Asymmetric noise probed with a Josephson junction

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Fluctuations of the current through a tunnel junction are measured using a Josephson junction. The current noise adds to the bias current of the Josephson junction and affects its switching out of the supercurrent branch. The experiment is carried out in a regime where switching is determined by thermal activation. The variance of the noise results in an elevated effective temperature, whereas the third cumulant, related to its asymmetric character, leads to a difference in the switching rates observed for opposite signs of the current through the tunnel junction. Measurements are compared quantitatively with recent theoretical predictions.

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The current through voltage-biased electrical conductors exhibits fluctuations, which, in contrast to equilibrium Johnson-Nyquist noise, are not symmetric with respect to the average current. This translates into finite odd cumulants in the distribution of the number of electrons transfered through the conductor in a given time. Whereas the full counting statistics of this number can be calculated for arbitrary conductors [1, 2], up to now high order cumulants have been measured in very few experiments. The third cumulant has been successfully accessed by signal processing the time-dependent current [3, 4, 5], but with set-ups that are restricted either to low impedance samples, which leads to large environmental effects [3], or to low frequencies [4, 5]. Another strategy, put forward by Tobiska and Nazarov [6], consists in using a Josephson junction (JJ) as a large bandwidth on-chip noise detector [7, 8, 9]. It has a high intrinsic sensitivity, and can be coupled to noise sources over a large range of impedances. The detection principle relies on the exponential current sensitivity of the switching of a JJ from a metastable zero-voltage branch to a dissipative one. When biased at a current I_J slightly below its critical current I_0, the rate of switching is therefore very sensitive to noise in the current. The first detection of asymmetric noise with a JJ was reported in Refs. [7, 8]. However, the detector JJ, which was placed in an inductive environment, had a very large plasma frequency, and the dynamics of the junction changed regime as the noise intensity increased, from macroscopic quantum tunneling (MQT) to retrapping [10] through thermal activation. The measured asymmetry in the escape rates could only be compared to an adiabatic model [11], using empirical parameters. For a detector to be of practical use it must have a well characterized and a simple enough response, so that information on the noise can be reliably extracted. As quantitative theories have been developed for a JJ in the thermal regime placed in a resistive environment [12, 13, 14], we designed an experiment in this framework, allowing for a detailed, quantitative comparison with theory.

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The principle of our experiment is to add the current noise from a noise source to the DC bias current of a JJ (see Fig. 1). The dynamics of a JJ placed in a resistive environment are described by the celebrated RCSJ model [15], with the voltage related to the average velocity of a fictitious particle placed in a tilted washboard potential. The tilt of the potential is determined by the reduced parameter $s = I_J/I_0$. At $s < 1$ the potential presents local minima where the particle can be trapped. The voltage is then zero: this corresponds to the supercurrent branch. The frequency of small oscillations is called the plasma frequency ω_p. Johnson-Nyquist current noise related to the finite temperature T of the environment of the junction is modeled as a fluctuating force on the particle, which triggers escape from the local minimum ("switching"). When $k_BT > \hbar\omega_p/2\pi$, the switching rate Γ is given by Kramer's formula [16] $\Gamma = A \exp -B_2(T)$ with $A \simeq \omega_p/2\pi$ for moderate quality factors Q, $\omega_p = \omega_{p0}\left(1-s^2\right)^{1/4}$ the plasma frequency in the tilted potential, $\omega_{p0} = \sqrt{I_0/\varphi_0 C_J}$ the bare plasma frequency determined by the capacitance C_J and critical current I_0 of the junction, with $\varphi_0 = \hbar/2e$, and

$$B_2(T) = \left(4\sqrt{2}I_0\varphi_0/3k_BT\right)(1-|s|)^{3/2}. \quad (1)$$

Recently, this result was extended to the situation in which an additional delta-correlated noise $\delta I_N(t)$, characterized by a finite third cumulant S_3 defined by $\langle\delta I_N(t)\delta I_N(t')\delta I_N(t'')\rangle = S_3\delta(t'-t)\,\delta(t''-t')$ and a second cumulant $\langle\delta I_N(t)\delta I_N(t')\rangle = S_2\delta(t'-t)$ adds to current through the JJ [12, 13, 14]. The effect of higher order cumulants is assumed to be weak. The corresponding fluctuating force leads to a modification of the rate: $\Gamma = A\exp-(B_2(T_{\text{eff}})+B_3)$. The second cumulant yields an increased effective temperature T_{eff} given by

$$2k_BT_{\text{eff}}/R_\parallel = 2k_BT/R + S_2. \quad (2)$$

Here, R is the parallel combination of all the resistances which produce Johnson-Nyquist noise across the junction. The resistance $R_\parallel$ characterizes the friction acting

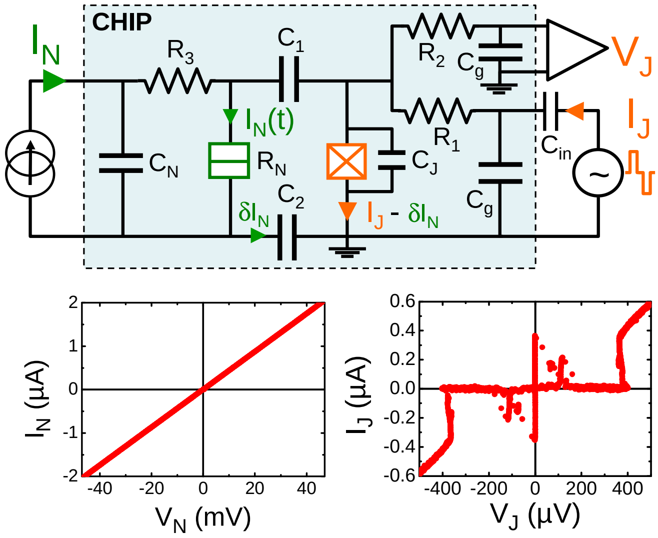

FIG. 1: (color online) Top: Experimental setup. Noise from a tunnel junction (green double box) biased at DC with a current I_N couples through capacitors $C_1 = 230\,\mathrm{pF}$ and $C_2 = 345\,\mathrm{pF}$ to a Josephson junction detector (JJ, in orange), which is current-biased on its supercurrent branch. The voltage V_J across the junction monitors the switching to the dissipative state. Capacitor $C_J = 12.5\,\mathrm{pF}$ lowers the JJ plasma frequency to $\omega_{p0}/2\pi \simeq 1.5$ GHz. Capacitors $C_N = 190\,\mathrm{pF}$ and $C_g = 140\,\mathrm{pF}$ shunt the external impedance at ω_{p0}, so that the impedance across the JJ is determined only by on-chip elements. Resistors $R_1 = R_3 = 215\,\Omega$ and $R_2 = 515\,\Omega$ were fabricated with thin Cr films. Bottom left: IV characteristic of the tunnel junction, linear at this scale, with inverse slope $R_N = 22.9\,\mathrm{k}\Omega$. Bottom right: IV characteristic of the detector JJ, with critical current $I_0 = 437\,\mathrm{n}A$. We attribute a resonance near $V_J \sim 120\,\mu\mathrm{V}$ to a mode of the electromagnetic environment of the junction.

on the fictitious particle and is, in a simple model, given by the total resistance across the junction, including both R and the resistance R_N of the noise source. This expression indicates that the second cumulant of noise from the noise source S_2 simply adds to the Johnson-Nyquist noise of the rest of the circuit. The third cumulant gives rise to the additional term

$$B_3 = -S_3 \left(\varphi_0/k_B T_{\mathsf{eff}}\right)^3 \omega_{p0}^2 \; j(s) \tag{3}$$

with $j(s)$ a function of the tilt that depends on the quality factor [14]. When reversing the sign of the average current I_N through the noise source, S_2 remains unchanged whereas S_3 changes sign. Therefore, the departure from 1 of the rate ratio

$$R_\Gamma = \Gamma\left(+I_N\right)/\Gamma\left(-I_N\right) = \exp\left(2\left|B_3\right|\right) \tag{4}$$

is a measure of non-symmetric noise $(S_3 \neq 0)$.

The experimental setup is shown schematically in Fig. 1. As it is well established that current noise through a tunnel junction is Poissonian ($S_2 = e\left|I_N\right|$ and $S_3 = e^2 I_N$, with e the electron charge), we use such a device (green double box) as a benchmark noise source. The detector JJ (orange crossed box) is coupled to it through capacitors C_1 and C_2. The finite frequency part δI_N of the current through the tunnel junction $I_N(t)$ flows through the detector JJ, owing to the high-pass filter formed by R_3, C_1 and C_2 (3 dB point at 5 MHz). The switching of the JJ current-biased at I_J is signaled by the appearance of a voltage V_J across it. The low plasma frequency of 1.5 GHz guarantees $k_B T > \hbar\omega_p/2\pi$ even at the lowest temperature of our experiment (20 mK) [17]. In the relevant range of frequencies slightly below $\omega_{p0}/2\pi$, numerical simulations of the actual circuit indicate that the quality factor of the Josephson oscillations Q is close to 5, insuring an underdamped dynamics, and no effect of retrapping [7, 10] as long as $s \gg 4/\pi Q \simeq 0.25$.

The sample was fabricated on a thermally oxidized high resistivity (10^3 to $10^4\,\Omega\,\mathrm{cm}$) Si wafer. All on-chip resistors are 10 nm-thick Cr layers, with $215\,\Omega/\square$ sheet resistance at 4 K, placed between mm-size pads. Capacitors were obtained from parallel aluminum films separated by 29 nm-thick sputtered silicon nitride as an insulator [18]. The tunnel junction and the detector JJ were fabricated at the same time by shadow evaporation of 20 nm and 80 nm-thick aluminum films. Their current-voltage characteristics are shown in Fig. 1. The tunnel junction has an area of $0.09\,\mu\mathrm{m}^2$ and a tunnel resistance $R_N = 22.9\,\mathrm{k}\Omega$. It was biased at voltages larger than twice the superconducting gap $2\Delta/e = 0.4\,\mathrm{mV}$ (which corresponds to $I_N = 0.02\,\mu\mathrm{A}$), so that it behaves as a normal metal junction, with Poissonian noise. The detector JJ, with area $1\,\mu\mathrm{m}^2$, has a supercurrent $I_0 = 0.437\,\mu\mathrm{A}$. It was biased in series with a resistor $R_1 = 215\,\Omega$ through a $50\,\Omega$ coaxial line equipped with attenuators. When switching occurs at a supercurrent I_{sw}, the voltage across the junction jumps to $(R_1 + 50\,\Omega)\, I_{\mathrm{sw}} < 2\Delta/e$, so that the current through it drops to zero and no quasiparticles are generated. Moreover, gold electrodes in good contact with the Al films were fabricated a few μm away from the junctions in order to act as traps for spurious quasiparticles [19] that could be excited by the high frequency noise. Apart from the Cr resistors and the Au traps, all conductors on the chip are superconducting aluminum films.

The sample was thermally anchored to the mixing chamber of a dilution refrigerator. The tunnel junction was biased by a floating voltage supply through two $1.5\,\mathrm{M}\Omega$ resistors. The on-chip capacitance $C_N = 190\,\mathrm{pF}$ on the bias line is large enough to maintain the voltage across the tunnel junction at $V_N = R_N I_N$ for all relevant frequencies. Escape rates of the JJ were measured using 2×10^5 current pulses of duration $\tau = 0.53\,\mu s$ with alternatively positive $(+I_J)$ and negative $(-I_J)$ amplitude, separated by $9\,\mu$s. They were fed through a non-polarized capacitor $C_{\mathrm{in}} = 200\,\mu\mathrm{F}$ placed at room temperature, which prevents DC thermoelectric currents from unbalancing the pulses. The switching rates Γ_+ and Γ_- for the two signs of I_J were deduced from the switching probability $P = 1 - e^{-\Gamma\tau}$ measured as the fraction of the

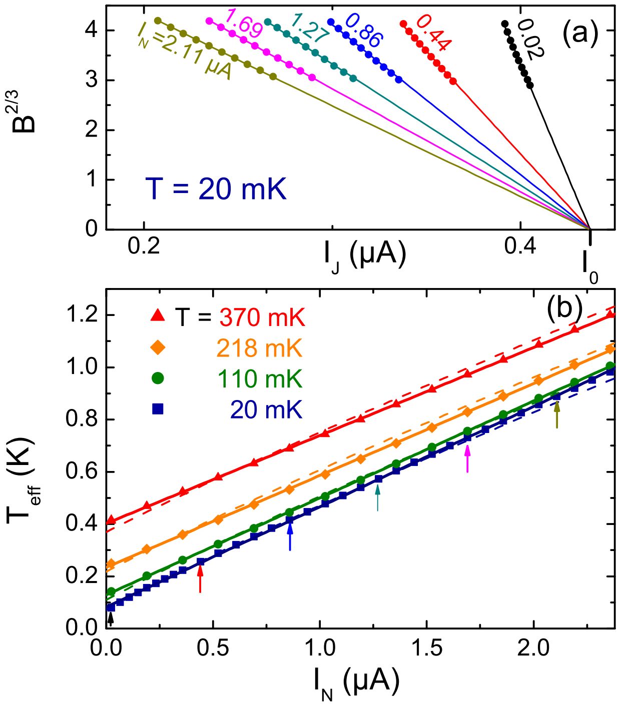

FIG. 2: (color online) (a) Dependence of $B^{2/3} = (-\log(\Gamma/A))^{2/3}$ on the current I_J through the detector JJ, for data taken at $T = 20\,\mathrm{mK}$ and currents through the tunnel junction $I_N = 0.02$ to $2.11\,\mu\mathrm{A}$, by steps of 0.42 μA. Linear dependence is a signature of the thermal activation regime. (b) Effective temperature extracted from the slope of datasets as in (a), as a function I_N, for various temperatures T. Arrows indicate the datapoints corresponding to the plots in (a). Solid lines are linear interpolations. Dashed line are the predictions from full theory, taking into account the frequency dependence of the admittance $Y(\omega)$ across the JJ and of the transfer function $\alpha(\omega)$ from noise source to JJ detector.

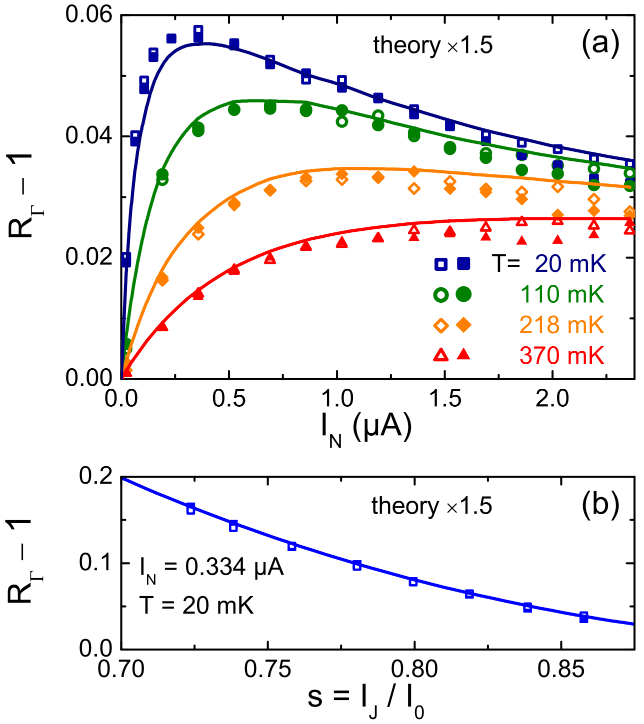

FIG. 3: (color online) (a) Rate asymmetry $R_\Gamma - 1$ as a function of current in noise source I_N for various temperatures T. Filled (open) symbols correspond to measurements with positive (negative) current through the detector JJ. Solid lines are theoretical predictions scaled by a factor 1.5. (b) Dependence of $R_\Gamma - 1$ on $s = I_J/I_0$, at $I_N = 0.334\,\mu A$, obtained with measurement pulses of various lengths. Solid line is theoretical prediction scaled by 1.5.

current pulses which led to a voltage pulse.

We first demonstrate that the switching of the detector junction is well described by the model of thermal activation whatever the current in the noise source. Figure 2(a) shows, for various currents $I_N > 0$, the $s-$dependence of $B^{2/3} = [-\ln(\Gamma/A)]^{2/3}$. Data fall on straight lines that extrapolate to 0 for $I_J = I_0$, as expected from Eq. (1). This allows us to extract an effective temperature T_{eff}, whose dependence on I_N is shown in Fig. 2(b), with data taken at four different base temperatures T. We do find a linear dependance with correct extrapolations at $I_N = 0$ (values slightly above T are attributed to imperfect filtering), as expected from Eq. (2) with $S_2 = eI$. Understanding the slope quantitatively requires an accurate model of the actual circuit at microwave frequencies: the RCSJ model assumes that the JJ is simply connected to a capacitor, a resistance $R_\parallel$ and a current source, in parallel. In the limit $Q \gg 1$, $R_\parallel$, which describes friction, has to be replaced with $R_\parallel(\omega_p) \equiv 1/\operatorname{Re}(Y(\omega_p))$, with $Y(\omega)$ the total admittance of the circuit across the JJ. Microwave simulations indicate that $R_\parallel(\omega_p)$ varies almost linearly from 63 Ω at 1 GHz to 36 Ω at 1.5 GHz, and that a current $I_N(\omega)$ through the tunnel junction leads to a current $\alpha(\omega)I_N(\omega)$ through the detector JJ, with a transfer function $\alpha(\omega)$ varying from 1.1 at 1 GHz to 1.27 at 1.5 GHz. Since escape is determined essentially by the noise at ω_p, we replace S_2 by $\alpha^2(\omega_p)eI_N$. Altogether, the prediction $T_{\text{eff}} \simeq T + \alpha^2(\omega_p)R_\parallel(\omega_p)eI_N/2k_B$ is in agreement with the data (see dashed lines in Fig. 2; to fit the 20 mK-data, we used $T = 72\,\mathrm{mK}$), apart from the slight change in slope when varying T which could be attributed to variations in the kinetic inductance of the superconducting electrodes.

We now discuss the effect of noise asymmetry. The $B^{2/3}$ plots for opposite signs of the current through the noise source are undistinguishable within the symbol size, demonstrating that the effect of the second cumulant S_2 is dominant. In the limit $eV_N \gg k_BT$, theory predicts that the effect of S_3 is to shift the curves by $\Delta I_J \sim 0.6B(\omega_{p0}/Q)(S_3/S_2) = 0.6Be/R_\parallel C_J \sim 0.2\% \times I_0$, which is difficult to measure reliably [20]. In our experiment, we measured directly the asymmetry ratio R_Γ defined by Eq. (4), which varies by several % (see Fig. 3). We first set the amplitude I_J of the current pulses at a value corresponding to a switching probability $P \sim 0.6$, for which the statistical precision on the rates is good [21]. We then measured 100 times Γ_+ and Γ_-, with alternatively $+I_N$ and $-I_N$ through the noise source. This allows for two independent measurements of R_Γ : $R_\Gamma^+ = \Gamma_+(-I_N)/\Gamma_+(+I_N)$ and $R_\Gamma^- = \Gamma_-(+I_N)/\Gamma_-(-I_N)$ [22]. In Fig. 3(a), we show with full and open symbols the corresponding measurements. The rate ratio R_Γ differs from 1, a signature of asymmetric noise, as soon as $I_N \neq 0$. The statistical uncertainty on R_Γ is smaller than the symbols. Small differences between R_Γ^+ and R_Γ^-, in particular around $I_N = 2\,\mu$A, are not understood. As for the comparison with theory, a difficulty arises because of the frequency dependence of the transfer function $\alpha(\omega)$, which results in a colored third cumulant at the detector $S_3(\omega_1, \omega_2) = \alpha(\omega_1)\alpha(\omega_2 - \omega_1)\alpha(-\omega_2)e^2 I_N$. In the following, and in the absence of indication as to which frequencies are important, we compare however with the only existing theory, which assumes white noise ($S_3 = e^2 I_N$). The corresponding predictions, Eqs. (3,4) with $j(s) \simeq 0.81\,(1-s)^{2.14}$[23], are shown as solid lines in Fig. 3(a), scaled by an arbitrary factor 1.5. R_Γ exhibits a maximum as a function of I_N due to the opposite variations of S_3 and T_{eff} with I_N. For T_{eff}, we used interpolations between the measured values shown in Fig. 2. When scaled up by 1.5, which might be due to frequency dependent transmission ($\alpha(\omega)$), theory accounts well for the experimental data. Feedback corrections due to the detector, described in [13], are neglected since $R_\parallel/R_N \ll 1$ [14]. Note that there is no feedback associated to the series resistance R_3 like in Ref. [3] because the current noise associated to R_3 does not flow through the noise source, but through the detector JJ. In Fig. 3(b), we also compare with theory the $s-$dependence of R_Γ. In order to perform this measurement, we used pulses of various durations (0.53 to 21 μs), which allows to obtain the switching rates at different values of s. For the longest pulses, the rate asymmetry is as large as 16%. Here also, theory scaled by 1.5 accounts precisely for the data.

Qualitative agreement between experiment and theory gives confidence for the use of the JJ as a measuring device for S_3, even if the application to a wider range of systems requires some theory for colored noise. A limitation concerns situations with strong non-linearities in the voltage dependence of the cumulants, where feedback effects could become sizeable [24, 25]. For quantitative measurements of S_3 on other systems, it is not only important to tune the plasma frequency of the junction in the GHz range as done in this work, but also to improve the microwave design, in particular with more compact electrodes, so as to avoid frequency dependent factors in the analysis. Proposals to access the full counting statistics with a JJ embedded in more complex circuits [6] remain to be investigated.

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[21] The expression for the uncertainty on R_Γ in Ref. [9] should be divided by $\sqrt{2}$.
[22] The signs in front of I_N account for the fact that in our setup, the noise current *subtracts* from the bias current.
[23] This analytical expression interpolates between the results of numerical calculations of $j(s)$ performed for $Q \sim 5$ and in the working interval $0.5 < s < 0.9$, along the lines of Ref. [14] (H. Grabert, private communication).
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