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Abstract: We consider a group or committee that faces a binary decision under uncertainty. Each member holds some private information. Members agree which decision should be taken in each state of nature, had this been known, but they may attach different values to the two types of mistake that may occur. Most voting rules have a plethora of uninformative equilibria, and informative voting may be incompatible with equilibrium. We analyze an anonymous randomized majority rule that has a unique equilibrium. This equilibrium is strict, votes are informative, and the equilibrium implements the optimal decision with probability one in the limit as the committee size goes to infinity. We show that this also holds for the usual majority rule under certain perturbations of the behavioral assumptions: (i) a slight preference for voting according to one's conviction, and (ii) transparency and a slight preference for esteem. We also show that a slight probability for voting mistakes strengthens the incentive for informative voting.

Key Words : Voting, Condorcet, committee, judgement aggregation.

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1. INTRODUCTION

Many important decisions are not taken by individuals but by groups, committees or electorates. We here analyze a class of such situations. The decision is binary and there are only two states of nature. All group members agree which decision is optimal in each state. However, the true state of the world is unknown. Group members have a common prior probability over states of nature, a prior that may be based on *ex ante* public information, such as evidence or expert reports presented to the whole group or committee. In addition, each committee member also has some private information, a private “signal” about the true state of nature. Group members may also differ in their valuations of the costs associated with each of the two types of mistake that may occur. What decision rule should the group use in order to aggregate their private information and valuations? How should each member act under such a group decision rule? What if group members have a slight preference for acting in accordance with their conviction, even when this may run against their strategic incentives, and/or care about their individual public esteem in case individual committee members’ votes become publicly known *ex post*? What if there is a positive probability that a group member makes a mistaken voting decision? These are the questions that we here address, within a stylized and abstract game-theoretic framework.

The topic is not new. Condorcet’s (1785) so-called jury theorem essentially establishes that if (a) each member’s information is positively correlated with the true state of nature (the defendant being innocent or guilty, respectively), (b) distinct members’ information is conditionally independent (given the state of nature), and (c) all jury members vote according to their own private information only, then aggregation by way of the majority rule is asymptotically efficient in the sense that the probability for a mistaken jury decision (convicting an innocent defendant or acquitting a guilty) tends to zero as the number of jury members goes to infinity.

However, the modern strategic analysis of committee voting, pioneered by Austen-Smith and Banks (1996), has pointed out a major weakness of the classical result. While Condorcet’s hypothesis (c) — that jury members base their votes on their own private information without regard to other jury members’ potential information and votes — may seem innocuous, a careful game-theoretic analysis shows that such voting behavior may not be individually rational, even when all members have identical preferences and never make mistakes. More exactly, Austen-Smith and Banks made the remarkable discovery that, if the number of voters is large, informative voting is generically not a Nash equilibrium of a Bayesian game that formally represents

Condorcet's setting. The game-theoretic reasoning runs as follows: an individual vote makes a difference to the outcome only if it is pivotal.¹ Hence, as a voter under majority rule, one should reason *as if* the other votes were in a tie. But if the number of votes is large and all the others vote informatively, the (hypothetical) fact that they are tied is very informative, perhaps "drenching" the individual voter's own private information. Under such weak "evidence," perhaps the individual voter should not vote according to his or her private information. If the group, committee, jury or electorate is large enough, this argument against informative voting becomes overwhelming, for generic probabilities. Consequently, informative voting is then not a Nash equilibrium and Condorcet's judgement-aggregation argument fails.

This reasoning builds on the consideration of a low-probability event — the occurrence of an exact tie — that, in addition, is non-trivial to compute. Experimental evidence concerning decision-making in groups suggests that there indeed is a strategic element in individual voting behavior in small groups, while the standard strategic-voting model does not very well predict the collective voting outcomes observed in somewhat larger groups, see Guarnaschelli, McKelvey and Palfrey (2000). This suggests that a voting model should maintain the strategic element but allow for a richer description of voters' motives and allow for some degree of bounded rationality. In particular, committee members may have a deontological preference for voting according to their own conviction (based on their prior and private information), and/or be concerned about their future esteem in case their individual votes will be publicly known *ex post*, and individual members may occasionally make mistaken voting decisions.

We here generalize the standard strategic-voting model in these three dimensions. We also analyze a new and alternative voting rule, a slightly randomized majority rule. We show that under this rule, informative voting is a Nash equilibrium, that this equilibrium is strict (hence also essential and perfect) and asymptotically efficient in the sense that it aggregates all information in the limit as the number of committee members goes to infinity. We also show that, unlike the usual voting rules, this rule has sincere voting as its *unique* Nash equilibrium. The above-mentioned variations of the behavioral hypotheses turn out to play essentially the same technical role as this variation of the institutional structure. In the standard voting model, the probability

¹The notion of a pivotal event for a player is not restricted to voting games; Al-Najjar and Smorodinsky (2000) defined, in a general setting, the *influence* of a player in a mechanism as the maximum difference this player's action can make to the expected value of a collective result. They show that, in a precise sense, the mechanisms that maximise the number of influential players are closely related to majority rule.

of a tie under sincere voting is small when the number of voters is large. Therefore, the strategic incentive against sincere voting becomes weak when there are many voters. Hence, small behavioral or institutional variations easily turn over the negative result concerning sincere voting in the standard model. By contrast, both Condorcet's hypothesis — informative voting — and conclusion — asymptotic efficiency — hold in strategic voting models that only differ slightly, in terms of institutional and/or behavioral assumptions, from the standard one.

Before embarking upon our analysis, let us briefly comment on some related research. The two seminal papers on incentives for informative voting are Austen-Smith and Banks (1996), mentioned above, and Feddersen and Pesendorfer (1996). In the latter paper, the so-called swing voter's curse is analyzed. It refers to the following phenomenon. If voters, among whom there are partisans for each alternative as well as non-partisans, are allowed to abstain from voting, then poorly informed non-partisans may use the following mixed strategy. They probabilistically balance their votes in such a way that they collectively compensate for the presence of partisan voters (who support a given candidate in any case) and leave room for the better informed non-partisan voters. This mixed strategy of poorly informed non-partisan voters involves abstention with positive probability. By contrast, in the present analysis we do not allow our voters to abstain — this is one of the two senses in which our model is more relevant for committees than general electorates.

Subsequent theoretical research on committee behavior has mainly concern the relative merits of different voting rules, see Feddersen and Pesendorfer (1998), and the role of straw-votes or debates before voting, see Coughlan (2000) and Austen-Smith and Feddersen (2005). When voters are identical, the picture is very different with and without debate or straw vote. If voters with identical preferences share their private information in the debate or straw vote, which they are in certain equilibria, then votes are unanimous in the decisive vote, and all majoritarian voting rules (including unanimity) are equivalent, see Gerardi and Yariv (2007). However, in general there is a plethora of other, uninformative equilibria, and truthful reporting in the straw vote is incompatible with equilibrium if the committee is sufficiently heterogeneous in terms of values/preferences. The second sense in which our model concerns committees and not electorates is that we assume that the number of voters is fixed and known. By contrast, in general elections, this number is usually unknown by individual voters at the time of voting. See Myerson (1998), Feddersen and Sandroni (2002) and Krishna and Morgan (2007) for theoretical models of abstention and electorates of random size.

The rest of the paper is organized as follows. In section 2 we dress the table for the subsequent analysis by way of spelling out the base-line model. Section 3 analyzes optimality of deterministic and anonymous voting rules. Section 4 is devoted to equilibrium considerations under diverse majoritarian voting rules, while Section 5 examines a two-stage voting procedure where the first stage is a straw vote. Then we come to our main results. Section 6 develops a slightly randomized majority rule. We show that informative voting is the unique Nash equilibrium under this rule, that it is a strict equilibrium, and that the unique equilibrium outcome is asymptotically efficient. In section 7 we analyze arguably plausible perturbations of the behavioral assumptions, such as a slight preference for voting according to one's conviction, a slight preference for esteem, and a slight probability of mistakes in voting. In the case of esteem, we study the incentive effect upon voting behavior of increased transparency — of making individual committee members votes known to the public, and hence making esteem considerations more important. Section 8 concludes. Mathematical proofs are relegated to an appendix at the end of the paper.

2. THE MODEL

2.1. Notation and basic setup. There are n committee members, where n is a positive integer, $n \in \mathbb{N}$. The committee has to make a binary decision, $x \in \{0, 1\} = X$. All committee members agree what is the right decision in each state of nature. However, they do not know the state of nature $\omega \in \{0, 1\} = \Omega$. Each committee member i receives a private “signal” $s_i \in \{0, 1\}$, a random variable that is positively correlated with the true state of nature ω :

$$\begin{cases} \Pr [s_i = 0 \mid \omega = 0] = q_0 \\ \Pr [s_i = 1 \mid \omega = 1] = q_1 \end{cases}$$

for $q_0, q_1 > 1/2$. Hence, all committee members are “equally competent” in the sense of having the same conditional probability of receiving the “correct” signal. Signals received by different committee members are, however, conditionally independent, given the state of nature. The committee members share a prior belief about the actual state of nature (prior to the receipt of their private signals). This common prior may be based in part on a common signal, received by all committee members, as, for example, in the proceedings during a trial, during a committee hearing, or in a public debate before a general election.² Let $\mu = \Pr [\omega = 1]$ be the common prior, for $0 < \mu < 1$.

²See Dixit and Weibull (2007) for an analysis of how judgmental polarization can arise among voters with distinct priors who receive a common signal.

All committee members agree that the right decision in state ω is $x = \omega$. However, they may differ in the von Neumann-Morgenstern utilities that they assign to the four possible decision-state pairs. For each committee member i , these utilities are given by the following table:

	$\omega = 0$	$\omega = 1$
$x = 0$	u_{00}^i	u_{01}^i
$x = 1$	u_{10}^i	u_{11}^i

(1)

where $u_{01}^i = u_{11}^i - \alpha_i$ and $u_{10}^i = u_{00}^i - \beta_i$ for $\alpha_i, \beta_i > 0$. For each committee member i , these two parameters are the disutilities or “costs” that the committee member attaches to the two types of mistake, namely, of taking the wrong decision in each of the two states. A committee member’s von Neumann-Morgenstern utilities may represent his or her personal values or those of some constituency that the member represents. We will sometimes refer to the first mistake (decision $x = 0$ in state $\omega = 1$) as a mistake of type I (accepting the false hypothesis that the state is 0) and the second mistake (decision $x = 1$ in state $\omega = 0$) as a mistake of type II (rejecting the true hypothesis that the state is 0). For many purposes, the relevant data about each committee member’s values, as given in (1), can be summarized in a single number, namely

$$\gamma_i = \frac{\mu\alpha_i}{(1 - \mu)\beta_i} \tag{2}$$

where $\gamma_i > 0$ follows from our assumptions. Note that $\gamma_i = 1$ if and only if committee member i attaches the same *ex ante* expected “cost” to both types of mistake. Before receiving his or her signal, the probability that a committee member attaches to state 1 is μ and the “cost” of a mistake in that state (a mistake of type I) is α_i . Hence, the *ex-ante* expected cost of a mistake of type I, according to committee member i ’s values, is $\mu\alpha_i$. Likewise, the probability attached to state 0 is $1 - \mu$ and the “cost” of a mistake then, that is, of type II, is, in i ’s view, β_i . Hence, the *ex-ante* expected cost of a mistake of type II, according to committee member i , is $(1 - \mu)\beta_i$. The summary parameter γ_i is the ratio between these two *ex-ante* expected costs, as evaluated by committee member i .

When studying asymptotic properties of increasingly large committees, we will assume that all parameter pairs (α_i, β_i) belong to the same compact set in the interior of the positive orthant:

$$(\alpha_i, \beta_i) \in \Theta = A \times B \subset \mathbb{R}_{++}^2 \tag{3}$$

We will refer to this condition as *the (uniform) value-boundedness condition*. This condition is trivially met if all committee members in ever larger committees are

identical, and it is also met under replication of a given finite preference profile, and under independent sampling from a fixed probability distribution with support in Θ .

In the base-line setting, each committee member i casts a vote $v_i \in \{0, 1\}$, a vote which may, but need not, be guided by i 's private signal, and the collective decision x is determined by way of some pre-specified rule f that maps each *vote profile* $v = (v_1, \dots, v_n)$ to a probability $f(v) \in [0, 1]$ that the decision will be $x = 1$. The probability for decision $x = 0$ is $1 - f(v)$. Formally, a *voting rule* is a function $f : \cup_{n \in \mathbb{N}} \{0, 1\}^n \rightarrow [0, 1]$. In particular, *majority rule* is the voting rule f defined by $f(v) = 1$ if $\sum_{i=1}^n v_i > n/2$, $f(v) = 0$ if $\sum_{i=1}^n v_i < n/2$ and $f(v) = 1/2$ otherwise. A *voting strategy* for committee member i in the base-line setting is a function $\sigma_i : \{0, 1\} \rightarrow [0, 1]$ that maps i 's signal s_i to a probability $\sigma_i(s_i)$ for a vote v_i on alternative 1: $\Pr[v_i = 1 \mid s_i] = \sigma_i(s_i)$.³ In others words, a voting strategy prescribes with what probability the committee member will vote for decision alternative 1. We assume that abstention is not an alternative, so the probability that i will vote on alternative 0 is $1 - \sigma_i(s_i)$.⁴ By a *pure* voting strategy we mean a strategy σ_i such that $\sigma_i(s_i) \in \{0, 1\}$ for both signals s_i . In this case, $v_i = \sigma_i(s_i)$. In the voting literature, the pure strategy to always vote according to one's signal, $\sigma_i(s_i) \equiv s_i$, is usually called *informative* voting, while voting for the alternative that maximizes the voter's expected utility, conditional on his or her own signal, and only on that piece of information, is called *sincere* voting.

2.2. Condorcet's jury theorem. Condorcet's Jury Theorem asserts that if all committee members vote informatively, then the probability of a mistaken collective decision under majority rule tends to zero as the committee size tends to infinity. The result hinges on the assumption that the signals are positively correlated with the true state and that they are conditionally independent. The result does not explicitly depend on committee members' values, since their voting behavior is assumed:

Theorem 1 [Condorcet]. *Suppose that all committee members vote informatively under majority rule. Let $X_n(\omega) \in \{0, 1\}$ be the collective decision when there are n committee members and the true state is ω . Then*

$$\lim_{n \rightarrow \infty} \Pr[X_n(\omega) \neq \omega] = 0$$

³We will later analyze behavioral voting strategies under two-stage voting rules.

⁴In some committees abstention is indeed not permitted while in others it is. We exclude the latter case for analytical convenience and brevity.

2.3. Signal informativeness. A hypothesis in Condorcet's theorem is thus that all committee members vote informatively. Clearly, this is not always a reasonable assumption, not even for $n = 1$, the case of a single decision-maker. To clarify this aspect, suppose, that one committee member has been selected to make the decision single-handedly, based only on his or her private signal. If the signal is noisy and her prior and valuation of mistake costs favor one alternative over the other, the right decision may well be to disregard the signal. An application of Bayes' rule gives the following posterior probability for state 0 after signal 0 has been received:

$$\Pr[\omega = 0 \mid s_i = 0] = \frac{(1 - \mu) \Pr[s_i = 0 \mid \omega = 0]}{\Pr[s_i = 0]} = \frac{(1 - \mu) q_0}{(1 - \mu) q_0 + \mu(1 - q_1)}$$

and likewise for the signal $s_i = 1$. Consequently, the strategy to vote informatively — when the decision is in i 's hands — is optimal if and only if $(1 - \mu) q_0 \beta_i \geq \mu(1 - q_1) \alpha_i$ and $\mu q_1 \alpha_i \geq (1 - \mu)(1 - q_0) \beta_i$, or, equivalently, if and only if $(1 - q_0)/q_1 \leq \gamma_i \leq q_0/(1 - q_1)$. We assume henceforth that both inequalities hold strictly for all committee members:

$$\frac{1 - q_0}{q_1} < \gamma_i < \frac{q_0}{1 - q_1} \quad \forall i, \quad (4)$$

a condition we will refer to as *the signal-informativeness condition*. It follows from our assumption $q_0, q_1 > 1/2$ that the lower (upper) bound in (4) is below (above) unity.

3. OPTIMAL VOTING RULES

What voting rules are optimal for the committee? We here briefly consider the optimality of *deterministic* one-stage voting rules, that is, voting rules $f : \cup_{n \in \mathbb{N}} \{0, 1\}^n \rightarrow \{0, 1\}$ that map vote profiles $v = (v_1, \dots, v_n)$ to decisions. As for normative criteria by which to define optimality, the following two seem most relevant: maximization of the probability for taking the right decision, or, alternatively, maximization of the sum of the committee members' expected (vonNeumann-Morgenstern) utilities from the decision. While the first criterion does not discriminate between mistakes of type I and II, the second does, which makes sense in many contexts. For instance, if most or all members of a jury (hiring committee) consider it a worse error to convict an innocent (hire an incompetent job candidate) than to acquit a guilty defendant (not hire a competent job candidate), then it is desirable that the voting rule accounts for this asymmetry of values. We here focus on this latter, utilitarian criterion.⁵ We

⁵This criterion is clearly cardinal in the sense that it is affected by individual linear transformations of committee members' vonNeumann-Morgenstern utilities (but not by addition of constants to these).

call a deterministic voting rule f (*first-best*) *optimal* if, given all committee members' private information, there exists no other such function that yields higher expected welfare. Formally, the criterion thus is to maximize

$$W(f) = \sum_{i,x,\omega} \Pr[(x,\omega)] \cdot u_{x\omega}^i$$

when $x \equiv f(s_1, \dots, s_n)$. Hence, we evaluate welfare by applying the voting rule f in question directly to the signal vector, or, equivalently, under (Condorcet's) hypothesis that all committee members vote informatively. In other words: a (first-best) optimal voting rule is an optimal deterministic *direct mechanism* with no requirement of incentive-compatibility.⁶

Of great practical relevance are the majoritarian rules. Formally, let \mathbb{N} be the nonnegative integers and for any $k \in \mathbb{N} \cap [0, n + 1]$, let $f^k : \{0, 1\}^n \rightarrow \{0, 1\}$ be the k -majority rule defined by $f^k(v_1, \dots, v_n) = 1$ iff $\sum_{i=1}^n v_i \geq k$. For n odd, majority rule is thus the special case $k = (n + 1)/2$. For arbitrary n , $k = 1$ and $k = n$ are the two *unanimity rules* (requiring n votes for decision 0 and 1, respectively), $k = 0$ the rule to take decision 1 irrespective of the votes and $k = n + 1$ the rule to take decision 0 irrespective of the votes. It is not difficult to verify that if the signal-informativeness condition (4) holds and a certain k -majority rule is optimal among such rules, then necessarily $1 \leq k \leq n$. Moreover, the k -majority rule in question is then optimal among all deterministic voting rules.⁷

Hence, without loss of generality we may restrict the quest for optimal rules to k -majority rules, where $1 \leq k \leq n$. Since the number n of committee members is finite, existence of an optimal voting rule is guaranteed. The following result provides a necessary and sufficient condition for optimality. Let

$$\bar{\alpha}(n) = \frac{1}{n} \sum_{i=1}^n \alpha_i, \quad \bar{\beta}(n) = \frac{1}{n} \sum_{i=1}^n \beta_i \quad \text{and} \quad \bar{\gamma}(n) = \frac{\mu \bar{\alpha}(n)}{(1 - \mu) \bar{\beta}(n)}.$$

⁶Using another definition of collective welfare, Chwe (2007) analyzes which deterministic voting rule maximizes welfare under the constraint that voters should have no incentive to vote insincerely. The optimal voting rule is then non-monotonic (a large majority in favor of one alternative leads to the adoption of the opposite decision) and under this rule all voters are indifferent between sincere and insincere voting.

⁷To see this, suppose that $f : \{0, 1\}^n \rightarrow \{0, 1\}$ is optimal. Since all voters' signals have the same precision, there exists some symmetric function $g : \{0, 1\}^n \rightarrow \{0, 1\}$ such that $W(g) = W(f)$. Then $g(s_1, \dots, s_n)$ is a function h of the signal sum $\sum s_i$. Since f maximizes W , so does g , and then h has to be increasing, since $q_0, q_1 > 1/2$ by assumption; nothing can be gained by disregarding a signal. Since h is increasing, g is a k -majority rule for some $k \in \{0, 1, \dots, n, n + 1\}$. If the signal informativeness condition (4) holds strictly, and $\alpha_i, \beta_i > 0$, it is never optimal to disregard all signals, so then $k \in \{1, \dots, n\}$.

The parameter pair $(\bar{\alpha}(n), \bar{\beta}(n))$ can be thought of as the values of a (*synthetic*) *representative voter*. For arbitrary positive integers n and k , let

$$g(k, n) = \left[\frac{(1 - q_0)(1 - q_1)}{q_0 q_1} \right]^k \left(\frac{q_0}{1 - q_1} \right)^n$$

and note that the factor in square brackets is less than 1 while the factor in round brackets exceeds 1. Hence, $g(k, n)$ is decreasing in k and increasing in n . The following result characterizes the optimal voting rule, for a given committee, in terms of its representative voter:⁸

Theorem 2. *Suppose that the signal-informativeness condition (4) holds. For any positive $n \in \mathbb{N}$ and $k \in \mathbb{N} \cap [1, n]$, k -majority rule is optimal among all deterministic voting rules if and only if*

$$g(k, n) \leq \bar{\gamma}(n) \leq g(k - 1, n). \tag{5}$$

For n odd, majority rule is deterministic and corresponds to $k = (n + 1)/2$. Hence, by (5), majority rule is optimal if and only if

$$\frac{1 - q_0}{q_1} \left[\frac{q_0(1 - q_0)}{(1 - q_1)q_1} \right]^{\frac{n-1}{2}} \leq \bar{\gamma}(n) \leq \frac{q_0}{1 - q_1} \left[\frac{q_0(1 - q_0)}{(1 - q_1)q_1} \right]^{\frac{n-1}{2}} \tag{6}$$

In the special case of equally precise signals, $q_0 = q_1$, the factor in square brackets is 1, and then (6) follows from the signal informativeness condition (4).⁹ In sum, irrespective of the committee members' values.¹⁰

Corollary 1. *Majority rule is optimal whenever n is odd and $q_0 = q_1$.*

Consider a sequence of committees of ever larger size $n = 1, 2, \dots$, all with the same signal precisions q_0 and q_1 , for arbitrary $q_0, q_1 \in (1/2, 1]$ and assume that the value-boundedness condition (3) holds. For each positive integer n , let $k^*(n)$ be optimal, and write $\rho^*(n)$ for $k^*(n)/n$, the optimal vote ratio.¹¹ It is not difficult to verify that $\rho^*(n)$ converges as the committee size n goes to infinity, and, perhaps surprisingly,

⁸Note that $q_0/(1 - q_1) \leq g(0, n) < g(n, n) \leq (1 - q_0)/q_1$. Hence, if the signal-informativeness condition (4) holds, then condition (5) below holds for some $k \in \{1, \dots, n\}$.

⁹By (4): $(1 - \mu)\beta_i(1 - q_0)/q_1 \leq \mu\alpha_i \leq (1 - \mu)\beta_i q_0/(1 - q_1)$ for all i . Addition of these n inequalities implies (6).

¹⁰The same result is obtained within a different model framework in Sah and Stiglitz (1988).

¹¹Generically, $k^*(n)$ is unique.

that the limit is independent of the committee members' values/preferences, as long as these meet the value-boundedness condition: the limit of $\rho^*(n)$, as n tends to infinity, then depends only on the precision of the two signals. In particular, the limit value is $1/2$ if the two signals are equally precise. In other words, as a special case we obtain that majority rule is optimal for large committees or electorates, independently of voters' preferences, as long as their two signals are equally precise. Moreover, by continuity, this holds approximately for approximately equally precise signals.

Corollary 2. *Suppose that the signal-informativeness condition (4) and the value-boundedness condition (3) hold. For any $q_0, q_1 \in (\frac{1}{2}, 1]$:*

$$1 - q_0 \leq \lim_{n \rightarrow \infty} \rho^*(n) = \frac{\ln\left(\frac{q_0}{1-q_1}\right)}{\ln\left(\frac{q_0}{1-q_1}\right) + \ln\left(\frac{q_1}{1-q_0}\right)} \leq q_1 \tag{7}$$

The reason why the optimal voting rule is asymptotically independent of committee members' values is, roughly, that for large committees the probability of a mistake is vanishingly small. It turns out that all that matters is the relative precision of the two signals, captured by the two parameters q_0 and q_1 . To see this, suppose that, for a given committee size n , a certain k -majority rule f^k is optimal: $k^*(n) = k$. Then we necessarily have $W(f^k) \geq W(f^{k+1})$, that is, it should not be welfare improving to instead use the $(k + 1)$ -majority rule. Let N_1 be the (random) number of signals 1 among the n signals received. Comparing voting rules f^k and f^{k+1} , it is clear that the first is (weakly) better than the second if and only if it is better when $N_1 = k$, this being the only event in which the two voting rules differ. Moreover, if $N_1 = k$, then f^k is (weakly) better than f^{k+1} if and only if erring under voting rule f^{k+1} (taking decision 0 when the state is 1) incurs no smaller social cost than erring under f^k (taking decision 1 when the state is 0), which amounts to the inequality

$$\bar{\alpha}(n) \Pr[N_1 = k \mid \omega = 1] \geq \bar{\beta}(n) \Pr[N_1 = k \mid \omega = 0].$$

For very large n and proportionately large k , $k = k^*(n) \approx \rho^*(n) n$, the ratio between the probabilities either tends to zero or to plus infinity (since $q_0, q_1 > 1/2$). However, the ratio $\bar{\gamma}(n) = \bar{\alpha}(n) / \bar{\beta}(n)$ is by hypothesis bounded away from zero and plus infinity (uniformly in n). Hence, asymptotically, it does not matter exactly what values the parameters $\bar{\gamma}(n)$ take, as long as they all belong to a bounded interval of positive numbers. Consequently, the limit ratio $\rho^*(n)$ does not depend on the committee members' values. (See appendix for a formal proof.)

4. EQUILIBRIUM

Suppose that the collective decision is to be taken according to k -majority rule among n committee members, for some positive integer $k \leq n$, and as described above. This k may be, but need not be optimal. Is sincere voting then a Nash equilibrium? In force of the signal informativeness condition (4), sincere voting is identical with informative voting, and we will use these two attributes interchangeably. In this voting game, each voter i first observes her private signal and then casts her vote $v_i \in \{0, 1\}$, simultaneously with all other voters. The collective decision $x = 1$ results if at least k voters cast the vote 1, while the collective decision $x = 0$ results in the opposite case.

In Nash equilibrium, each voter maximizes his or her expected utility, given his or her private signal, and given all other voters' strategies. Clearly, there is a plethora of (pure and mixed) uninformative Nash equilibria whenever $n \geq 3$. For example, to always vote 0 (or 1), independently of one's private signal, constitutes a Nash equilibrium. For if others vote according to such a strategy, then my vote will never be pivotal and hence I can just as well use the same uninformative voting strategy as the others. Under what conditions, if any, will sincere voting constitute an equilibrium?

Theorem 3. *Suppose that the signal-informativeness condition (4) is met. For any positive integers n and k with $1 \leq k \leq n$, sincere voting under k -majority rule constitutes a Nash equilibrium if and only if*

$$g(k, n) \leq \gamma_i \leq g(k-1, n) \quad \forall i \quad (8)$$

Some remarks are in place. First, if committee members have identical values, then conditions (8) and (5) are identical. Hence, in this special case, a k -majority rule is optimal (for a committee of given size n) if and only if sincere voting under this rule is a Nash equilibrium. This was first proved by Austen-Smith and Banks (1996, Lemma 2), see also Costinot and Kartik (2006) for more findings under the same hypothesis of *ex ante* identical committee members. Secondly, for $k = n = 1$ — the case of a single decision-maker — condition (8) is, as one would expect, identical with the signal informativeness condition (4).

Thirdly, if n is odd and $k = (n + 1)/2$ — *majority* rule — then (8) coincides with the weak-inequality version of (4) when $q_0 = q_1$. Hence, for n odd and equally informative signals, sincere voting is a Nash equilibrium under majority rule, irrespective of individual valuations γ_i , as long as these meet condition (4). This is not surprising, since in the knife-edge case of equally precise signals, a tie among an even

number of other votes does not affect the odds for one state over the other.¹² Generically, however, q_0 and q_1 are not identical.¹³ Suppose, thus, that $q_0 \neq q_1$ and consider majority rule in a committee with an odd number of members. Condition (8) then becomes

$$\frac{1 - q_0}{q_1} \left[\frac{q_0 (1 - q_0)}{(1 - q_1) q_1} \right]^{\frac{n-1}{2}} \leq \gamma_i \leq \frac{q_0}{1 - q_1} \left[\frac{q_0 (1 - q_0)}{(1 - q_1) q_1} \right]^{\frac{n-1}{2}} \quad \forall i \quad (9)$$

If q_0 and q_1 differ even the slightest, then the factor in square brackets is distinct from unity. Hence, as n tends to infinity, this factor either converges to zero (if $q_0 > q_1$) or to plus infinity (if $q_0 < q_1$). Inevitably, for any given positive γ_i -value, one of the two inequalities in (8) is thus violated for all n sufficiently large. We have obtained the following slight generalization of Theorem 1 in Austen-Smith and Banks (1996):¹⁴

Corollary 3. *Suppose that $q_0 \neq q_1$. For any positive sequence $(\gamma_i)_{i \in \mathbb{N}}$ there exists an $n_0 \in \mathbb{N}$ such that sincere voting is a Nash equilibrium under majority rule for no $n \geq n_0$.*

This result is intuitively plausible. For suppose that state 0 is more likely to give rise to signal value 0 than state 1 is likely to give rise to signal value 1, that is, $q_0 > q_1$. In such a case, signal 0 is *less* informative than signal 1 in the sense that signal 0 is more likely in state 1 than signal 1 is in state 0. If n is large, a tie among the others is then quite a strong indication of state 1, even if a voter's own signal is 0, since in total there are just about as many signals 0 as signals 1, quite an unlikely event in state 0. Hence, even if I, as a voter, believed that the others vote sincerely, I should nevertheless vote on alternative 1, irrespective of my own signal.

Remark 1. *The corollary can also be explained in terms of Corollary 2: If $q_0 \neq q_1$, then majority rule is not asymptotically optimal. Hence, by Theorem 3 there exists a committee size n_0 beyond which sincere voting is not a Nash equilibrium.*

We finally explore the relations between (i) optimality of a voting rule and (ii) sincere voting being an equilibrium under a voting rule. It follows immediately from Theorems 2 and 3 that if sincere voting is a Nash equilibrium under some k -majority

¹²Then $\Pr[\omega = 0 \mid \mathcal{T} \wedge s_i = 0] = \Pr[\omega = 0 \mid s_i = 0]$ and likewise for $\omega = s_i = 1$.

¹³The probability that an innocent defendant will appear innocent may well be different from the probability that a guilty defendant will appear guilty, and the same can be said about signals for good and bad investment opportunities or for different states of a market or whole economy.

¹⁴Their result concerns the special case $\alpha_1 = \alpha_2 = \dots = \alpha_n = \beta_1 = \beta_2 = \dots = \beta_n$.

rule, then this rule is optimal. What about the converse? We already noted that the converse holds if committee members have identical values. What if they do not? We proceed to show that if a k -majority rule is optimal, then sincere voting is an equilibrium under this rule for sufficiently homogenous committees. To this end, it is useful to first ask which collective decision $x \in \{0, 1\}$ individual i then would like to see taken, if i had known the total number N_1 of signals 1 received among all committee members. As shown in the appendix, voter i will deem decision $x = 1$ to be better than decision $x = 0$ if and only if :

$$\Pr[N_1 \mid \omega = 0] \leq \gamma_i \Pr[N_1 \mid \omega = 1] \tag{10}$$

(The condition is self-evident in the special case when $\gamma_i = 1$.) After some algebraic manipulation, this condition can be re-written as $N_1 \geq \lambda_i$, where

$$\lambda_i = \frac{n \ln \frac{q_0}{1-q_1} - \ln \gamma_i}{\ln \frac{q_0}{1-q_1} + \ln \frac{q_1}{1-q_0}} \tag{11}$$

We note that both terms in the denominator are positive and that the critical value λ_i decreases with γ_i . Without loss of generality we may assume that $\gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_n$ and hence $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. For generic parameter values q_0, q_1 and γ_i , the parameters λ_i are not integers. We here focus on this generic case. We will call $\lambda_i \in \mathbb{R}$ the *threshold* of i ; voter i needs more than λ_i signals 1 to prefer decision 1 (in the jury case, a “guilty” verdict). Write $m_i = \lfloor \lambda_i \rfloor + 1$ for the smallest integer exceeding λ_i . Committee member i thus prefers decision 1 (over decision 0) if and only if the number of signals 1 is at least $m_i \in \mathbb{N}$. We call the committee *homogeneous* if these integer thresholds are the same for all members,

$$m_i = m_j \quad \forall i, j \tag{12}$$

while otherwise the committee will be called *heterogeneous*.¹⁵

Proposition 1. *If sincere voting is a Nash equilibrium under k -majority rule, then k -majority rule is optimal. Conversely, if k -majority rule is optimal and the committee is homogeneous, then sincere voting under this rule is a Nash equilibrium.*

¹⁵A committee is thus heterogeneous if $\lambda_1 > \lfloor \lambda_{m+1} \rfloor + 1$ or $\lambda_n < \lfloor \lambda_{m+1} \rfloor$ or both.

5. STRAW VOTE

It is plausible that communication before voting can probabilistically improve the outcome when the committee is homogenous, since then no member has an incentive to misreport his or her private information. Coughlan (2000) considers the following two-stage voting procedure. In stage one, all committee members simultaneously report their private signals to a “center.” These reports may be truthful or false. The total counts of reported zeros and ones are made public to the whole committee. In stage two, there is simultaneous voting under some k -majority rule as described above, but now with the total number of reports of each type being common knowledge. Coughlan shows that under such a straw vote procedure, truthful revelation of one’s signal in the communication stage is compatible with sequential equilibrium if voters are *ex ante* identical. However, this is but one of a plethora of equilibria, many of which are uninformative. For example, even if everyone sends truthful reports, it is optimal to vote on alternative 0 irrespective of the information available, if all others to do likewise. Moreover, as Coughlan (2000) shows, truthful reporting is not compatible with equilibrium if the committee is heterogeneous in terms of values. Members with extreme values will truthfully report their information only if it “points in the right direction.”

We here generalize Coughlan’s result, in the special case of majority rule. Consider a committee consisting of an arbitrary odd number n of committee members. Without loss of generality, suppose again that $\gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_n$. For each committee member i , let λ_i be as defined in equation (11). We will call committee member $i = (n + 1) / 2$ *the median voter* — this is the committee member who has equally many committee members on her “left” (in the weak sense) as on her “right” (in the weak sense) on the value scale. Let s_i be the signal received by committee member i . Denote by $t_i \in \{0, 1\}$ the *straw vote* of i in stage one of this voting mechanism (abstentions not allowed) and call these votes *reports*. Let the random variable N_1 be the number of reports “1” in the first stage: $N_1 = \sum_{i=1}^n t_i$. In the second stage, the committee votes again, now “for real,” and the collective decision x is taken according to majority rule. The vote cast by individual i at this second stage is denoted v_i . When deciding which vote to cast, voter i knows her own private signal s_i and report t_i , as well as the realization of N_1 . Hence, a *pure strategy* for each committee member i is a pair $(\tau_i, \tilde{\sigma}_i)$, where $\tau_i : \{0, 1\} \rightarrow \{0, 1\}$ assigns a report $t_i = \tau_i(s_i)$ to each signal s_i received, and $\tilde{\sigma}_i : \{0, 1\}^2 \times \{0, 1, 2, \dots, n\} \rightarrow \{0, 1\}$ assigns a vote $v_i = \tilde{\sigma}_i(s_i, t_i, N_1)$ to each own signal s_i received, own report t_i delivered, and observed count N_1 of reports “1” in the straw vote. Is truthful reporting compatible with sequential equilibrium?

Proposition 2. *Let n be odd and suppose that $\lambda_{(n+1)/2} \notin \mathbb{N}$. Truthful reporting in the straw vote is compatible with sequential equilibrium if and only if the committee is homogeneous.*

The same result, for the special case $q_0 = q_1$, can be found in Coughlan (2000), who also considers other majoritarian voting rules.

6. RANDOMIZED MAJORITY RULE

Consider first the voting rule according to which all n members of the committee simultaneously cast their votes, whereafter a random sample of $n' \leq n$ of these votes is drawn and the collective decision is made by way of some k -majority rule applied to this random sample of size n' . If each vote has a fixed positive probability of being sampled and the sample size n' is small enough, then we know from the preceding analysis that sincere voting will be a Nash equilibrium. More precisely, consider a committee of n members for which the signal informativeness condition (4) holds. Let the positive integer $n^* \leq n$ be maximal with the property that condition (8) holds for all subsets of the committee of sizes $n' \leq n^*$ (with n' in the place of n in (8)). From the signal-informativeness condition it follows that such an integer $n^* \geq 1$ exists. Let f^* be the randomized voting rule according to which all n committee members vote simultaneously and the collective decision x is determined by majority rule applied to a sample of odd size $n' \leq \min\{n^*, n\}$ of these votes, the sample being drawn at random from among all subsets of size n' , with equal probability for each such subset. Assume that the sample draw is statistically independent of the state of nature and of the signals and votes. The following observation follows immediately from Theorem 3:

Corollary 4. *Sincere voting is a Nash equilibrium under voting rule f^* .*

An evident drawback of this anonymous voting rule is that it does not aggregate the private information in an efficient way when n is large, since the sample size may remain bounded even if n increases. Hence, the collective decision under f^* may remain bounded away from full informational efficiency when $n \rightarrow +\infty$. However, there is a straight-forward remedy: combine this randomized majority rule with the usual majority rule. Instead of always letting a randomly selected subset of votes determine the collective decision, use a binary randomization device to determine whether the collective decision x be determined by the majority of the random sample or by the majority of all n votes. Under this doubly randomized majority rule,

Condorcet’s claim can be restored: if the binary randomization device is carefully calibrated, the collective decision will be correct with probability one in equilibrium in the limit as the committee size n tends to infinity. Moreover, we will show that, for every committee size n , this equilibrium is strict and unique. As is well known from analyses of other voting mechanisms, uniqueness is not trivial to obtain. The following example shows that this is also true for the present mechanism; even if the probability for random delegation is large enough to render sincere voting a (strict) Nash equilibrium, there may still exist other, less informative, equilibria.

Example 1. Consider a committee with three members with a uniform prior, $\mu = 1/2$, equally precise signals, $q_0 = q_1 = q$, distinct values, $\gamma_1 = 1/c$, $\gamma_2 = 1$ and $\gamma_3 = c$ for some $c > 1$, such that the signal-informativness condition (4) is strictly met for all committee members. Since the two signals are equally precise, sincere voting is a strict Nash equilibrium under majority rule. Consider the following randomized majority rule: with probability $1 - \varepsilon$ the decision is made according to majority rule, with probability ε it is delegated to $h = 1$ of the individual votes. Since an increase in ε enhances all voters’ incentive for sincere voting, sincere voting is a strict Nash equilibrium for all $\varepsilon \in [0, 1]$. However, for certain values of ε , there also exists a mixed equilibrium in which (a) voter 1 votes sincerely for sure when receiving signal 0 and with probability x when receiving signal 1, (b) voter 2 always votes sincerely, and (c) voter 3 votes sincerely for sure when receiving signal 1 and with probability y when receiving signal 0. The probability y should render voter 1 indifferent when receiving signal 1 and the probability x should render voter 3 indifferent when receiving signal 0. It is not difficult to verify that this amounts to the following requirements:

$$x = \frac{3(1 - \varepsilon)(c - 1)q(1 - q) - \varepsilon[(c + 1)q - c]}{3(1 - \varepsilon)(c + 1)(2q - 1)(1 - q)q}$$

$$y = \frac{3(1 - \varepsilon)(c - 1)q(1 - q) - \varepsilon[(c + 1)q - 1]}{3(1 - \varepsilon)(c + 1)(2q - 1)(1 - q)q}$$

For instance, for $c = 2$, $q = 0.7$ and $\varepsilon = 0.2$ we obtain $x^* \approx 0.80026$ and $y^* \approx 0.46958$, a Nash equilibrium in which the “extreme” voters 1 and 3 randomize. We also note that the mixed equilibrium exists ($0 < x, y < 1$) if and only if ε is sufficiently small:

$$\varepsilon < \frac{3(c - 1)q(1 - q)}{3(c - 1)q(1 - q) + (c + 1)q - 1}$$

In particular, for $c = 2$ and $q = 0.7$, the upper bound on ε is approximately 0.36.

For the sake of definiteness and ease of notation, we establish the claimed result for the special case of majority rule, n odd and $n' = 1$. For any committee with an odd number n of members: let all votes be cast simultaneously, and then, with a pre-specified (and by the voters known) probability, the decision is either taken according to majority rule, applied to all n votes, or, alternatively, according to a randomly drawn single vote. Let $1 - \varepsilon_n$ be the probability for the first event and ε_n the probability for the second. These probabilities are known by all committee members when they cast their votes. Denote by f^ε such a doubly randomized voting rule for committees with odd numbers of members, for any sequence $\varepsilon = (\varepsilon_n)_{n \in \mathbb{N}}$ with $0 \leq \varepsilon_n \leq 1$ for all $n \in \mathbb{N}$.

We first investigate the condition on the delegation probability ε_n for sincere voting to be a Nash equilibrium under such a voting rule f^ε , for a given committee size n . Suppose that committee member i has received the signal $s_i = 0$. Denote by $\Delta u_i^{\varepsilon_n}$ the difference in expected utility, for that member, when casting the sincere vote $v_i = 0$ rather than the insincere vote $v_i = 1$. Committee member i will become the “ex-post dictator” with probability ε_n/n , while majority rule will be applied to all n votes with probability $1 - \varepsilon_n$. If another committee member’s vote is sampled, then i ’s vote does not matter. It follows from the proof of Theorem 3 in the appendix that:¹⁶

$$\begin{aligned} \Delta u_i^{\varepsilon_{2m+1}} &= \frac{\varepsilon_n (1 - \mu) q_0 \beta_i - \mu (1 - q_1) \alpha_i}{n (1 - \mu) q_0 + \mu (1 - q_1)} + \\ &+ (1 - \varepsilon_n) \binom{2m}{m} \frac{(1 - \mu) q_0^{m+1} (1 - q_0)^m \beta_i - \mu q_1^m (1 - q_1)^{m+1} \alpha_i}{(1 - \mu) q_0 + \mu (1 - q_1)} \end{aligned} \quad (13)$$

The first term on the right-hand side — the probability that i ’s vote is randomly sampled times the conditionally expected utility difference when this happens — is non-negative if and only if $\gamma_i \leq q_0/(1 - q_1)$. The corresponding inequality for signal 1 is $\gamma_i \geq (1 - q_0)/q_1$. As expected, both inequalities are satisfied (strictly) under the signal-informativeness condition (4). It follows that $\Delta u_i^{\varepsilon_n}$ is positive for all $\varepsilon_n < 1$ close enough to 1. Moreover, for ε_n fixed, it is not difficult to show that the second term on the right-hand side — the probability that majority rule will be applied to all $n = 2m + 1$ votes times the conditional utility difference when this happens — tends to zero as n tends to infinity. This is not surprising since the probability for a tie tends to zero and the utilities in question are bounded. Under our value boundedness condition (3), this can be shown to hold uniformly for all i (see Appendix). Hence,

¹⁶Set $n = 2m + 1$ and $k - 1 = m$ in equation (17).

ε_n can be made small when n is large (and odd). The incentive for voting sincerely is then strict, so sincere voting is in fact a strict Nash equilibrium. In general, there may exist other Nash equilibria, along with this strict one, as we saw in Example 1 above. However, if ε_n is not reduced too fast as n increases, then it can be proved that the strict and informative equilibrium is unique: no other Nash equilibrium, neither pure nor mixed, then exists.

In the appendix we prove these claims under the hypothesis that the signal informativeness condition is *uniformly* met in the sense that there exists some $\eta < 1$ such that

$$\frac{1 - q_0}{\eta q_1} < \gamma_i < \frac{\eta q_0}{1 - q_1} \quad \forall i \tag{14}$$

Theorem 4. *Suppose that the value-boundedness condition (3) holds and that the signal-informativeness condition is uniformly met. There exist a sequence of positive $\bar{\varepsilon}_n \rightarrow 0$ such that for any voting rule f^ε with $\varepsilon_n \geq \bar{\varepsilon}_n$ for all odd $n \in \mathbb{N}$:*

- (i) *sincere voting is a strict Nash equilibrium*
- (ii) *there exists no other Nash equilibrium.*

In force of this result, the claim in Condorcet’s jury theorem is valid for a suitably specified sequence of doubly randomized majority rules. For each committee size n , let the probability for random delegation to a single vote, ε_n , remain above the critical threshold value $\bar{\varepsilon}_n$ so that sincere voting is the unique Nash equilibrium. Since the threshold $\bar{\varepsilon}_n$ decreases to zero as the committee increases in size, we may also let the randomization probability ε_n tend to zero, thereby reducing the informational inefficiency asymptotically to zero. Formally, let $(\varepsilon_n)_{n \in \mathbb{N}}$ be any sequence of positive numbers tending to zero, such that $\varepsilon_n \geq \bar{\varepsilon}_n$ for all n , where $(\bar{\varepsilon}_n)_{n \in \mathbb{N}}$ satisfies Theorem 4. The associated sequence of randomized voting rules, $(f^{\varepsilon_n})_{n \in \mathbb{N}}$, is asymptotically efficient:

Corollary 5. *Suppose that the value-boundedness condition (3) holds and that the signal-informativeness condition is uniformly met. Let $X_n \in \{0, 1\}$ be the committee decision under a voting rule f^{ε_n} such as just described, for each $n \in \mathbb{N}$. Then*

$$\lim_{n \rightarrow \infty} \Pr [X_n \neq \omega] = 0$$

Remark 2. *Although sincere voting is a strict and unique Nash equilibrium for all n and $\varepsilon \geq \bar{\varepsilon}_n$ under the hypotheses of the corollary, sincere voting is not a dominant strategy for all such n and ε . We show this in five steps. First, for any voter*

type $(\alpha, \beta) \in \Theta$, let $k^*(\alpha, \beta) \in \mathbb{N}$ be the minimal $k \in \mathbb{N}$ such that the conditional expected utility to a committee member of type (α, β) from decision $x = 1$ is higher than from decision $x = 0$, conditional upon k signals 1 and 1 signal 0. Let $k^* = \max_{(\alpha, \beta) \in \Theta} k^*(\alpha, \beta)$. Since Θ is compact with positive lower bounds, $k^* \in \mathbb{N}$. Second, fix $k \geq k^*$ and, consider committees of odd sizes n such that $(n - 1)/2 > k$. For each such n , consider any committee member i and let $\tilde{\sigma}_{-i}^{n,k}$ be the following strategy combination for the others: $(n - 1)/2$ of them always vote 0, $(n - 1)/2 - k$ always vote 1, and the remaining k voters vote sincerely. Third, according to the voting rule f^ε , committee member's vote is decisive with probability ε/n and pivotal with probability $(1 - \varepsilon)p$, where p is the probability for a tie among the other $n - 1$ votes. Under $\tilde{\sigma}_{-i}^{n,k}$, p is the probability that the k sincere voters all receive signal 1, a probability that depends on k but not on n (as long as $(n - 1)/2 > k$). Under $\tilde{\sigma}_{-i}^{n,k}$, i being pivotal is thus a strong indication that the state of nature is $\omega = 1$, so strong that the conditionally expected utility, given that i is pivotal, is maximized when i votes 1 irrespective of his or her own signal. Fourth, by continuity there exists an $\tilde{\varepsilon}_n > 0$ such that, against $\tilde{\sigma}_{-i}^{n,k}$, always voting 1 is a better reply for i than voting sincerely, for all $\varepsilon \in (0, \tilde{\varepsilon}_n)$. Moreover, $\tilde{\varepsilon}_n$ is increasing in n . Fifth, and finally: there exists a $n^* \in \mathbb{N}$ such that $\bar{\varepsilon}_n < \tilde{\varepsilon}_n$ for all $n \geq n^*$. For each $\varepsilon \in (\bar{\varepsilon}_n, \tilde{\varepsilon}_n)$, voting on alternative 1 irrespective of i 's signal is a better reply for i , against $\tilde{\sigma}_{-i}^{n,k}$, than sincere voting. Hence, for such n and ε , sincere voting is not a dominant strategy or voter i under f^ε .

Remark 3. We conclude by noting that claim (i) in Theorem 4 holds also under incomplete information concerning committee members's values. Suppose, thus, that there is a probability measure ν_n on \mathbb{R}_+^{2n} , for each $n \in \mathbb{N}$, such that, for each committee size n , "nature" first draws the value vector $\theta = \langle (\alpha_1, \beta_1), \dots, (\alpha_n, \beta_n) \rangle \in \mathbb{R}_+^{2n}$ according to ν_n , whereafter each committee member i gets to know his or her own value pair (α_i, β_i) , and then the above voting game is played. If there exists a $\eta < 1$ such that (14) holds with probability one for all n , then sincere voting is a sequential equilibrium; other committee members' values are irrelevant for a committee member's voting decision when all others vote informatively. In order to know the expected utility associated with each of i 's four pure local strategies, given her value pair, i only needs to know everybody's signal precisions (which are taken to be commonly known and the same for all committee members, q_0 and q_1 , respectively).

7. PERTURBING THE BEHAVIORAL ASSUMPTIONS

7.1. A slight preference for sincerity. Suppose that committee members have a slight preference for voting according to their personal conviction. In addition to the expected utility from the collective decision, such a committee member i receives additional (psychological) utility if he or she votes on the alternative that — given the member’s values α_i and β_i , prior μ and signal s_i — is the right alternative. Such voters are not “irrational,” it is only that they also care (a bit) about the sincerity of their own vote. (Most humans arguably feel some discomfort when acting against their personal conviction.) How are the above results affected, if at all, if all committee member have a slight preference for sincerity *per se*? We maintain the signal-informativeness condition (4) and focus on majority rule. Suppose that each committee member i who receives the signal 0 obtains additional utility $\delta_i^0 > 0$ from voting 0, and likewise if the committee member receives the signal 1. For any odd committee size n , the expected utility difference between voting 0 and 1, for a committee member i who has received the signal 0, now is:

$$\Delta u_i^0 = \delta_i^0 + \binom{n-1}{(n-1)/2} \frac{(1-\mu)q_0^{\frac{n+1}{2}}(1-q_0)^{\frac{n-1}{2}}\beta_i - \mu q_1^{\frac{n-1}{2}}(1-q_1)^{\frac{n+1}{2}}\alpha_i}{(1-\mu)q_0 + \mu(1-q_1)} \quad (15)$$

and likewise when the same committee member has received the signal 1. Clearly, sincere voting is a Nash equilibrium for all $\delta_i^0, \delta_i^1 > 0$ sufficiently large. Moreover, if there exists a positive lower bound δ on all δ_i^0 and δ_i^1 , then sincere voting is a Nash equilibrium granted n is large enough, since the probability for a tie under sincere voting goes to zero as $n \rightarrow +\infty$ and hence the strategic incentive against sincere voting vanishes asymptotically as the size of the committee tends to infinity. In this sense, the negative result in Corollary 2 is not robust. The reason for this non-robustness is that the second term in (15) tends to zero as n tends to plus infinity. Hence, no matter how small $\delta_i^0 > 0$ is, Δu_i^0 is positive for n large enough, and likewise for the expected utility difference between voting 1 and 0 for voters who have received the signal 1.

More generally, consider a sequence of committees with ever larger size n , such that the value-boundedness condition (3) holds and the signal informativeness condition is uniformly met. The utility from sincere voting may depend on the committee size. In order to allow for this possibility, let $\delta_{i,n}^0$ and $\delta_{i,n}^1$ be positive for each member i and committee size n . Granted these parameter values do not decrease too fast with n , informative voting is a strict Nash equilibrium, and it is the unique Nash equilibrium for each committee size n . In particular, the whole plethora of uninformative Nash

equilibria that exist in the standard voting model in sections 4 and 5 vanishes:

Theorem 5. *Suppose that the value-boundedness condition (3) holds and the signal-informativeness condition is uniformly met. There exist a sequence of positive numbers $\bar{\delta}_n \rightarrow 0$ such that for each $n \in \mathbb{N}$, if $\delta_{i,n}^0, \delta_{i,n}^1 \geq \bar{\delta}_n$ for all $i \in \{1, \dots, n\}$:*

- (i) *sincere voting is a strict Nash equilibrium*
- (ii) *there exists no other Nash equilibrium.*

The claim in Condorcet's jury theorem thus holds for rational and strategically voting committee members when these have a slight preference for sincerity:

Corollary 6. *Suppose that the value-boundedness condition (3) holds, that the signal-informativeness condition is uniformly met, and that $\delta_{i,n}^0, \delta_{i,n}^1 \geq \bar{\delta}_n > 0$ for all $n \in \mathbb{N}$ and $i \in \{1, \dots, n\}$. Let $X_n(\omega) \in \{0, 1\}$ be the collective decision in pure-strategy Nash equilibrium under majority rule with n voters. Then*

$$\lim_{n \rightarrow \infty} \Pr [X_n(\omega) \neq \omega] = 0.$$

The following example illustrates the possibility of a mixed Nash equilibrium when the preference for sincerity is not strong enough.

Example 2. *Consider a committee, similar to that in Example 1, with three members with a uniform prior, $\mu = 1/2$, equally precise signals, $q_0 = q_1 = q$, distinct values, $\gamma_1 = 1/c$, $\gamma_2 = 1$ and $\gamma_3 = c$ for some $c > 1$, and an equally strong preference for sincerity, $\delta_i^0 = \delta_i^1 = \delta \geq 0$ for $i = 1, 2, 3$, such that the signal-informativeness condition is met for all committee members. Since the two signals are equally precise, sincere voting is then a Nash equilibrium under majority rule, for all $\delta \geq 0$. However, there may also exist a mixed (anti-symmetric) equilibrium in which (a) voter 1 votes sincerely when receiving signal 0 and votes sincerely with probability x when receiving signal 1, (b) voter 2 always votes sincerely, and (c) voter 3 votes sincerely when receiving signal 1 and votes sincerely with probability x when receiving signal 0. Does there exist an $x \in (0, 1)$ for which this constitutes a Nash equilibrium? The probability x should render voter 1 indifferent when receiving signal 1 and voter 3 indifferent when receiving signal 0. It is not difficult to show that the "anti-symmetry, of values makes these two indifference conditions coincide and boil down to the following equation:*

$$x = \frac{q(c-1)(1-q) - \delta}{(c+1)(2q-1)(1-q)q} \quad (16)$$

For example, for $q = 0.8$, $\delta = 0.1$ and $c = 2$, the signal-informativeness condition is met and we obtain $x \approx 0.31$. More generally, the equilibrium randomization x is decreasing in δ , the preference for sincerity. For $\delta \geq q(c-1)(1-q)$, no mixed equilibrium of this sort exists. For example, for $q = 0.8$ and $c = 2$ this is true for all $\delta \geq 0.16$.

How are the other results for the standard model affected if committee members have a slight preference for sincerity? Condorcet's original result, Theorem 1 is clearly unaffected, since it *presumes* sincere voting. The characterization of optimal voting rules in Theorem 2 is also unaffected. By contrast, our characterization of when sincere voting is an equilibrium, Theorem 3, is affected. With a slight preference for sincerity, condition (8) is still sufficient for sincere voting to be an equilibrium, but it is no longer necessary. Because committee members with a (slight) preference for sincerity have a (slightly) stronger incentive to vote sincerely and hence sincere voting may be an equilibrium even when condition (8) is violated. By the same token, the negative result in Corollary 3 no longer holds up without qualification. Its validity hinges on the how the preference for sincerity depends on the size of the committee. If it decreases very fast, then the claim in the corollary holds up, while if it does not decrease too fast, then the claim in the corollary does not hold (the latter follows from Theorem 5). The first half of proposition 1 — the implication from equilibrium to optimality — is not generally valid when committee member have a preference for sincerity, but the second half of the proposition — the implication from optimality to equilibrium — holds *a fortiori*. Likewise, only half of the result concerning the straw vote procedure, Proposition 2, holds up when committee members have a preference for sincerity. Homogeneous committees will still report truthfully in the straw votes, but members of a heterogeneous committee may do so too; they have to balance their strategic motive to not report truthfully against their disutility from false (insincere) reporting. The positive result for randomized majority rules, Theorem 4 and its corollary, hold *a fortiori* if committee members have a preference for sincerity.

7.2. Transparency and a slight concern for esteem. Another important source for motivation is esteem: self-esteem, esteem by peers (here the other committee members) and/or by society at large.¹⁷ This motive may become stronger the more transparent the committee's decisions are. Indeed, some central banks (Bank of England and the Sveriges Riksbank, for instance) have in recent years introduced

¹⁷We are grateful to Torsten Persson for pointing out this possibility.

transparency rules whereby individual board members' votes are made public afterwards. Hence, individual votes can after some time be evaluated against the backdrop of later incoming information about the economy's true state in the period in question. A board member who's vote, with hindsight, appears to have been correct (wrong) may enhance (diminish) that member's public esteem.¹⁸ With increased transparency, anticipation of the possibility of state revelation may influence voting incentives. See Gersbach and Hahn (2008), Hahn (2008) and Swank, Swank and Visser (2008) for studies of the effects of increased transparency on decision-making in central banks. We here sketch how transparency and such concerns for esteem can be incorporated in the present model.

In each state of nature ω , let $\tau_\omega \in [0, 1]$ be the probability that both the true state of nature and all committee members' votes will become publicly known after the decision has been made. An increase in τ_0 and τ_1 thus represents increased transparency. Arguably, being a committee member who voted for the ex-post right decision, when other committee members were erring, may enhance the committee member's esteem in the general public. Similarly, some social stigma may be attached to an ex-post mistaken vote when other committee members voted for the ex-post correct decision. Let $\rho_i^+(\omega, k/(n-1)) > 0$ be the additional esteem-utility for committee member i for having voted correctly in state ω , when k of the $n-1$ other committee members voted correctly (in a committee of size n). Likewise, let $\rho_i^-(\omega, k/(n-1)) > 0$ be the stigma-disutility for committee member i for having voted incorrectly in state ω , when k of the $n-1$ other committee members voted correctly. Arguably, $\rho_i^+(\omega, k/(n-1))$ is decreasing and $\rho_i^-(\omega, k/(n-1))$ increasing, in the second argument, the share of correct votes.

The expected additional utility from voting sincerely ($v_i = 0$) upon signal $s_i = 0$ can be written as:

$$\begin{aligned} & \sum_{k=0}^{n-1} \rho_i^+(0, k/(n-1)) \Pr [k \text{ votes } 0 \wedge \text{revelation} \wedge \omega = 0 \mid s_i = 0] \\ & - \sum_{k'=0}^{n-1} \rho_i^-(1, k'/(n-1)) \Pr [k' \text{ votes } 1 \wedge \text{revelation} \wedge \omega = 1 \mid s_i = 0], \end{aligned}$$

where "revelation" denotes the event in which the true state and the votes become

¹⁸An interesting extension of the present model would be to allow for the possibility that signal precisions differ between committee members, where a member's signal precision is his or her private information, see Section 8. In such a richer model, changes in esteem could be derived from the public's Bayesian updating of beliefs about board members' competence.

publicly known. Given the signal 0, the difference in expected additional utility to committee member i from voting sincerely ($v_i = 0$) instead of insincerely ($v_i = 1$) is then:

$$\begin{aligned} \delta_i^0 &= \tau_0 \sum_{k=0}^{n-1} \rho_i^*(0, k/(n-1)) \Pr [k \text{ votes } 0 \wedge \omega = 0 \mid s_i = 0] \\ &\quad - \tau_1 \sum_{k=0}^{n-1} \rho_i^*(1, k/(n-1)) \Pr [k \text{ votes } 1 \wedge \omega = 1 \mid s_i = 0]. \end{aligned}$$

where

$$\rho_i^*(\omega, k/(n-1)) = \rho_i^+(\omega, k/(n-1)) + \rho_i^-(\omega, k/(n-1)).$$

We note that it is not clear, from a psychological point of view, whether $\rho_i^*(\omega, k/(n-1))$ is increasing or decreasing in its second argument; that depends on how quickly one's esteem falls and one's stigma increases, when the share of correct votes among the others increases. If all other committee members vote sincerely, then

$$\begin{aligned} \delta_i^0 &= \frac{\tau_0(1-\mu)q_0}{(1-\mu)q_0 + \mu(1-q_1)} \sum_{k=0}^{n-1} \rho_i^*(0, k/(n-1)) \binom{k}{n-1} q_0^k (1-q_0)^{n-1-k} \\ &\quad - \frac{\tau_1\mu(1-q_1)}{(1-\mu)q_0 + \mu(1-q_1)} \sum_{k=0}^{n-1} \rho_i^*(1, k/(n-1)) \binom{k}{n-1} q_1^k (1-q_1)^{n-1-k}. \end{aligned}$$

The expression for δ_i^0 is involved but can be interpreted. The first sum is the expected value of $\rho_i^*(0, k/(n-1))$, when the binomial random variable k has mean value $(n-1)q_0$. Likewise, the second sum is the expected value of $\rho_i^*(1, k/(n-1))$, when the binomial random variable k has mean value $(n-1)q_1$. For large n , and granted that $\rho_i^*(0, k/(n-1))$ and $\rho_i^*(1, k/(n-1))$ are sufficiently smooth functions of k and n , these sums are approximately equal to $\rho_i^*(0, q_0)$ and $\rho_i^*(1, q_1)$, respectively. These quantities can be interpreted as the net values of the social esteem to committee member i for being in the non-erring majority in the two states respectively, when the size of this majority is the one expected in that state, $k = (n-1)q_0$ and $k = (n-1)q_1$, respectively. For large n , we thus obtain the approximation

$$\delta_i^0 \simeq \frac{\tau_0(1-\mu)q_0\rho_i^*(0, q_0) - \tau_1\mu(1-q_1)\rho_i^*(1, q_1)}{(1-\mu)q_0 + \mu(1-q_1)}.$$

Thus, δ_i^0 is positive if and only if $\tau_0(1-\mu)q_0\rho_i^*(0, q_0) > \tau_1\mu(1-q_1)\rho_i^*(1, q_1)$. Reasoning likewise for the other signal, the incentive is, for both signals, in favor of sincerity

if

$$\frac{1 - q_0}{q_1} < \frac{\mu \tau_1 \rho_i^*(1, q_1)}{(1 - \mu) \tau_0 \rho_i^*(0, q_0)} < \frac{q_0}{1 - q_1}.$$

This condition has the same form as the signal informativeness condition (4). The disutilities α_i and β_i associated with mistakes of type I and II are replaced by the social esteem factors $\tau_0 \rho_i^*(0, q_0)$ and $\tau_1 \rho_i^*(1, q_1)$. This is, then, a sufficient condition for our “usual” conclusion to hold: under this condition, even a small dose of transparency ensures that, in large enough committees, sincere voting is the unique and strict Nash equilibrium.

7.3. Noise voters. Experimental evidence from laboratory studies suggests that human subjects in committee decision problems of the kind analyzed here sometimes make mistakes, see Guarnaschelli *et al.* (2000). For instance, even when sincere voting is a Nash equilibrium under majority rule in a three-person committee setting, some subjects do not vote according to their informative signal. Awareness of a positive error rate in others’ voting clearly influences the voting incentives of a rational committee member. We here extend our model to allow for this possibility.¹⁹ Consider a committee consisting of n members who may, but need not, have a slight preference for sincerity and/or concern for esteem. Assume that with probability $\lambda \in [0, 1]$ exactly one of these members suddenly becomes a *noise voter*, defined as a voter who votes randomly according to an exogenous probability distribution, irrespective of his or her private signal. Assume, moreover that such a noise voter’s vote is statistically independent of the state and all private signals. A committee member who is not a noise voter is called an *informed voter*.

We analyze a special case of this set-up in full detail. Assume thus, that n is an odd number, that the voting rule is majority rule, that the prior is uniform, that the two signals are equally precise and that a noise voter randomizes uniformly. It is not difficult to then show that the possibility of a noise voter in the committee increases each informed committee member’s incentive to vote sincerely when other informed committee members vote sincerely. The reason for this is two-fold: a noise voter among the others increases the probability for an informed voter of becoming pivotal, and it enhances the conditionally expected net utility gain from sincere voting (over insincere voting) if the informed voter is pivotal. It is as if the strategic counter-force against sincere voting is mitigated by the potential noise voter — an uninformed vote

¹⁹Eliasz (2002) analyses the mechanism-design implementation problem with k faulty players among n players. Blais *et al.* (2008) use the 1 faulty player model to analyse experimental data on voting.

among the others' votes make it rational to place more weight on one's own signal. Formally

Proposition 3. *Consider majority rule in a committee with n odd, $\mu = 1/2$, $1/2 < q_0 = q_1 < 1$, and with a probability $\lambda \in [0, 1]$ for the presence of a noise voter who randomizes uniformly. The probability that a given committee member's vote will be pivotal under sincere voting is strictly increasing in λ . Moreover, conditional upon being pivotal, the expected-utility difference between sincere and insincere voting is strictly increasing in λ .*

It follows that, at least in the special case hypothesized in the proposition, the possibility of a noise voter increases informed voters's incentive for sincere voting. Consequently, the above results for committees without noise voters, for the randomized voting rule and for voters with a preference for sincere voting and/or concern for esteem, hold *a fortiori* in the presence of noise voters. We conjecture that this qualitative conclusion is valid more generally. Indeed, the more noise voters there are in a committee, the stronger should the incentive for sincere voting be.

8. CONCLUSION

The above analysis is restricted to a committee of equally "competent" members who receive private information of exogenously fixed precision and face a binary collective decision problem with no possibility of abstention. Despite these heroic simplifications, we believe that the qualitative conclusions hold more generally.

First, suppose that the committee members are unequally "competent" in the sense that some members receive more precise signals than others. If the competence differences are known by all members, then weighted majoritarian rules, whereby more competent voters are given higher weights than less competent ones, may be superior the k -majority rules studied here. For a survey of results of this sort, see Grofman, Owen and Feld (1983), Owen, Grofman and Feld (1989) and Ben-Yashar and Milchtaich (2006). Under the usual majority rule, but with differing competence among the committee members, what can be said about equilibrium voting? Two main cases appear relevant for such a consideration. In the first case, each member i has precision parameters $q_0^i, q_1^i > 1/2$ and these are known by all committee members. In the second case, each member i has precision parameters $q_0^i, q_1^i > 1/2$ but these are known only by member i himself.²⁰ Let us briefly re-consider the statement and proof

²⁰Visser and Swank (2007) assume that committee members do not even know their own competence.

of Theorem 3. The quantities $g(k, n)$ have to be re-defined and will, in general, also depend on i . More precisely, each such quantity $g_i(k, n)$ will no longer be a simple product of two factors raised to powers k and n , but will be a complex multinomial sum; instead of just counting the number of signals of each type, one has to keep track of which signal was received by which member, and consider all permutations. With so defined quantities $g_i(k, n)$ in condition (3), the claim of Theorem 3 would remain true, and the quantities $g_i(k, n)$ would be continuous in the parameter vector $\langle (q_0^1, q_1^1), \dots, (q_0^n, q_1^n) \rangle$. Hence, Theorem 3 would be approximately correct for approximately equally competent committee members. Similar considerations apply to other equilibrium results. The second case, that of incomplete information concerning competence, appears to be particularly interesting for analyses of the incentive effects of transparency. For studies of such settings, see Visser and Swank (2007), Gersbach and Hahn (2008), Swank, Visser and Swank (2008) and Hahn (2008).

A second direction for generalization, which would be valuable and challenging to explore, concerns the binary nature of both signals and choices. What can be said if the choice is binary but there are more than two signal values (perhaps just three, or a whole continuum)? What if there are more than two choice alternatives? New results have recently been obtained for more general collective decision problems of this sort, see McLennan (2007).

A third direction would be to analyze equilibrium outcomes if abstention is an option and/or the number of voters is unknown by the voters. Such aspects may be less relevant for some committees but may play a major role in other committees and certainly in general elections. Krishna and Morgan (2007) undertake an investigation of precisely these two aspects, in a setting where the number of voters is a Poisson distributed random variable and each voter draws a random cost for casting a vote. Each voter only observes his or her own signal and voting cost. Krishna and Morgan assume that the voters are *ex-ante* identical, that the two states of nature are equally likely and that the two signals are equally precise. They show that sincere voting then is the unique Nash equilibrium under super-majority rules when the expected number of voters is large. Moreover, equilibrium participation rates are such that the outcome is asymptotically efficient. While their model thus is cast more in the mold of general elections, it would be interesting to explore whether (strategic) abstention, allowed for in their framework, can be introduced in our framework for a committee of fixed and known size, and whether the kind of preference heterogeneity that we here permit can be introduced into their framework. Here, we only note that sincere voting under our randomized majority rule will remain a strict Nash equilibrium also

when abstention is allowed. However, our uniqueness claim may then fail.

A fourth and final avenue for future work would be to endogenize voters' signal precision. Before a committee meets, individual members usually make (typically unobserved) efforts to study the question at hand, so that they will be well informed at the meeting. However, as is well-known both by practitioners and theorists, this gives rise to a free-rider problem, whereby committee members tend to under-invest and arrive at the meeting less informed than what would be collectively desirable. For recent analyses of this moral hazard phenomenon in various models, see Mukhopadhyaya (2003), Persico (2004), Gerardi and Yariv (2008) and Koriyama and Szentes (2007).

9. APPENDIX

We here provide mathematical proofs of claims not proved in the main text.

9.1. Theorem 1. Suppose that $\omega = 0$ and consider any positive integer n . The probability that voter i votes $v_i = s_i = 1$, when voting informatively, is $1 - q_0$. Under majority rule, the probability of a wrong decision in this state is thus

$$\Pr[X_n = 1 \mid \omega = 0] \leq \Pr\left[\frac{1}{n} \sum_{i=1}^n s_i \geq \frac{1}{2} \mid \omega = 0\right]$$

Conditional upon $\omega = 0$, the random variables $\{s_i\}_{i=1}^n$ are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem (see, for example, Theorem 27.1 in Billingsley, 1995), their average, $\frac{1}{n} \sum_{i=1}^n s_i$ (given $\omega = 0$), converges in distribution towards the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $1 - q_0 < \frac{1}{2}$:

$$\lim_{n \rightarrow \infty} \Pr\left[\frac{1}{n} \sum_{i=1}^n s_i \geq \frac{1}{2} \mid \omega = 0\right] = 0$$

The same argument applies to the state $\omega = 1$, and the result follows from the identity

$$\Pr[X_n(\omega) \neq \omega] = (1 - \mu) \Pr[X_n = 1 \mid \omega = 0] + \mu \Pr[X_n = 0 \mid \omega = 1]$$

9.2. Theorem 2. Write $W(f^k)$ in the following way, where the random variable N_1 is the number of signals 1 received, $U_0 = (1 - \mu) \sum_{i=1}^n u_{00}^i$ and $U_1 = (1 - \mu) \sum_{i=1}^n u_{11}^i$, two real numbers:

$$\begin{aligned} W(f^k) &= -(1 - \mu) \bar{\beta}(n) \Pr[x = 1 \mid \omega = 0] - \mu \bar{\alpha}(n) \Pr[x = 0 \mid \omega = 1] + U_0 + U_1 \\ &= -(1 - \mu) \bar{\beta}(n) \Pr[N_1 \geq k \mid \omega = 0] - \mu \bar{\alpha}(n) \Pr[N_1 < k \mid \omega = 1] + U_0 + U_1 \end{aligned}$$

Hence,

$$W(f^{k+1}) - W(f^k) = (1 - \mu) \bar{\beta}(n) \Pr[N_1 = k \mid \omega = 0] - \bar{\alpha}(n) \Pr[N_1 = k \mid \omega = 1]$$

and thus

$$\begin{aligned} W(f^{k+1}) \leq W(f^k) &\iff \bar{\gamma} \geq \frac{\Pr[N_1 = k \mid \omega = 0]}{\Pr[N_1 = k \mid \omega = 1]} \\ &\iff \bar{\gamma} \geq \frac{(1 - q_0)^k q_0^{n-k}}{q_1^k (1 - q_1)^{n-k}} = g(k, n) \end{aligned}$$

Likewise:

$$\begin{aligned} W(f^{k-1}) \leq W(f^k) &\iff \bar{\gamma} \leq \frac{\Pr[N_1 = k-1 \mid \omega = 0]}{\Pr[N_1 = k-1 \mid \omega = 1]} \\ &\iff \bar{\gamma} \leq \frac{(1-q_0)^{k-1} q_0^{n-k+1}}{q_1^{k-1} (1-q_1)^{n-k+1}} = g(k-1, n) \end{aligned}$$

Since $g(k, n)$ is decreasing in k , $W(f^k) \geq W(f^h)$ for all $h = k+1, k+2, \dots, n$ if and only if $\bar{\gamma} \geq g(k, n)$. Likewise, $W(f^k) \geq W(f^h)$ for all $h = k-1, k-2, \dots, 1$ if and only if $\bar{\gamma} \leq g(k-1, n)$. Hence, as k increases from 1 to n , $W(f^k)$ reaches its maximum value either at a unique k or (non-generically) at two adjacent values, $k-1$ and k . As noted in footnote 6, $k=0$ and $k=n+1$ are never optimal.

9.3. Corollary 1. Condition (5) is equivalent with

$$\left[\frac{(1-q_0)(1-q_1)}{q_0 q_1} \right]^k \left(\frac{q_0}{1-q_1} \right)^n \leq \bar{\gamma}(n) \leq \left[\frac{(1-q_0)(1-q_1)}{q_0 q_1} \right]^k \left(\frac{q_1}{1-q_0} \right) \left(\frac{q_0}{1-q_1} \right)^{n+1}$$

or

$$\left(\frac{q_0}{1-q_1} \right)^n \leq \bar{\gamma}(n) \cdot \left[\frac{q_0 q_1}{(1-q_0)(1-q_1)} \right]^k \leq \left(\frac{q_1}{1-q_0} \right) \left(\frac{q_0}{1-q_1} \right)^{n+1}$$

Taking logarithms and dividing through with n , we obtain

$$\begin{aligned} \ln \left(\frac{q_0}{1-q_1} \right) &\leq \frac{1}{n} \ln \bar{\gamma}(n) + \frac{k}{n} \ln \left[\frac{q_0 q_1}{(1-q_0)(1-q_1)} \right] \\ &\leq \frac{1}{n} \ln \left(\frac{q_1}{1-q_0} \right) + \left(1 + \frac{1}{n} \right) \ln \left(\frac{q_0}{1-q_1} \right) \end{aligned}$$

As $n \rightarrow \infty$, the upper bound converges to the lower bound, $\ln \left(\frac{q_0}{1-q_1} \right)$, and $\frac{1}{n} \ln \bar{\gamma}(n)$ tends to zero since, in force of the value-boundedness condition (3). This establishes the equality in (7).

In order to establish the claimed inequalities, let

$$B = \frac{\ln \left(\frac{q_0}{1-q_1} \right)}{\ln \left(\frac{q_0}{1-q_1} \right) + \ln \left(\frac{q_1}{1-q_0} \right)}.$$

Suppose first that $q_1 \leq q_0$. Then $\frac{q_1}{1-q_0} \leq \frac{q_0}{1-q_1}$, from which we deduce that $B \geq 1/2$ and $1-q_0 \leq B$. To obtain the second claimed inequality, $B \leq q_1$, note that this can be re-written, after some manipulation, as

$$q_1 \ln(1-q_0) + (1-q_1) \ln q_0 \leq q_1 \ln q_1 + (1-q_1) \ln(1-q_1).$$

The right-hand side is independent of q_0 , while the left-hand side is decreasing in q_0 . Thus, the claimed inequality $B \leq q_1$ holds for all $q_0 \in [q_1, 1]$ if and only if it holds for $q_0 = q_1$. Writing the inequality for that special case, one obtains

$$q_1 \ln(1 - q_1) + (1 - q_1) \ln q_1 \leq q_1 \ln q_1 + (1 - q_1) \ln(1 - q_1),$$

or, equivalently,

$$(2q_1 - 1) \ln(1 - q_1) \leq (2q_1 - 1) \ln q_1,$$

an inequality which clearly holds since $q_1 \geq 1/2$.

Now suppose that $q_1 \leq q_0$. Then the above reasoning (switching q_0 and q_1) gives us $1 - q_1 \leq 1 - B \leq q_0$, which is equivalent with the claimed inequality $1 - q_0 \leq B \leq q_1$.

9.4. Theorem 3. For any $n, k \in N$ such that $1 \leq k \leq n$, denote by \mathcal{T} the event of a tie among the others, that is, that exactly $k - 1$ of the other voters receive the signal 1 and thus $n - k$ receive the signal 0. Suppose, for instance, that i received the signal $s_i = 0$. Should i then vote on alternative 0? The probability for the joint event that $s_i = 0$ and that there is a tie among the others, conditional on the state $\omega = 0$, is

$$\Pr[\mathcal{T} \wedge s_i = 0 \mid \omega = 0] = \binom{n-1}{k-1} q_0^{n-k} (1 - q_0)^{k-1}$$

Likewise, conditional on the state $\omega = 1$, we have

$$\Pr[\mathcal{T} \wedge s_i = 0 \mid \omega = 1] = \binom{n-1}{k-1} q_1^{k-1} (1 - q_1)^{n-k}$$

Therefore, the probability for the joint event that i receives the signal 0 and there is a tie among the others is

$$\Pr[\mathcal{T} \wedge s_i = 0] = \binom{n-1}{k-1} \left[(1 - \mu) q_0^{n-k} (1 - q_0)^{k-1} + \mu q_1^{k-1} (1 - q_1)^{n-k} \right]$$

Since the probability of receiving the signal 0 is $\Pr[s_i = 0] = (1 - \mu) q_0 + \mu (1 - q_1)$, committee member i attaches the following conditional probability of a tie among the others, conditional upon $s_i = 0$:

$$p_0(k) = \Pr[\mathcal{T} \mid s_i = 0] = \binom{n-1}{k-1} \frac{(1 - \mu) q_0^{n-k} (1 - q_0)^{k-1} + \mu q_1^{k-1} (1 - q_1)^{n-k}}{(1 - \mu) q_0 + \mu (1 - q_1)}$$

We are now in position to compute the difference in expected utility for voter i between casting the sincere vote $v_i = 0$ instead of the insincere vote $v_i = 1$, conditional upon the signal $s_i = 0$:

$$\Delta u_i = \mathbb{E}[u_i \mid s_i = v_i = 0] - \mathbb{E}[u_i \mid s_i = 0 \wedge v_i = 1]$$

Because i 's vote affects the collective decision x only in the event \mathcal{T} , we have

$$\Delta u_i = p_0(k) \cdot (\mathbb{E}[u_i | \mathcal{T} \wedge s_i = v_i = 0] - \mathbb{E}[u_i | \mathcal{T} \wedge s_i = 0 \wedge v_i = 1])$$

where

$$\begin{aligned} \mathbb{E}[u_i | \mathcal{T} \wedge s_i = v_i = 0] &= \kappa_i - \alpha_i \Pr[\omega = 1 | \mathcal{T} \wedge s_i = 0] \\ \mathbb{E}[u_i | \mathcal{T} \wedge s_i = 0 \wedge v_i = 1] &= \kappa_i - \beta_i \Pr[\omega = 0 | \mathcal{T} \wedge s_i = 0] \end{aligned}$$

and κ_i is the conditionally expected utility of taking the right decision, $x = \omega$, conditional on the event $\mathcal{T} \wedge s_i = 0$.²¹ By Bayes' law (factorials cancel):

$$\begin{aligned} \Pr[\omega = 0 | \mathcal{T} \wedge s_i = 0] &= \frac{(1 - \mu) \Pr[\mathcal{T} \wedge s_i = 0 | \omega = 0]}{\Pr[\mathcal{T} \wedge s_i = 0]} \\ &= \frac{(1 - \mu) q_0^{n-k+1} (1 - q_0)^{k-1}}{(1 - \mu) q_0^{n-k} (1 - q_0)^{k-1} + \mu q_1^{k-1} (1 - q_1)^{n-k+1}}. \end{aligned}$$

Hence:

$$\Delta u_i = \binom{n-1}{k-1} \frac{(1 - \mu) q_0^{n-k+1} (1 - q_0)^{k-1} \beta_i - \mu q_1^{k-1} (1 - q_1)^{n-k+1} \alpha_i}{(1 - \mu) q_0 + \mu (1 - q_1)}. \quad (17)$$

The condition for Δu_i to be nonnegative, that is, for i to rationally want to vote according to her signal $s_i = 0$, is thus

$$(1 - \mu) q_0^{n-k+1} (1 - q_0)^{k-1} \beta_i \geq \mu q_1^{k-1} (1 - q_1)^{n-k+1} \alpha_i,$$

which can be written as

$$\gamma_i \leq \left(\frac{1 - q_0}{q_1} \right)^{k-1} \left(\frac{q_0}{1 - q_1} \right)^{n-k+1} = g(k-1, n). \quad (18)$$

Hence, sincere voting on alternative 0 (that is, to chose $v_i = 0$ when $s_i = 0$) is optimal if and only if the right inequality in (8) is met. Likewise, sincere voting on alternative 1 is optimal if and only if

$$\gamma_i \geq \left(\frac{1 - q_0}{q_1} \right)^k \left(\frac{q_0}{1 - q_1} \right)^{n-k} = g(k, n) \quad (19)$$

²¹

$$\kappa_i = u_{00}^i \Pr[\omega = 0 | \mathcal{T} \wedge s_i = 0] + u_{11}^i \Pr[\omega = 1 | \mathcal{T} \wedge s_i = 0]$$

9.5. Proposition 1. Suppose that sincere voting is an equilibrium under k -majority rule, then, by Theorem 3, $g(k, n) \leq \gamma_i \leq g(k-1, n)$ for all i . Hence

$$(1 - \mu) \beta_i g(k, n) \leq \mu \alpha_i \leq (1 - \mu) \beta_i g(k-1, n) \quad \forall i$$

so $\bar{\beta}(n) g(k, n) \leq \bar{\alpha}(n) \leq \bar{\beta}(n) g(k-1, n)$ or, equivalently, $g(k, n) \leq \bar{\gamma}(n) \leq g(k-1, n)$. Thus, by Theorem 2, the k -majority rule is optimal.

Conversely, suppose that the committee is homogeneous. There exists an integer m such that $m-1 \leq \lambda_i \leq m$ for all i . By definition (11) of λ_i , this is equivalent to

$$\left[\frac{q_0}{1-q_1} \right]^{n-m} \left[\frac{1-q_0}{q_1} \right]^{m-1} \leq \gamma_i \leq \left[\frac{1-q_0}{q_1} \right]^{m-1} \left[\frac{q_0}{1-q_1} \right]^{n-m} \quad \forall i \quad (20)$$

This implies the same inequality for $\bar{\gamma}(n)$ and thus k -majority rule is optimal for $k = m$, by Theorem 2. But since (20) holds, sincere voting is an equilibrium, by Theorem 3.

9.6. Proposition 2. Suppose that all committee members report truthfully in the first stage: $t_i = s_i$ for all i . In stage two, all committee members are then essentially in the same “information set” in stage two, before casting their “real” votes: they all know the total number N_1 of signals 1 among the n signals received. No voter knows exactly who received what signal, except for their own, but this is of no consequence since all voters by assumption receive signals of the same “quality.” Suppose that each committee member votes for his or her preferred decision alternative in stage two, given his or her information. Will each voter i then have an incentive to report truthfully in the first stage? A single voter can change N_1 by only one unit.

Case 1: Voter i has received the signal $s_i = 1$ and that there are N_0 other signals 1. Then $N_1 = N_0 + 1$ if i will truthfully report $t_i = 1$ while $N_1 = N_0$ if i would falsely report $t_i = 0$. It follows that i 's report will affect the final decision x if and only if $N_0 < \lambda_{(n+1)/2} < N_0 + 1$, or, equivalently, if and only if $N_0 = \lfloor \lambda_{(n+1)/2} \rfloor$.²² Therefore any committee member i who receives the signal $s_i = 1$ can reason conditionally on this event, namely, that the total number of signals $s_j = 1$ received by the other committee members is exactly $\lfloor \lambda_{(n+1)/2} \rfloor$. The probability for this event does not depend on the identity of member i and it does not depend on i 's signal or report. This probability is positive whenever $0 \leq \lfloor \lambda_{(n+1)/2} \rfloor \leq n$, a condition that can be written as

$$\gamma_{(n+1)/2} \leq \left(\frac{q_0}{1-q_1} \right)^n .$$

²²The reasoning is based on the assumption that λ_{m+1} does not happen to be an integer.

But this inequality is implied by the informativeness condition (4) that we already imposed.

Case 2: Voter i has instead received the signal $s_i = 0$, while the others have still together received N_0 signals 1. Then i 's report will affect the final decision x if and only if $N_0 - 1 < \lambda_{(n+1)/2} < N_0$, or, equivalently, if and only if $N_0 = \lfloor \lambda_{(n+1)/2} \rfloor + 1$. Again, this event has positive probability.

9.7. Claim (i) in Theorem 4. Write $n = 2m + 1$, that is, for any committee member, m is half of the rest of the committee. To see that sincere voting under f^ε is a strict Nash equilibrium, first note that $\Delta u_i^\varepsilon > 0$ if and only if

$$\frac{\varepsilon}{1 - \varepsilon} > \frac{2m + 1}{B_i} \cdot \binom{2m}{m} [\alpha_i \mu q_1^m (1 - q_1)^{m+1} - \beta_i (1 - \mu) q_0^{m+1} (1 - q_0)^m] \quad (21)$$

where the factor $B_i = \beta_i (1 - \mu) q_0 - \alpha_i \mu (1 - q_1)$ is positive by (4). By Stirling's formula,

$$\binom{2m}{m} = \frac{(2m)!}{(m!)^2} = \frac{4^m}{\sqrt{\pi m}} (1 + o(m))$$

so the right-hand side of (21) is approximated by

$$\begin{aligned} & (1 + o(m)) \cdot \frac{4^m}{B_i (2m + 1) \sqrt{\pi m}} \cdot [\alpha_i \mu q_1^m (1 - q_1)^{m+1} - \beta_i (1 - \mu) q_0^{m+1} (1 - q_0)^m] \\ & \leq (1 + o(m)) \cdot \frac{2\alpha_i \mu (1 - q_1)}{B_i \sqrt{\pi}} [4q_1 (1 - q_1)]^m \sqrt{m} \leq (1 + o(m)) \cdot \frac{C_i}{B_i} \cdot a^m \sqrt{m} \end{aligned}$$

where $C_i = 2\alpha_i \mu (1 - q_1) / \sqrt{\pi}$ and $a = 4q_1(1 - q_1) < 1$. Hence, (21) is met if

$$\frac{\varepsilon}{1 - \varepsilon} > \frac{C_i}{B_i} (1 + o(m)) a^m \sqrt{m}$$

A sufficient condition for this to hold is that

$$\varepsilon > \frac{C_i}{B_i} (1 + o(m)) a^m \sqrt{m} \quad (22)$$

The preference boundedness condition (3) together with the hypothesis that the signal informativeness condition is uniformly met implies that C_i/B_i is uniformly bounded: there exists a $D \in \mathbb{R}$ such that $C_i/B_i < D$ for all i .²³ Let $\varepsilon = b^m$ with

²³To see this, note that $C_i/B_i \leq D$ iff

$$\frac{1}{\gamma_i} \cdot \frac{q_0}{1 - q_1} - 1 \geq \frac{1}{D}$$

and let $\eta = D/(D + 1)$.

$a < b < 1$. Then $\varepsilon \rightarrow 0$ as $m \rightarrow +\infty$. Moreover, since

$$\left(\frac{b}{a}\right)^m \frac{1}{\sqrt{m}} \rightarrow +\infty \quad \text{as } m \rightarrow \infty,$$

(22) holds for all m large enough, irrespective of how large D is.

The same reasoning applies to the expected utility upon receiving the signal $s_i = 1$. This proves claim (i) for $\varepsilon = b^m$, for any b such that

$$\max\{4q_0(1 - q_0), 4q_1(1 - q_1)\} < b < 1$$

where we note that lower bound indeed is less than 1 since $q_0, q_1 > 1/2$.

9.8. Corollary 4. Suppose first that $\omega = 0$ and consider informative voting under f^n , for $n = 2m + 1 \in \mathbb{N}$ fixed. The probability that committee member i votes $s_i = 1$ is, by definition $1 - q_0$. If the collective decision is taken by majority rule applied to all n votes, the probability of a wrong decision, $X_n = 1$, is some number Q_n . So the probability of a wrong decision, given $\omega = 0$, is

$$\Pr[X_n = 1 \mid \omega = 0] = \varepsilon_n(1 - q_0) + (1 - \varepsilon_n)Q_n$$

The probability of a wrong decision in state $\omega = 0$ thus tends to 0 if $Q_n \rightarrow 0$ as $n \rightarrow \infty$ since $\varepsilon_n \rightarrow 0$. It thus remains to prove that $Q_n \rightarrow 0$. We proceed just as in the proof of Condorcet's jury theorem. First note that, since n is odd:

$$Q_n = \Pr \left[\sum_{i=1}^n s_i > \frac{n}{2} \mid \omega = 0 \right]$$

Conditional upon $\omega = 0$, the signals s_i are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem, $\frac{1}{n} \sum_{i=1}^n s_i$, given $\omega = 0$, converges in distribution to the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $q_0 > \frac{1}{2}$:

$$\lim_{m \rightarrow \infty} \Pr \left[\frac{1}{n} \sum_{i=1}^n s_i > \frac{1}{2} \mid \omega = 0 \right] = 0.$$

The same argument applies to the case $\omega = 1$.

9.9. Claim (i) in Theorem 5. In order to establish that informative voting constitutes a strict Nash equilibrium, consider, first, a voter who has received the signal 0. Under majority rule applied to $n = 2m + 1$ voters, the expected utility difference between voting 0 and 1 is given by (17). Focusing on large n and applying Stirling's formula,

$$m! = \sqrt{2\pi m} \cdot (m/e)^m \cdot (1 + o(m)),$$

we have

$$\binom{2m}{m} = \frac{(2m)!}{(m!)^2} = \frac{4^m}{\sqrt{\pi m}}(1 + o(m))$$

and obtain

$$\begin{aligned} \lim_{m \rightarrow \infty} \Delta u_i^0 &= \delta_i^0 + \lim_{m \rightarrow \infty} \frac{4^m}{\sqrt{\pi m}} \cdot \frac{\beta_i(1-\mu)q_0[q_0(1-q_0)]^m - \alpha_i\mu(1-q_1)[q_1(1-q_1)]^m}{(1-\mu)q_0 + \mu(1-q_1)} \\ &\geq \delta_i^0 - \lim_{m \rightarrow \infty} \frac{1}{\sqrt{\pi m}} \cdot \frac{\alpha_i\mu(1-q_1)[4q_1(1-q_1)]^m}{(1-\mu)q_0 + \mu(1-q_1)} \\ &= \delta_i^0 - \lim_{m \rightarrow \infty} \frac{1}{\sqrt{\pi m}} \cdot \frac{\alpha_i\mu(1-q_1)}{(1-\mu)q_0 + \mu(1-q_1)} \cdot a^m = \delta_i^0 - 0 = \delta_i^0 \geq \delta > 0, \end{aligned}$$

because $1/2 < q_1 < 1$ implies that $a = 4q_1(1-q_1) < 1$. The same holds for a voter who has received the signal 1. Claim (i) is thus obtained in much the same way as claim (i) in Theorem 4, namely, for a sequence of δ -values decreasing in n at the rate b^m , for $m = (n-1)/2$, where $\max\{4q_0(1-q_0), 4q_1(1-q_1)\} < b < 1$.

9.10. Claim (ii) in Theorems 4 and 5. We here show that all equilibria are pure, under the hypotheses of Theorems 4 and 5, respectively. Consider voter strategies $\sigma_i : \{0, 1\} \rightarrow [0, 1]$, for $i = 1, \dots, n$, that map voter i 's signal s_i to a probability $p_i = \sigma_i(s_i)$ for i voting on alternative 1 (and voting on alternative 0 with the complementary probability, $1 - \sigma_i(s_i)$). Sincere voting thus is the strategy $\sigma_i(s_i) \equiv s_i$.

Consider now a given voter i who has received signal $s_i = 0$. Denote by \mathcal{T}_i the event of a tie among all other votes. Such a tie may arise by chance, even for given signals, if other voters randomize their votes. However, since signals, and hence also votes, are statistically independent conditionally upon the state ω , we have, under any strategy profile $(\sigma_1, \dots, \sigma_n)$:

$$\Pr[\mathcal{T}_i \wedge s_i = 0 \mid \omega] = \Pr[\mathcal{T}_i \mid \omega] \cdot \Pr[s_i = 0 \mid \omega]$$

for $\omega = 0, 1$. In the base-line model, game $G(m)$, the difference in expected utility

for voter i between voting 0 and 1, conditional on having received signal 0, is

$$\begin{aligned}\Delta u_i^0 &= \beta_i \Pr[\mathcal{T}_i \wedge \omega = 0 \mid s_i = 0] - \alpha_i \Pr[\mathcal{T}_i \wedge \omega = 1 \mid s_i = 0] \\ &= \beta_i \frac{(1 - \mu) q_0}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \frac{\mu(1 - q_1)}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 1]\end{aligned}$$

In the game $G_\delta(m)$ perturbed with the sincerity bonus δ , voting sincerely when receiving signal 0 is optimal if and only if $\Delta u_i^0 + \delta \geq 0$, or, equivalently,

$$\delta \Pr[s_i = 0] + \beta_i (1 - \mu) q_0 \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \mu (1 - q_1) \Pr[\mathcal{T}_i \mid \omega = 1] \geq 0, \quad (23)$$

and mixing is optimal if and only if this last equation is an equality.

Likewise, the difference in expected utility for voter i between voting 1 and 0, conditional on having received signal 1, is

$$\Delta u_i^1 = \alpha_i \frac{\mu q_1}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i \frac{(1 - \mu)(1 - q_0)}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 0],$$

and thus sincere voting in this case is optimal if only if

$$\delta \Pr[s_i = 1] + \alpha_i \mu q_1 \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i (1 - \mu)(1 - q_0) \Pr[\mathcal{T}_i \mid \omega = 0] \geq 0, \quad (24)$$

and mixing is optimal if and only if this last equation is an equality.

Summing the right hand sides of (23) and (24) yields:

$$\delta + \beta_i (1 - \mu)(2q_0 - 1) \Pr[\mathcal{T}_i \mid \omega = 0] + \alpha_i \mu (2q_1 - 1) \Pr[\mathcal{T}_i \mid \omega = 1],$$

Because q_0 and q_1 are larger than $1/2$, this is a strictly positive number as soon as δ is strictly positive. Therefore at least one of the two inequalities (23) and (24) is strict, which means that each voter must be voting sincerely on (at least) one signal. In particular no voter can be strictly mixing on both signals. The same argument works in games $\Phi_\varepsilon(m)$ and it is worth noticing this fact:

Fact: Each voter is voting sincerely on at least one signal.

From this it follows that there exists one signal, say signal $s = 0$, such that at least half of the population vote sincerely when receiving this signal. Without loss of generality we may take the point of view of individual $i = 2m + 1$ and suppose that individuals $j = 1, \dots, m$ vote $v_j = 0$ when receiving signal $s_j = 0$. Let \mathcal{N}_0 denote the

random variable “number of votes 0 among voters $1, \dots, 2m$, conditionally on $\omega = 0$ ”. Then:

$$\Pr[\mathcal{T}_i \mid \omega = 0] = \Pr[\mathcal{N}_0 = m].$$

One can decompose the variable \mathcal{N}_0 as:

$$\begin{aligned} \mathcal{N}_0 &= \mathcal{X}_0 + \mathcal{Y}_0 \\ \mathcal{X}_0 &= \sum_{j=1}^m \mathbf{1}\{s_j = 0 \mid \omega = 0\} \\ \mathcal{Y}_0 &= \sum_{j=1}^m \mathbf{1}\{s_j = 1 \wedge v_j = 0 \mid \omega = 0\} + \sum_{j=m+1}^{2m} \mathbf{1}\{v_j = 0 \mid \omega = 0\} \end{aligned}$$

Notice that

$$\begin{aligned} \Pr[\mathcal{N}_0 = m] &= \sum_{k=0}^m \Pr[\mathcal{X}_0 = k] \cdot \Pr[\mathcal{Y}_0 = m - k] \\ &\leq \max_{0 \leq k \leq m} \Pr[\mathcal{X}_0 = k] \end{aligned}$$

We do not know the probability distribution of \mathcal{Y}_0 , because of possible mixing, but we know that \mathcal{X}_0 is binomial with parameter q_0 and m . Therefore $\max_{0 \leq k \leq m} \Pr[\mathcal{X}_0 = k]$ is equal to $\Pr[\mathcal{X}_0 = \lfloor q_0 m \rfloor]$ where $\lfloor q_0 m \rfloor$ denotes the integer part of $q_0 m$. If $q_0 m$ is an integer, then we obtain:

$$\Pr[\mathcal{N}_0 = m] \leq \Pr[\mathcal{X}_0 = q_0 m] = \binom{q_0 m}{m} q_0^{q_0 m} (1 - q_0)^{m - q_0 m},$$

and, after using Stirling’s formula:

$$\Pr[\mathcal{X}_0 = q_0 m] \sim \frac{1}{\sqrt{2\pi m q_0 (1 - q_0)}}.$$

This last property can be shown to actually hold even if $q_0 m$ is not an integer but we leave this technical point to the interested reader. To have a majorization, we may note for instance that it follows that there exists an A (which only depends on q_0) such that for all $m > A$, $\Pr[\mathcal{T}_{2m+1} \mid \omega = 0] < B/\sqrt{m}$, for $B = 1/\sqrt{q_0(1 - q_0)}$. Thus, looking again at the condition (24) one can see that, for $m > A$ and

$$\delta > \frac{1}{\Pr[s = 1]} \beta_i (1 - \mu) (1 - q_0) \frac{B}{\sqrt{m}} = \frac{\beta_i (1 - \mu) (1 - q_0)}{(1 - q_1) \mu + (1 - \mu) (1 - q_0)} \frac{B}{\sqrt{m}}$$

(24) is a strict inequality, which means that sincere voting on signal $s = 1$ is strictly optimal for the considered voter $i = 2m + 1$. Then it follows that there exists a B' (which depends on all the parameters q, α, β, μ) such that for $m > A$ and $\delta > B'/\sqrt{m}$, sincere voting on signal $s = 1$ is strictly optimal for all voters. The values of the parameters α_i, β_i, μ for different voters are bounded, so we can take B' to be a constant of the model, independent of the population size.

If all voters vote sincerely on signal $s = 1$, the number \mathcal{N}_1 of votes 1 among voters $j = 1, \dots, 2m$, conditionally on $\omega = 1$ can be decomposed as:

$$\begin{aligned}\mathcal{N}_1 &= \mathcal{X}_1 + \mathcal{Y}_1 \text{ for} \\ \mathcal{X}_1 &= \sum_{j=1}^{2m} \mathbf{1}\{s_j = 1 \mid \omega = 1\} \\ \mathcal{Y}_1 &= \sum_{j=1}^{2m} \mathbf{1}\{s_j = 0 \wedge v_j = 1 \mid \omega = 1\}\end{aligned}$$

with \mathcal{X}_1 binomial $(2m, q_1)$. Again we note that

$$\begin{aligned}\Pr[\mathcal{T}_i \mid \omega = 1] &= \Pr[\mathcal{N}_1 = m] = \Pr[\mathcal{X}_1 + \mathcal{Y}_1 = m] \\ &= \sum_{k=0}^m \Pr[\mathcal{X}_1 = k] \cdot \Pr[\mathcal{Y}_1 = m - k] \\ &\leq \max_{0 \leq k \leq m} \Pr[\mathcal{X}_1 = k]\end{aligned}$$

The mode of the binomial distribution of \mathcal{X}_1 is reached at the integer part of the real number $2q_1m$, a number that exceeds m . It follows that

$$\max_{0 \leq k \leq m} \Pr[\mathcal{X}_1 = k] = \Pr[\mathcal{X}_1 = m] = \binom{m}{2m} q_0^m (1 - q_0)^m.$$

Using Stirling's approximation formula, one finds again that this number is decreasing (this time exponentially) with m . The same reasoning as before can now take place with respect to equation (23): the negative term, $-\alpha_i \mu (1 - q_1) \Pr[\mathcal{T}_i \mid \omega = 1]$, is asymptotically small and we conclude that there exist numbers A' and B'' such that if $m > A'$ and $\delta > B''/\sqrt{m}$, inequalities (23) and (24) are both strict for all i , which means that all voters vote sincerely on both signals. Point (ii) in Theorem 4 follows immediately. The reasoning is the same for Theorem 5.

9.11. Proposition 3. Let $n = 2m + 1$, $\mu = 1/2$ and $q_0 = q_1 = q$. In order to prove the first claim in the proposition, let $p(\lambda)$ be the conditional probability for any given committee member i 's vote to be pivotal, conditional upon the event that i is not a noise voter. For $\lambda = 0$ we have, from the calculations in our baseline model,

$$p(0) = \binom{2m}{m} \cdot [q(1 - q)]^m$$

For $\lambda > 0$:

$$p(\lambda) = \frac{\lambda}{(1 - \lambda)n + \lambda} p(1) + \frac{(1 - \lambda)n}{(1 - \lambda)n + \lambda} p(0)$$

so it remains to identify $p(1)$. We obtain

$$\begin{aligned} p(1) &= \frac{1}{2} \binom{2m - 1}{m} [q^m (1 - q)^{m-1} + q^{m-1} (1 - q)^m] \\ &= \binom{2m - 1}{m} \cdot \frac{[q(1 - q)]^m}{2} \cdot \left(\frac{1}{1 - q} + \frac{1}{q} \right) = \\ &= \binom{2m - 1}{m} \cdot \frac{[q(1 - q)]^{m-1}}{2} \end{aligned}$$

Hence, for any $m \geq 1$:

$$\frac{p(1)}{p(0)} = \frac{1}{4q(1 - q)}$$

and thus $p(\lambda)/p(0) \geq 1$ for all $q \in [1/2, 1]$ with strict inequality when $q > 1/2$. This proves the first claim in the proposition.

In order to prove the second claim, suppose that voter i is an informed voter with signal $s_i = 0$. Conditional upon being pivotal under majority rule, what is the conditional probability for each state? Assume first that $\lambda = 0$. We are then back in the standard model and the conditional probability for state $\omega = 0$, conditional upon i 's signal $s_i = 0$ and being pivotal under sincere voting, is q (the m other signals 0 cancel the m other signals 1, because $\mu = 1/2$ and $q_0 = q_1$). Secondly, assume that $\lambda = 1$. Being pivotal, i knows that there are either m signals 0 and $m - 1$ signals 1, or $m - 1$ signals 0 and m signals 1, with equal probability for both events (since the noise voter randomizes uniformly). The conditional probability for state $\omega = 0$, conditional upon i 's signal being $s_i = 0$ and upon i 's vote being pivotal under sincere voting, is no less than q . To see this, let \mathcal{T} be the event of a tie among the $2m$ other committee members (including the noise voter), let N_0 and N_1 be the (random)

numbers of signals 0 and 1 among the other $2m - 1$ informed voters:

$$\begin{aligned}
 \Pr[\omega = 0 \mid \mathcal{T} \wedge s_i = 0] &= \frac{1}{2} \Pr[\omega = 0 \mid s_i = 0 \wedge N_0 = m \wedge N_1 = m - 1] + \\
 &\quad + \frac{1}{2} \Pr[\omega = 0 \mid s_i = 0 \wedge N_0 = m - 1 \wedge N_1 = m] \\
 &= \frac{1}{2} \Pr[\omega = 0 \mid (m + 1 \text{ signals } 0) \wedge (m - 1 \text{ signals } 1)] + \\
 &\quad + \frac{1}{2} \Pr[\omega = 0 \mid (m \text{ signals } 0) \wedge (m \text{ signals } 1)] \\
 &= \frac{1}{2} \Pr[\omega = 0 \mid (m + 1 \text{ signals } 0) \wedge (m - 1 \text{ signals } 1)] + \frac{1}{2}
 \end{aligned}$$

Moreover,

$$\Pr[\omega = 0 \mid m + 1 \text{ signals } 0 \wedge m - 1 \text{ signals } 1] = \frac{q^{m+1} (1 - q)^{m-1}}{2 \cdot \Pr[(m + 1 \text{ signals } 0) \wedge (m - 1 \text{ signals } 1)]}$$

and

$$\Pr[(m + 1 \text{ signals } 0) \wedge (m - 1 \text{ signals } 1)] = \frac{1}{2} [q^{m+1} (1 - q)^{m-1} + (1 - q)^{m+1} q^{m-1}]$$

Hence,

$$\begin{aligned}
 \Pr[\omega = 0 \mid \mathcal{T} \wedge s_i = 0] &= \frac{1}{2} + \frac{1}{2} \cdot \frac{q^{m+1} (1 - q)^{m-1}}{q^{m+1} (1 - q)^{m-1} + (1 - q)^{m+1} q^{m-1}} \\
 &= \frac{1}{2} + \frac{1}{2} \cdot \frac{q^2}{q^2 + (1 - q)^2}
 \end{aligned}$$

It is not difficult to verify that the quantity on the right-hand side exceeds q whenever $1/2 \leq q < 1$, and equals q when $q = 1$.

A similar calculation, quantitatively identical due to the assumed symmetry, holds for the case when $s_i = 1$. Hence, the potential presence of a noise voter among the others enhances the predictive power of one's own signal in case of a tie among the others. Hence, the expected-utility difference between sincere and insincere voting is increased.

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