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► **To cite this version:**

Wisama Khalil, Philippe Lemoine, Maxime Gautier. Autonomous calibration of Robots using planar points. International Symposium on Robotics and Manufacturing, ISRAM'96, May 1996, Montpellier, France. pp. 383-388. hal-00362605

**HAL Id: hal-00362605**

**<https://hal.science/hal-00362605>**

Submitted on 17 Mar 2009

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# AUTONOMOUS CALIBRATION OF ROBOTS USING PLANAR POINTS

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## ABSTRACT

This paper presents two methods for the calibration of the geometric parameters of serial robots. They are based on putting the terminal point of the robot in a plane, only the position sensors of the joints are required. The methods can be put in practice quickly and precisely. Some practical issues concerning their practical use are given.

**KEYWORDS :** Robot calibration, geometric parameters, identification.

## I. INTRODUCTION

Geometric calibration of robots is the process by which the parameters defining the base parameters, link parameters and tool parameters are precisely identified. The absolute accuracy of a robot depends to a large extent on the calibration of its geometric parameters. Classically, the geometric calibration is carried out by solving a system of linear or non-linear equations which is a function of the end-effector pose measurements and of the joint positions [1,...,4]. In practice all the industrial robots are equipped with joint sensors, thus the real problem is to find a precise external sensor to give the pose values precisely and quickly because a lot of configurations are generally needed. Recently, some papers have proposed autonomous calibration methods which do not need an external sensor :

-in [5,6,7,8] the observation matrix and the error vector have been constructed using the fact that different configurations of the robot could give the same pose of the terminal effector .

- in [9] the observation matrix is based on equating to zero the vector product of a set of vectors in the same plane. Second order terms appearing in the vector products are neglected, thus the obtained observation matrix contains more approximation than the classical one, and the problem of the identifiable parameters in this method is not considered.

In this paper we use a set of points in a plane to carry out the calibration of the geometric parameters, the observation matrix is constructed using the equation of the plane, two methods are used. In the first, the plane coefficients are supposed known, while they will be identified in the second. The solution is obtained using a linearized model which is solved iteratively using least squares method.

It is to be noted that the idea of using points in a plane has been also used in [10], but we do not classify this method as autonomous one, because it is based on putting the end effector point in a plane which must be the fixed x-y plane, thus only the equation  $z=0$  is used. In our method the plane may be put any where and the constraint equation of the plane is used and its coefficients can be identified.

## II. CLASSICAL CALIBRATION

We consider serial robots consisting of  $n$  joints and  $n+1$  links. Link 0 is the base and link  $n$  is the terminal link, frame  $j$  is defined fixed on link  $j$ . We denote :

frame -1 : the fixed reference frame, frame  $n+1$ : the frame fixed to the tool.

The end-effector pose can be calculated with respect to the reference frame by the direct geometric model (DGM) :

$${}^{-1}\mathbf{T}_{n+1} = {}^{-1}\mathbf{T}_0 {}^0\mathbf{T}_n(\mathbf{q}) {}^n\mathbf{T}_{n+1} \quad (1)$$

where  ${}^i\mathbf{T}_j$  is the 4x4 transformation matrix defining frame  $j$  with respect to frame  $i$ .

The definition of the link frames will be carried out by Khalil and Kleinfinger notation [11]. Frame  $j$  is defined with respect to frame  $j-1$  by the matrix  ${}^{j-1}\mathbf{T}_j$ , function of the four parameters  $(\alpha_j, d_j, \theta_j, r_j)$ .

It have been seen that, the calculation of  ${}^{-1}\mathbf{T}_{n+1}$  can be obtained as a function of the geometric parameters of  $n+2$  frames represented by four parameters for each frame  $(\alpha_j, d_j, \theta_j, r_j)$  [7,12] ( $\Delta\alpha_0$  and  $\Delta d_0$  are identically equal to zero). The joint variable  $q_j$  is equal to  $\theta_j$  if joint  $j$  is rotational and  $r_j$  if joint  $j$  is prismatic.

In the calibration process, we have to identify the deviation of the real parameters from the nominal values, thus to identify :  $\Delta\alpha_j, \Delta d_j, \Delta\theta_j, \Delta r_j$  (for  $j = 0, n+1$ , except  $\Delta\alpha_0$  and  $\Delta d_0$ ).

If the axis of joint  $j$  is parallel to the axis of joint  $j-1$ , an additional parameter  $\Delta\beta_j$  must be considered, the nominal value of  $\beta_j$  is equal to zero [13].

The error in the transmission gain of the joints  $\Delta K_j$  can be also taken into account [14].

### The identification model

The tool frame pose can be calculated by the direct geometric model given as  ${}^{-1}\mathbf{T}_{n+1}$  which is a non-linear function of the geometric parameters. Using a first order Taylor development of the parameters errors, a linear differential model will be used in the calibration. If the errors are big, an iterative procedure can be applied.

The differential vectors defining the deviation of the end effector due to the differential error in the geometric parameters can be given by [7,14] :

$$\Delta\mathbf{y} = \mathbf{J}(\mathbf{q}) \Delta\mathbf{X} \quad (2)$$

where:  $\Delta\mathbf{y}$  represents the (6x1) vector of the position and orientation error.

$\mathbf{J}$  is a (6xNp) matrix,

$N_p$  = number of the parameters to be calibrated.

-  $\Delta \mathbf{X}$  defines the ( $N_p \times 1$ ) vector of the errors of the geometric parameters. It is equal to :

$$\Delta \mathbf{X} = \mathbf{X}_r - \mathbf{X}$$

where :  $\mathbf{X}_r$  is the vector of the real unknown values of the geometric parameters,

$\mathbf{X}$  is the vector of the nominal values of the geometric parameters,

The expressions of the columns of  $\mathbf{J}$  are calculated as given in [7,12,14].

To identify  $\Delta \mathbf{X}$ , equation (2) will be used for a sufficient number of configurations  $\mathbf{q}^1, \dots, \mathbf{q}^m$ , the corresponding poses will be measured and the  $\Delta \mathbf{y}^i$  will be calculated. The resulting linear system of equations will be represented by :

$$\Delta \mathbf{Y} = \mathbf{W} \Delta \mathbf{X} \quad (3)$$

where:

$$\mathbf{W} = \begin{bmatrix} \mathbf{J}(\mathbf{q}^1) \\ \dots \\ \mathbf{J}(\mathbf{q}^m) \end{bmatrix}, \Delta \mathbf{Y} = \begin{bmatrix} \Delta \mathbf{y}^1 \\ \dots \\ \Delta \mathbf{y}^m \end{bmatrix} \quad (4)$$

Equation (3) will be solved to get the least squares errors solution.

**Remark :** If the orientation of the terminal link is not measurable, only the equations corresponding to the position error will be taken into account.

### The identifiable parameters

It can be seen that if some columns of  $\mathbf{J}$  are dependent in all configurations. Relation (2) can be reduced to [12,15]:

$$\Delta \mathbf{y} = \mathbf{J}_b(\mathbf{q}) \Delta \mathbf{X}_b \quad (5)$$

where  $\mathbf{J}_b(\mathbf{q})$  contains the  $b$  independent columns of  $\mathbf{J}$ , the corresponding parameters are known as identifiable parameters or base parameters they will be denoted by the vector  $\Delta \mathbf{X}_b$ .

The determination of the identifiable parameters  $\Delta \mathbf{X}_b$  must be done before the identification process. They can be obtained numerically using the QR decomposition of a matrix  $\mathbf{W}$  like that given in (3) in which  $\mathbf{q}^1, \dots, \mathbf{q}^m$  represent random values [15].

The choice of  $b$  independent columns to be kept is not unique. We choose the columns corresponding to the parameters which can be updated easily in the control system and do not change the existing inverse and direct geometric models

### III. THE NEW CALIBRATION METHODS

The main problem in the classical method, is the need to have an accurate, fast and not expensive equipment to measure the end-effector pose.

The methods, presented in this section, will be carried out using a set of configurations of the robot, where the position of the terminal point of the robot is in the same plane. Practically this set of points can be easily done [9].

The general equation of a plane is supposed as :

$$a x + b y + c z + 1 = 0 \quad (6)$$

where a, b, c represent the plane coefficients,

Since the terminal point of the robot is in the plane, thus :

$$a P_x(\mathbf{q}) + b P_y(\mathbf{q}) + c P_z(\mathbf{q}) + 1 = 0 \quad (7)$$

where  $P_x$ ,  $P_y$ ,  $P_z$  represent the coordinates of the terminal point in the world frame.

Equation (7) will be exploited to carry out the calibration of the robot parameters. Two methods are developed :

The first supposes that the coefficients of the plane (a, b, c) are known, while the second method supposes that the coefficients of the plane (a, b, c) are unknown.

### **The first method**

Assuming the coefficients of the plane are known, this can be practically useful in the case of interchanging two robots, such that a plane in the working area is already identified by the first robot. The equations of the plane can be also obtained easily using an external sensor where a very limited number of points is needed (theoretically only three points are needed) to determine the plane coefficients. If the values of the geometric parameters in the model  $\mathbf{X}$  are different than the real values  $\mathbf{X}_r$ , then using a first order development for  $\mathbf{X}$  in equation (7), we obtain :

$$a (P_x(\mathbf{q}) + \mathbf{J}_x(\mathbf{q}) \Delta \mathbf{X}) + b (P_y(\mathbf{q}) + \mathbf{J}_y(\mathbf{q}) \Delta \mathbf{X}) + c (P_z(\mathbf{q}) + \mathbf{J}_z \Delta \mathbf{X}) + 1 = 0 \quad (8)$$

Relation (8) can be written as :

$$[a \mathbf{J}_x(\mathbf{q}) + b \mathbf{J}_y(\mathbf{q}) + c \mathbf{J}_z(\mathbf{q})] \Delta \mathbf{X} = -a P_x(\mathbf{q}) - b P_y(\mathbf{q}) - c P_z(\mathbf{q}) - 1 \quad (9)$$

where :

$P_x(\mathbf{q})$ ,  $P_y(\mathbf{q})$ ,  $P_z(\mathbf{q})$  represent the coordinates of the terminal point calculated as a function of  $\mathbf{q}$  using the direct geometric model and the nominal values of the geometric parameters,

$\mathbf{J}_x$  is the first row of the Jacobian matrix defined in (2),  $\mathbf{J}_y$  is the second row of the Jacobian matrix defined in (2),  $\mathbf{J}_z$  is the third row of the Jacobian matrix defined in (2),

Equation (9) gives a linear equation in  $\Delta \mathbf{X}$ , for each configuration.

Sufficient number of configurations must be used to obtain a system of equations on the standard form ( $\Delta \mathbf{Y} = \mathbf{W} \Delta \mathbf{X}$ ). This system of equation will be solved iteratively to identify  $\Delta \mathbf{X}$ , the geometric parameters will be updated after each iteration.

The calculation of the identifiable parameters can be carried out numerically using a procedure similar to that given in [15], by studying the QR decomposition of a matrix  $\mathbf{W}$  calculated from Eq.(9) using m random points in a given plane.

### **The second method**

Assuming that the coefficients of the plane are unknown, from equation (7) the following equation can be obtained :

$$(a+\Delta a) (P_x(\mathbf{q})+J_x(\mathbf{q}) \Delta \mathbf{X})+(b+\Delta b) (P_y(\mathbf{q})+J_y(\mathbf{q}) \Delta \mathbf{X})+(c+\Delta c)(P_z(\mathbf{q})+J_z(\mathbf{q})\Delta \mathbf{X})+1=0 \quad (10)$$

neglecting the second order terms, we get :

$$[P_x(\mathbf{q}) \quad P_y(\mathbf{q}) \quad P_z(\mathbf{q}) \quad a \quad J_x(\mathbf{q})+b \quad J_y(\mathbf{q})+c \quad J_z(\mathbf{q})] \begin{bmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta \mathbf{X} \end{bmatrix} = -aP_x(\mathbf{q})-b P_y(\mathbf{q})-c P_z(\mathbf{q})-1 \quad (11)$$

we have to repeat this process for sufficient number of configurations to get a linear system as in the standard form .

The coefficients of the plane are initialized by calculating the equation of the nearest plane to the terminal points of the given configurations. The Cartesian coordinates of the terminal point are calculated using the direct geometric model of the robot. The solution of the following equation gives the initial values of a, b, c :

$$\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} P_x^1 & P_y^1 & P_z^1 \\ \dots & \dots & \dots \\ P_x^m & P_y^m & P_z^m \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (12)$$

with  $P_x^j$  ,  $P_y^j$  ,  $P_z^j$  are the coordinates of the terminal point calculated by the DGM for the configuration  $\mathbf{q}(j)$ .

The calculation of the identifiable parameters can be carried out numerically, by studying the QR decomposition of a matrix  $\mathbf{W}$  calculated using m random points in a given plane, where each row represents an equation similar to (11).

### Practical issues

The following remarks are reported as a result of experiments and simulation of different robots using the given planar methods :

- The first method, which supposes that a, b, c are known, is the best. The convergence to the real parameters always takes place even if the initial errors in the geometric parameters are very big.
- The second method works well for moderate initial errors in the parameters such that the error pose is about 0,30 meters, which is far away of the performances of industrial robot.

### IV. EXAMPLE

In this section we present the non-identifiable parameters, of a PUMA 560 robot with arbitrarily base location and terminal end effector, using four different methods of calibration. In the following bold letters gives the parameters having no effect on the model while the others are regrouped to some other parameters :

i- classical method using position measure the non identifiable parameters are :  $\Delta\theta_7$ ,  $\Delta r_3$ ,  $\Delta r_7$ ,  $\Delta\alpha_7$

- ii- position link method without external sensor [7], the non identifiable parameters are :  $\Delta\theta_1, \Delta d_1, \Delta r_0, \Delta\alpha_1, \Delta\theta_0, \Delta\theta_7, \Delta r_1, \Delta r_6, \Delta r_7, \Delta\alpha_7, \Delta r_3,$
- iii- using known plane, the non identifiable parameters are :  $\Delta\theta_7, \Delta r_0, \Delta r_6, \Delta r_7, \Delta\alpha_7, \Delta\theta_0, \Delta r_1, \Delta r_3.$
- iv- using unknown plane, the non identifiable parameters are :  $\Delta\theta_7, \Delta\theta_1, \Delta d_1, \Delta r_0, \Delta r_6, \Delta r_7, \Delta\alpha_1, \Delta\alpha_7, \Delta\theta_0, \Delta r_1, \Delta r_3.$

It is to be noted that in methods 2, 3 and 4 most of the non-identifiable parameters belong to the parameters whose axes are fixed, such as :  $\Delta r_0, \dots, \Delta\theta_1$ . The use of an external sensor will permit to identify these parameters in a frame fixed with respect to this sensor, which is in general not the desired world frame. Thus actually, the problem of identifying these parameters exists also in the case of external sensor. To solve this problem, the coordinates of some known poses in the environment must be available in order to put the robot in these poses and use the classical method to identify them.

## V. CONCLUSION

This paper presents new methods for the calibration of the geometric parameters of serial robots. They are based on putting the terminal point of the robot in a plane, only the position sensors of the joints are required. The methods can be put in practice quickly and precisely. The identifiable parameters given by the proposed method are slightly different than those of the classical method using an external position sensor. A software package "GECARO" (GEometric CALibration of RObots) has been developed to carry out the calibration of real robots or to study the simulation of the calibration process using the classical methods and the the autonomous methods presented in this paper and those presented in [7].

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