plot  
 319 the electrical impedance for the two loudspeakers at two different fixed fre-  
 320 quencies. One of them is at the resonance frequency and the other one is at  $200$   
 321  $\text{Hz}$ . In Fig. (8), the electrical impedance modulus of the woofer which is run  
 322 in is a function of time. The fixed frequency equals  $200\text{Hz}$  and the input cur-  
 323 rent equals  $100\text{mA}$ . This figure shows that the electrical impedance modulus  
 324 increases in time. In Figure (9), we still plot the electrical impedance modulus

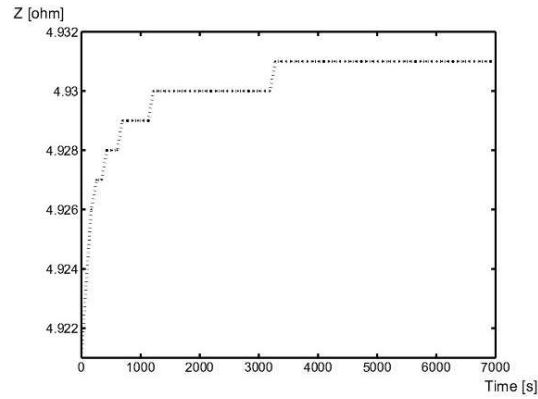


Fig. 8. Electrical impedance modulus of the woofer which is run in. The frequency equals  $200Hz$  and the input current equals  $100mA$ . The electrical impedance modulus is a function of time.

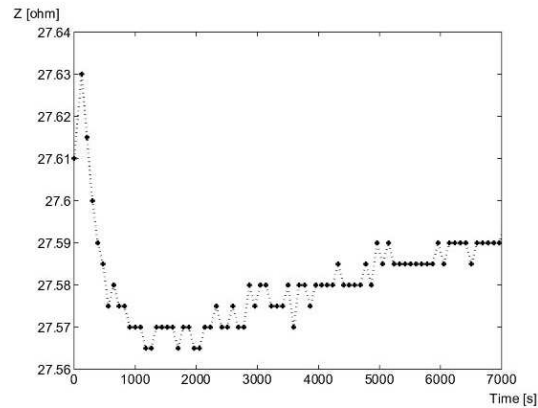


Fig. 9. Electrical impedance modulus of the boomer which is run in. The frequency equals the resonance frequency and the input current equals  $100mA$ . The electrical impedance modulus is a function of time.

325 of the woofer which is run in, but the fixed frequency equals the resonance  
 326 frequency. The temporal behaviour of the electrical impedance is very different  
 327 according to the frequency measurement. Actually, the electrical impedance of  
 328 the woofer which is run in decreases a lot at the beginning and increases only  
 329 after three hour measurement. Moreover, the electrical impedance modulus  
 330 varies more in time at the resonance frequency than another frequency (here:  
 331  $200Hz$ ).

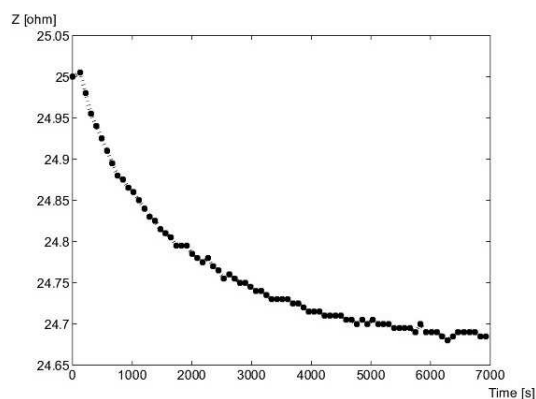


Fig. 10. Electrical impedance modulus of the woofer which is not run in. The frequency equals the resonance frequency and the input current equals  $100mA$ . The electrical impedance modulus is a function of time

332 The same experimental measurements are done with the electrodynamic loud-  
 333 speaker which is not run in. Again, an experimental measurement is realized  
 334 with a fixed frequency which equals the resonance frequency. Such an experi-  
 335 mental measurement is represented in Fig.10 This figure shows that the elec-  
 336 trical impedance decreases in time. The behavior of the electrical impedance  
 337 is very different according to the electrodynamic loudspeaker used at the res-  
 338 onance frequency. The figure (10) shows the electrical impedance modulus of  
 339 the woofer which is not run in as a function of time. The fixed frequency equals  
 340  $200Hz$  and the input current equals  $100mA$ .

341 As seen previously in the case of the run in electrodynamic loudspeaker, the  
 342 electrical impedance modulus increases in time. In Fig.(11), the electrical  
 343 impedance modulus decreases in time. Moreover, we see that the electrical  
 344 impedance modulus varies more at the resonance frequency than another fre-  
 345 quency (here:  $200Hz$ ).

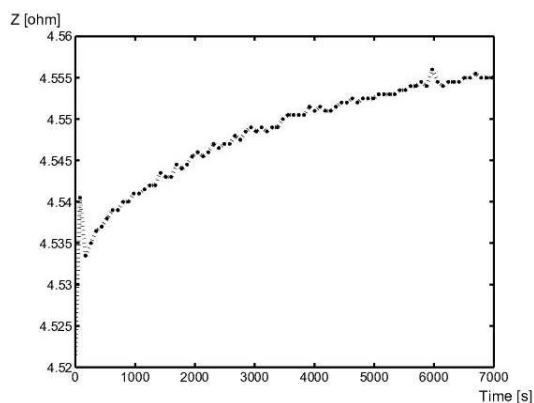


Fig. 11. Electrical impedance modulus of the woofer which is not run in. The frequency equals  $200Hz$  and the input current equals  $100mA$ . The electrical impedance modulus is a function of time.

#### 346 4.6 *Running in an electrodynamic loudspeaker*

347 The aim of this section is to show the time necessary to consider that an  
 348 electrodynamic loudspeaker is run in. For this purpose, we use the electrical  
 349 impedance modulus of the electrodynamic loudspeaker. We take a frequency  
 350 which equals the resonance frequency, an input current which equals  $100mA$   
 351 and we plot the electrical impedance modulus at each instant. Such an elec-  
 352 trical impedance modulus is plotted in Fig.(12). This figure shows that the  
 353 electrical impedance modulus does not vary after  $10^4s$ , which corresponds to  
 354 about three hours. It can be concluded that this is the time necessary for  
 355 breaking in this electrodynamic loudspeaker.

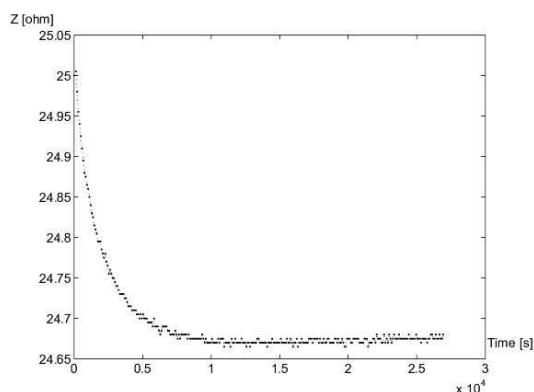


Fig. 12. Electrical impedance modulus of the woofer which is not run in. The frequency equals the resonance frequency and the input current equals  $100mA$ . The electrical impedance modulus is a function of time.

## 356 **5 Analytical study of the distortions created by an electrodynamic** 357 **loudspeaker**

### 358 *5.1 Obtaining the experimental impedance*

359 The way of obtaining the experimental impedance is similar to the one de-  
 360 scribed previously. In order to derive the input current dependence of Small  
 361 signal parameters, the first step is to use the experimental impedance layer.  
 362 The time dependence of Small signal parameters is neglected and the input  
 363 current varies from  $20mA$  to  $200mA$ . The Wayne Kerr wedge cannot deliver  
 364 currents higher than  $200mA$ . The electrodynamic loudspeaker used is sup-  
 365 posed to be run in. The lower measurement frequency equals  $50Hz$  and the  
 366 upper measurement frequency equals  $650Hz$ . The experimental impedance  
 367 layer is represented in Fig.(13).

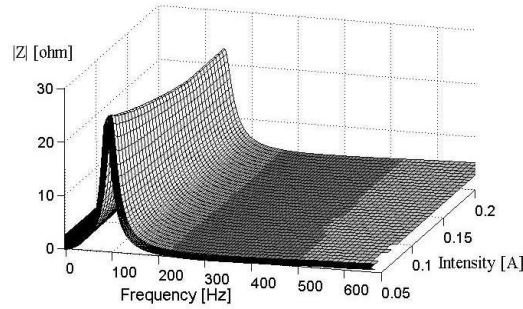


Fig. 13. Experimental three-dimensional representation of the electrical impedance modulus of the electrodynamic loudspeaker (x:0.05A to 0,2A) (y: 0Hz to 650Hz) (z: 0Ω to 25Ω)

368 *5.2 Obtaining the parameters sensitive to the input current*

369 *5.2.1 Error sheet between the experimental impedance and the theoretical*  
 370 *impedance*

371 In the previous section, the experimental impedance layer is determined with  
 372 the measurement algorithm presented in section (3). In this section, the experi-  
 373 mental impedance is compared to the theoretical one calculated with the Small  
 374 signal model using lumped parameters. For this purpose, the difference  $\Delta Z_e^2(i)$   
 375 between the experimental impedance modulus and the theoretical impedance  
 376 modulus is calculated for each frequency and at each intensity. The intensity  
 377 step is 10mA. This difference  $\Delta Z_e^2(i)$  is represented in Fig.(14). We define  
 378 the mean difference  $\overline{\Delta Z_e}$  as the difference  $\Delta Z_e^2(i)$  divided by the number of  
 379 points necessary to plot the experimental impedance layer. By using the Small  
 380 signal model using lumped parameters with constant parameters, the mean  
 381 difference  $\overline{\Delta Z_e}$  equals 2,04Ω.

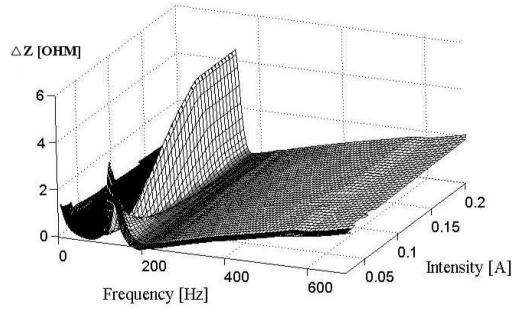


Fig. 14. Three-dimensional representation of the difference  $\Delta Z_e^2(i)$  between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with constant parameters (x:0,05A to 0,2A) (y: 0Hz to 200Hz) (z: 0Ω to 6Ω)

### 382 5.2.2 Parameters sensitive to the input current

383 To reduce  $\overline{\Delta Z_e}$ , the Symplex algorithm is used and five nonlinear parameters  
 384 are taken into account to reduce  $\overline{\Delta Z_e}$ . In Fig.(15), we represent the differ-  
 385 ence  $\Delta Z_e^2(i)$  between the experimental impedance modulus and the theoretical  
 386 impedance modulus which takes into account the Small signal parameter vari-  
 387 ations. This difference is a function of both the input current and frequency.  
 388 The mean difference  $\overline{\Delta Z_e}$  equals 0,39Ω.

389 In table (1), all the parameters and their expansions are described and the  
 390 sensitivity to the least square is precise. This table shows that the parameter  
 391 which is the more sensitive to the input current is the equivalent damping  
 392 parameter  $R_{ms}$ .

### 393 5.3 Obtaining the time-varying nonlinear differential equation

394 This section presents the time-varying nonlinear differential equation of the  
 395 electrodynamic loudspeaker which is run in. For this purpose, we take into

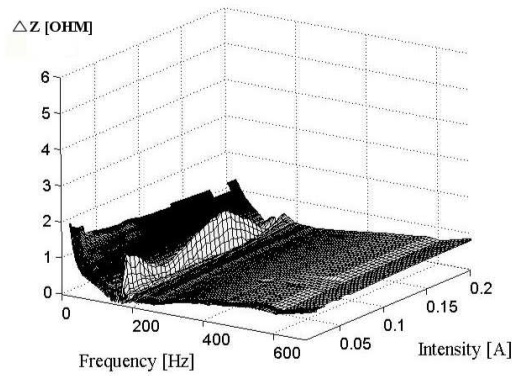


Fig. 15. Three-dimensional representation of the difference between the experimental impedance and the theoretical impedance ; the theoretical impedance is based on the Small signal model using lumped parameters with variable parameters (x: 0A to 0,2A) (y: 0Hz to 200Hz) (z: 0Ω to 25Ω)

Ranking	Parameter	Law of variation	$\overline{\Delta Z_e}[\Omega]$	Sensitivity
1	$R_{ms}$	$1.1(1 + 4.09i - 8.36i^2)$	1.24	33%
2	$Bl$	$5.5(1 + 0.33i - 1.02i^2)$	1.67	18%
3	$M_{ms}$	$0.009(1 + 0.56i - 0.22i^2)$	1.74	14%
4	$k$	$7440(1 - 0.2i + 0.9i^2)$	1.86	8%
5	$L_e$	$0.0017(1 - 1.68i + 7.58i^2)$	1.98	3%
6	$R_\mu$	2, 28	2.04	0%
7	$R_e$	3, 17	2.04	0%

Table 1

Ranking of the parameters according to their sensitivity to the least square algorithm

396 account the nonlinear parameters defined in the previous section and we also  
 397 take into account the time variation of the mechanical stiffness  $k$ . The time-  
 398 varying nonlinear differential equation is defined by Eq.(27) in the case when  
 399 we also take into account the variation of the electrical resistance  $R_e$  in time.

$$400 \quad a(i) \frac{d^3 x(t)}{dt^3} + b(i, t) \frac{d^2 x(t)}{dt^2} + c(i, t) \frac{dx(t)}{dt} + d(i, t)x(t) = u(t) \quad (27)$$

401 with

$$402 \quad a(i) = \frac{(Mms(1 + \mu_{Mms}i + \mu_{Mms}^2 i^2))(Le(1 + \mu_{Le}i + \mu_{Le}^2 i^2))}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} \quad (28)$$

$$b(i, t) = \frac{\left( (Mms(1 + \mu_{Mms}i + \mu_{Mms}^2 i^2))R_e(1 + \nu_{R_e}t + \nu_{R_e}^2 t^2) \right)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} + \frac{R_{ms}(1 + \mu_{R_{ms}}i + \mu_{R_{ms}}^2 i^2)L_e(1 + \mu_{Le}i + \mu_{Le}^2 i^2)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} \quad (29)$$

403

$$c(i, t) = \frac{R_e(1 + \nu_{R_e}t + \nu_{R_e}^2 t^2)(Rms(1 + \mu_{Rms}i + \mu_{Rms}^2 i^2))}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} + \frac{(Le(1 + \mu_{Le}i + \mu_{Le}^2 i^2))k(1 + \nu_k t + \nu_k^2 t^2)(1 + \mu_k i + \mu_k^2 i^2)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} + \frac{(Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2))^2}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} \quad (30)$$

$$404 \quad d(i, t) = \frac{R_e(1 + \nu_{R_e}t + \nu_{R_e}^2 t^2)(k(1 + \nu_k t + \nu_k^2 t^2)(1 + \mu_k i + \mu_k^2 i^2))}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} \quad (31)$$

#### 405 5.4 Solving the time-varying nonlinear differential equation

406 We explain here how to solve the equation defined in the previous section. We  
 407 can point out that the coefficient  $a(i)$  defined in Eq.(28) is the only coefficient  
 408 which is constant in time. We use the notation  $Re_t$  and  $k_t$  to indicate that these

409 parameters depend on time. To solve the time-varying nonlinear differential  
 410 equation, a Taylor series expansion is used.

#### 411 5.4.1 Discussion about the time-varying differential equation

412 It is noticeable that the temporal variations of the Small signal parameters  
 413 do not create any important distortions. Indeed, if we assume all the Small  
 414 signal parameters to be constant with the input current, the general differential  
 415 equation of the electrodynamic loudspeaker is written:

$$416 \quad \tilde{a} \frac{d^3 x(t)}{dt^3} + \tilde{b}(t) \frac{d^2 x(t)}{dt^2} + \tilde{c}(t) \frac{dx(t)}{dt} + \tilde{d}(t)x(t) = u(t) \quad (32)$$

417 with

$$418 \quad \tilde{a} = \frac{M_{ms} L_e}{Bl} \quad (33)$$

$$419 \quad \tilde{b}(t) = \frac{M_{ms} R_e (1 + \nu_{R_e} t + \nu_{R_e}^2 t^2)}{Bl} + \frac{R_{ms} L_e}{Bl} \quad (34)$$

$$420 \quad \tilde{c}(t) = \frac{R_e (1 + \nu_{R_e} t + \nu_{R_e}^2 t^2) R_{ms} + Bl^2 + k(1 + \nu_k t + \nu_k^2 t^2) L_e}{Bl} \quad (35)$$

$$421 \quad \tilde{d}(t) = \frac{k(1 + \nu_k t + \nu_k^2 t^2) R_e (1 + \nu_{R_e} t + \nu_{R_e}^2 t^2)}{Bl} \quad (36)$$

422 The time-varying differential equation defined in Eq.(32) is a hypergeometric  
 423 equation and can be solved in the general case by using the theory of the Power  
 424 Series Method [12]. However, if we take  $u(t) = Ae^{j\omega t}$  where  $A$  is a term of  
 425 amplitude, the response does not contain terms in  $e^{(j2\omega t)}, e^{(j3\omega t)}, \dots$ . In con-  
 426 sequence, we deduct that the time dependence of the Small signal parameters  
 427 does not generate any distortions.

428 5.5 Solving the nonlinear differential equation

429 The nonlinear differential equation can be solved at each time. By assuming  
430 the electrical resistance to be constant in time, the only parameter sensitive to  
431 time is the mechanical stiffness. To simplify the resolution of the time-varying  
432 nonlinear differential equation, we can write that at each time, the nonlinear  
433 differential equation is stationary. The distortions predicted by the nonlinear  
434 differential equation depend on time but can be solved at each time. The  
435 study of the nonlinear small signal parameters can be done with either the  
436 input current or with the position of voice coil. In fact, the relation between  
437 the input current  $i$  and the position  $x(t)$  of the voice coil is linear. Indeed,  
438 by using the classical approach, Laplace Law describes the movement of the  
439 voice coil at first order.

440 
$$M_{ms} \frac{d^2x(t)}{dt^2} = Bli(t) \quad (37)$$

441 If we consider that the current is varying sinusoidally in time, above the fre-  
442 quency resonance, the displacement of the voice coil is proportional to the  
443 Laplace force and in opposed directions. The displacement of the voice coil  
444 can be described by:

445 
$$x = -\frac{Bli}{M_{ms}w^2} \quad (38)$$

446 where  $w$  is the radian frequency of the input current. In consequence, it exists  
447 a parameter  $\alpha$  which verifies:

448 
$$x = \alpha i \quad (39)$$

449 where  $\alpha = -\frac{Bl}{M_{ms}w^2}$ . All the Small signal parameters can be expressed as a  
450 Taylor series expansion. By inserting all these expansion series in Eq.(28),  
451 we obtain a classical nonlinear differential equation. Its solution is given by  
452 Eq.(40). The solution is developed until the order 2 ( $\mu_2$ ).

$$453 \quad x(t) = x_0(t) + \mu x_1(t) + \mu^2 x_2(t) + \dots \quad (40)$$

454 where  $x_0(t)$  is the solution of the nonlinear differential equation of the elec-  
455 trodynamic loudspeaker when the terms with orders higher than zero are  
456 neglected,  $x_1(t)$  is the solution of the nonlinear differential equation when the  
457 terms with orders higher than one and smaller than one are neglected,  $x_2(t)$  is  
458 the solution of the nonlinear differential equation when the terms with orders  
459 higher than two and smaller than two are neglected. In short, the solution of  
460 the nonlinear differential equation of the electrodynamic loudspeaker is given  
461 by:

$$462 \quad x(t) = A \cos(\omega t) + B \sin(\omega t) + C \cos(2\omega t) + D \sin(2\omega t) + \dots \quad (41)$$

463 The terms  $A$  and  $B$  can be found by inserting  $A \cos(\omega t) + B \sin(\omega t)$  in Eq.(27)  
464 with an excitation  $u(t)$  which equals  $P \sin(\omega t)$  where  $P$  is an amplitude. The  
465 terms  $C$  and  $D$  can be found by taking the terms with orders higher than one  
466 and smaller than one into account, etc...

### 467 5.6 *Experimental and theoretical displacement spectrums*

468 This section presents the experimental and the theoretical displacement spec-  
469 trums of the electrodynamic loudspeaker which is run in. The theoretical dis-  
470 placement spectrum is obtained by calculating the Fourier transform of the

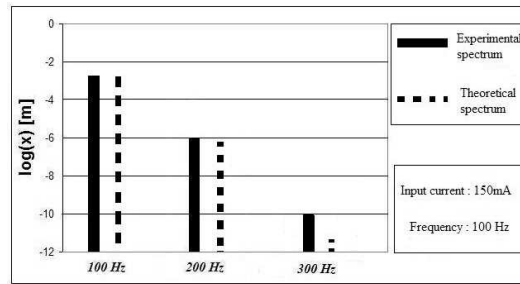


Fig. 16. Experimental and theoretical spectrums of the electrodynamic loudspeaker which is run in. The input current equals  $100mA$  and the input frequency equals  $100Hz$ .

471 solution given in Eq.(41). The experimental displacement spectrum is obtained  
 472 by using a laser Doppler velocimeter. The theoretical displacement spectrum is  
 473 consistent with the experimental displacement spectrum. The theoretical and  
 474 experimental first-harmonic and second-harmonic shows a very good agree-  
 475 ment. However, the theoretical third-harmonic is lower than the experimental  
 476 one. This discrepancy between the theoretical third-harmonic and the exper-  
 477 imental one shows the limit of the use of a series Taylor expansion. It can be  
 478 noted that the experimental spectrums have been measured at low frequen-  
 479 cies. For higher frequencies, the theoretical model should take into account  
 480 membrane modes.

## 481 6 Conclusion

482 The aim of this paper is the study of the time-varying effects and nonlinear ef-  
 483 fects of electrodynamic loudspeakers. A temporal study based on a very precise  
 484 measurement shows the time dependence of the membrane mechanical stiffness  
 485  $k$ . However, this time dependence does not create any distortions. Moreover,  
 486 two identical electrodynamic loudspeakers are compared and important time

487 discrepancies are discussed. The resonance frequency between an electrody-  
488 namic loudspeaker which is run in and one which is not is extremely different  
489 and does not vary in time in the same way. Then, the time-varying nonlinear  
490 differential equation of the electrodynamic loudspeaker is solved by using a se-  
491 ries Taylor expansion. For this purpose, the time-varying effects are neglected  
492 but can be taken into account by solving the nonlinear differential equation  
493 at different instants. The theoretical displacement spectrum is consistent with  
494 the experimental displacement spectrum.

## References

- [1] R. H. Small, "Closed-boxloudspeaker systems, part 1: Analysis," *J. Audio Eng. Soc.*, vol. 20, pp. 798–808, 1972.
- [2] A. J. M. Kaizer, "Modeling of the nonlinear response of an electrodynamic loudspeaker by a volterra series expansion," *J. Audio Eng. Soc.*, vol. 35, pp. 421–433, June 1987.
- [3] W. Klippel, "Dynamic measurement and interpretation of the nonlinear parameters of electrodynamic loudspeakers," *J. Audio Eng. Soc.*, vol. 38, pp. 944–955, 1990.
- [4] W. Klippel, "Loudspeaker nonlinearities - cause, parameters, symptoms," *J. Audio Eng. Soc.*, vol. 54, pp. 907–939, 2006.
- [5] J. W. Noris, "Nonlinear dynamical behavior of a moving voice coil," in *105th convention, San Francisco*, no. 4785, Audio Eng. Soc., 1998.
- [6] C. S. A. Dobrucki, "Nonlinear distortions of woofers in fundamental resonance region," in *80th convention, Montreux*, no. 2344, Audio Eng. Soc., 1986.

- [7] M. Berkouk, "Contribution a l'étude des actionneurs électrodynamiques." These de doctorat de l'Université de Bretagne Occidentale, 2001.
- [8] M. R. Gander, "Dynamic linearity and power compression in moving-coil loudspeaker," *J. Audio Eng. Soc.*, pp. 627–646, September 1986.
- [9] J. Vanderkooy, "A model of loudspeaker driver impedance incorporating eddy currents in the pole structure," *J. Audio Eng. Soc.*, vol. 37, pp. 119–128, March 1989.
- [10] G. Lemarquand and M. Bruneau, "Large bandwidth loudspeaker emitting coherent acoustic waves: nonlinear inter-modulation effects," *J. Audio Eng. Soc.*, 2007.
- [11] M. R. Gander, "Moving-coil loudspeaker topology as an indicator of linear excursion capability," *J. Audio Eng. Soc.*, vol. 29, 1981.
- [12] E. Kreyszig, *Advanced Engineering Mathematics*. Wiley, 2006.