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## Procyclical International Capital Flows, Debt Overhang and Volatility

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# Procyclical International Capital Flows, Debt Overhang and Volatility\*

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**Abstract:** In this paper, I show how procyclical capital flows originate boom-bust and sunspot episodes in a neoclassical growth model of a small, open economy. All markets are perfect, with the exception of the fact that some upper, endogenous limit is imposed on how much the economy can borrow from foreign creditors, due to potential debtor default. It is shown that the steady state is locally indeterminate when the credit multiplier is larger than some threshold level, whereas saddle-point stability prevails when the credit multiplier

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is low enough. As a consequence, high levels of the credit multiplier lead to both booms followed by busts and sunspot-driven volatility near the steady state, while, in contrast, low levels ensure monotonic convergence. Compared with saddle-path equilibria, boom-bust and sunspot equilibria are associated with both lower welfare and debt overhang, that is, a *crowding-out* effect of credit: when the economy is highly leveraged, it uses savings to cut down foreign debt, at the expense of both human and physical investment. Numerical examples show that volatility arises at rather low values of financial development and for debt-to-GDP ratios that fall within the range of available estimates. Finally, the effects of shocks to the world interest rate on output and consumption are amplified and persistent in the debt overhang regime.    Keywords: international financial markets, endogenous borrowing constraints, small open economy, debt overhang, business cycles, indeterminacy, sunspot equilibria.    *Journal of Economic Literature* Classification Numbers: E22, E44, F34, F41, G15.

# 1 Introduction

In the present paper, I show how volatility of consumption, investment and output arises in a simple growth model of a small, open economy that attracts foreign financial flows. Human capital is immobile while physical capital flows across borders. All markets are perfect (and returns to scale are constant), with the exception of the fact that some upper limit is imposed on how much the economy can borrow from foreign creditors. This borrowing limit is assumed to arise because of potential debtor default and to depend positively on agents' wealth, which gives rise to a credit multiplier such that debt is proportional to the physical capital stock. Such a formulation, which combines insights by Barro *et al.* [4] and Aghion *et al.* [1], is used as a simple way to capture the well documented evidence that international capital flows are procyclical (see Kaminsky *et al.* [11, Table 8]).

The main result of this paper, in section 2.2, states that both booms followed by busts and sunspot equilibria drive the dynamics of such a small open economy when international financial markets are “too” generous in the sense that foreign borrowing exceeds significantly the physical capital stock. More precisely, I show that the steady state is locally a saddle when the credit multiplier - that is, the loan-to-wealth ratio - is lower than a threshold level. This happens, in particular, in Barro *et al.* [4] which obtains here as a particular case. However, local indeterminacy prevails when the credit multiplier is (greater than one and) large enough. As a consequence, booms followed by busts and stationary sunspot equilibria are expected near the steady state when the credit multiplier is sufficiently high, that is, when the foreign debt-to-GDP ratio is large enough. In other words, a “debt overhang regime” arises at high enough levels of international financial integration.

In section 3, I underline three macroeconomic implications of the main result. First, I show that equilibria not only exhibit consumption, investment and output volatility in the debt overhang regime, but they are also associated with consumption allocations that have *lower welfare* than equilibrium allocations converging to saddle-point steady states. Second, I study the adjustment process to shocks to the world interest rate and I show that the impact of such shocks is both amplified and persistent in the debt overhang regime. As

emphasized in Section 2.3, volatile dynamics of consumption, investment and output arise when the credit multiplier is large enough because of a *crowding-out* effect of credit: in the indeterminacy regime, there is debt overhang, that is, the economy uses savings to cut down foreign debt, at the expense of both human and physical investment. Consumption growth then decreases savings which eventually turn negative. Capital investment financed by foreign credit then starts off again, leading the economy at the outset of a new cycle. I should stress that such a cyclical pattern does not involve expectations. However, stationary sunspot equilibria involving self-fulfilling movements of expectations do also exist near steady state. Finally, I extend the basic model to incorporate domestic collateral and show that boom-bust and sunspot episodes may occur for values of the foreign debt-to-GDP ratio around 50%, which fall within the range of available estimates provided by Lane and Milesi-Ferretti [14]. Such an extension also provides an intriguing prediction, as it shows that, absent risk sharing, domestic financial development may be detrimental to macroeconomic stability when the economy opens up to capital flows. It turns out that a large domestic credit multiplier implies a lower threshold value of the international debt multiplier above which the economy is vulnerable to indeterminacy. In other words, an economy with a highly developed domestic credit market is likely to be susceptible to boom-bust and sunspot episodes at lower levels of international financial integration.

The main empirical motivation behind the analysis comes from the mounting evidence that access to international financial markets does not ensure perfect consumption smoothing. The suspicion is that, because capital flows are procyclical in most countries, they tend to act as a significant source of aggregate volatility (see, e.g., the reviews by Kaminsky [10] and Prasad *et al.* [20]). In addition, reported evidence suggests both that lending booms often precede financial crashes (Tornell and Westermann [24]) and that many countries have been located on the downward-sloping part of the debt Laffer curve (Imbs and Ranciere [12]). The model is broadly consistent with those empirical patterns. In particular, the main lesson one may draw from the analysis is that an economy which is saddle-point stable at low levels of financial integration is, in sharp contrast, vulnerable to (welfare dominated) boom-bust and sunspot episodes at levels of the borrowing-investment ratio

beyond a threshold value. In particular, the model predicts that lending booms may, if they increase the credit multiplier beyond the critical level, give birth to macroeconomic volatility. This is consistent with observed lending booms preceding financial crashes. My main result also accords with recent evidence on debt overhang and adds to this literature by stressing the importance of volatility as a theoretical channel through which leverage may be detrimental to growth if it is too high.

In a nutshell, the model generalizes Barro *et al.* [4] by endogeneizing the debt-to-capital ratio, as in Aghion *et al.* [1]. It relies upon the analysis of Cohen and Sachs [8], who show that credit limits arise endogenously in a growth model with potential debt repudiation *à la* Eaton and Gersovitz [9]. A related model based on collateral-backed credit has been studied by Barro *et al.* [4], who impose a stricter borrowing constraint such that the credit multiplier is set to one. In both settings, however, indeterminacy and sunspots do not appear. Aghion *et al.* [1] have used a similar formulation of imperfect credit markets and shown that endogenous cycles occur at intermediate levels of the credit multiplier.<sup>1</sup> Besides some differences in modeling assumptions, it should be noticed that Aghion *et al.* [1] focus on deterministic cycles (not on indeterminacy and sunspot equilibria), the occurrence of which relies on complementary inputs. Aghion *et al.* [1, Section 3.2] provide some simulations of their model showing that cycles exist only if the elasticity of input substitution is lower than one third. In contrast, I assume a Cobb-Douglas technology and prove that deterministic cycles do not occur near steady state. Moreover, I stress the importance of debt overhang and show both that the debt constraint is binding along the transition to the steady state and that a locally indeterminate steady state is associated with allocations that have lower welfare. Such a welfare analysis is absent in Aghion *et al.* [1], whose paper assumes that the credit constraint binds. On the other hand, my economy is subject to booms followed by busts and sunspot-driven volatility at rather low levels of financial development - the threshold value is in fact close to one - for which cycles would be typically absent in the Aghion *et al.* [1] economy. Numerical examples provided by Aghion *et al.* [1, p. 1090] set

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<sup>1</sup>Related research also includes the overlapping generations models of Boyd and Smith [6] and Matsuyama [16], who concentrate on the issue of cross-country inequality.

the credit multiplier to 4.<sup>2</sup> In contrast, section 2.2 below shows that such large values are not feasible in my model and that boom-bust and sunspot episodes arise at lower values around 1.75. To put it differently, booms and busts occur when the credit-investment ratio is larger than 60% but disappear if one considers the range compatible with cycles in the Aghion *et al.* [1] economy, which require that borrowing accounts for 80% of investment (more than 90% in Caballé *et al.* [7]).<sup>3</sup> My result is not inconsistent with recent evidence suggesting that low to moderate levels of financial integration lead to large volatility (see Prasad *et al.* [20]).

So as to ease comparison between both settings, Section 2.1 displays some microeconomic foundations of the credit multiplier based on potential debtor default, adapted from Aghion *et al.* [2, App. 1]. From this exercise, we learn that the debt overhang regime relies on low values of financial development. Here again, this is because the threshold parameter value above which debt overhang prevails is much smaller than the cut-off level leading to cycles in Aghion *et al.* [1]. More precisely, booms followed by busts and sunspots occur for low values of either the penalty of default imposed on borrowers or the probability of debt collection, for which cycles are typically excluded in Aghion *et al.* [1]. In summary, while the analysis of Aghion *et al.* [1] suggest that low or high financial development may protect open economies from deterministic cycles, my results contrast by showing that boom-bust and sunspot-driven business-cycles occur at rather low levels of the credit multiplier.

This work is also related to recent research by Martin and Rey [15] and Schneider and Tornell [22] on the mechanisms leading to financial crises. My paper is arguably complementary to theirs, which emphasize different channels that may exacerbate aggregate volatility. Unlike this paper, for instance, both contributions study demand-side aspects of financial crashes in finite-horizon settings without capital accumulation. Finally, Martin and Rey [15] stress the importance of incomplete financial markets, while Schneider and Tornell [22] point at asymmetries in corporate finance across sectors. Differently, I both

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<sup>2</sup>Caballé *et al.* [7] have recently introduced a Cobb-Douglas technology into the Aghion *et al.* [1] model and shown that cycles then occur for values of the credit multiplier that are larger than 10.

<sup>3</sup>As a benchmark, notice that Barro *et al.* [4] assume a stricter borrowing limit such that credit represents only 50% of investment.

underline the importance of debt overhang and study welfare issues. Another related strand of the literature has shown how economies that are open to international financial markets may be susceptible to local indeterminacy (see, e.g., Lahiri [13], Weder [26], Nishimura and Shimomura [18], Meng and Velasco [17], Aloi and Lloyd-Braga [3]). In contrast with the present work, these contributions rely on increasing returns to scale at the aggregate level and on the assumption that financial markets are perfect. As argued by Rochet [21], however, models of foreign borrowing with perfect capital markets do not accord with the empirical evidence, as they imply complete consumption smoothing and countercyclical capital flows. Weder [26, pp. 351-6] incorporates imperfect credit markets, as an extension of his main result, by assuming that the world interest rate is an increasing function of the debt/capital ratio but his formulation leads to a four-dimensional dynamical system that is not easily amenable to analytical study.

The rest of the paper is organized as follows. Section 2 presents the open-economy model, derives its dynamics and conditions leading to local indeterminacy, while Section 3 discusses the main macroeconomic implications. Finally, some concluding remarks and directions for future research are collected in Section 4, while appendices gather the main proofs.

## **2 Boom-Bust and Sunspot Episodes in a Debt-Constrained Open Economy**

This section sets up the model, which generalizes Barro *et al.* [4] by endogeneizing the credit multiplier arising from potential debtor default, as in Aghion *et al.* [2]. For simplicity, I abstract from taxes, population growth and technological progress.

## 2.1 The Model with Credit Multiplier

The economy produces a tradeable good  $Y$  by using physical capital  $K$ , human capital  $H$ , and raw labor  $L$ , according to the following technology:

$$Y = AK^\alpha H^\eta L^{1-\alpha-\eta}, \quad (1)$$

where  $A > 0$  is total factor productivity,  $\alpha \geq 0$ ,  $\eta \geq 0$  and  $\alpha + \eta < 1$ .

The Ramsey-type households have preferences represented by:

$$\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \quad (2)$$

where  $C > 0$  is consumption,  $\theta \geq 0$  is the inverse of the elasticity of intertemporal substitution in consumption, and  $\rho \geq 0$  is the discount rate. The representative consumer owns the three inputs and rent them to firms through competitive markets. The budget constraint is:

$$\dot{H} + \dot{K} - \dot{D} = \omega L + (R_K - \delta)K + (R_H - \delta)H - rD - C, \quad (3)$$

where  $\omega$  is the real wage,  $R_i$  is the return on capital (with  $i = K, H$ ),  $\delta \geq 0$  is the depreciation rate for both types of capital,  $D$  is the amount of net foreign debt, and  $r \geq 0$  is the world interest rate that is taken as given by the small open economy. The initial stocks  $K(0) > 0$ ,  $H(0) > 0$ ,  $D(0) > 0$  and the labor endowment  $L^* = 1$  are given to the households. Labor and human capital are not mobile.

Households' decisions follow from maximizing (2) subject to the budget constraint (3), using equation (5), given the initial capital stocks. In particular, standard computations yield the following Euler conditions:

$$\theta \dot{C}/C = R_H - \delta - \rho. \quad (4)$$

As Aghion *et al.* [1] do, one can interpret  $R_H$  - the price of human capital in units of the output (tradeable) good - as the real exchange rate. It is useful to describe what happens in the two following benchmark cases (see Barro *et al.* [4, Sections II and III] for details). If the economy is closed, human capital and physical capital earn the same return, which is endogenously determined by the equality between savings and investment. In this case,

straightforward analysis shows that such a closed economy converges monotonically towards its unique steady state. At the other end of the spectrum, if the economy opens up to capital markets and if there is perfect capital mobility, the economy jumps instantaneously to the steady state. This is happening because the world interest rate is given and equals the returns on both human capital and physical capital, which fixes both capital stocks and output. The important point is that both economies do not experience volatility, which turns out to differ drastically from what happens when capital mobility is limited.

I assume that human capital is immobile and that international borrowing is possible, up to some limit. More precisely, a credit multiplier effect relates the amount borrowed to the agent's wealth, that is,  $D = \lambda K$  where  $\lambda > 0$ . One should think of the latter equality as a debt constraint  $\lambda K \geq D$  that is binding along the transition towards the steady state (I will indeed prove this fact later on, in Proposition 2.3). Barro *et al.* [4] have set  $\lambda = 1$  and shown that the speed of convergence is larger when compared with the case of no borrowing (that is,  $\lambda = 0$ ). In contrast, I follow Cohen and Sachs [8] and let the credit multiplier take any positive values. It is worth noting that, in the analysis of Cohen and Sachs [8] which relies on potential debt repudiation, the debt limit is a linear function of the capital stock only when technology is linear. Although this is not the most general debt policy, I retain the linear borrowing multiplier because it is simpler and consistent with the linearization of the model that is performed below. The analysis could be easily extended to cover the formulation such that  $D = \lambda(K)$  is a non-linear function, as shown by Cohen and Sachs [8] to be generally the case. Not surprisingly, similar results emerge if the function  $\lambda(K)$  is neither too inelastic nor too elastic at steady state. As Barro *et al.* [4] emphasize, one can interpret  $H$  as a productive input that is *not* accepted as collateral for foreign capital, whereas  $K$  is. Therefore, thinking of  $H$  as human capital or, more generally, as any form of capital that is not easily transferable to foreign creditors (like buildings) is useful to outline the story behind the model. However, in section 3.3 where I discuss an extension of the model, it will be useful to interpret  $H$  in the most general way and not strictly as skilled labor.

**Assumption 2.1** *Foreign borrowing is subject to a limit such that  $D = \lambda K$ , with  $\lambda > 0$ .*

Several interpretations may be associated to the credit multiplier  $\lambda$ . For instance, one may define  $1 \geq \sigma \geq 0$  as the fraction of investment that must be self-financed, that is,  $\sigma \equiv K/(K + D) = 1/(1 + \lambda)$ . Then  $1 - \sigma = D/(K + D) = \lambda/(1 + \lambda)$  is the fraction of investment that is borrowed. When  $\lambda = 1$ , as in Barro *et al.* [4, Section IV], the credit constraint is so tight that it limits to 50% the loan-to-investment ratio. As in Cohen and Sachs [8], I allow  $\lambda$  to be larger than one, that is, I consider values such that  $50\% \geq \sigma$  (or  $1 - \sigma \geq 50\%$ ): for one dollar invested, one may borrow more than half of a dollar.

The next task is to discuss what determines the credit multiplier  $\lambda$  when there is potential debtor default, as in Eaton and Gersovitz [9], Cohen and Sachs [8]. The idea is to ease comparison between my results and those of Aghion *et al.* [1] by adopting common microeconomic foundations of the credit multiplier, that are adapted from Aghion *et al.* [2, App. 1]. Appendix B shows that  $\lambda \equiv \tau R / (r(1 - p(r)) - \tau R)$ , where  $R > 0$  is the return on investable funds,  $r > 0$  is the debtor interest rate,  $0 < \tau < 1$  is the borrower's marginal cost of default and  $0 < p < 1$  is the probability of debt recollection chosen by the lender (which is a function of  $r$ ). As in Cohen and Sachs [8],  $\lambda$  increases with  $\tau$  and decreases with  $r$  (as long as  $p(r)$  is not too elastic). Let us further simplify by assuming  $R = r$ , so that  $\lambda = \tau / (1 - p - \tau)$ . Under the condition that  $\tau < 1 - p$ ,  $\lambda$  is positive. The latter inequality states that borrowing is bounded above only if  $\tau / (1 - p)$  is not too large, that is, if credit market imperfections are strong enough. When, in contrast,  $\tau > 1 - p$ , there is no risk of strategic default and no borrowing limit so that one gets back to the case with perfect mobility (and no volatility) described above. In other words, the self-financed fraction of investment is now  $\sigma = 1 / (1 + \lambda) = 1 - \tau / (1 - p)$ , while  $1 - \sigma = \tau / (1 - p)$  represents the loan-investment ratio, which is less than one only if  $\tau < 1 - p$ .

Such a formulation of the credit multiplier aims at capturing the documented evidence that international capital flows are procyclical, as reported by Kaminsky *et al.* [11, Table 8] for most OECD and developing countries over the period 1960-2003. I now show that large values of  $\lambda$  lead to local indeterminacy, hence to booms followed by busts and sunspot

equilibria near the steady state.

Under physical capital mobility, the net return on physical capital  $\alpha Y/K - \delta$  has to equal the world interest rate  $r$ , which yields  $K = \alpha Y/(r + \delta)$  and, therefore, by using equation (1) and the fact that  $L = L^* = 1$  in equilibrium,

$$Y = BH^\varepsilon, \quad (5)$$

where  $B = A^{\frac{1}{1-\alpha}}[\alpha/(r + \delta)]^{\frac{\alpha}{1-\alpha}}$ ,  $\varepsilon = \eta/(1 - \alpha)$ , with  $\alpha + \eta > \varepsilon \geq 0$ . Therefore,  $\varepsilon$  is simply the share of human capital relative to the share of effective labor, that is, human capital and raw labor.<sup>4</sup>

So as to simplify the budget constraint (3), let us rewrite  $Y - \delta K - rD$  as  $\nu Y$ , with  $\nu \equiv 1 - \alpha(\lambda r + \delta)/(r + \delta)$ . Requiring income net of (physical capital) depreciation and interest payments to be positive, i.e.  $\nu > 0$ , therefore imposes the following:

**Assumption 2.2** *The credit multiplier  $\lambda$  is such that  $\lambda < [r + \delta(1 - \alpha)]/(\alpha r) \equiv \bar{\lambda}$ , where  $r > 0$ ,  $\delta \geq 0$ ,  $0 < \alpha < 1$  and  $\bar{\lambda} > 1$ .*

Notice that for  $\delta$  close to zero and  $\alpha = 1/3$ , one has that  $\bar{\lambda} \approx 3$ . Therefore, values such that  $\lambda > 4$  - as considered by Aghion *et al.* [1] and Caballé *et al.* [7] - are *not* feasible in the present model. It is important to keep this in mind when comparing the two types of economies.

One easily gets that  $\dot{H} + \dot{K} - \dot{D} = f(H)\dot{H}$ , where  $f(H) \equiv 1 + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)$ . To derive the latter equality, one uses that  $\dot{D} = \lambda\dot{K}$ ,  $\dot{K} = \dot{Y}\alpha/(r + \delta)$  and, by differentiating (5),  $\dot{Y} = \dot{H}B\varepsilon H^{\varepsilon-1}$ . Collecting all these facts, one therefore gets that equation (3) boils down to:

$$f(H)\dot{H} = \nu BH^\varepsilon - \delta H - C. \quad (6)$$

Moreover, equation (4) now reads:

$$\theta\dot{C}/C = \nu B\varepsilon H^{\varepsilon-1} - \delta - \rho. \quad (7)$$

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<sup>4</sup>It is easily shown that a similar, reduced-form, production function (5) is obtained in the case of a CES technology.

Equations (6)-(7) characterize the dynamics of intertemporal equilibria. It is not difficult to check that the associated transversality constraint is met in the following analysis, as we consider orbits that converge towards the interior steady state: both the human capital stock and the co-state variable converge. Direct inspection of equations (6)-(7) reveals that the dynamics arising in the credit-rationed economy are somewhat similar to the laws of motion describing *closed* economies with capital accumulation and inelastic labor. The major difference is that human (instead of total) capital is here the key state variable. Note that the Euler condition in equation (7) implies that the marginal return on human capital equals  $\rho + \delta$  in steady state, which also equals the marginal return of physical capital when  $r = \rho$ .

## 2.2 Procyclical Credit Flows and the Debt Overhang Regime

In Appendix A, I linearize equations (6)-(7) around the interior steady state  $H^*, C^* > 0$  which is shown to be unique. Straightforward computations yield the following expressions for trace  $Tr$  and determinant  $Det$  of the Jacobian matrix associated with equations (6)-(7):

$$Tr = \rho/f(H^*), \quad Det = \mu/f(H^*), \quad (8)$$

with  $\mu \equiv (\varepsilon - 1)(\delta + \rho)[\rho + \delta(1 - \varepsilon)]/(\theta\varepsilon) < 0$  and  $f(H^*) = 1 + \alpha(1 - \lambda)(\delta + \rho)/[\nu(r + \delta)]$ .

I focus, as most papers in the literature do, on the case for which the steady state is locally either a saddle (such that  $Det < 0$ ) or a sink (such that  $Tr < 0$  and  $Det > 0$ ). In the latter configuration, the steady state is locally indeterminate and one can show that sunspot equilibria exist nearby (see, for instance, Benhabib and Farmer [5] for a survey of closed-economy models).

Direct inspection of equations (8) shows that  $Tr$  and  $Det$  have opposite sign and that  $signTr = signf(H^*)$ . It is not difficult, using  $\nu \equiv 1 - \alpha(\lambda r + \delta)/(r + \delta)$  and  $f(H^*) = 1 + \alpha(1 - \lambda)(\delta + \rho)/[\nu(r + \delta)]$ , to show that  $f(H^*) > 0$  if and only if  $\lambda < \hat{\lambda}$ , where  $\hat{\lambda} \equiv (r + \delta + \alpha\rho)/[\alpha(r + \delta + \rho)] > 1$  and  $\hat{\lambda} < \bar{\lambda}$ . The immediate consequence is that  $Det > 0$  and  $Tr < 0$  when  $\lambda$  is larger than  $\hat{\lambda}$ . The latter configuration will be called the “debt overhang” regime, for reasons that will soon become apparent.

### Proposition 2.1 (Local Indeterminacy and the Credit Multiplier)

Under Assumptions 2.1-2.2, the dynamics of human capital  $H$  and consumption  $C$  are given by equations (6)-(7) and have a unique positive steady state  $H^*, C^* > 0$  which is, locally:

- (i) a saddle (hence determinate) when  $0 < \lambda < \hat{\lambda}$ ,
- (ii) a sink (hence indeterminate) when  $\hat{\lambda} < \lambda < \bar{\lambda}$ ,

where  $\hat{\lambda} \equiv (r + \delta + \alpha\rho)/[\alpha(r + \delta + \rho)] > 1$  is the threshold value of the credit multiplier above (resp. below) which local indeterminacy (resp. local determinacy) prevails.

Therefore, booms followed by busts and sunspot equilibria near steady state occur when  $\hat{\lambda} < \lambda < \bar{\lambda}$  (the “debt overhang regime”; see figure 2). In contrast,  $0 < \lambda < \hat{\lambda}$  ensures saddle-path convergence towards the steady state (see figure 1).

*Proof:* See Appendix A.

Note that an important corollary follows from Proposition 2.1. In contrast with Aghion *et al.* [1], our economy does *not* have closed orbits (that is, deterministic cycles) near steady state in the debt overhang regime. An easy way to show this is to use the Bendixson criterion, which applies here to the neighborhood of the steady state: from the proof contained in Appendix A, one gets that the trace of the Jacobian matrix is not identically zero and does not change sign near steady state. In addition, in case (ii) of Proposition 2.1, one can use the techniques developed in Shigoka [23] to construct sunspot equilibria near the indeterminate steady state.

I now restate Proposition 2.1 in terms of the micro-parameters that underlie the credit multiplier. One can arguably relate the level of financial integration to the borrowing-investment ratio  $1 - \sigma = \tau/(1 - p)$ . The interpretation is straightforward: the larger either the borrower’s penalty for default  $\tau$  or the lender’s probability of debt collection  $p$ , the more generous the loan market - that is, the larger the credit multiplier  $\lambda$ . When  $\tau/(1 - p)$  increases from zero to one, the debt limit goes up from zero to infinity. Alternatively, one can interpret  $\sigma = 1 - \tau/(1 - p)$  as a measure of credit market imperfections. When  $1 - \tau/(1 - p)$  increases from zero to one, the debt limit goes down from infinity to zero.

Proposition 2.2 states that the economy is vulnerable, in the debt overhang regime, to boom-bust and sunspot episodes whenever  $\tau/(1-p)$  is sufficiently (but not too) high.

**Proposition 2.2 (Local Indeterminacy and Capital Market Imperfections)**

*Under the assumptions of Proposition 2.1, the small open economy is not credit-constrained and it jumps instantaneously to the steady state when  $\tau/(1-p) > 1$ , where  $0 < \tau < 1$  is the borrower's marginal cost of default and  $0 < p < 1$  is the probability of debt recollection chosen by the lender. Borrowing with potential debtor default is not feasible in the small open economy if  $\bar{\lambda}/(1+\bar{\lambda}) < \tau/(1-p) < 1$ . In addition, the steady state is locally:*

- (i) *a saddle (hence determinate) when  $0 < \tau/(1-p) < \hat{\lambda}/(1+\hat{\lambda})$ ,*
- (ii) *a sink (hence indeterminate) when  $\hat{\lambda}/(1+\hat{\lambda}) < \tau/(1-p) < \bar{\lambda}/(1+\bar{\lambda})$ .*

*It follows that the debt overhang regime - associated with boom-bust and sunspot episodes - occurs when either the debtor's defaulting cost or the lender's probability of debt collection is neither too small nor too large.*

*Proof:* See Appendix B.

In order to compare with the predictions of Aghion *et al.* [1], I now consider some numerical examples (see table 1). As a benchmark, Barro *et al.* [4] set  $\lambda = 1$ , hence  $\tau/(1-p) = 0.5$ . Setting  $r = \rho$  to any value,  $\delta = 0$ , and  $\alpha = 0.4$ , one gets from Proposition 2.1 that  $\hat{\lambda} = 1.75 < \bar{\lambda} = 2.5$ . From Proposition 2.2, one immediately gets that local indeterminacy prevails when  $0.6 < \tau/(1-p) < 0.7$ . On the other hand, Aghion *et al.* [1, p. 1090] fix  $\lambda$  at 4, so that periodic cycles occur when  $\tau/(1-p) = 0.8$ . In the formulation of Caballé *et al.* [7, p. 1267] with a Cobb-Douglas technology, it follows that  $\lambda > 10$ , or  $\tau/(1-p) > 0.9$ , is necessary for cycles to occur. To put it differently, indeterminacy, boom-bust and sunspot episodes occur when the self-financed fraction of investment is lower than 40%, whereas cycles occur in the Aghion *et al.* [1] economy when this ratio equals 20% (is lower than 10% in Caballé *et al.* [7]).

In summary, one may learn from this exercise that the debt overhang regime occurs at low values of financial development, for which cycles are typically ruled out in Aghion *et*

*al.* [1]. More precisely, indeterminacy relies either on a lower penalty of default imposed on borrowers or on a lower probability of debt collection, in comparison with the cycles in Aghion *et al.* [1].

Insert Table 1 here.

The above examples, that are summarized in table 1, illustrate how the two types of results differ. My predictions are that booms and busts occur at rather low values of the credit multiplier (that is, with international credit markets that are strongly imperfect). In contrast, for such low values, endogenous cycles are typically ruled out in the analysis of Aghion *et al.* [1] that shows how low (or large) levels of financial integration stabilize the economy. The numerical examples derived above suggest that open economies that are financially constrained are subject to boom-bust and sunspot episodes for values of the loan-investment ratio  $\tau/(1-p)$  that are much lower and, in fact, incompatible with the occurrence of cycles (see table 1). Conversely, the large values of the credit multiplier considered by Aghion *et al.* [1] are simply not feasible in the present paper.

### 2.3 Interpreting the Results: the Debt Overhang Regime

To get some further understanding of how the credit multiplier affects the dynamics around steady state, it is useful to construct the phase diagram arising from the dynamical system associated with equations (6)-(7). In Appendix C, I show that three cases occur, depending on the value of  $\lambda$ . When  $1 \geq \lambda > 0$  (case (i) in Proposition 2.1), the phase diagram is the same as that occurring in the standard Ramsey (closed-economy) model in which the steady state is (globally) a saddle point and has a stable manifold going through the origin. This is consistent with the intuition that an economy borrowing only a little behaves as a closed economy, in the sense that the dynamics are qualitatively equivalent. In other words, the phase diagram is almost as in figure 1, except that the vertical line of  $H = \hat{H}$  should be translated to the left and made to coincide with the  $C$ -axis.

A similar configuration occurs when  $1 < \lambda < \hat{\lambda}$  (case (i) in Proposition 2.1), in figure 1. The path followed by the credit-constrained economy, that has now access to more generous credit markets, is then altered in the following way. When the initial human capital stock is such that  $H(0) > \hat{H}$ , where  $\hat{H}$  is now positive, the economy converges in a monotonic way, along the stable manifold, to the steady state. Paths such that  $H(0) < \hat{H}$  are not feasible, as they imply unbounded consumption growth. Finally, the dynamics differ dramatically when  $\hat{\lambda} < \lambda < \bar{\lambda}$  (case (ii) in Proposition 2.1), as one sees in figure 2. Paths such that  $H(0) > \hat{H}$  are not feasible, while orbits starting at  $0 < H(0) < \hat{H}$  converge to the steady state that is asymptotically stable (that is, a sink), hence locally indeterminate.

The following intuitive account may shed some light on the mechanisms at work in both regimes of Proposition 2.1. I should stress that non-monotonic convergence in the debt overhang regime does not involve movements in expectations, as it will soon become clear. Not surprisingly, the usual saddle-path property applies to case (i) with low borrowing. Suppose that the economy has initial conditions located on that part of the stable manifold which converges to the steady state from below (see point *B* in figure 1). The initial human capital stock being low, its marginal return is large and savings (net of interest payments) are positive, which allows both consumption and human capital to grow and converge towards the saddle steady state. Different dynamics arise under large borrowing (debt overhang regime; case (ii)), when the economy starts at point *B* in figure 2. Here again, net savings are initially positive and consumption grows over time. However, *savings are now used to cut down foreign debt, which crowds out both physical and human capital accumulation*. In other words, there is debt overhang. When net savings are positive (resp. negative), capital investment is negative (resp. positive). Consumption growth decreases savings, which then turn negative so that capital investment financed by foreign debt starts off again. When human capital reaches its steady state level, consumption is too high so that it starts decreasing until savings become positive again. Human capital then decreases; consumption continues to go down until it reaches a minimum and then starts to increase,

leading the economy at the outset of a new cycle. In short, booms and busts happen in that regime. The preceding account should have made clear that no expectations are involved in the non-monotonic dynamics that arise in the debt overhang regime. However, although such a cyclical pattern dies out in the absence of shocks, stationary sunspot equilibria involving self-fulfilling movements of expectations do exist near steady state. Therefore, the main lesson one may draw from the analysis is that an economy which is saddle-point stable at low levels of financial integration is, in sharp contrast, vulnerable to boom-bust and sunspot episodes at levels of the borrowing-investment ratio beyond a threshold value. For instance, the model predicts that lending booms may, if they increase the credit multiplier beyond the critical value, give birth to macroeconomic volatility. This is consistent with observed lending booms preceding financial crashes (see, e.g., Tornell and Westermann [24]) and debt overhang (Imbs and Ranciere [12]).

The last task one faces is to show that the debt constraint  $\lambda K \geq D$  is actually binding along the transition to the steady state in both cases pictured in figures 1 and 2. The next statement proves that this is true over the relevant range of parameters, that is, when  $\lambda > 1$ .

**Proposition 2.3 (Binding Debt Constraint)**

*Under the assumptions of Proposition 2.1, the debt constraint is binding - that is,  $D = \lambda K$  - along the transition to the steady state whenever  $\lambda > 1$ .*

*Proof:* See Appendix D.

## 3 Macroeconomic Implications

### 3.1 Welfare Implications of Financial Integration

In this section, it is shown that the indeterminacy regime (case (ii) of Proposition 2.1) is not only associated with aggregate volatility, but it is also characterized by lower welfare. Therefore, an important conclusion of my analysis is that financial integration pushes the economy into low-welfare equilibria if  $\lambda > \hat{\lambda}$ . More precisely, the following proposition establishes that the steady state consumption and capital allocations are decreasing functions of the credit multiplier  $\lambda$ . Therefore, *equilibria in the debt overhang regime are converging to a lower-welfare steady state in a non-monotonic fashion.*

#### Proposition 3.1 (Welfare)

*Under the assumptions of Proposition 2.1, both  $H^*$  and  $C^*$  are decreasing function of the credit multiplier  $\lambda$ . Consider local equilibria that are located in a neighborhood of, and converge towards, the steady state  $H^*, C^*$ . Then local equilibria that are associated with  $\lambda > \hat{\lambda}$  (the debt overhang regime) have lower welfare than local equilibria such that  $1 < \lambda < \hat{\lambda}$ .*

*Proof:* In Appendix A, equation (9) delivers that  $H^*$  is a decreasing function of  $\lambda$ . Direct inspection of equation (9), that is,  $\nu B \varepsilon (H^*)^{\varepsilon-1} = \rho + \delta$ , shows that the larger  $\lambda$ , the lower  $\nu \equiv 1 - \alpha(\lambda r + \delta)/(r + \delta)$ , and, therefore, the lower  $H^*$ . Moreover, from the latter observation and equation (10), that is,  $C^* = H^*[\rho + \delta(1 - \varepsilon)]/\varepsilon$ , one concludes that steady-state consumption  $C^*$  is also a decreasing function of  $\lambda$ . Therefore, case (ii) of Proposition 2.1 is associated with lower consumption at steady state, in comparison with case (i). In addition, while convergence towards the steady state is monotonic in case (i), it is not in case (ii). Because of concavity in utility, therefore, there is a second-order loss that is due to the fact that paths converge with oscillations in case (ii), that is, when  $\lambda > \hat{\lambda}$ . This proves that equilibria converging to the steady state have lower welfare in case (ii) than in case

(i).

□

According to Proposition 3.1, one should regard figures 1 and 2 as pictured at different scales: in figure 1, the steady state levels of capital  $H^*$  and consumption  $C^*$  (associated with small  $\lambda$ 's) are larger than the corresponding values in figure 2 (with large  $\lambda$ 's). On the contrary,  $\hat{H}$  in figure 1 is smaller than the corresponding level in figure 2. Finally, note that comparing allocations with different  $\lambda$ 's such that  $1 < \lambda < \hat{\lambda}$  delivers an ambiguous conclusion regarding welfare: with a higher  $\lambda$ , the economy converges more rapidly towards a steady state with lower consumption.

### 3.2 Amplification and Persistence of World Interest Rate Shocks

The knowledge of the phase diagram corresponding to each regime of equations (6)-(7) may be fruitfully applied to understand how specific shocks affect open economies. Of some importance are variations of the world interest rate. For instance, Uribe and Yue [25] document some evidence, based on VAR estimates, that innovations in the US interest rate explains about 20% of movements in aggregate activity at business-cycle frequency, in a sample of emerging economies. Drawing from this observation, let us now focus on a permanent increase in the world interest rate  $r$  and show that its impact is both amplified and persistent in the debt overhang regime (figure 2), therefore exacerbating volatility, in comparison with what happens in the saddle regime (figure 1). Under the usual assumption that sets  $r = \rho$ , one can check, by manipulating the steady state expressions derived in Appendix A, that the stationary solution with a larger  $r$  has lower  $H^*$  and  $C^*$  (provided that  $\delta$  is small enough): steady state consumption, human capital, and output are lower when the interest rate is larger. In other words, if the economy was at steady state prior to the interest rate increase, it will be located north-east of the steady state after  $r$  has gone up. For example, let point  $A$  in figures 1 and 2 represent such an ex-post situation following the interest rate hike. Figure 1 suggests that the impact of the initial shock to  $r$  dies out

without generating much volatility when the economy is in the saddle regime (when  $\lambda$ , or  $\tau/(1-p)$  for that matter, is small enough): consumption and output converge monotonically towards their new steady state values. Moreover, it turns out that convergence is more rapid the larger the credit multiplier, hence the adjustment is faster the closer  $\lambda$  is to  $\hat{\lambda}$ .

Insert Figure 3 here.

The adjustment process differs in the debt overhang regime depicted in figure 3 (when  $\lambda$  is large enough). As shown in figure 3, the effect of the interest rate shock on output is more persistent (in the sense that its half-life is longer) and may be amplified in the indeterminacy regime, in comparison with what happens in the saddle regime. In particular, output and consumption occasionally stay below their new steady state levels, therefore displaying some amplification (or overshooting) effect and hump-shaped impulse response functions. Moreover, output may increase at impact following the interest rate hike, if the economy starts at point A in figure 2. This is likely to be the case if the shock occurs after agents have committed to a consumption plan. Output goes up because, as explained in section 2.3, foreign borrowing finances capital accumulation when savings are negative. If, as one expects, consumption goes down at the time the interest rate increases, then the economy starts, say, at point  $A'$  in figure 2 and output falls initially, as depicted in figure 3.<sup>5</sup>

It is also worth observing that the threshold value  $\hat{\lambda}$  is, when  $r = \rho$ , a decreasing function of  $r$ . Therefore, it is possible that an increase of the interest rate involves a switch from the saddle regime to the indeterminacy regime. Not only a world interest rate hike may have amplified and persistent effects, but it may also change the stability properties of the steady state in the credit-constrained economy. Setting here again  $r = \rho$  and assuming that  $\lambda$  is inelastic with respect to  $r$ , such a change in stability happens when  $r$  goes, from below, through the critical value  $\delta(1 - \alpha\lambda)/(2\alpha\lambda - 1 - \alpha)$ , which is likely to be small for small  $\delta$ 's.

---

<sup>5</sup>The configuration appearing in figure 3 also represents the qualitative effect of a decrease in  $A$  - that is, a negative shock to total factor productivity.

### 3.3 Extending the Model: International and Domestic Collateral

From Section 2.2, one learns that local indeterminacy occurs for values of the credit multiplier that are larger than a threshold level. In that basic version of the model, this turns out to happen for large values of the debt-to-GDP ratio. More precisely, the latter fraction, given by  $D/Y = \alpha\lambda/(r + \delta)$  is bound to be large when  $\lambda > 1$  and when parameter values are calibrated using quarterly data, which implies that both  $r$  and  $\delta$  are close to zero. For example, suppose we follow Barro *et al.* [4, p. 107] and set  $\alpha = 0.3$ ,  $r = \rho = 0.02$ ,  $\delta = 0.05$ . Then indeterminacy occurs when  $\lambda > \hat{\lambda} \approx 2.8$ , that is, when  $D/Y > 12$ . Such values for the ratio of foreign debt to GDP are hardly realistic. Obviously, the debt-to-GDP ratio has dimension  $t^{-1}$  with respect to time so it is expected to be larger the lower the length of the period. In addition, it may well be that in a discrete-time (say, quarterly) version of the very same model, the debt overhang regime occurs at lower values. Instead of checking that, which is left for future research, I now take a first step towards amending the model in a realistic way. The extension is simple and therefore mainly suggestive. More precisely, I now show that incorporating *domestic* collateral delivers a lower (and plausible) threshold level for  $\lambda$ . Most importantly, such an extension allows the debt overhang regime to be compatible with values of the debt-to-GDP ratio that falls within the range of estimates. Suppose we relax Assumption 2.1 by allowing the second type of capital,  $H$ , to be accepted as collateral only for domestic borrowing. That is, Assumption 2.1 is replaced by:

**Assumption 3.1** *Borrowing is subject to a limit such that  $D = \lambda K + \phi H$ , with  $\lambda, \phi > 0$ .*

The idea is that foreign debt is still limited, as before, to  $\lambda K$ . In other words,  $K$  is accepted as collateral for foreign borrowing. In addition, domestic lending is now available, up to  $\phi H$ . Of course, one should think of  $H$  as a type of capital that is immobile and that serves as financial guarantee for domestic borrowing (like buildings), because it is not easily transferable to foreign creditors. In that interpretation,  $H$  should not be thought of as human capital, for obvious reasons. In Appendix E, I show that the previous analysis is virtually unchanged, with the exception that the threshold level now depends on  $\phi$ , as

shown in the following statement that is the analog of Proposition 2.1.

**Proposition 3.2 (Local Indeterminacy with International and Domestic Collateral)**

*Under Assumptions 2.2-3.1, the dynamics of human capital  $H$  and consumption  $C$  have a unique positive steady state  $H^*, C^* > 0$  which is, locally:*

- (i) *a saddle (hence determinate) when  $0 < \lambda < \hat{\lambda}$ ,*
- (ii) *a sink (hence indeterminate) when  $\hat{\lambda} < \lambda < \bar{\lambda}$ ,*

*where, under the assumption that  $\phi < (r + \delta + \alpha\rho)/[(1 - \alpha)(\delta + r)]$ ,  $\hat{\lambda} \equiv [r + \delta + \alpha\rho - \phi(1 - \alpha)(\delta + r)]/[\alpha(r + \delta + \rho)] > 0$  is the threshold value of the credit multiplier above which the debt overhang regime prevails.*

*Proof:* See Appendix E.

One important implication of Proposition 3.2 is that local indeterminacy occurs for *small* values of the *foreign* debt/GDP ratio, provided that the domestic credit multiplier is large enough. To see this, denote  $D_f = \lambda K$  as the level of foreign borrowing. Then adopting the same parameter values as those at the outset of this section and setting  $\lambda = \hat{\lambda}$ , one finds that  $D_f/Y \approx 50\%$  provided that  $\phi \approx 1.5$ . Note that in this example, one has  $\lambda + \phi \approx 1.6$  so that the “total” credit multiplier equals 60% of the corresponding level without domestic collateral where  $\lambda$  has to be larger than 2.8 for indeterminacy to prevail. Alternatively,  $D_f/Y = 1$  when  $\phi \approx 1.4$  and  $\lambda + \phi \approx 1.6$ . Moreover, note that a trade-off exists in the sense that indeterminacy with low values of  $\lambda$  requires large  $\phi$ 's.

In consequence, the corresponding value of the external debt-to-GDP ratio falls within the range of estimates of, for instance, Lane and Milesi-Ferreti [14]. Figure 8 in their paper reveals that most low-income, indebted countries cluster around a value of the debt-to-GDP ratio that is about one-half. Therefore, the key result of this section is that indeterminacy is compatible with low values of the foreign credit multiplier, provided that domestic borrowing implies a large leverage effect. Therefore, such an extension of the analysis suggest that boom-bust and sunspot episodes obtain for values of the fraction

of foreign debt in GDP that are not unrealistic. Although such an extension is highly stylized, it suggests that high domestic financial development may create a new channel that is detrimental to macroeconomic stability when the economy opens up to capital flows. This is because a large domestic credit multiplier implies a *lower* threshold value of the *international* debt multiplier above which the economy is vulnerable to the debt overhang regime. In other words, an economy with a highly developed domestic credit market is more likely to be susceptible to boom-bust and sunspot episodes at lower levels of international financial integration. Of course, one should observe that such a conclusion depends on risk sharing being ruled out in the present deterministic model.

## 4 Conclusion

This paper has studied a simple open-economy growth model with endogenous constraints on international borrowing arising from potential debtor default. It has been shown that local indeterminacy, booms followed by busts, and sunspot equilibria drive the dynamics when the credit multiplier is sufficiently larger than in the benchmark case considered by Barro *et al.* [4]. It follows that opening the capital account may push a saddle-point stable economy on a low-welfare, volatile path. In particular, the model predicts that lending booms may, if they increase the credit multiplier beyond a critical value, give birth to macroeconomic volatility. This may occur at rather low levels of financial development and for values of the debt-to-GDP ratio that fall within the range of available estimates. The main mechanism through which boom-bust and sunspot episodes occur is the presence of a crowding-out effect of foreign credit. In the indeterminacy regime, there is debt overhang: when highly leveraged, the economy uses savings to cut down foreign debt, at the expense of both human and physical investment. Such a framework is not inconsistent with recent empirical evidence on procyclical net capital inflows, debt overhang, borrowing constraints and debt-to-GDP ratios, positive credit-investment correlation, and the impact of world interest rate movements in emerging economies.

Relevant extensions of the analysis would be to take into account uncertainty, foreign direct investment, and policy designed to mitigate volatility at large levels of financial integration. In addition, a more detailed model with domestic and international collateral is called for. Finally, currency mismatch and systemic bail out guarantees stand as important features to be incorporated in the modeled economy. In an amended version of the analysis with endogenous interest rate, it is expected that the mechanisms of this paper would be reinforced: during booms, the interest rate would go down, which would increase the credit multiplier. In such a framework, shocks to the world interest rate could trigger regime switching. It remains to be seen whether such a more realistic model is able to account, in a satisfactory way, for the business-cycle facts observed in emerging countries. To address such an issue, one would have to write down the discrete-time analog of the model and ask whether or not the debt overhang regime occurs for lower values of the credit multiplier at high (say, quarterly) frequency. Finally, although the analysis is easily modified to account for exogenous growth, it would be interesting to introduce endogenous growth and investigate the conditions under which volatility reduces growth, as some evidence suggests it is the case.

## A Proof of Proposition 2.1

In this appendix, I derive and linearize, around the steady state, the dynamical system describing intertemporal equilibria which consists of equations (6)-(7). It is straightforward to show that, under our assumptions, the differential equations (6)-(7) possess a unique interior steady state  $H^*, C^* > 0$ . More precisely,  $\dot{H} = 0$  yields, from equation (6):

$$\nu B \varepsilon (H^*)^{\varepsilon-1} = \rho + \delta. \quad (9)$$

The latter equality delivers, under Assumption 2.2, a unique steady state value for the human capital stock  $H^* > 0$ . Moreover, equation (6) gives  $C^* = H^*(\nu B (H^*)^{\varepsilon-1} - \delta)$ , that is, using (9):

$$C^* = H^*[\rho + \delta(1 - \varepsilon)]/\varepsilon, \quad (10)$$

so that  $C^* > 0$ .

Now rewrite (6)-(7) as:

$$\begin{cases} \dot{H} &= (\nu BH^\varepsilon - \delta H - C)/f(H), \\ \dot{C} &= C(\nu\varepsilon BH^{\varepsilon-1} - \delta - \rho)/\theta, \end{cases} \quad (11)$$

with  $f(H) \equiv 1 + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)$ . Straightforward computations lead to the following expressions of the Jacobian matrix associated with equations (11), evaluated at the steady state  $H^*, C^* > 0$ :

$$J = \begin{pmatrix} \rho/f(H^*) & -1/f(H^*) \\ (\varepsilon - 1)(\delta + \rho)[\rho + \delta(1 - \varepsilon)]/(\theta\varepsilon) & 0 \end{pmatrix}, \quad (12)$$

where  $f(H^*) = 1 + \alpha(1 - \lambda)(\delta + \rho)/[\nu(r + \delta)]$  and  $\nu \equiv 1 - \alpha(\lambda r + \delta)/(r + \delta)$ . It is easy to get, from (12), the expressions of  $Tr$  and  $Det$ , respectively the trace and determinant of the Jacobian matrix associated with equations (11), evaluated at the steady state  $H^*, C^* > 0$ :

$$\begin{aligned} Tr &= \rho/f(H^*), \\ Det &= \mu/f(H^*), \end{aligned} \quad (13)$$

with  $\mu \equiv (\varepsilon - 1)(\delta + \rho)[\rho + \delta(1 - \varepsilon)]/(\theta\varepsilon) < 0$  and  $f(H^*) = 1 + \alpha(1 - \lambda)(\delta + \rho)/[r + \delta - \alpha(\lambda r + \delta)]$ .

Therefore, one has that  $signTr = -signDet$ , with  $signTr = signf(H^*)$ . Straightforward computations show that  $f(H^*) > 0$  if and only if  $\lambda < \hat{\lambda}$ , with  $\hat{\lambda} \equiv (r + \delta + \alpha\rho)/[\alpha(r + \delta + \rho)] > 1$ . This gives us that the steady state is a saddle (with  $Det < 0$  and  $Tr > 0$ ) if and only if  $\lambda < \hat{\lambda}$ , while the steady state is a sink (with  $Det > 0$  and  $Tr < 0$ ) if and only if  $\lambda > \hat{\lambda}$ .

In both cases, the transversality condition holds, as  $\lim_{t \rightarrow +\infty} e^{-\rho t} H(t)(C(t))^{-\theta} = 0$  when  $H(t)$  and  $C(t)$  converge respectively to  $H^*$  and  $C^*$ .  $\square$

## B Microeconomic Foundations of the Credit Multiplier: Proof of Proposition 2.2

The purpose of this appendix is to adapt the formulation with ex-post moral hazard and costly-state verification proposed by Aghion *et al.* [2, App. 1] so as to give some

microeconomic foundations to the credit multiplier formulation such that  $\lambda = D/K$ . The borrower has wealth  $K$  and he requests a loan  $D$  that he will pay back if and only if:

$$R(K + D) - rD \geq (1 - \tau)R(K + D) - prD, \quad (14)$$

where  $R > 0$  is the return on investable funds,  $r > 0$  is the debtor interest rate,  $0 < \tau < 1$  is the borrower's marginal cost of default and  $0 < p < 1$  is the probability of debt recollection that is chosen by the lender. Equation (14) may be rewritten as:

$$K \geq D(r(1 - p) - \tau R)/(\tau R). \quad (15)$$

On the lender's side, the effort cost of recollecting debt is a function  $\pi(p)$ , with  $\pi'(p) > 0$  and  $\pi''(p) \geq 0$  for all  $0 < p < 1$ . Therefore, the lender will set  $p$  so that his expected profits  $[prD - D\pi(p)]$  are maximized. Such a value, say  $\bar{p}$ , solves  $r = \pi'(\bar{p})$ , or equivalently,  $\bar{p} = p(r)$ , where  $p(r) \equiv (\pi')^{-1}(r)$  is well defined under the usual boundary conditions (e.g. Inada conditions).

Assuming that the creditor borrows as much as he can, one gets from equation (15) that:

$$K = D(r(1 - p(r)) - \tau R)/(\tau R) \text{ or } D/K = \lambda \equiv \tau R/(r(1 - p(r)) - \tau R). \quad (16)$$

Assuming that  $r/R > \tau/(1 - p(r))$  (which is achieved by a small enough  $\tau$ , the cost of stalling away), one gets that  $\lambda > \hat{\lambda}$  only if  $\tau/(1 - p(r)) > r\hat{\lambda}/[R(1 + \hat{\lambda})]$ . When  $r = R$ , the latter inequality simplifies to  $\tau/(1 - p(r)) > \hat{\lambda}/(1 + \hat{\lambda})$ , as stated in Proposition 2.2. Assumption 2.2 further imposes that  $\tau/(1 - p(r) - \tau) < \bar{\lambda} \equiv [r + \delta(1 - \alpha)]/(\alpha r)$  or, equivalently, that  $\tau/(1 - p(r)) < \bar{\lambda}/(1 + \bar{\lambda})$ .

In equation (16), it is assumed that  $r(1 - p(r)) > \tau R$ . If, in contrast,  $r(1 - p(r)) < \tau R$ , then equation (14) always holds, that is, there is repayment whatever is the level of debt. This happens if, for instance, either  $p$  is close to one or  $\tau R/r$  is close to one. It follows that borrowing is then unconstrained, because either the probability of debt collection is large or the cost of default is large. On the contrary, the condition that  $r(1 - p(r)) > \tau R$  leads to the case we focus on, for which debt is constrained as described in equation (15).  $\square$

## C Constructing Phase Diagrams

The aim of this section is to construct the phase diagram arising from the dynamical system associated with equations (6)-(7). The sign of  $f(H) \equiv 1 + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)$  plays here a critical role and several cases have to be considered, depending on the value of  $\lambda$ . The most simple configuration occurs when  $1 \geq \lambda > 0$ . In that case,  $f(H) > 0$  for all  $H > 0$  and one gets back to the standard phase diagram of the Ramsey model involving a unique saddle-point stable steady state. When  $\lambda > 1$ , two new diagrams emerge, each related to the conditions in Proposition 2.1, as we now show.

The first step is to observe that, when  $\lambda > 1$ ,  $f(H) > 0$  if and only if one has that  $H > [B\varepsilon\alpha(\lambda - 1)/(r + \delta)]^{1-\varepsilon} \equiv \hat{H}$ . It follows from equation (7) that  $\dot{H} > 0$  when either  $C < \nu BH^\varepsilon - \delta H$  and  $H > \hat{H}$  or  $C > \nu BH^\varepsilon - \delta H$  and  $H < \hat{H}$ , while  $\dot{H} < 0$  when either  $C < \nu BH^\varepsilon - \delta H$  and  $H < \hat{H}$  or  $C > \nu BH^\varepsilon - \delta H$  and  $H > \hat{H}$ . Moreover, one has that  $\hat{H} < H^*$  if and only if  $\lambda < \hat{\lambda}$ . Finally, equation (6) shows that  $\dot{C} > 0$  if and only if  $H < H^*$ .

Collecting all these facts, one can draw the phase diagrams that appear in figures 1 and 2, corresponding to cases (i) (with the additional assumption that  $\lambda > 1$ ) and (ii) of Proposition 2.1, respectively.

## D Binding Debt Constraint: Proof of Proposition 2.3

In this appendix, I show that the debt constraint  $\lambda K \geq D$  is binding when  $\lambda > 1$ , along the transition to the steady state. As emphasized by Barro *et al.* [4, p. 110],  $\lambda K \geq D$  is binding if and only if the net wealth is lower than the steady state level of human capital, that is, when  $H + K - D < H^*$ : when its equity is low, the economy has then incentives to borrow as much as possible in order to accelerate convergence towards the steady state. As  $H + K - D < H^*$  implies that  $D = \lambda K$ , the former inequality can be rewritten as  $H + (1 - \lambda)K < H^*$  or, using  $K = \alpha Y/(r + \delta)$  and equation (5), as  $H(1 + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)) < H^*$ . In summary, the debt constraint  $\lambda K \geq D$  is binding

if and only if the following inequality is satisfied:

$$H(1 + (1 - \lambda)\alpha BH^{\varepsilon-1}/(r + \delta)) < H^*. \quad (17)$$

Then it is easy to see that two cases have to be considered. When  $1 < \lambda < \hat{\lambda}$ , then condition (17) is satisfied for all  $H < H^*$ . Therefore, the debt constraint is binding whenever the economy is initially poor enough in capital and follows the saddle path, as depicted in figure 1. On the other hand, consider now that  $\lambda > \hat{\lambda}$  (see figure 2). Define  $g(H) \equiv 1 + (1 - \lambda)\alpha BH^{\varepsilon-1}/(r + \delta)$ . Remembering that  $f(H) \equiv 1 + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)$ , one has that  $g(H) < f(H)$  for all  $H > 0$  under the assumption that  $\varepsilon < 1$ . One knows from Appendix C that  $f(H) < 0$  when  $H < \hat{H}$ . Therefore,  $H < \hat{H}$  implies that  $g(H) < 0$  and, consequently, that equation (17) is met. Along the transition to the steady state, the debt constraint is binding in that case too.  $\square$

## E Proof of Proposition 3.2

In this appendix, I derive and linearize, around the steady state, the dynamical system describing intertemporal equilibria when the model incorporates domestic collateral. Equations (6)-(7) are replaced by:

$$\begin{cases} \dot{H} &= (\nu BH^\varepsilon - (\delta + r\phi)H - C)/f(H), \\ \dot{C} &= C(\nu\varepsilon BH^{\varepsilon-1} - (\delta + r\phi) - \rho)/\theta, \end{cases} \quad (18)$$

with  $f(H) \equiv 1 - \phi + (1 - \lambda)\alpha B\varepsilon H^{\varepsilon-1}/(r + \delta)$ . It is straightforward to show that, under our assumptions, the differential equations (18) have a unique interior steady state  $H^*, C^* > 0$ . More precisely,  $\dot{H} = 0$  yields, from the first equation in (18):

$$\nu B\varepsilon(H^*)^{\varepsilon-1} = \delta + r\phi + \rho. \quad (19)$$

The latter equality delivers, under Assumption 2.2, a unique steady state  $H^* > 0$ . Moreover, the second equation in (18) gives  $C^* = H^*(\nu B(H^*)^{\varepsilon-1} - \delta - r\phi)$ , that is, using (19):

$$C^* = H^*[\rho + (\delta + r\phi)(1 - \varepsilon)]/\varepsilon, \quad (20)$$

so that  $C^* > 0$ .

Straightforward computations lead to the following expressions of the Jacobian matrix associated with equations (18), evaluated at the steady state  $H^*, C^* > 0$ :

$$J = \begin{pmatrix} \rho/f(H^*) & -1/f(H^*) \\ (\varepsilon - 1)(\delta + r\phi + \rho)[\rho + (\delta + r\phi)(1 - \varepsilon)]/(\theta\varepsilon) & 0 \end{pmatrix}, \quad (21)$$

where  $f(H^*) = 1 - \phi + \alpha(1 - \lambda)(\delta + \rho + r\phi)/[\nu(r + \delta)]$  and  $\nu \equiv 1 - \alpha(\lambda r + \delta)/(r + \delta)$ . It is easy to get, from (21), the expressions of  $Tr$  and  $Det$ , respectively the trace and determinant of the Jacobian matrix associated with equations (18), evaluated at the steady state  $H^*, C^* > 0$ :

$$\begin{aligned} Tr &= \rho/f(H^*), \\ Det &= \mu/f(H^*), \end{aligned} \quad (22)$$

with  $\mu \equiv (\varepsilon - 1)(\delta + \rho + r\phi)[\rho + (\delta + r\phi)(1 - \varepsilon)]/(\theta\varepsilon) < 0$  and  $f(H^*) = 1 - \phi + \alpha(1 - \lambda)(\delta + \rho + r\phi)/[r + \delta - \alpha(\lambda r + \delta)]$ . Therefore, one has that  $signTr = -signDet$ , with  $signTr = signf(H^*)$ . Straightforward computations show that  $f(H^*) > 0$  if and only if  $\lambda < \hat{\lambda}$ , with  $\hat{\lambda} \equiv [r + \delta + \alpha\rho - \phi(1 - \alpha)(\delta + r)]/[\alpha(r + \delta + \rho)] > 0$  if  $\phi < (r + \delta + \alpha\rho)/[(1 - \alpha)(\delta + r)]$ . This proves that the steady state is a saddle (with  $Det < 0$  and  $Tr > 0$ ) if and only if  $\lambda < \hat{\lambda}$ , while the steady state is a sink (with  $Det > 0$  and  $Tr < 0$ ) if and only if  $\lambda > \hat{\lambda}$ .

In both cases, the transversality condition holds, as  $\lim_{t \rightarrow +\infty} e^{-\rho t} H(t)(C(t))^{-\theta} = 0$  when  $H(t)$  and  $C(t)$  converge respectively to  $H^*$  and  $C^*$ .  $\square$

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$\tau/(1-p)$	$(0, \hat{\lambda}/(1+\hat{\lambda}))^*$	$(\hat{\lambda}/(1+\hat{\lambda}), (\bar{\lambda}/(1+\bar{\lambda})))$	$(\bar{\lambda}/(1+\bar{\lambda}), 1)^{**}$	$(1, +\infty)$
<b>Stability of steady state</b>	saddle* (monotonic convergence)	sink (boom-bust and sunspot dynamics)	<i>not possible**</i>	economy jumps at steady state
<b>Foreign borrowing</b>	constrained*	constrained	<i>not possible**</i>	unconstrained

Table 1: Stability of steady state depending on the loan-investment ratio  $\tau/(1-p)$ .

\* Barro *et al.* (1995) assume parameter values that belong to this range.

\*\* Range of parameter values for which cycles occur in Aghion *et al.* (2004).

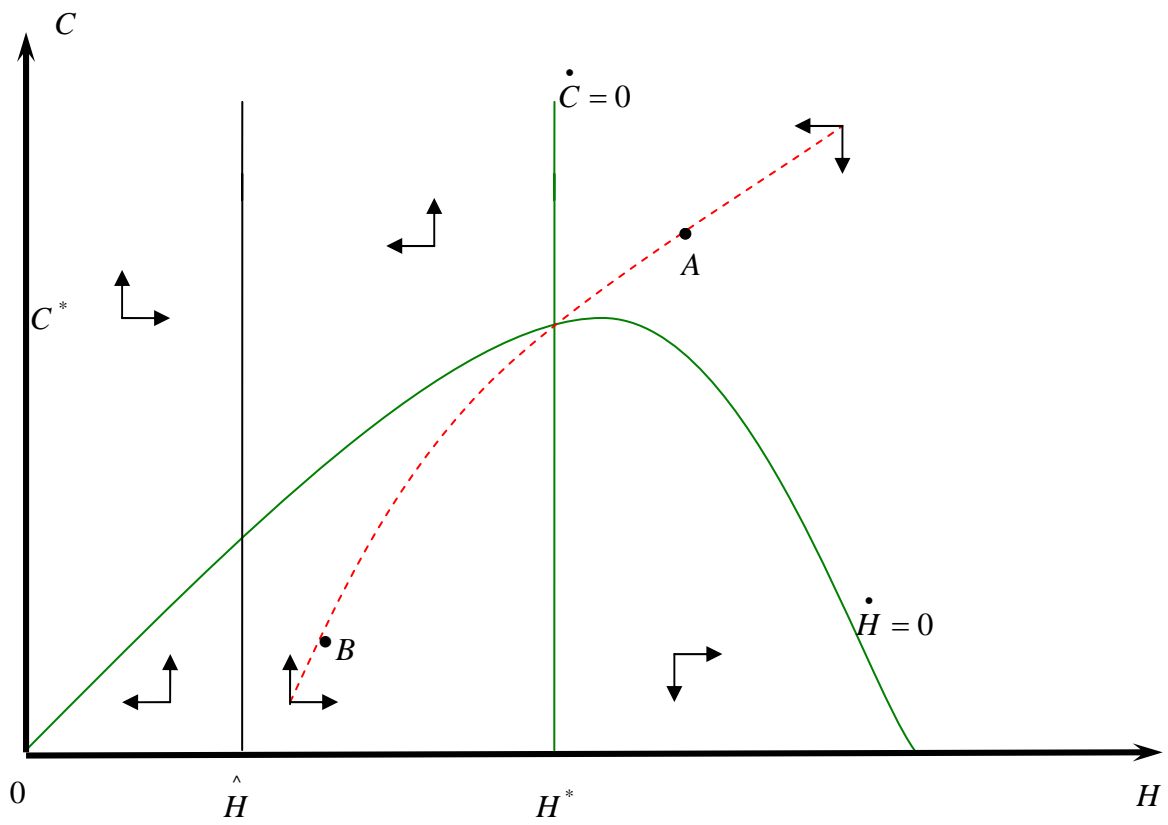


Fig.1: Phase diagram in the determinacy regime ( $1 < \lambda < \hat{\lambda}$ ).

The dashed line is the stable manifold of the saddle-point steady state.

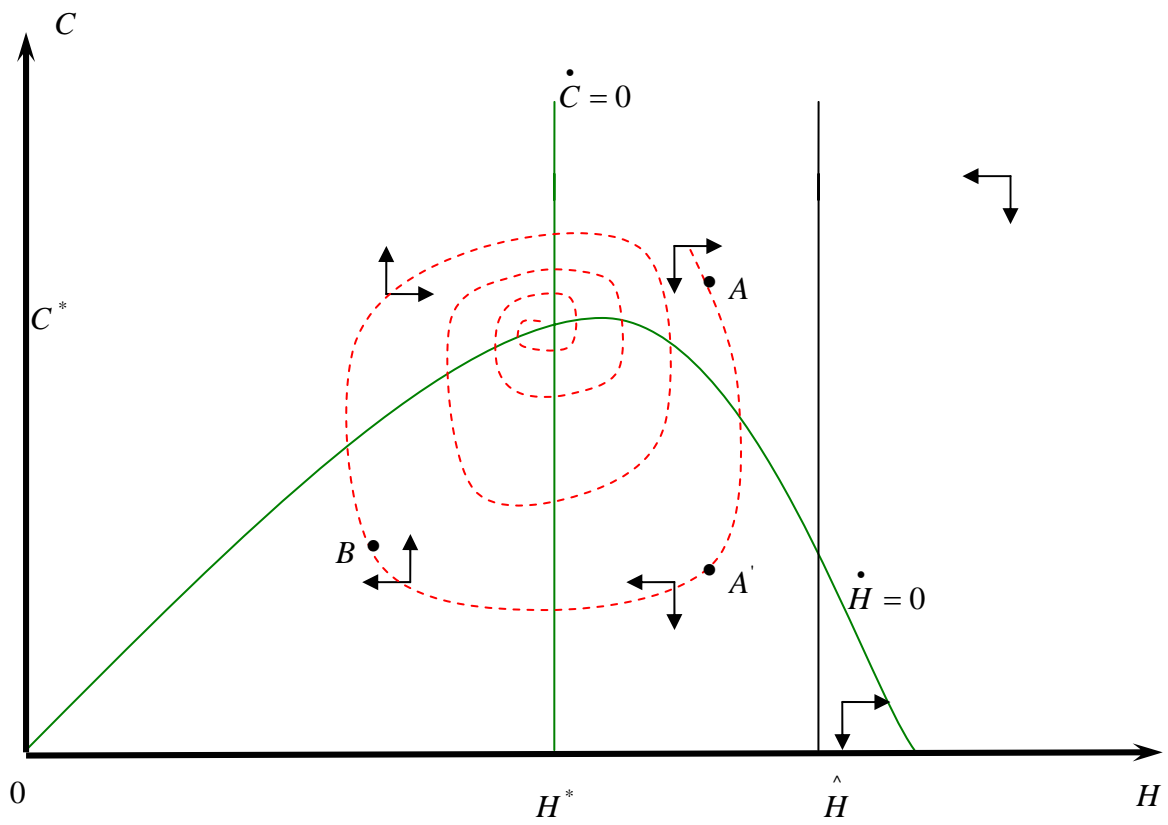


Fig.2: Phase diagram in the debt overhang regime ( $\hat{\lambda} < \lambda < \bar{\lambda}$ ).

The dashed line represents one possible stable path converging to the steady state, which is a sink.

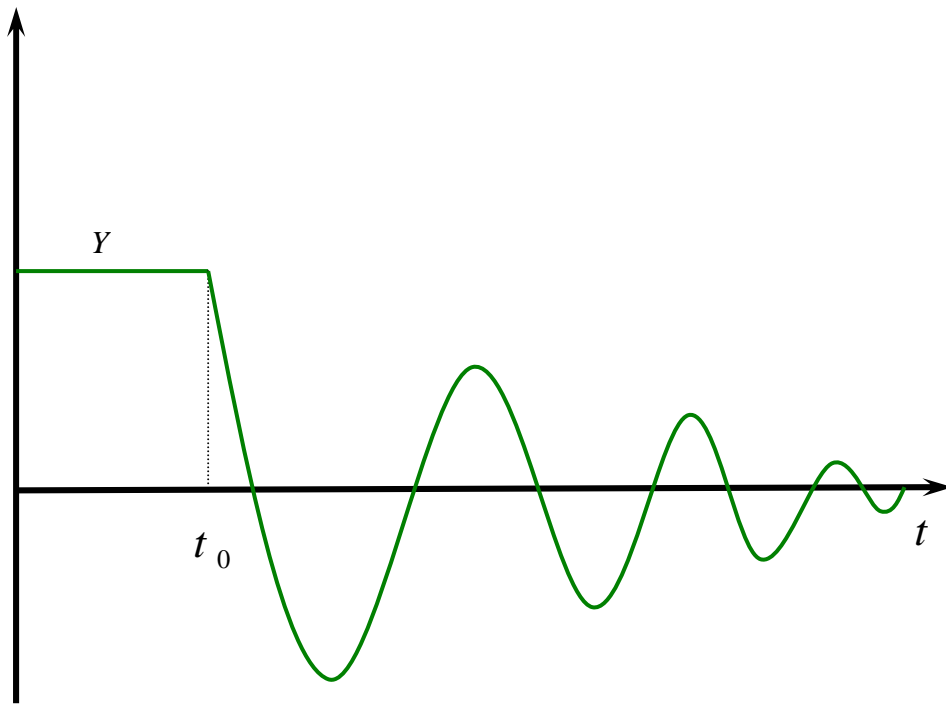


Fig.3: Time series of output after an increase of the world interest rate at date  $t_0$  (debt overhang regime, that is,  $\hat{\lambda} < \lambda < \bar{\lambda}$ ).